DEVELOPMENT OF VOLUME EQUATIONS USING DATA OBTAINED BY UPPER STEM DENDROMETRY WITH MONTE CARLO INTEGRATION: PRELIMINARY RESULTS FOR EASTERN REDCEDAR

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Abstract—Preliminary results are given for development of an eastern redcedar (*Juniperus virginiana*) cubic-volume equation based on measurements of redcedar sample tree stem volume using dendrometry with Monte Carlo integration. Monte Carlo integration techniques can be used to provide unbiased estimates of stem cubic-foot volume based on upper stem diameter measurements obtained at randomly selected stem heights. Monte Carlo integration with importance sampling and antithetic variates was used to obtain sample tree volume estimates in this study. Importance sampling is a variance reduction technique that uses a proxy taper function (in this case a paraboloid) to randomly select the most influential upper stem sample diameters. Antithetic variates use negatively correlated upper stem measurements to further reduce the variance of stem volume estimates. The estimator was revised to use tree d.b.h. rather than stump diameter. Since volume estimates from these techniques are unbiased, they can be used as dependent variables to estimate the parameters in a standard volume equation for eastern redcedar. Data were obtained from more than 30 sample trees using a Wheeler Pentaprism to obtain two upper stem diameter measurements for each sample tree. Sample tree total height and d.b.h. were also measured. Preliminary results indicate an R^2 (0.85), which is somewhat lower than that usually obtained in studies where trees are felled and measured deterministically. However, since the Monte Carlo integration estimates are unbiased the mean regression line is probably well located even though the variance about the line may be somewhat greater than would be the case for felled tree data.

INTRODUCTION

A cubic-foot volume equation for eastern redcedar (*Juniperus virginiana*) was desired for use in the Payne County, OK, area. Since no suitable equations had been developed, it was decided to use upper stem dendrometry on standing redcedar trees to develop a cubic-foot volume equation. Monte Carlo integration with importance sampling and antithetic variates (Van Deusen and Lynch 1987) provides an unbiased and efficient method of estimating individual tree stem volumes from only two randomly chosen upper stem measurements. Importance sampling selects upper stem diameter measurements using a proxy taper function so that measurements representing greater volume are more likely to be chosen (generally these are lower on the bole). Through use of the antithetic variates technique, the two randomly chosen upper stem diameter measurements are negatively correlated. This results in variance reduction of the volume estimate because the variance of the sum of two negatively correlated random variables is less than the sum of their variances.

For this application it was desired to modify the estimating equations described by Van Deusen and Lynch (1987) so that the proxy taper function is based on tree total height and d.b.h. rather than stump diameter. As suggested by Van Deusen and Lynch (1987) a paraboloid was used as a proxy taper function. The following formula can be used to compute the cubic-foot volume of a paraboloid having the same height and d.b.h. as the tree of interest:

$$V_{par} = \frac{\pi D^2}{2 \times 576}\left(\frac{H}{1-\frac{4.5}{H}}\right)$$

where

V_{par} = cubic-foot volume of paraboloid
D = d.b.h. (4.5 feet) in inches
H = total tree height in feet

The estimation process begins by selecting a random variable u which is uniformly distributed between zero and 1. This number is used to select two antithetic random heights at which upper stem diameters will be measured. The following formula is used to generate these two heights:

$$h_1 = H\ \left[1-\sqrt{u}\right]$$
$$h_2 = H\ \left[1-\sqrt{(1-u)}\right]$$

where

h_1 = upper stem height 1 in feet
h_2 = upper stem height 2 in feet
u = random number uniformly distributed between 0 and 1
H = total tree height in feet

A Wheeler Pentaprism was used to measure upper stem heights on each redcedar sample tree at heights h_1 and h_2.

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The paraboloid volume given above was then adjusted by using ratios of measured squared diameter to paraboloid squared diameter:

$$\hat{V} = \frac{V_{par}}{2}\left[\frac{d_{m1}^2}{d_{p1}^2} + \frac{d_{m2}^2}{d_{p2}^2}\right]$$

where

$d_{p1} = D\sqrt{\frac{H-h_1}{H-4.5}}$, upper stem diameter (inches) of paraboloid at h_1

$d_{p2} = D\sqrt{\frac{H-h_2}{H-4.5}}$, upper stem diameter (inches) of paraboloid at h_2

d_{m1} = is measured upper stem diameter (inches) at h_1

d_{m2} = is measured upper stem diameter (inches) at h_2

h_2 = is upper stem height 2 in feet

$\hat{V}$ = estimated redcedar cubic-foot total stem volume

RESULTS AND DISCUSSION

At two locations in Payne County, OK, a total of 38 redcedar trees were subsampled on BAF = 10-factor point sample plots to provide a test of these methods. Upper stem diameters were measured at two randomly chosen antithetic heights so that estimated stem volumes could be obtained by using the equation above. Linear regression techniques were then used to estimate the parameters in the following combined variable (Avery and Burkhart 2002 p. 173) total volume equation:

$$\hat{V} = \underset{(0.535629)}{0.859397} + \underset{(0.000188)}{0.002674}\, D^2H$$

where

$\hat{V}$ = predicted cubic-foot volume outside bark

D = d.b.h. (inches)

H = total height (feet)

Numbers in parentheses under the equation above are the standard errors of estimated coefficients.

Fit statistics for the equation above were: coefficient of determination $R^2 = 0.85$, standard error $S_{y.x} = 1.81$, number of observations $n = 38$, calculated "F" statistic is $F = 201$, (significant at levels <.0001)

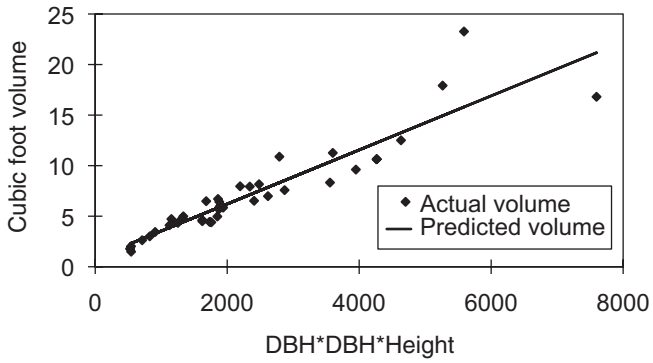

Figure 1—Actual vs. predicted cubic-foot volume for 38 redcedar sample trees.

The plot of the regression relationship vs. sample data indicated reasonable fit (fig. 1).

SUMMARY AND CONCLUSIONS

Estimation of sample tree volume by Monte Carlo integration with importance sampling and antithetic variates was adequate for preliminary development of a combined variable volume equation on this small redcedar dataset. The $R^2 = 0.85$ was not as good as that usually seen in datasets that use felled tree data, but since Monte Carlo estimation is unbiased, this method should give adequate results in terms of fitting the mean volume line. This method of obtaining volume observations for volume equation studies may be of interest in situations where the cost and time required to obtain felled tree data is prohibitive.

LITERATURE CITED

Avery, T.E.; Burkhart, H.E. 2002. Forest measurements. 5th ed. New York: McGraw-Hill. 456 p.

Van Deusen, P.C.; Lynch, T.B. 1987. Efficient unbiased tree-volume estimation. Forest Science. 33: 583–590.