# Forecasting Productivity in Forest Fire Suppression Operations: A Methodological Approach Based on Suppression Difficulty Analysis and Documented Experience[1]

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## Abstract

The abandonment of land, the high energy load generated and accumulated by vegetation covers, climate change and interface scenarios in Mediterranean forest ecosystems are demanding serious attention to forest fire conditions. This is particularly true when dealing with the budget requirements for undertaking protection programs related to the state of current and future forest landscapes. Based on economic analysis, suppression costs represent a significant proportion of the total budget available for forest fire protection programs. The need to make efficient use of available budgets requires directing funding towards areas with the lowest costs and maximum benefits. Accordingly, incorporating econometric tools enables establishing criteria to optimize budget allocation.

In trying to optimize budget allocation experience capitalization[4] takes on great importance. Modeling the fire suppression process and budget allocation requires knowing how suppression is managed, how resources are dispatched and how costs are incurred. Advances in the design of fire management tools have benefited from current techniques, (e.g., knowledge capitalization, utility analysis, efficiency analysis) for economic analysis of the potential hazard posed by forest ecosystems conditions and related fire suppression operations difficulties based on fire behavior expectations. Incorporation of econometric tools based on efficiency, productivity, and utility functions help fire managers define budgetary requirements to ensure ecosystem protection compatible with the value of the resources being protected and associated suppression difficulties and costs. The methodological approach presented here allows forecasting fire suppression operations

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[4] A set of recorded and organized experiences of suppression plans and associated organizations used in different fires available to fire incident commanders as references or *lessons learned* from prior fires.

productivity, based on suppression difficulty and cost, as well as records from documented fire suppression operation plans from prior fires.

*Keywords: fire economics, econometrics, fire management, fire program planning, operational plans, fire budgets*

## Acquiring knowledge and experience of suppression operations

Capitalizing on the experience of documented operations/attack plans allows information exchanges among fire managers. It provides validation of lived events and helps improve operational decisions and human behavior in suppression work. Capitalization converts experiences from being individual to collective, isolated to shared, and forgotten to reused; in this way, the experience becomes available, locatable, and accessible (Fig. 1) (Rodríguez y Silva and others 2007).

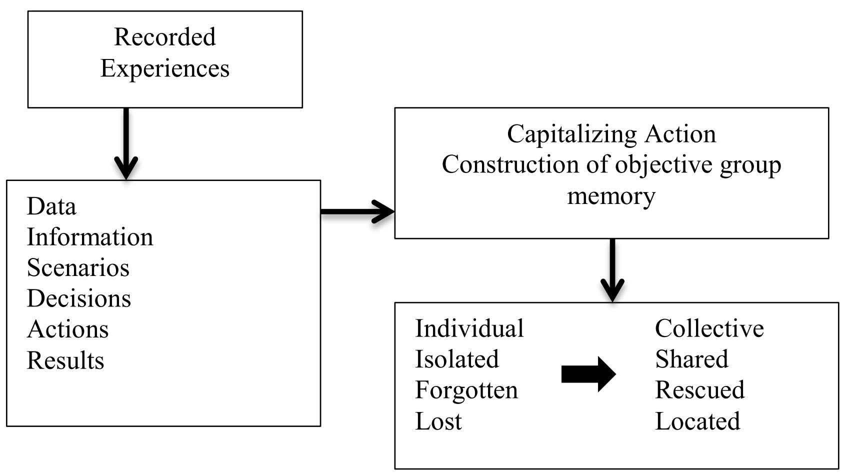


**Figure 1—**Experience capitalization procedure.

The operations/attack plans for an incident can be described as the objective group memory of those managing the fire at the time. The primary reason for capitalization (organization of prior incidents fire management operations/attack plans information in a lesson learned type files) is to share the data extracted from the experience and obtained through the fire events (Rodriguez y Silva and others 2007, MacGregor and González-Cabán 2008). Sharing events and knowledge acquired from real-life situations facilitates exchanges of experience regarding the behaviors during development of operations/attack plans. The benefit is access to knowledge among various officials and firefighters regardless of whether they had been eyewitnesses to the event or not.

The officials involved in organizing and directing the suppression work play a pivotal role in the capitalization function because they are the main actors and witnesses to the events. These officials are in the best positions to report on the actions taken and review their own experiences in the process of developing an operational attack plan.

During the capitalization process the managers and firefighters involved in the incident analyzed are essentially the producers of information. However, they may also be receivers of information when discussing information or events unknown to them during the incident; thus, incorporating a new set of descriptive information on the actions included in attack plans (Rodríguez y Silva and others 2007).

When managing suppression work, the requested resources are managed by the Incident Commander and its Staff at all different levels. These managers gather all possible information on the current fire situation to evaluate and analyze it to make appropriate decisions regarding the most appropriate operations/ attack plan; thus implementing the most efficient and safe distribution of available suppression resources.

In the process of analyzing the potential operation/attack plan for a current incident, the capitalized experience can play a significant role. For example, knowing and understanding how firefighting resources and operational plans were implemented in similar situations (i.e., objective group memory information) allows managers to use previous lessons learned information to develop an operation/attack plan for the current incident.

The ***objective group memory*** is held in an updatable database of operations/attack plans. These databases include behavior patterns (human factors), environmental conditions at the time of incident, and operations/attack strategies adopted. This gives access to past experiences, facilitating knowledge sharing. This enables building of an ***experience exchange network*** among the different people in the organization responsible for controlling and suppressing forest fires.

The databases described above provide the necessary information for selection of the variables and parameters that enables studying, analyzing and evaluating the productivity and efficiency of the suppression operations. Basically the following variables should be considered:

- Fireline production rate (fireline produced by unit of time; measured as m/min)
- Area contraction factor, or ACF (difference between the unit and the ratio between the actual area affected by the fire and the potential area affected by a free-burning fire, i.e., without suppression operations; considering the time elapsed from detection until the fire is controlled) (Rodríguez y Silva and others 2007), (Rodríguez y Silva and González-Cabán 2010).
- Average area affected per recorded forest fire (ha/fire)

- Average suppression costs per affected area ($/ha, €/ha); Ratio between suppression cost and natural resources net value change

## Choosing suppression resources and the associated utility function

Utility theory (Fishburn 1970, Mongin 1997) can be used to help decide the amount and type of suppression resources for incorporation into an organizational forest fire suppression model, or an individualized treatment according to suppression operations. Using a maximization technique gives us the benefit derived from the consumption of a certain amount of goods previously selected. Application of utility theory to fire suppression operations requires consideration of the law of diminishing marginal utility. The fundamental tenet of this law is that the first unit dispatched (first unit consumed in terms of personal satisfaction) provides the highest level of utility for the fire suppression organization (highest level of utility for a consumer). Each of the next units dispatched provides a lower utility level to the suppression efforts (or to a consumer satisfaction); until the point in which one additional unit dispatched to a suppression organization does not contribute anything or nearly anything to the suppression capability (or utility) of the organization (i.e., the marginal utility of the unit is very small or zero). At this point the total utility does not appreciably increase (and might even decrease); yet operational costs continue to rise.

Given a fire suppression/attack plan for an incident and the variety and combinations of firefighting resources that can be dispatched, the plan must rank them according to the utility that each generates (contribution to total fire suppression efforts) through a preference ordering system. The consumer preference ordering system that best meets the above conditions is the *indifference curve* (Muñoz 2006).

An *indifference curve* shows the same level of utility for all the combinations of resources along that curve (see figure 2). That is, the manager is indifferent to whether there are, for example, 10 20-persons hand crews and 5 1,000-gallons fire engines or only 3 1,000-gallons engines and 12 20-persons hand crews because both combinations produce the same level of utility (i.e., same level of protection capability). Given this definition, an *indifference curve* to the right of and higher than any previous curve will represent a higher utility level. The slope of the *indifference curve* measures the marginal rate of substitution (MRS) between the resources(Y and X)[1] (Eq. 1). That is, how many units of one resource a consumer (suppression /attack

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[1] We understand that all numbers used in the paper have associated estimation errors. However, for ease of presentation and computation we have chosen to ignore sample and measurement errors here. However, future application of the methodology should provide information on the error associated with parameters estimates to indicate their precision.

plan) needs to obtain the same level of utility (suppression capability) from another type of resource or combination of resources thereof.

$$MRS = \Delta Y / \Delta X \tag{1}$$

Then, we can integrate the set of forest fire suppression resources ($X_i$, $Y_j$) or possible combinations thereof into the utility function (U) (Eq. 2),

$$U = f(X_i), \text{ and } MRS = \lfloor \partial Y_j / \partial X_i \rfloor U \tag{2}$$

Given the existing utility function, changes in the utility of the suppression system can be determined for changes in the amounts of each of the fire suppression resources assigned (dispatched). That is, the marginal utility of each suppression resource, or the resulting percentage increase in utility level (suppression efficiency in terms of decrease in area affected per fire attacked) resulting from increasing the specific fire suppression resource used in the marginal analysis by one unit (Eq. 3).

$$\partial U = \sum (\partial U / \partial X_i) \quad \partial X_i = 0 \tag{3}$$

The solution to the problem of choosing between available resources or a combination thereof requires having the necessary information. This consists of the suppression resources to choose from, the budget constraint (maximum budget available to cover operational costs) and the *indifference curves* map (Fig. 2). The budget constraint tangential to the *indifference curve* at point (A) represents the marginal rate of substitution ($MRS$) between resource $X_i$ and resource $X_j$; this is exactly the ratio between the relative prices of $X_i$ ($P_i$) and $X_j$ ($P_j$) (Fig. 2). This can be expressed as follows:

$$MRSX_i / P_i = MRSX_j / P_j \tag{4}$$

At this point, when measuring the productivity and efficiency of the system we must make an important conceptual change relative to the *indifference curves*. Because the *suppression capacity*, as well as the *operational constraints* inherent to *each resource* the MRS between firefighting resource $X_i$ and $X_j$ cannot be considered only as a ratio relationship between their relative prices. Accordingly, it is necessary to construct a new variable that identifies each resource, including their suppression capacity and inherent operational constraints. The definition of each resource is given by the following expression (Eq. 5):

$$Y^e = \alpha P + \varphi R + \mu L, \qquad (5)$$

where α, φ, μ represent weighting coefficients of each one of the parameters, P represents the resource price per unit of time (e.g., \$/h, €/h), R represents the suppression capability, expressed in terms of the corresponding production rate by unit of time (e.g., m/min), and L represents the operational constraints.

Thus, to choose between two firefighting resources or combinations thereof, given an allocated budget, a manager will distribute the budget between them so that the marginal utilities of the resources are proportional to their respective prices. Therefore, at equilibrium, the system (the consumer in economic terms) gets the same marginal utility per monetary unit spent on resource $X_i$ (or combination thereof) as on resource $X_j$.

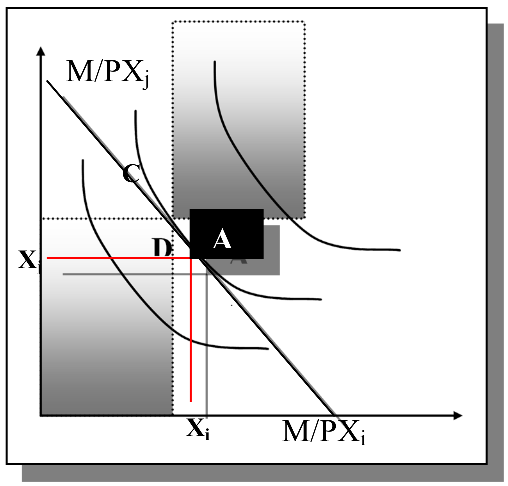


**Figure 2—**Graph showing marginal utility isoquants or *indifference* curves for two products.

The algebraic solution to the problem of choice can be determined by using Lagrange multipliers. To do this using the utility function (for example, minimum area affected), the available budget (M), the amounts of each one of the resources ($X_i$) and their corresponding prices ($P_{xi}$) are transformed algebraically through the composition of the Lagrange function ($t_i$ represents the use time of the resource (i) (Eqs. 6-9), (Rodríguez y Silva and others 2007).

$$M > \sum Px_i X_i t_i \left(Y^e\right) \qquad (6)$$

$$V = u\left(X_{1,2,3\ldots n}\right) + \lambda\left(M - \sum Px_i X_i t_i \left(Y^e\right)_i\right) \qquad (7)$$

$$\partial V / \partial X_i = \partial U / \partial X_i - Px_i = 0 \qquad (8)$$

$$\partial V / \partial \lambda = M - \sum Px_i X_i = 0 \tag{9}$$

The Lagrange multiplier indicates how utility varies with increasing monetary spending on $X_i$; therefore, it measures the marginal utility of the available suppression budget.

## The management index, a tool for assessing efficiency

Assessing management efficiency is always necessary for responding to inquiries about investments made and the results obtained. Applying econometric tools to the evaluation of investments in forest fire protection is always difficult due to the lack of variables that define suppression costs and effectiveness in suppression operations in an integrated way. To help clear the uncertainties arising from the lack of integrated variables, an integrative variable was defined that enables including information on costs and effectiveness in suppression operations; we call this variable the *Management Index* (MI) (Carmona and Rodriguez y Silva 2009). MI is defined as the ratio between ACF and the cost per hectare

$$MI = ACF / Ctot / Aq - 100[(1 - Aq / Av) Aq] / Ctot \tag{10}$$

where MI is the management index, ACF is the area contraction factor, $C_{tot}$ is the total suppression cost, $A_q$ is the area affected, and $A_v$ is the area affected in a simulated free-burning fire (without suppression operations) for the time between detection and control.

Conducting a differential analysis of the model, relative to the behavior of each of the three variables ($A_q$, $A_v$, $C_{tot}$), permits us to obtain the expression that determines the maximum condition for the behavior of MI (Eq. 11).

(11)

Solving for the first derivative to be zero then the values that make the second derivative of the determining function of MI negative will provide the sufficient condition for MI to reach its highest value under those conditions; though not necessarily a single global maximum.

Using this procedure, one can study each fire individually or analyze a given district or province for a particular time period. The comparison between periods allows tracking MI behavior. With this system of equations, the efficiency analysis permits determination of a point at which MI is most efficient. As seen in figure 3, the joint graphic representation of the regressions between MI versus Ca and ACF shows that there is a corresponding cut point below which Ca exceeds the efficiency of the resources represented by ACF. The further to the right of the Point of ***Minimum Efficient Management*** (Pme, zone B), the greater the efficiency; therefore, the more positive the assessment of fire management. On the contrary, MI values located to the left of Pme (zone A) are poorly managed fires, in which the ACF is below the Ca.

The mathematical representation of the minimum efficient management point takes the following general form (Eqs. 12 and 13):

$$Ca = a.e^{b.MI} \qquad (12)$$

$$ACF = e.MI^{d} \qquad (13)$$

The values for the coefficients (a) and (c), as well as for the powers (b) and (d), are obtained from historical records of fire occurrence for a period of time and district or region in which the analysis is performed.

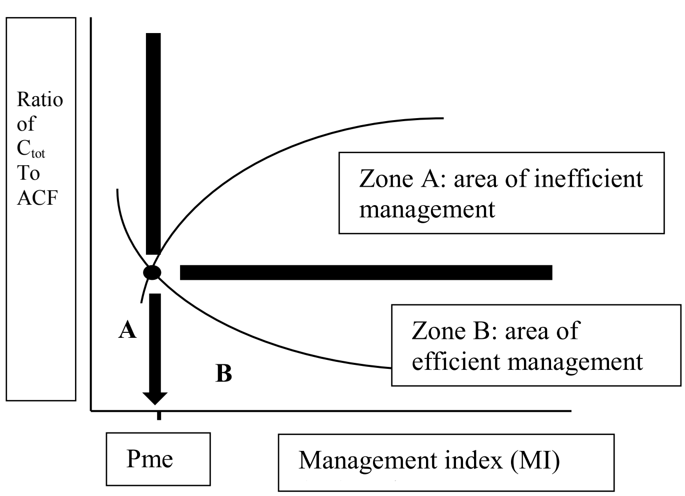

**Figure 3—**Graph showing the most efficient management program. Zone A represents an inefficient management area, while zone B represents an efficient management area.

The MI utilities range is very broad within the analytical framework that integrates the results of suppression operations and associated costs. Regular monitoring of MI helps identify operational deficiencies in the different districts where the analysis is applied. Internally the ACF provides information regarding the effectiveness of suppression operations by including a comparison ratio between the area affected and the area that the fire could have reached if suppression actions had not been undertaken. This comparison is done for the period of time between the start of the fire and the time fire is declared controlled. Applying the model to different provinces provides numerical information for the coefficients and powers of equations 12 and 13. The example that follows is for a study of the province of Cadiz, Spain, for the years 2001 to 2011.

Based on the MI for the area, estimates for Ca and AFC are shown in equations 14 and 15 respectively.

$$CA = 3754{,}25e^{-13{,}991MI} \quad (14)$$

$$ACF = 297{,}16MI^{0{,}508} \quad (15)$$

## Estimation of the efficiency of suppression methods with nonparametric methods; Data envelopment analysis

Determining the efficiency of production systems can be understood as an econometric technique through which the existing relationship between the resources employed or assigned and the results obtained, or the target previously set as a goal to be reached, is addressed. That is, it can be understood as the relationship between revenues and expenditures or inputs and outputs generated in a given production system. Conceptually and when evaluating suppression operations, efficiency can be associated with estimating a ratio that allows getting information from the results obtained from using a particular set of suppression resources and their incurred costs on a specific incident or fire.

The study and determination of efficiency can be approached either through parametric methods by determining the stochastic production possibility frontier[2],

[2] This refers to the maximum possible production of a product given a set amount of resources. See chapter 2 in Kumbhakar and Lowell (2000).

which is a deterministic procedure, or by nonparametric methods using linear programming techniques (Parra and others 2009).

For simplicity, consider the case of two types of suppression resources whose costs or prices derived from their use are ($X_1$) and ($X_2$), respectively. Moreover, Y, can be defined as the final product obtained by their marginal utility. In other words, as the final product obtained after fire control (e.g., area controlled per unit of time, ACF reached, etc.). A graphical representation of the efficiency analysis permits us to define two important concepts that allow obtaining explanatory components of efficiency, these are ***technical efficiency*** (TE) (maximum output with a given set of inputs) and ***allocative efficiency*** (AE) (reduction in production costs if the unit produced used its production inputs most efficiently) (Fig. 4; adapted from Farrell 1957).

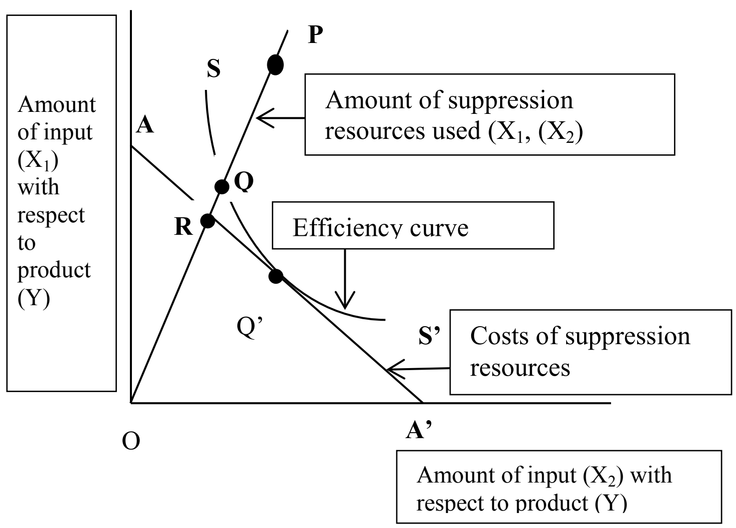


**Figure 4**—Graphic representation of efficiency analysis, relating the amount of inputs to their prices and the amount thereof used in suppression.

The curve SS' represents the production isoquant for the combination of suppression resources ($X_1$) and ($X_2$). The segment (QP) determines both the ***technical inefficiency*** (TIE) and the reduction in the amounts of the suppression resources ($X_1$) and ($X_2$), without producing changes in the output target (or initially agreed final suppression variable). From the graph it is clear that the amounts used are above the efficient production isoquant curve (SS'). The value of TIE is determined by the ratio between the distances defined by the segments (QP) and (OP) (Eq. 16).

Based on the above expression, ***technical efficiency*** (TE) is determined as one minus the value of *TIE*.

$$TE = 1 - TIE = OP - QP/OP = OQ/OP \qquad (17)$$

Knowing the price variation between suppression resources with the same classification or type (e.g., transport and attack helicopters), one can determine AE (reduction in production costs, if the unit produced (assigned) was efficient in the allocation of suppression resources). AE is represented by the segment (RQ) and determines the cost reduction by switching the efficiency point (Q) to position (Q') (Fig. 4). Finally, ***economic efficiency*** (EEI) can be obtained by the product of TE and AE (Eq. 18)

$$EEI = (OQ/OP)(OR/OQ) = OR/OP \qquad (18)$$

The efficiency isoquant curve is estimated using data from the fire occurrence database in the forest region or district in which the efficiency analysis is performed.

Once the econometric components of the efficiency and its conceptual identification have been achieved the analysis and study of the results of forest fire control and suppression operations can proceed. Using the analysis method presented here for determining efficiency requires the prior definition of the following econometric model applied to forest fire suppression operations (Eq. 19).

*Inputs* refer to the economic value of the area that has been successfully protected based on suppression activities defined when planning the suppression operations.

$$I = Vr - Vr(Fd) = Vr(1 - Fd), \qquad (19)$$

where I represents the inputs (total economic value saved from the impact of the fire within the fire perimeter), Vr is the economic value of each of the resources in the area where the wildfire has evolved (area bounded by the final fire perimeter), expressed in monetary units (€, $), and Fd is the depreciation factor of the economic value due to the effect or impact of the fire (Molina and others 2009, Rodríguez y Silva and González Cabán 2010, Rodríguez y Silva and others 2012).

The total input or value for all existing resources is given by equation 20.

$$\sum Vr - \sum Vr(Fd) - \sum Vr(1 - Fdi) \qquad (20)$$

On the other hand, the outputs or suppression costs are the sum of the individual costs of all suppression resources dispatched to the fire. In terms of efficiency analysis, *outputs* are the resulting solutions from the combination of suppression resources with a production rate level allocated to the economic value of the resources at risk saved from the impact of the fire. This is the output or suppression costs of the operational suppression system. Accordingly, the efficiency expression can be written as follows, where ET is the efficiency estimator, and all other variables as previously defined:

$$ET = 1 - \sum_{j=1}^{j=m} Cej \Bigg/ \sum_{i=1}^{i=n} Vrix(1 - Fdi) \qquad (21)$$

From equation 21, ET values result from the ratio between revenues (outputs) and expenditures (inputs); therefore, it can generalize that:

If $0<ET<1$, then the efficiency is low to very high
If $ET=1$, then the efficiency is in balance

To establish measurement consistency, adjustments are necessary to the ET equation (Eq. 21). To do this, weights are introduced that normalize the measurement heterogeneity among the variables in the equation. The weights help put all variables in terms of the ratio of its value to the total overall value for that variable. Thus, for the case of the different suppression resources used (outputs), the weights ($\beta j$) represent the time each resource was used in relation to the total elapsed time from the start of suppression actions until the fire is under control. For the *inputs or* economic value of each natural resource, the weights ($\alpha_i$) represents the economic value of each resource relative to the total economic value of all natural resources in the area affected by the fire. By applying the weighting criteria to ET, we obtain the following equation:

$$ET = 1 - \sum_{j=1}^{j=m} \beta j(Cej) \Bigg/ \sum_{i=1}^{i=n} \alpha i Vrix(1 - Fdi) \qquad (22)$$

For simplicity, we suggest a somewhat arbitrary qualitative classification of the results to establish an efficiency ranking (Table 1) by associating the ET estimates to four categorical levels: Low, Moderate, High, and Very High.

**Table 1**—*Qualitative classification to establish an efficiency ranking*

| Efficiency value interval (ET) | Qualitative classification |
|---|---|
| 0<ET<0.26 | LOW |
| 0.26≤ET<0.6 | MODERATE |
| 0.6≤ET<0.7 | HIGH |
| 0.7≤ET<1 | VERY HIGH |

We used the Almonaster fire that occurred in 2008 in the Huelva Province, Spain as a case study to test the applicability of the methodology. Determining the economic valuation of the natural resources and their economic depreciation as a result of a fire enables obtaining their residual economic value and therefore, the equivalent output volume that forms the numerator of eq. 18 that provides the economic efficiency value EEI (Table 2).

**Table 2**—*Economic valuation of the natural resources and losses due to the impact of the fire*

| **Resource\Economic Value (€)** | **Existing Value** | **Value Lost** | **Value Saved** | **Weighting ($\alpha_i$)** |
|---|---|---|---|---|
| **TIMBER** | 246.658,20 | 145.321,64 | 101.336,56 | 0,4485 |
| **FIREWOOD** | 65.426,34 | 31.734,26 | 33.692,08 | 0,1491 |
| **FRUIT** | 38.195,25 | 12.726,71 | 25.468,54 | 0,1127 |
| **BIODIVERSITY** | 75.436,24 | 45.386,75 | 30.049,49 | 0,1323 |
| **RECREACTION** | 35.124,62 | 25.761,24 | 9.363,38 | 0,0414 |
| **HUNTING** | 87.356,42 | 61.324,15 | 26.032,27 | 0,1152 |
| **TOTAL** | 548.197,07 | 322.254,75 | 225.942,32 | |

For the costs of suppression resources, a descriptive table (Table 3) was prepared that identifies for each suppression resource its cost, intervention time and weighting value ($\beta_j$).

**Table 3**—*Costs of suppression resources dispatched*

| No. of units | Type of suppression resource | Suppression cost (€) | Intervention time in minutes | Weighting (βj) |
|---|---|---|---|---|
| 2 | CL-215T Aircraft | 62.940,09 | 826.0 | 0,043 |
| 3 | Bell 412 Helicopter | 38.430,40 | 1.261 | 0,066 |
| 1 | KAMOV K32 Helicopter | 25.423,18 | 726.0 | 0,038 |
| 2 | Air Tractor 802 Aircraft | 15.247,33 | 1.401 | 0,074 |
| 10 | Crew (15 components) | 73.990,37 | 8.057 | 0,420 |
| 1 | Bulldozer | 1.155,56 | 946.0 | 0,050 |
| 8 | Pumper truck | 8.781,86 | 5.605 | 0,290 |

The resulting Efficiency Factor (ET) for the 2008 Almonaster fire from applying the weighted efficiency expression EEI, as shown in equation 23, is 0,33.

$$ET = 1 - \sum_{j=1}^{j=m} \beta j(Cej) \Big/ \sum_{i=1}^{i=n} \alpha iVrix(1 - Fdi) = 1 - 41017/60728 = 0.33 \quad (23)$$

The results from equation 23 show that the efficiency achieved with the combination of suppression resources listed in Table 3 was 0.33 or 33%. As shown in Table 2 this efficiency ratio enabled saving 41%[3] of the economic value of the natural resources in the area affected by the fire. In our qualitative ranking (Table 1) 33% efficiency corresponds to a classification of moderate efficiency.

Efficiency analysis can be directed beyond the individualized study of forest fires. Indeed, it enables comparative evaluations between fires and even determines the efficiency of a given forest fire protection program in a forest district. For example, we can assume that each of the (N) fires has (K) inputs (suppression resources) and (M) outputs (revenues or benefits obtained by protecting the economic values of natural resources). Then, the i$^{th}$ fire is represented by the vectors $X_i$ and $Y_j$, respectively. The matrix (KxN) of inputs X and the matrix (MxN) of outputs Y represent the data of the (N) fires considered in the analysis. The purpose of applying data envelopment analysis is to build through a nonparametric method a production possibility frontier on the reference points, such that all observed points are either on or inside the frontier.

The efficiency for each fire is obtained from the ratio of all the outputs over all the inputs ($U'y_i/V'x_j$), where (U') is the vector of output weights (dimension Mx1) and V'is the vector of input weights (dimension Kx1). The selection of the optimum weights is obtained from equation 24.

$$Maxuv(U^{'}y_i / V^{'}x_j), \text{ subject to } U^{'}y_i / V^{'}x_j \leq 1, U, V \geq 0, \text{ and } j = 1,2,3...N \quad (24)$$

[3] Computed as Value saved/Existing value or 225.942,32/548.197,07.

This implies determining the values of U (weight associated with the generic i^th^ output) and V (weight associated with the i^th^ generic input), such that the efficiency of the i^th^ forest fire of the total fires (N) under study is maximized, subject to the efficiency measures being less than or equal to one; considering the following restriction:

$$Vx_j = 1, \quad \max_{(u,v)}(U' y_j), \text{ subject to } U' y_j - V' x_j \leq 0 \qquad (25)$$

Applying linear programming, the above expressions are as follows,

$$-y_i + Y\lambda \geq 0 \qquad (26)$$

$$\Theta x_i - X\lambda \geq 0 \qquad (27)$$

$$\lambda \geq 0 \qquad (28)$$

where **λ** is the efficiency of the i^th^ fire**.** A linear programming solution is necessary for each of the N fire records being analyzed, at the spatial-geographic or temporal scale.

The methodological approach presented here provides options for the combined study of suppression costs and residual economic valuation of natural resources after the impact of a forest fire. For example, the information produced permit managers not only to analyze and classify the results of agreed upon and applied fire suppression options, but also to make adjustments in the combination of suppression resources assigned if necessary. They also allows forecasting fire suppression operations productivity, based on suppression difficulty and cost, as well as records from documented fire suppression operation plans from prior fires. This is an objective tool for use in fire economic planning and decision making.

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