REINEKE'S STAND DENSITY INDEX: A QUANTITATIVE AND NON-UNITLESS MEASURE OF STAND DENSITY

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Abstract—When used as a measure of relative density, Reineke's stand density index (SDI) can be made unitless by relating the current SDI to a standard density but when used as a quantitative measure of stand density SDI is not unitless. Reineke's SDI relates the current stand density to an equivalent number of trees per unit area in a stand with a quadratic mean diameter (*Dq*) of either 10 inches (English units) or 25.4 cm (metric units). Thus, when used as a quantitative measure, SDI is in fact unit dependent on two levels, one is whether English or metric units are used, and the second is whether the unit area is 1 acre (ha) or two, three, etc. Foresters should express SDI as either number of trees per acre or /ha to clearly indicate the unit. When viewing SDI as a quantitative measure, it is legitimate to use the slope of the linear portion of an individual stand's size-density trajectory.

INTRODUCTION

Measures of stand density help managers identify levels of competition and site utilization and to determine necessary management scenarios to meet objectives. Reineke (1933) developed a stand density index (SDI) that relates the current stand density to an equivalent density in a stand with a quadratic mean diameter (*Dq*) of 10 inches. Reineke's SDI can be expressed as:

$$SDI = N(Dq/10)^b \quad (1)$$

where
- SDI = Reineke's stand-density index
- N = trees per acre
- Dq = quadratic mean diameter (inches)
- b = exponent of Reineke's equation, often reported to equal –1.605

Reineke's SDI can be expressed on the ln-ln scale as:

$$\ln(SDI) = \ln N + b \ln Dq - b \ln 10 \quad (2)$$

where
- ln = natural logarithm
- b = is the slope of the relationship between ln$N$ and lnDq in fully stocked stands, equivalent in magnitude to the exponent of equation (1), and other variables as previously defined.

This measure of stand density has been widely used in the development of density management diagrams and as a measure of stand density reported in scientific articles as well as in forest inventories. Papers have presented a variety of summary measures with accompanying units but failed to include units when presenting SDI (e.g., Cochran and Barrett 1999, Curtis and Marshall 2002, Williams 1994). For example, Curtis and Marshall (2002) included a figure presenting basal area (their figure 7) with accompanying units but the figure presenting SDI (their figure 14) did not include an accompanying unit. In this current paper, it is shown that SDI is not unitless and that units should accompany any report of SDI.

Reineke's SDI can be used either as a relative measure or a quantitative measure of stand density (Avery and Burkhart 2002, p. 321–324). Stocking refers to using a quantitative measure to relate the current stand density to some optimum stand density thought to best meet management objectives. When viewing Reineke's SDI strictly as a relative measure, the slope in equation (2) must be estimated exclusively using stands that are fully stocked (Clutter and others 1983, p. 72), or that are at the maximum level of ln$N$ for a specific lnDq for a particular species. Thus, the optimum stand densities are those of fully stocked stands and the measure of stocking is comparing Reineke's SDI of any stand to the fully stocked stands. Measures of stocking are often quantified using a ratio producing a unitless measure ranging from zero to 1, when using SDI this is referred to as relative SDI or "percentage stocking" (Reineke 1933). The terms "relative" and "relate" have caused confusion in the understanding of SDI though. Many foresters believe SDI is exclusively a relative measure to the maximum N per acre for a given Dq in even-aged stands of a certain species (Clutter and others 1983, p. 72 and 73).

When using SDI as a quantitative measure of stand density, this measure is relative not because it is compared to some optimum stand density and is thus unitless or strictly a relative measure, but because it relates the current stand density to an equivalent N per acre of a stand with a Dq of 10 inches (fig. 1). Thus, in some sense the standard becomes a stand with a Dq of 10 inches, but in fact any value of Dq could be used as the standard. A better word than "relates" when using SDI as a quantitative measure of stand density would be "equates." Nonetheless, this relation should not be confused as to imply that SDI is unitless. In fact, Reineke (1933) clearly specified that his SDI could be used as a quantitative measure, "It is deemed more desirable, however, to use the

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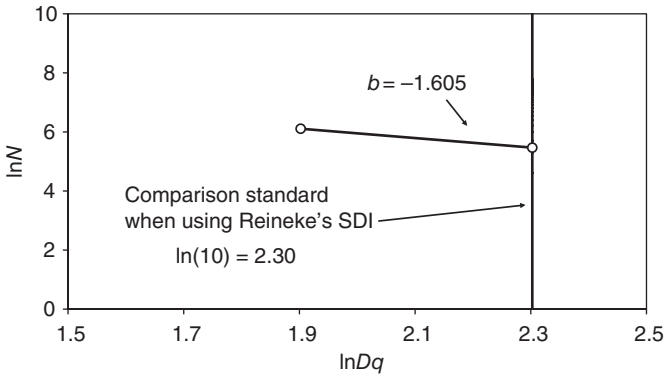

Figure 1—For Reineke's stand-density index, a slope of –1.605 is often used to relate (equate) the current stand density, e.g., 450 trees per acre, *Dq* = 6.7 inches, to an equivalent trees per acre (exp[5.47] = 237 trees per acre) for a stand with a *Dq* equal to the standard measure of 10 inches.

number of trees as the index. This is a quantitative, not a relative, measure and permits a better visualization of stand conditions."

What is advantageous about viewing Reineke's SDI as a quantitative measure is that the slope of equation (2) can be estimated for each individual stand. This approach eliminates having to estimate the slope exclusively using stands thought to be "fully stocked," or that are at the maximum level of ln*N* for a specific ln*Dq* for a particular species. Thus, the maximum size-density relationships (MSDR) of an individual stand become the optimum and we can consider these MSDRs of an individual stand as "fully stocked." One only needs to determine what observations are within the linear portion of self-thinning for an individual stand [VanderSchaaf and Burkhart (2008)—defined as the MSDR dynamic thinning line] and then estimate the slope between ln*N* and ln*Dq*, defined as the MSDR dynamic thinning line slope. Alternatively, when viewing Reineke's SDI as a quantitative measure, an estimate of the population average MSDR dynamic thinning line slope (VanderSchaaf and Burkhart 2007) can be used in Reineke's equation. Remember, from a quantitative perspective, we are only interested in equating the current stand density to a stand with a *Dq* of 10 inches. Estimating the slope of an individual stand to calculate SDI is not as foreign as it may appear. It has been well accepted that the MSDRs of each individual species can serve as defining "fully stocked" whereby we can estimate species-specific slopes in equation (2) allowing us to compare SDI among a variety of species. Using an individual stand is no different, we are simply estimating the slope of equation (2) at a much finer level.

The use of trees per acre implies SDI is not unitless. Equation (1) can also be expressed as:

$$SDI = N(Dq/25.4)^b \quad (3)$$

where

SDI = Reineke's stand-density index
N = trees/ha
Dq = quadratic mean diameter (cm)
b = exponent of Reineke's equation, often reported to equal 1.605

It is obvious that whether one uses *N* and *Dq* in metric or English units a different value of SDI will be obtained. However, the actual on-the-ground stand density is still the same, only the units of measure have been changed. When using metric units, an SDI of 500 implies a much different level of competition and site occupancy as compared to when using English units. As opposed to equating the current stand density to an equivalent stand density with a *Dq* of 10 inches, equation (3) determines what trees/ha would equate the current stand density to a stand with a *Dq* of 25.4 cm. This is the first level where SDI is not unitless, the units of measure will impact the meaning of the stand-density index measure.

The second level where SDI is not unitless is whether *N* represents the number of trees per 1 acre (ha) or the number of trees per 2; 3; 1,000; etc., acres (ha). Rather than equating the current stand density to an equivalent number of trees with a *Dq* of 10 inches on a per-acre basis, we could also equate stand densities by using the number of trees per 2 acres, per 3 acres, etc. An SDI of 500 would represent a much different level of competition and site occupancy if *N* was on a per-acre (ha) basis or if *N* was on a 1,000-acre (ha) basis. This is the second level where SDI is not unitless.

In conclusion, SDI should not be considered unitless, a fact that Reineke himself pointed out and others have clearly noted (Husch and others 2003, p. 179). Additionally, SDI should not exclusively be considered a relative measure, it can also be interpreted as a quantitative measure. In the future, foresters should report SDI as trees per acre or trees/ha to clearly indicate the unit being used and, as many foresters already do, specify if Reineke's relative SDI is being reported.

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