PREDICTING DIAMETER AT BREAST HEIGHT FROM TOTAL HEIGHT AND CROWN LENGTH

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Abstract—Tree diameter at breast height (d.b.h.) is often predicted from total height (model 1a) or both total height and number of trees per acre (model 1b). These approaches are useful when Light Detection and Ranging (LiDAR) data are available. LiDAR height data can be employed to predict tree d.b.h., and consequently individual tree volumes and volume/ha can be obtained for the tract. In this paper, we will examine alternative methods of predicting d.b.h. from total height and crown length (model 2a), or from total height, crown length, and number of trees per acre (model 2b), based on the uniform stress theory. The uniform stress theory hypothesizes that stems behave like tapered cantilever beams to equalize bending stress across their length. The four models were evaluated based on the mean difference between observed and predicted diameters, mean absolute difference, and fit index. Results revealed that the two models based on the uniform stress theory (models 2a and 2b) were more appropriate for predicting d.b.h., which is needed to compute tract volume using LiDAR data.

INTRODUCTION

Airborne laser scanning or Light Detection And Ranging (LiDAR) has been used in many forestry applications (Lefsky and others 1999, Means and others 2000, Nelson and others 1988, Nilsson 1996, Parker and Evans 2004, Parker and Mitchell 2005) and can provide measurements of height and crown dimensions. The vertical distribution of forest canopy can be characterized with LiDAR (Arp and others 1982, Dean and others 2009, Drake and others 2002, Harding and others 2001, Lefsky and others 1999, Ritchie and others 1993). In an application of LiDAR in forest inventory, Parker and Evans (2004) evaluated different functions to predict diameter at breast height (d.b.h.) from total tree height and number of trees per unit area. The predicted diameter is needed for calculation of individual tree volumes and ultimately of stand volume.

Stem diameter can also be predicted from the uniform stress theory. This theory states that the taper of tree boles allows them to equalize bending stress (produced mainly by wind pressure on the crown foliage) across their length (Dean and Long 1986). Evidence exists showing a strong relationship between foliage distribution and stem size and taper (Dean 2004, Dean and Long 1986, Dean and others 2002, West and others 1989). Dean and Long (1986) applied the uniform stress model to predict stem diameter anywhere on the tree bole, based on the length of the lever arm and total leaf area above that point. Therefore, for a fixed height such as breast height, diameter can be predicted from total height, crown length, and total leaf area. Total tree height and crown length can be obtained from LiDAR data, and total leaf area can be predicted from total height and crown length by use of a regression equation (Jerez 2002, Roberts and others 2003). Therefore, the uniform stress model allows d.b.h. to be predicted from just two parameters—total height and crown length.

The conventional method so far has been to predict d.b.h. from either total height or from total height and number of trees/ha. The objective of this study was to determine if adding crown length to the above predictor variables improves the prediction.

DATA

Data collected from a loblolly pine (*Pinus taeda*) plantation at the Hill Farm Research Station, Homer, LA, were used in this study. Twenty 0.1-ha (0.25-acre) plots were established with seedlings planted at 1.83- by 1.83-m (6- by 6-foot) spacing. The plots were thinned to 2,470, 1,482, 741, 494, and 247 trees/ha (1,000, 600, 300, 200, and 100 trees per acre) in a stepwise thinning procedure, completed by age 7. Measurements for each tree include d.b.h., total height, and height to the base of live crown. Measurements from 278 trees in 14 plots at age 21 constitute the fit dataset, used for estimation of coefficients of the regression models.

The validation dataset comprised 454 trees at age 28 from another study, also at the Hill Farm Research Station. These trees came from 26 plots of size 0.1 ha (0.25 acres), which underwent thinning (to 741, 494, and 247 trees/ha at age 11) and pruning (once at age 6, and twice at ages 6 and 11) treatments. Summary statistics for the fit and validation datasets are shown in table 1.

MODELS

Conventional Models

Conventional models were developed to predict d.b.h. from total height, or from total height and number of trees/ha. The following models were selected as most appropriate for the fit data, based on an evaluation of numerous models:

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Table 1—Summary statistics of stand and tree variables in the fit and validation datasets

Variable	n	Mean	Std. dev.	Min.	Max.
Fit dataset					
D.b.h. (cm)	278	23.3	6.3	11.4	48.3
Total height (m)	278	17.6	2.5	9.1	23.4
Crown ratio	278	0.41	0.12	0.15	0.83
Number of trees per ha	14	849	593	237	2303
Validation dataset					
D.b.h. (cm)	454	28.1	6.1	15.5	48.0
Total height (m)	454	21.7	2.4	13.9	27.5
Crown ratio	454	0.37	0.09	0.04	0.73
Number of trees per ha	26	440	187	194	717

Std. dev. = standard deviation; Min. = minimum; Max. = maximum.

Model 1a—This model performed slightly better than Parker and Evans' (2004) model, which used the natural logarithm of H instead of H as the independent variable.

$$D = b_1 + b_2 H^{b_3} \tag{1}$$

where
D = diameter at breast height in cm
H = total height in m
b_i's = regression coefficients

Model 1b—Stand density in terms of number of trees/ha (N) was added to equation (1) to form model 1b:

$$D = b_1 + b_2 H^{b_3} N^{b_4} \tag{2}$$

Parker and Evans (2004) evaluated four models and found that model 1b performed best in five out of six datasets.

Models Based on The Uniform Stress Theory

Dean and Long (1986) proposed the following taper model to predict tree diameter in cm (d_h) at height h in m:

$$d_h = b_1 (A_h L_h)^{b_2} \tag{3}$$

where
A_h = total leaf area (m^2) above d_h
L_h = distance in m between the center of leaf area above d_h and the point at height h

Model 2a—By fixing h at 1.37 m or 4.5 feet, one can predict d.b.h. using the above equation:

$$D = b_1 x^{b_2} \tag{4}$$

where
$x = A\ L_h$
A = total leaf area (m^2)
$L_h = H_{MC} - 1.37$
$H_{MC} = H_T - CL/2$ = height to the center of the crown
H_T = total tree height in meters
CL = crown length in meters

Total leaf area was predicted from an equation developed by Roberts and others (2003) and refitted by Jerez (2002):

$$\log(A) = -2.19715 + 7.5437 \log(H_T) - 5.422006 \log(H_{MC}) \tag{5}$$

where
$\log(A)$ = logarithm base 10 of A

Model 2b—Similar to model 1b, model 2b was obtained by adding number of trees/ha to equation (4):

$$D = b_1 x^{b_2} N^{b_3} \tag{6}$$

It is evident that we had two groups of models: models 1a and 2a required only heights, whereas models 1b and 2b required both heights and number of trees/ha as predictor variables. The regression coefficients in these models were obtained with nonlinear regression.

RESULTS AND DISCUSSION

The four models were evaluated based on three statistics: mean difference (MD) between observed and predicted diameters, mean absolute difference (MAD), and fit index (FI), which is computationally similar to R^2 in linear regression. Table 2 shows the evaluation statistics for the four models, based on the fit and validation datasets.

Diameter Models Based on Heights

Model 1a had a bias MD close to zero for the fit data, but its bias increased to –3.058 cm for the validation dataset. On the other hand, model 2a produced an MD value of only 0.252 cm for the validation data. For both the fit and validation data, MAD was lower and FI was higher for model 2a than for model 1a. For the validation data, model 2a lowered the MAD value from 4.914 cm to 3.192 cm and increased the FI value from 0.008 to 0.560. Trees in the validation dataset were older and, on the average, larger and taller than those in the fit dataset. This might explain why model 1a failed to adequately represent the validation data (fig. 1). On the other hand, model 2a characterized both the fit and validation data equally well (fig. 1). This suggests that the uniform stress model was reasonably reliable and could be employed with confidence to describe a larger segment of the population.

Diameter Models Based on Heights and Stand Density

The evaluation statistics were slightly better for model 1b and model 2b for the fit dataset; however, its prediction capability drastically diminished for the validation data. The value of FI fell from 0.760 to 0.217 for the fit and validation datasets, respectively. Testing model 2b with the same validation data, we obtained the following statistics: FI = 0.645 cm, MAD = 2.957 cm (vs. 4.404 cm from model 1b), and MD = –0.214 cm (vs. –3.595 cm from model 1b). Figure 2 shows that the modified uniform stress model with the addition of number of trees/ha did a good job in both fitting the sample data and predicting for the population.

Use of Stand Density as an Additional Variable

Adding number of trees/ha as an extra predictor variable improved both the fit and predictive ability of models 1a and 2a. Parker and Evans (2004) obtained similar results. FI value for the validation data in this study increased from 0.008 to 0.217 for model 1b and from 0.560 to 0.645 for model 2b. Because tree counts are readily available from LiDAR data, this variable should be incorporated into model for predicting diameter from remotely sensed data.

Table 2—Evaluation statistics for the four models to predict d.b.h.

Model	Equation	MD	MAD	FI
		Fit dataset		
1a	$D = b_1 + b_2 H^{b_3}$	0.000	4.174	0.293
1b	$D = b_1 + b_2 H^{b_3} N^{b_4}$	0.000	2.472	0.760
2a	$D = b_1 x^{b_2}$	0.023	3.257	0.550
2b	$D = b_1 x^{b_2} N^{b_3}$	0.010	2.552	0.734
		Validation dataset		
1a	$D = b_1 + b_2 H^{b_3}$	–3.058	4.914	0.008
1b	$D = b_1 + b_2 H^{b_3} N^{b_4}$	–3.595	4.404	0.217
2a	$D = b_1 x^{b_2}$	0.252	3.192	0.560
2b	$D = b_1 x^{b_2} N^{b_3}$	–0.214	2.957	0.645

MD = mean difference between observed and predicted diameters; MAD = mean absolute difference; FI = fit index (computationally similar to R^2 in linear regression).

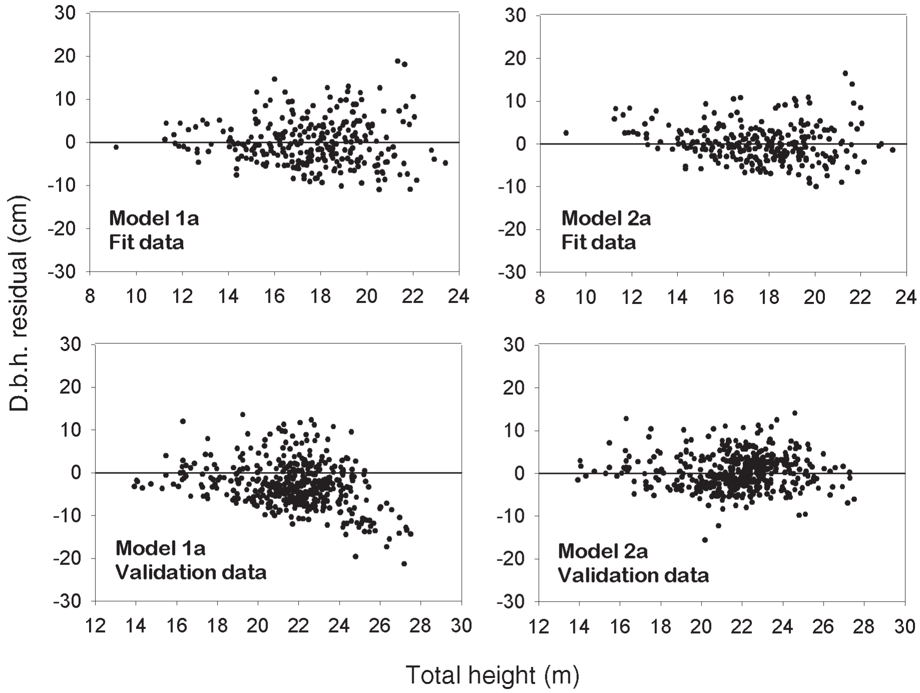

Figure 1—Fit data and validation data for model 1a and model 2a.

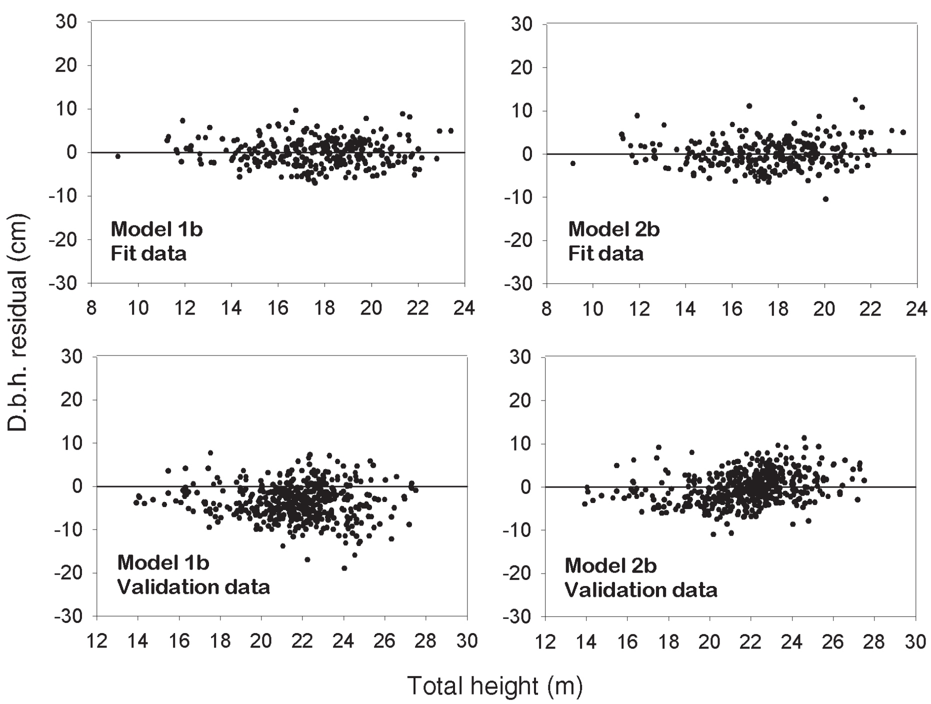

Figure 2—Fit data and validation data for model 1b and model 2b.

SUMMARY AND CONCLUSIONS

In this study, the conventional method of predicting d.b.h. from total height (model 1a) was evaluated against the new method of predicting d.b.h. from total height and crown length (model 2a). Evaluation statistics and residual plots revealed that model 2a, which was based on the uniform stress theory, was better at predicting diameters for the validation data. Similar results were obtained when number of trees/ha was included as a predictor variable. The addition of crown length (model 2b) drastically improved the d.b.h. prediction for the validation data. These results showed that the uniform stress theory can be successfully modified to predict d.b.h. from total height, crown length, and number of trees/ha, all of which can be obtained from LiDAR data.

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