

A Study of Flame Spread in Engineered Cardboard Fuelbeds Part I: Correlations and Observations

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Abstract

Wind tunnel laboratory fires spreading through laser-cut cardboard fuel beds were instrumented and analyzed for physical processes associated with spread. Flames in the span-wise direction appeared as a regular series of peaks-and-troughs that scaled directly with flame length. Flame structure in the stream-wise direction fluctuated with the forward advection of coherent parcels that originated near the rear edge of the flame zone. Thermocouples arranged longitudinally in the fuel beds revealed the frequency of these temperature fluctuations decreased with flame length but increased with wind speed. The downstream extent of these fluctuations scaled with Froude number and flame zone depth. These behaviors are remarkably similar to those of boundary layers, suggesting a dominant role for buoyancy in determining wildland fire spread.

Nomenclature

D horizontal flame zone depth (m)
 f frequency (Hz)
 g acceleration of gravity (9.81 m s^{-2})
 L flame length (m)
 t time (s)
 U horizontal wind speed (m/s)
 w fuel loading (kg m^{-2})

X horizontal stream-wise distance (m)
 Y transverse width of fuel bed (m)
 Z fuel bed depth, vertical (m)
 λ transverse wavelength of flames (m)
 R fire spread rate (m s^{-1})

Subscripts

r Pearson correlation coefficient
 f flame residence time

Introduction

In the study of wildfire spread, the heating and ignition of fuel particles by flame contact has been largely neglected in favor of radiation. This is unfortunate because research has recently suggested that radiant heating is insufficient alone to ignite fine fuel elements at fluxes common to wildland fires. [1] Fine particles ($<1 \text{ mm}$ diameter), such as grasses and pine needles that comprise most wildland fuel beds, cool efficiently by free (and forced) convection within the thin boundary layers of small particles. [2] [3]. Experiments imply that fine fuels remain well below nominal ignition temperature until impinged by flames, which takes place within the final centimeters before ignition. [4] [5] [6] Flame structure and variability in and near the fuel bed is, therefore, critical to the preheating and ignition process.

Flame structure in spreading fires, particularly near the leading edge, has received scant attention. Stationary fires of liquid or solid fuel (pool fires, crib fires) in the absence of wind are perhaps the most commonly researched and exhibit regular pulsing [7] [8]. Similar pulsatile behavior has been observed in fires on sloping surfaces [9] [10] [11]. Here we report observations of coherent flame structures in spreading wind-driven laboratory fires, finding that the flame dynamics were responsible for spread.

Experimental Methods

Fire spread experiments were conducted in uniform fuel beds made of laser-cut cardboard. This new technique is more practical at larger scales than laboratory fuel beds made of matchsticks [12] [13] [14] or toothpicks [15], and with more regular particle spacing than excelsior [16] or pine needles [4]. Cardboard and paper strips have been used previously [17] and offer advantages of known homogenous properties such as density and customizable physical dimensions of discrete particles (length, surface area etc.). Fuel beds were burned in the wind tunnel at the Missoula Fire Sciences Laboratory. The wind profiles of the 3m cross-section have been described previously [4] [16] and are laminar except along the bottom surface where an upstream trip-fence produces a turbulent boundary layer. Wind speeds were varied from 0.22 m s^{-1} to 1.5 m s^{-1} with relative humidity of approximately 25%.



Figure 1. Picture of laser-cut cardboard laboratory fuel bed.

We employed a commercial CO₂ laser system for cutting cardboard fuel elements that are connected at regular spacing along a common spine. The cards or “combs” could then be arranged in rows at various spacing to form a fuel bed with vertically standing particles (Figure 1). The cardboard used was brown “chip board” 1.27mm (0.05 inch) thick with approximately 60% recycled content. Fuel particles were created at different lengths and widths and arranged at different row spacing to achieve specific fuel bed properties (Table 1). The laser cutter/engraver system was a Universal Laser Systems Inc. ILS12.150D

model equipped with two 60W laser cartridges. The beams from both lasers were collimated for cutting. The table accommodates sheets of cardboard 0.61m X 1.22m (2 ft X 4 ft) so that multiple combs can be cut from the same sheet in one operation.

Fuel beds constructed of these cardboard combs were 1.22m to 2.45m in width and 3.05m to 6.1m in length (Table 1). The combs were supported and arranged on a foundation of cement-board strips (Hardy Board) 0.635cm x 5.08cm (1/4 in. x 2 in) each separated by a steel spacer 0.158cm x 2.54cm (1/16 in x 1.0 in). The steel spacers rested on the floor to preserve a slot at the upper surface which pinched the spine of the fuel combs such that only the vertical tines were exposed (Figure 1). Tine lengths of 2.54cm (1in), 10.1cm (4in), 20.3cm (8in), and 35.6cm (14in) were used in the burns. The longitudinal spacing of the combs could be adjusted every 1.43cm (5/16in). To limit inflow to the combustion zone along the lateral edges during burning, the

sides of the beds were lined with paper that was treated with the flame retardant Diammonium Phosphate, $(\text{NH}_4)_2\text{HPO}_4$. This technique was described by Byram et al. in a USDA report, where fire retardant limits independent flaming combustion but allows the paper to burn in conjunction

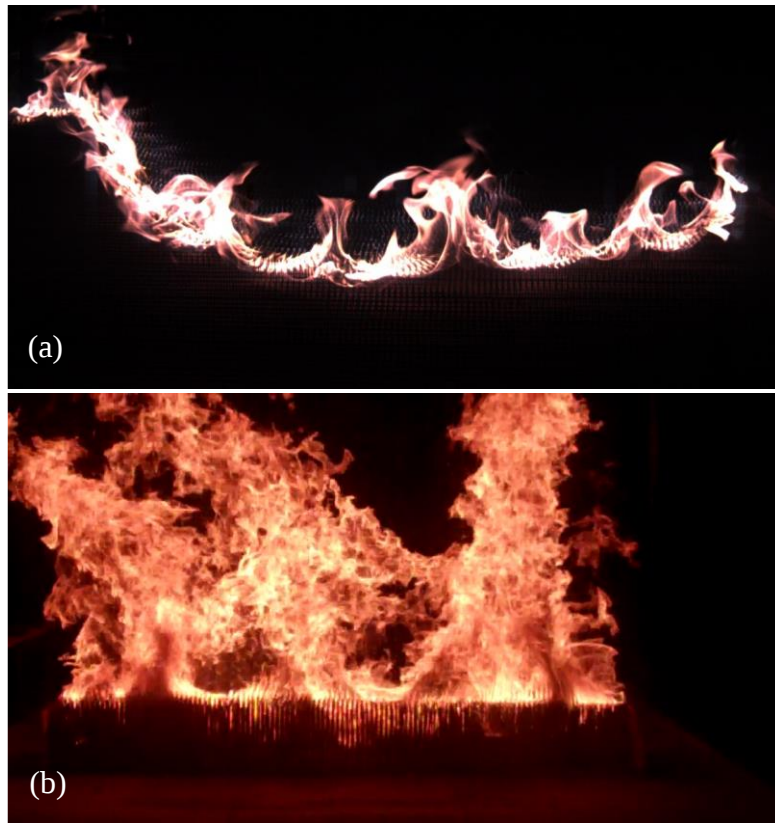


Figure 2. Peak and valley structure of flames looking upwind. In the top image, (a), flames are approximately 0.15m tall. In the bottom image, (b), flames approximately 2m tall.

with the advancing fire front. [18] The consumption of the paper sideliners at the trailing edge of the burning zone avoids channeling of air inflow to the rear of the fire which has been shown to affect fire spread on slopes [19]. Cutouts of the sideliner permitted filming of the ignition process within the fuelbed.

A series of preliminary burns of the cardboard fuelbeds in the wind tunnel were used to refine the instrumentation. The fires were filmed with digital video cameras from the top, sides, front, back, and various oblique angles. This footage revealed two principal dynamic features of the flame zone. First the flame zone became divided in the transverse or span-wise direction into convective peaks and troughs at fairly regular spacing. The ignition interface (at the leading edge of the combustion zone) was convoluted in association with this flame structure, with a concave segment located directly beneath these peaks (Figure 2) and convex segments in the troughs. Second, the flame zone exhibited clear instabilities, which when viewed at an angle from behind and above the bed, appeared as patches originating near the rear of the burning zone. The patches produced dish-shaped depressions on the upper flame surface which expanded in horizontal area as they advected coherently downwind through the troughs (Figure 3). After reaching the ignition interface at the leading edge of the burning zone, they impinged new fuels ahead. When the flame zone was viewed normal to the stream-wise direction, eddies appeared on the upper and lower flame edges which rotated in the opposite direction (Figure 4). Regardless of the local geometry of the fire edge, it progressed at the same rate across the fire front (the convolutions did not noticeably increase or decrease the local spread rate).

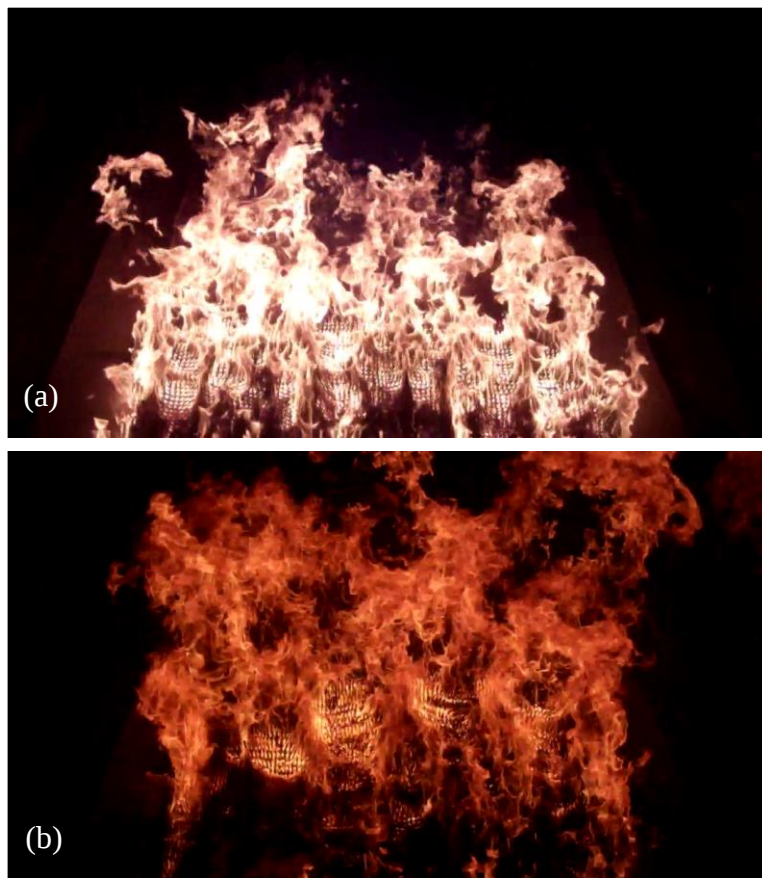


Figure 4. Flame zone structure viewed looking downwind (fire spreading away from camera) shows patches of instabilities (a) soon after the fire starts when the Görtler vortices have shorter wavelength and (b) after larger flames the steady fire front develops. Note the unstable portions expand in horizontal dimension as they advect downwind through the flame troughs.

To record temperature signals inferred from observed flame fluctuations within the fuelbed, a series of 64 thermocouples (0.012mm, type K, bead welded) were arranged in a single line in the direction of fire spread. The thermocouple junction was located at the same the height as the upper surface of the fuel bed. The first 32 thermocouples were spaced 1.5cm and the second 32 thermocouples spaced 3.0 cm apart. Data logging occurred at 500Hz using a National Instruments Inc. data acquisition system. The range of burning conditions reported here includes wind speeds from 0.22 to 1.5 m/s with relative humidity about 25%. Temperature time-series recorded by the 64 thermocouples were analyzed for frequency and correlation of the temperature signals among thermocouples. Each time series was divided into three periods, pre-ignition, burning, and glowing (Figure 5). Pre-ignition was defined from the first crossing of the 350°C



Figure 3. Flame structure normal to spread direction showing rotation of flame eddies.

temperature to where a spline smoothing of the temperature data reached its peak. The burning phase then continued until the spline again dropped below the 350°C level. The beginning time of the burning phase was used to calculate fire spread rate based on the fixed distances among thermocouples.

Frequency analysis of raw temperature signals for each period was conducted using a level-crossing of 350°C and summarized as the average frequency (number of up-crosses divided by the time period) for all 64 thermocouples. Despite temperature pulsing apparent in thermocouple traces (Figure 5), level crossing was used to estimate average frequency of each period because Fourier spectral analysis could not identify peak frequencies consistently among thermocouples. This may be because the peaks were obviously not sinusoidal and apparently not regular enough to produce consistent spectral peaks. The horizontal downwind distance that the temperature signals extend from the ignition interface prior to ignition was estimated by correlating the temperature time-series of the pre-ignition period with the temperatures on each down-wind thermocouple at that time interval. To find this “*flame correlation distance*”, the correlation coefficient was plotted by downwind distance from each thermocouple to identify the horizontal extent of temperature signals prior to ignition. We did not correct for transit time of the flame pulses since the objective was to explore relationships rather than estimate exact distances.

Table 1. Data obtained from experimental burns in the wind tunnel using cardboard fuel.

Experiment	D [m]	Z [m]	L [m]	U [m/s]	Y	Fr	Str	λ [m]	W [kg/m ²]	R [m/min]	t [s]
1	0.250	0.102	0.300	0.500	0.914	0.085	1.272	0.457	0.708	1.201	12
2	0.250	0.102	0.300	0.500	0.914	0.085	1.079	0.457	0.708	1.252	12
3	0.600	0.102	0.600	1.000	0.914	0.170	2.323	0.610	0.708	3.103	7
4	0.800	0.102	0.800	1.500	0.914	0.287	1.178	0.610	0.708	2.938	12
5	0.250	0.102	0.300	0.500	0.914	0.085	1.465	0.457	0.708	1.317	12
6	0.400	0.102	0.600	1.000	0.914	0.170	1.737	0.457	0.708	2.258	12
9	0.100	0.102	0.300	0.220	1.219	0.016	2.431	0.305	0.389	0.734	10
10	0.200	0.203	0.600	0.220	1.219	0.008	3.462	0.406	0.653	0.719	12
11	0.100	0.025	0.100	0.220	1.219	0.049	1.440	0.203	0.144	1.118	4
12	0.200	0.025	0.200	0.440	1.219	0.099	1.575	0.152	0.144	1.606	4
13	0.300	0.025	0.300	0.670	1.219	0.153	1.903	0.406	0.144	3.445	4
14	0.200	0.102	0.500	0.440	1.219	0.039	1.994	0.406	0.546	1.374	7
15	0.500	0.102	0.800	0.670	1.829	0.057	2.331	0.610	0.409	3.595	7
16	0.788	0.102	1.000	0.890	2.438	0.081	3.482	0.813	0.409	5.077	8
17	0.915	0.102	1.000	0.890	2.438	0.081	2.528	0.813	0.409	7.333	7
18	0.200	0.203	1.000	0.440	1.829	0.020	2.717	0.610	1.092	1.798	6
19	0.400	0.203	1.500	0.670	1.829	0.031	3.678	0.914	1.092	3.254	7
20	0.800	0.102	1.200	1.341	2.438	0.153	1.854	1.219	0.546	6.867	8
21	0.400	0.025	0.300	1.341	1.829	0.611	0.770	0.366	0.158	4.058	5
22	0.400	0.025	0.300	1.341	1.829	0.611	0.847	0.366	0.158	3.786	5
23	1.200	0.356	2.500	0.670	2.438	0.018	4.396	1.219	1.430	5.240	12
24	1.200	0.203	2.000	0.890	2.438	0.040	3.054	1.219	1.092	6.464	15
25	0.500	0.203	1.000	0.440	2.438	0.020	5.566	0.813	0.653	5.330	7
26	0.300	0.203	0.600	0.440	2.438	0.033	1.585	0.610	0.466	3.969	8
27	0.900	0.356	1.500	0.220	2.438	0.003	9.258	0.813	1.430	4.322	12
28	0.300	0.102	0.400	1.341	2.438	0.458	0.851	0.610	0.570	4.336	7

Results and Discussion

The 26 experimental burns reported here revealed features of the flame zone that were strongly indicative of flame spread by non-steady convective heating of fuel particles. Specifically, the

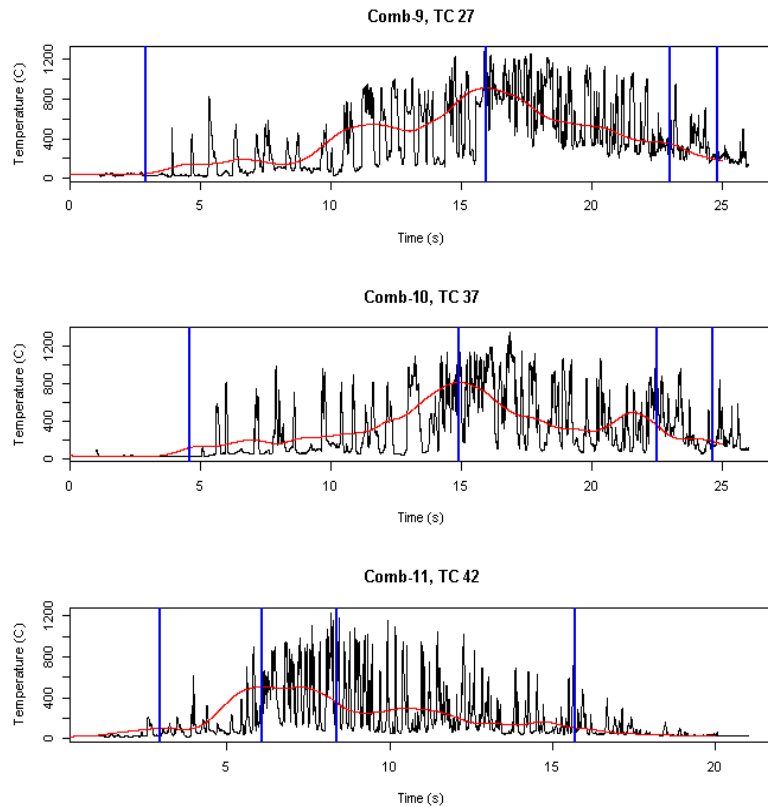


Figure 6. Temperatures recorded by individual thermocouples show non-steady and quasi-periodic flame contacts prior to ignition. Blue vertical lines delimit pre-ignition, burning, and glowing phases.

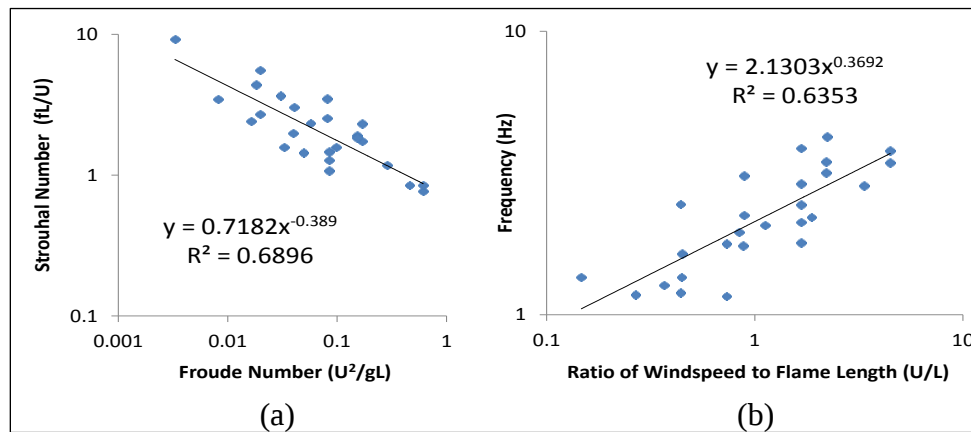


Figure 5. Average temperature fluctuation frequency from level crossing analysis suggests (a) Strouhal-Froude number relationship with negative power and (b) positive relationship with ratio of wind speed to flame length.

thermocouples recorded fluctuating flame presence that creates a temperature signal alternating from nearly ambient temperature to over 1200°C multiple times per second (Figure 5). The average frequency of fluctuations as determined by level crossing of 350°C (number of high temperature crossings per unit time) for the pre-ignition phase showed Strouhal-Froude number scaling (Figure 6). For comparison, the exponent -0.38 is less than the -0.5 for diameter-scaling of pulse frequency in axisymmetric pool fires [8] [20]. A single flame zone dimension did not correlate well to frequency in our spreading fires but frequency did show a strong relation to the ratio of horizontal wind speed and flame length (Figure 6). Our difficulty in detecting a dominant

frequency with Fourier spectral analysis suggests more complex buoyant instabilities similar to those of boundary layers during transition to turbulence [21] [22] [23].

As with boundary layer instabilities

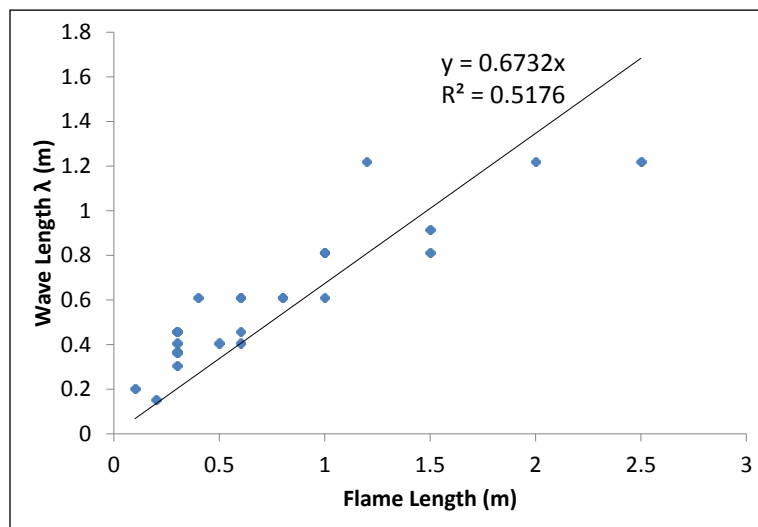


Figure 7. Linear relationship between average flame length and span-wise wavelength from Görtler vortices.

vortex spacing in our fires increases with flame zone dimensions (similar to the radius of curvature). The longitudinal vortices tilt somewhat to the vertical in the flame zone apparently due to buoyancy, and partition the flame front into alternating zones of downwash and upwash where the vortex pairs converge. The downwash zones channel the ambient air flow down and forward through the flame zone and downwind of the burning region as described by Beer in

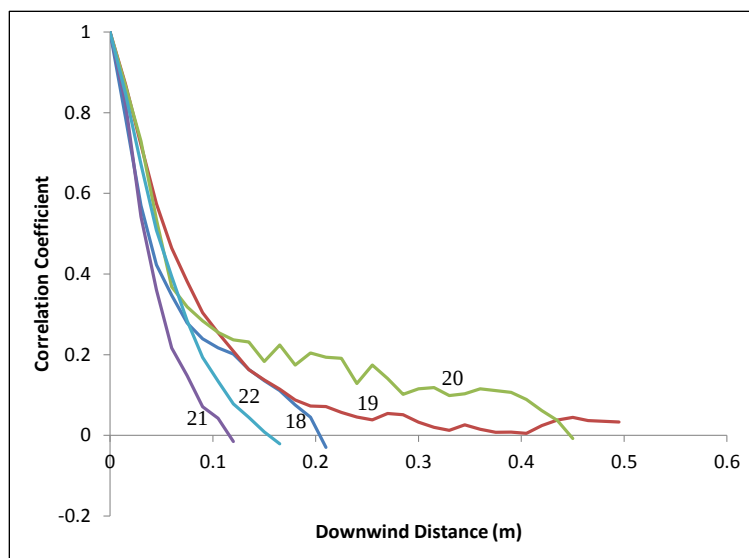


Figure 8. Temperature time-series during pre-ignition were found to be correlated among thermocouples for different distances ahead of the fire. Shown here are average correlations from 32 thermocouples for five fires with very different overall spread rates, wind speeds, and fuel configurations (see Table 1).

[22] we observed a regular transverse (span-wise) spacing of flame “towers” or alternating peaks and troughs (Figure 2) with a wavelength that was proportional to visually estimated flame length (Figure 7). In the flame zones of spreading fires these structures may be explained by the lift imparted to the ambient wind stream by the buoyant flame zone (1000-1200C) which creates instabilities driven by centrifugal forces. Similarly, centrifugal forces in flows over concave surfaces result in counter-rotating longitudinal (stream-wise) vortices at regular spacing, known as Görtler vortices. [24] [25] The vortex spacing in our fires increases with flame zone dimensions (similar to the radius of curvature). The longitudinal vortices tilt somewhat to the vertical in the flame zone apparently due to buoyancy, and partition the flame front into alternating zones of downwash and upwash where the vortex pairs converge. The downwash zones channel the ambient air flow down and forward through the flame zone and downwind of the burning region as described by Beer in 1991 [26]. The downwash also forces flame contact with fuel particles as the dish-shaped parcels related to secondary instabilities journey forward from the rear of the flame zone to the ignition interface where they impinge and ignite new fuel particles. Quasi-periodic frequencies of roller vortices in plane mixing layers [27] [28] [29] are remarkably similar to the flame fluctuations recorded by thermocouples in our spreading fires (Figure 3 and Figure 5). Flame towers move sideways erratically, similar to the sinuous instabilities noted for Görtler vortices [30] and possibly in association with the passing of the flame pulses in the troughs (contributing a lateral source of variability to the temperature signals recorded by each

thermocouple).

The longitudinal sequence of thermocouples permitted temperature patterns of the pre-ignition phase to be statistically correlated with thermocouples located at increasing distance downwind (Figure 8). Plots from all burns suggested that the downwind distance over which correlations

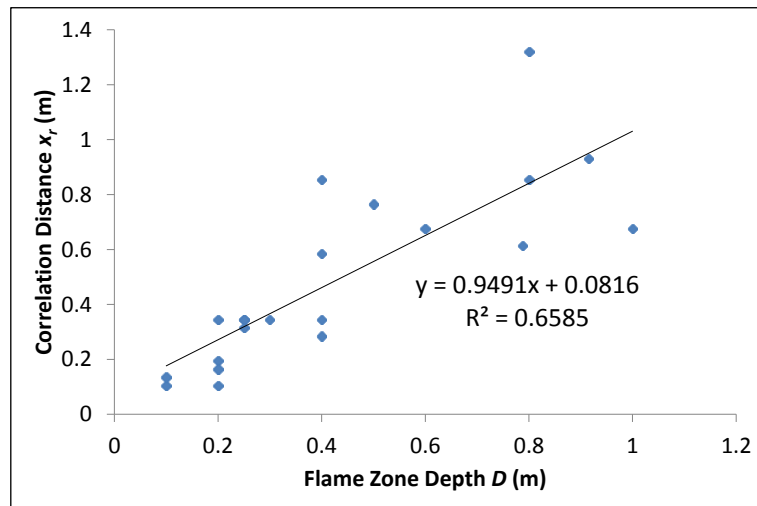


Figure 9. Relationship between flame correlation distance (distance downwind of ignition interface where flame temperatures are statistically correlated) and horizontal flame zone dimension (depth).

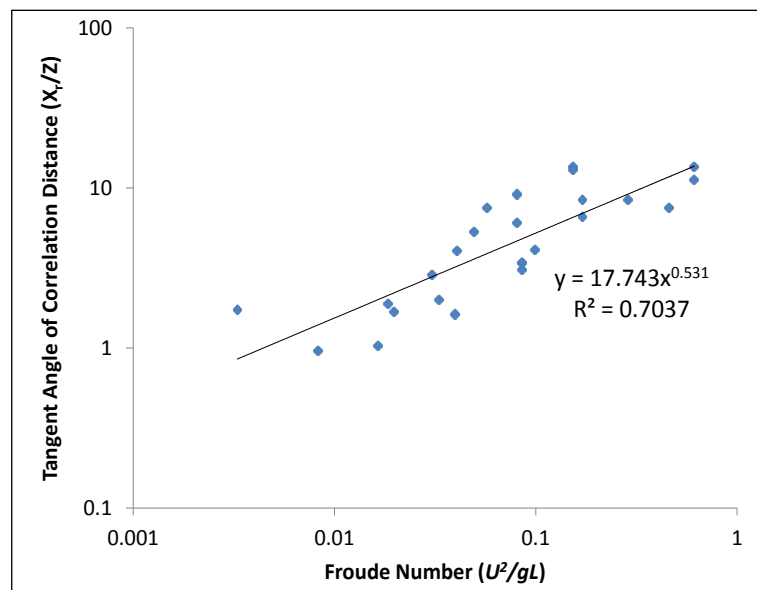


Figure 10. Relationship between the tangent of correlation distance (X_r/Z) and the Froude number.

remain positive was proportional to flame zone depth (Figure 9). This is consistent with analyses of flame drag (flame trailing) downwind of pool fires which typically scale with the diameter of the flame source. [31] [32] The implication here is that the downwind extent to which flames are intermittently heating fresh fuel ahead of the ignition interface related to factors producing flame drag. Also, the reported relationships of Froude number to the tangent of flame angle [33] [34] were supported by our data where the tangent was estimated from the ratio of correlation distance to fuel bed depth (Figure 10).

The upshot of this research is the suggestion that wildfire spread results from the trajectory and frequency of flame contacts which produce fuel particle ignition. Quasi-periodic flame behavior in spreading fires has been noticed previously for trench fires for pine needle beds, but the role of such non-steady flame impingement in igniting fuel particles and flame spread has not been considered. [9] [10] [11]. The Strouhal-Froude scaling suggested by our data is consistent with buoyant dynamics of stationary fire phenomena [35] but new to spreading fires [36] [37]. These findings suggest that the difficulty of identifying an integral length scale for convection related

to ignition [38] comes from obviating the time-dependency of particle heating. It also suggests by virtue of the inverse dependency of average pulse frequency on flame dimensions that slower buoyant dynamics of larger flame zones compensate for the increases in energy release and convective heating distance to avoid runaway spread. If buoyant instabilities are responsible for the flame behaviors and particle ignition, then it strongly suggests that laboratory-scale fire spread processes should extend readily to full-scale field proportions, by proper scaling laws [39], because flame temperature in diffusion flames (and thus buoyancy) remains approximately the same regardless of fire size. Much work is yet to be done to understand useful scaling relationships, but the ultimate goal is to someday incorporate these findings into practical tools for wildland fire managers.

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