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Econometric analysis of fire suppression production functions for large wildland fires

Thomas P. Holmes[A,C] *and David E. Calkin*[B]

[A]USDA Forest Service, Southern Research Station, PO Box 12254, Research Triangle Park, NC 27709, USA.
[B]USDA Forest Service, Rocky Mountain Research Station, Federal Building, 200 East Broadway, Missoula, MT 59807, USA.
[C]Corresponding author. Email: tholmes@fs.fed.us

Abstract. In this paper, we use operational data collected for large wildland fires to estimate the parameters of economic production functions that relate the rate of fireline construction with the level of fire suppression inputs (handcrews, dozers, engines and helicopters). These parameter estimates are then used to evaluate whether the productivity of fire suppression inputs during extensive fire suppression efforts are similar to productivity estimates derived from direct observation and used as standard rates by the US Forest Service. The results indicated that the production rates estimated with operational data ranged from ~14 to 93% of the standard rates. Further, the econometric models indicated that the productivity of all inputs taken together increases more than proportionally as their use is increased. This result may indicate economies of scale in fire suppression or, alternatively, that fire managers learn how resources may be deployed more productively over the course of a fire. We suspect that the identified productivity gaps are primarily due to unobserved factors related to fire behaviour, other resources at risk, firefighter fatigue, safety considerations and managerial decision-making. The collection of more precise operational data could help reduce uncertainty regarding the relative importance of factors that contribute to productivity shortfalls.

Additional keywords: efficiency, fireline productivity, fractal dimension, random parameters, returns to scale, selective rationality.

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Introduction

It is widely recognised that the cost of wildland firefighting in the United States has been trending upwards since the mid-1990s (Calkin *et al.* 2005). As a consequence, federal agencies charged with wildfire suppression responsibilities have been asked to identify means for reducing wildland fire suppression costs while continuing to protect lives, property and natural resources (GAO 2007). With the goal of improving efficiency, prior economic analysis has been applied to examine tradeoffs between fire suppression, fuel reduction and wildfire prevention education programs (Mercer *et al.* 2008; Butry *et al.* 2010) as well as identifying strategies that optimise the allocation of wildfire suppression resources (Donovan and Rideout 2003). Further, economists have argued that the current incentive structure faced by fire managers does little to constrain the commitment of firefighting resources or to cause fire managers to consider the beneficial effects of wildfires (Donovan and Brown 2005; Donovan and Brown 2007; Calkin *et al.* 2011). Despite these advances, one substantial and largely overlooked aspect of fire management that could be subjected to economic analysis is the problem of understanding on-the-ground operational efficiency in fighting large wildland fires. Because very little is known either about the productivity or efficiency of wildland-firefighting resources during lengthy episodes of extended attack, it is not possible to assess whether a change in firefighting strategies or tactics could help reduce fire suppression costs while meeting safety and resource protection goals.

In this paper, we use operational data to estimate the productivity of firefighting resources on large wildfires. Our basic hypothesis is that the complexity of suppressing large wildfires induces a gap between the standard fireline production rates used by the US Forest Service and rates estimated using economic production functions based on daily fire suppression data. We suggest that evidence of a quantitative gap between standard and econometrically estimated fireline production rates may be due to the stochastic, dynamic nature of wildland fire suppression, which induces less than optimal responses both by fire managers and firefighters in the suppression of large wildland fires. Further, evidence of a productivity shortfall would suggest the need for future research to develop more precise data and analyses for identifying sources of firefighting inefficiency with the goal of designing better wildfire suppression strategies and tactics.

To establish the context for our analysis, the following section provides a review of the literature on fireline productivity as well as a limited literature that focuses on fire suppression decision-making during wildfire events. Then we describe the operational data used for our empirical analysis and the method we used to estimate daily rates of fireline construction. A description of the econometric models used to quantify operational performance is followed by the presentation of the results of our empirical analysis. Finally, we present our conclusions and make recommendations for future research.

Literature review

In this section we review several studies that have reported quantitative measures of fireline productivity and that provide insight into the difficulties associated with establishing production standards, especially on large fires that may burn for several weeks. This is followed by an overview of what is known about decision-making on actual wildland fire operations.

Estimates of fireline productivity

Estimates of fireline productivity are necessary for planning purposes and for decisions regarding how many resources to deploy on wildfires of various sizes. Some of the earliest studies of fireline productivity were made by the US Forest Service in California during 1937 and 1938 (Lindquist 1970). These studies evaluated the productivity of three-man crews working a maximum of four 45-min shifts under non-fire situations. Estimates of the number of chains per hour that could be constructed under different resistance to control classes were published in the Forest Service 'Fire Control Handbook' in 1940, and provided the standard fireline productivity rates for 30 years.[A]

Recognising the potential importance of fatigue in constructing firelines over longer periods of time, Lindquist (1970) analysed fireline work logs obtained from fire-suppression activities in California during the 1966 to 1968 fire seasons. He found that fatigue was a significant factor in explaining fireline production rates: after 5 h, rates were 65% of the rate at 30 min. Lindquist (1970) appears to be the first to recommend that fireline construction rates be viewed as probability functions, and he fit a gamma (skewed right) function to estimate the probability of achieving a given rate of productivity.

In addition to these early, formal studies of fireline construction rates, anecdotal evidence of fireline productivity had been reported in several sources such as 'Fire Control Notes' (Hanson and Abell 1941; Stevenson 1951). Given the availability of several sources of handcrew productivity estimates, an effort was made to collect and synthesise all available data on handcrew production rates (Haven *et al.* 1982). The authors reported that fireline construction rates were highly variable, varying by as much as 500%.

Historically, it has been difficult to measure fireline productivity under actual conditions given the logistics of deploying observers, and concerns for their safety. An innovative approach that overcomes this limitation is to estimate fireline productivity using expert opinion. This approach was pioneered by Fried and Gilless (1989) who developed a probabilistic model of fireline productivity for initial attack using a β (skewed right) distribution. The authors found that, compared with rates revealed by expert opinion, previously published rates were quite optimistic, especially for handcrews and dozers. The authors suggest that published rates are generally based on conditions that do not incorporate any of the unlucky events that can, and do, affect actual production rates.

A comprehensive review of literature published in the United States, Canada and Australia regarding the productivity and effectiveness of initial attack fire crews was reported by Hirsch and Martell (1996). They take particular note of the conclusion from prior studies that fireline productivity rates appear to be stochastic and highly variable.

The limitations of prior published studies of fireline productivity, combined with the changes in fire management and fire behaviour since the most recent studies were undertaken, necessitated the development of a new study to evaluate fireline production rates. A comprehensive study of fireline production rates observed on actual wildfires was recently reported (Broyles 2011) and provides the best estimates of fireline productivity on actual fires. During the fire seasons from 2006 to 2009, trained observers collected systematic data on fireline distances, onsite weather, topography, number of personnel on the line, fire behaviour and fuel model. The author notes that there were numerous days for which crews received assignments to construct handlines, but were unable to engage the fire due to extreme fire behaviour.

In addition to recording observations on fireline productivity and using estimates based on expert opinion, a third and more recent approach to analysing wildfire containment has been to use operational data generated from actual wildland fire suppression operations. Finney *et al.* (2009) used daily operations data to estimate a statistical model to identify factors influencing the containment of large wildfires. A related analytical approach was developed by Hesseln *et al.* (2010) who used aggregate operational data to econometrically estimate the influence on wildfire area burned and cost of labour and capital inputs, weather and topographic variables.

Decision-making on wildland fires

The use of economic analysis to study wildland fire suppression differs from its use in the study of most economic enterprises because very little is known about the nature of the productive relationship between suppression inputs and fireline construction when viewed over the duration of extended attack episodes. This perspective was succinctly articulated by Finney *et al.* (2009): '…the effectiveness of suppression efforts on the progress or containment of large fires has not been modeled or even characterized, and it is presently not known what or how different factors are related to successful containment' (p. 249). More than 40 years ago, Leibenstein (1966) argued that the lack of a good understanding of the technical relationships between inputs and outputs can induce a loss of efficiency (which he labelled 'X-inefficiency'). Further, he proposed that

[A]Throughout the manuscript, we use metric units to report results and analysis. For completeness, we note that 1 chain = 66 feet = 20.1168 m. Further, we note that 1 acre = 0.4047 ha.

decision-makers deviate from maximising behaviour due to selective rationality, which he defined as the relative number of opportunities and constraints attended to by decision-makers (Leibenstein 1979). He argued that the degree to which people deviate from maximising behaviour is a function of both their personality and the decision-making context.

The idea that industrial firms exhibit various degrees of inefficiency due to limited or selective rationality may be anticipated to apply even more so in command and control situations exemplified by combat operations or in the control of structural and wildland fires. Decision making in this context is characterised by several critical factors including continually changing conditions, high stakes, time stress, missing data and multiple players (Klein 1993). In these situations, the necessity of making rapid decisions under varying degrees of ambiguity does not afford the luxury of fully analysing all options and tradeoffs. Rather, it is argued that decision makers are selectively rational and seek satisfactory (rather than optimal) decisions. This perspective reflects the concept of 'satisficing' proposed by Simon (1955), and is in concert with cue-based decision models in which decision-makers rely both on their experience and cues from the environment when making decisions in a dynamic environment (MacGregor and González-Cabán 2008).

Empirical testing of the selective rationality (or 'recognition-primed') decision model was undertaken on a series of large wildland fires that ultimately burned a total of 10 117.14 ha (25 000 acres) of timber (Taynor *et al.* 1990). Decisions were categorised as either functional (concerning placement of fire control lines or the mode of attacking a fire) or organisational (concerning the communication among members of the fire management team and between the team and other stakeholders). The authors reported that incident commanders on these large wildfires used selectively rational decision strategies for ~56% of the functional decisions and for ~39% of the organisational decisions. Analytical decision-making was used for the remainder of the decisions.

Data

The operational data we use in this study were drawn from Incident Command System (ICS-209) daily reports for the calendar year 2008 for fires equalling or exceeding 121.4 ha (300 acres). Despite the limitations of operational data, the ICS-209 reports contain a wealth of information that can be used to estimate production functions for large scale firefighting operations, including observations on the levels of resources available for deployment on a fire (crews, dozers, engines and helicopters), incident-management team type, fire size and percentage of the fire contained (see Appendix for ICS-209 resource definitions).

Three considerations were necessary to prepare the ICS-209 data for analysis. First, we determined that data records for fire complexes are ambiguous and, therefore, we only used data for single fires. Second, as we needed a continuous measurement of fire containment, we eliminated fires if more than one contiguous day was missing from the fire record (and used averages for the missing observations). Third, we truncated the data at the day in which fires were declared 100% contained so that final mop-up and line rehabilitation operations were not included in the analysis. After implementing these constraints, we had observations on 47 large fires spanning 650 total fire days.

Fireline construction is not reported in ICS-209 data and needed to be estimated. The estimation method we developed proceeded in three steps. First, we used a mathematical-statistical model, based on Mandelbrot's (1967) description of the fractal dimension (D) of an irregular curve, to evaluate the spatial pattern of final fire perimeters. A prior example of the use of the fractal dimension to estimate wildland fire perimeters was reported by McAlpine and Wotton (1993). In this paper, we used the slope method for estimating D (Lovejoy 1982; Díaz-Delgado *et al.* 2004) based on the following relationship between perimeters and areas:

$$P = \mathrm{C}^{1-D} A^{D/2} \tag{1}$$

where P is the perimeter, C is a constant, A is area and D is the fractal dimension. Taking the logarithm of both sides of Eqn 1, a linear equation is obtained.

$$\log P = (1 - D) \times \log \mathrm{C} + (D/2) \times \log A \tag{2}$$

Hence, given data on P and A, the fractal dimension can be estimated using linear regression. Because fuel types and topography may affect the fractal dimension of fire perimeters, separate statistical estimates of Eqn 2 were made for each US Forest Service region contained in our dataset. Further, given estimates of the intercept ($(1 - D) \times \log \mathrm{C}$) and slope ($D/2$) parameters shown in Eqn 2, the values of C and D can be computed.[B]

Second, estimates of C and D by US Forest Service region were used to construct estimates of daily fire perimeters (P_t) using data on daily fire area (A_t). For this step, it was necessary to assume that the fractal dimension of daily fire perimeters (for which sufficient data were not available) was the same as the fractal dimension of final fire perimeters. Although this assumption may introduce error into the estimation of daily fire perimeters, we suspect that the error is random rather than systematic.

Third, daily fire-perimeter estimates for each fire day t were multiplied by the percentage area contained (Π_t) to estimate the daily cumulative fireline (CFL_t):

$$CFL_t = P_t \times \Pi_t \tag{3}$$

The daily fireline (DFL_t) constructed was then computed as the difference.

$$DFL_t = CFL_t - CFL_{t-1} \tag{4}$$

[B]In particular, multiplying the parameter estimate for log(area) times 2 yields estimates of D. Further, by equating the estimated value of the constant with $(1 - D) \times \log \mathrm{C}$, the value of C can be recovered.

We recognise that DFL_t contains measurement error. In addition to the lack of data on daily fire perimeters, a second source of error stems from the reported estimates of the percentage of the fire that is contained. Typically, these data are not precisely measured and anecdotal evidence suggests that, in some cases, these numbers may be strategically manipulated in order to gain or hold resources. For example, if fire managers are concerned about having resources removed from a fire if it appears that the fire is nearly contained, they may report relatively low containment percentages until they are certain that the firelines will hold and the fire will in fact be contained. This type of strategy could reduce the apparent productivity of resources during the period for which reported containment is conservatively reported. Conversely, apparent productivity of resources would increase when managers are confident that the fire will be contained and containment numbers are adjusted upwards. We think that it is likely that these types of errors introduce random noise into our estimates of daily fireline constructed, but we do not suspect that they would introduce systematic bias.[C]

Although random measurement error in the dependent variable of a linear regression model is clearly undesirable, it does not bias parameter estimates but rather will appear in the equation error term. However, the presence of measurement error has a substantial effect on the possibility of estimating efficiency measures that rely on decomposing the error term into random and systematic components. Consequently, in this paper, we focus on estimates of input productivity and do not explore issues related to efficiency.

In addition to the data on purchased inputs, two variables related to the difficulty of fire suppression, measured on a daily basis, were included in the model specification. First, we included energy release component (ERC), which is an estimate of the heat released per unit area of fuel at the head of a moving fire. This variable is used in the National Fire Danger Rating systems to help determine preparedness levels and is a key input into several key institutional fire behaviour models such as Farsite (Finney 1998), FSPro (Finney *et al.* 2011*a*) and FSIM (Finney *et al.* 2011*b*). Both relative and cumulative ERC variables were computed for the centroid of each final fire perimeter. Cumulative ERC, which places the raw ERC values on a 1–100 scale representing the cumulative probability of observing an ERC value at a specific weather station, scales the variable relative to normal conditions and was included in the model specification. We also included data on average daily wind speed and maximum gust speed in the specification. These data were obtained from the weather station closest to each fire centroid.

We also used data representing heterogeneous conditions across fires, but which were fixed for any individual fire. Measures of mean elevation, standard deviation of elevation and dominant fuel type within each final fire perimeter were computed using GIS tools. Finally, we computed the estimated daytime population living and working within the final fire perimeter.[D] We anticipate that as the population at risk increases, the effort placed on point protection of homes and other structures will likewise increase. In turn, we anticipate that the deployment of resources to point protection is likely to decrease the rate of fireline production.

Econometric methods

Our econometric model is based on the well known Cobb–Douglas production function, which has smooth, convex isoquants and is homogeneous of a degree determined by the sum of the parameter estimates on purchased inputs (Douglas 1976). This function relates a vector of wildland fire suppression inputs (handcrews, dozers, engines, helicopters) to output (daily fireline constructed).

Recall that our data on fire suppression operations include observations over multiple contiguous days for each of several fire suppression operations (referred to as panel data). Given this setup, the Cobb–Douglas function can be written:

$$DFL_{it} = \gamma x_{it}{}^{\boldsymbol{\beta}} \tag{5}$$

where DFL_{it} is the fireline constructed on fire i and day t, x_{it} is a vector of production inputs, γ is a constant and $\boldsymbol{\beta}$ is a vector of parameters. One advantage of the Cobb–Douglas functional form is that, by taking the logarithm of both sides of Eqn 5, we obtain a linear equation that can be estimated by ordinary least-squares regression:

$$\ln(DFL_{it}) = \alpha + \boldsymbol{\beta} \ln(x_{it}) + \varepsilon_{it} \tag{6}$$

where α is a constant (ln γ) and ε_{it} is the equation error. The parameter estimates $\boldsymbol{\beta}$ are elasticities, and indicate the percentage change in output associated with a percentage change in input factors.

The literature review above indicated that fireline production on wildland fires is highly variable and dependent on a suite of exogenous factors that may influence the productivity of suppression inputs. In contrast to purchased inputs, these variables may be considered free inputs (Prestemon *et al.* 2008). Factors that vary within fire operations can be directly entered as inputs to the production process:

$$\ln(DFL_{it}) = \alpha + \boldsymbol{\beta} \ln(x_{it}) + \zeta \ln(\mathbf{w_{it}}) + \varepsilon_{it} \tag{7}$$

[C]Plots of regression residuals from the fitted models reported in the results failed to disclose any apparent systematic bias. However, we note that our procedure may artificially reduce the standard errors around the mean parameter estimates of fireline productivity.

[D]To estimate population within fire perimeters we used the 2008 Landscan USA 3-arc-second (~90 m) daytime and night-time residential population distribution dataset (Oak Ridge National Laboratory 2008). LandScan USA refines the distribution of the population derived from census block level data for each polygon by distributing the total block population to grid cells according to weights proportional to the calculated likelihood of being populated. The likelihood of being populated is based on proximity to landmarks and geographic features such as roads and water bodies, as well as geologic features such as slope (Bhaduri *et al.* 2007).

where $\mathbf{w_{it}}$ is a vector representing within-fire factors (such as daily weather conditions) and ζ are the associated parameters to be estimated.

The Cobb–Douglas model shown in Eqn 7 was estimated using both fixed-effects and random parameter models (described below), which utilise different approaches for incorporating free inputs into the model specification. Further, we recognised that for a small percentage (~4%) of the fire days included in our data, the reported fire containment percentage decreased. A shrinkage in percentage containment likely reflects what Fried and Gilless (1989) referred to as 'unlucky events', and it appears that a loss in fire containment percentage reflects either (1) days for which constructed fireline failed to contain the advancing fire or (2) days for which the fire perimeter grew faster than the containment perimeter. We refer to either of these conditions as extreme fire days and we estimate the Cobb–Douglas production function with and without the adverse days, which allows us to evaluate the effect of extreme fire conditions on productivity.

Fixed-effects model

Note that the model shown in Eqn 7 above does not include heterogeneous factors that remain constant during an entire fire suppression operation, but vary across fire operations. In order to include these factors, a fixed-effects model can be specified:

$$\ln(DFL_{it}) = \alpha_i + \boldsymbol{\beta}\ln(x_{it}) + \zeta\mathbf{w_{it}} + \varepsilon_{it} \quad (8)$$

where α_i is a fixed effect (dummy variable) associated with fire i.[E] Clearly, other factors that induce heterogeneity across fires cannot be included as explanatory variables in Eqn 8, as they would be perfectly collinear with the fixed effect.

Random parameters model

An alternative specification to the Cobb–Douglas panel data model is to treat the constant term as containing random, rather than fixed, effects. This approach has the advantage that variables representing heterogeneous conditions across fires can be included in the model specification (because they are no longer collinear with the constant terms):

$$\ln(DFL_{it}) = \alpha + \boldsymbol{\beta}\ln(x_{it}) + \zeta\mathbf{w_{it}} + \boldsymbol{\varphi}\mathbf{h_i} + \varepsilon_{it} \quad (9)$$

where $\mathbf{h_i}$ is a vector of variables representing heterogeneous conditions and $\boldsymbol{\varphi}$ is a vector of associated parameters. Further, the random effects model can be generalised by considering that some or all of the parameters in the model contain random variation. This so-called random parameters model is able to incorporate all observed and unobserved sources of heterogeneity in the data that may affect production parameter estimates. This model specification is in keeping with literature reviewed above suggesting that fireline production parameters are stochastic and highly variable.

The general form of the random parameters model permits the β parameter estimates associated with each input in the Cobb–Douglas production function to vary within and across fires:

$$\beta_{ki} = \beta_k^0 + \theta_k\omega_{ki} \quad (10)$$

where β_{ki} is the parameter for the kth input on fire i, β_k^0 is the mean (to be estimated) for the kth input, θ_k is the scale factor (to be estimated) and that captures the random variation in the parameter for kth input and ω_{ki} is a random variable associated with input k and fire i. Each ω_i is a random draw from a probability distribution that is specified by the researcher, and the model is estimated using simulated maximum likelihood (Gouriéroux and Monfort 1996).[F] Although we anticipate that the marginal productivity of fire suppression inputs will generally be positive, there may be some days when additional inputs have a negative marginal effect. Consequently, we chose not to restrict the productivity parameters to be strictly positive and thus specified the random parameters to be normally distributed.[G]

The random parameters model presents the opportunity to measure both the mean productivity of purchased inputs as well as the distribution of the productivity parameters within and across fires. Thus, we can estimate the productivity attained for, say, the 5% most productive fire days using the properties of the normal probability distribution.

$$\beta_{k,0.95} = \beta_k^0 + \theta_k \times (1.96) \quad (11)$$

where $\beta_{k,0.95}$ is the estimated productivity parameter for the 95th percentile (1.96 standard deviations above the mean) of the normal distribution for the kth input.

Results

Forty-seven wildland fires from 2008 were included in the analysis. These fires ranged in size from 121.41 to 65 892.44 ha, and the average number of (contiguous) fire days recorded per fire exceeded 2 weeks (Table 1). A type 1 incident management

[E]Schmidt and Sickles (1984) showed how the fixed-effects model could be reinterpreted and used to estimate production frontiers, where the frontier is the maximum of output that can be produced from a collection of inputs. However, as pointed out by Greene (2005), the fixed-effects model mixes time invariant heterogeneity with inefficiency, and this mixed effect cannot be decomposed.

[F]To compute the random parameter model, the true log-likelihood function to be maximised is the sum of the log-likelihoods for each fire, and three procedural steps are required to estimate the true log-likelihood. First, a draw of $\omega_{k,i}$ is made from the specified probability distribution. Second, given the draw of ω for each fire and input variable, the true log-likelihood function is maximised with respect to all of the parameters in the model. Third, the first two steps are repeated a large number of times (for our analysis, 500 replications were used). Fourth, the parameter estimates are averaged across all replications to obtain the simulated maximum-likelihood estimates of β_k^0 and θ_k. For a complete description of the technical details, see Greene (2007, pp. E17-54–E17-55).

[G]We note that sampling from an unrestricted triangular distribution yielded very similar results.

Table 1. Descriptive statistics
ERC, energy release component; IMT, incident management team

Variable	Mean	s.d.	Minimum	Maximum
Final fire size (ha)	10 569.91	15 339.91	121.41	65 892.44
Fire duration (days)	16.402	15.24	3	80
Handcrews (number of crews)	13.08	13.78	0	105
Bulldozers (number of machines)	3.67	5.55	0	39
Engines (number of machines)	22.67	30.43	0	185
Helicopters (number of machines)	4.35	3.87	0	20
Type 1 IMT (dummy variable)	0.23	0.42	0	1
Cumulative ERC	85.35	17.55	22	100
Max wind speed ($km\,h^{-1}$)	29.07	11.52	8.05	119.09

Table 2. Parameter estimates from regressions used to compute fractal dimension of fire perimeters
The dependent variable is the logarithm of final wildland fire perimeter. Standard errors reported in parentheses. Probabilities are significant at **, <0.05; ***, <0.01

Variable	Coefficient (standard error)	Adjusted R^2	Number of observations
All Regions			
constant	4.793*** (0.283)	0.89	47
log(area)	0.595*** (0.031)		
Region 3			
constant	4.057*** (0.683)	0.90	10
log(area)	0.661*** (0.073)		
Region 4			
constant	4.029** (0.633)	0.94	7
log(area)	0.671*** (0.069)		
Region 5			
constant	5.424*** (0.315)	0.95	16
log(area)	0.527*** (0.032)		
Region 6			
constant	5.370*** (0.741)	0.79	11
log(area)	0.538*** (0.087)		

team was engaged for approximately one-quarter of the days spent fighting the fires in our sample. Both daily variables used in the analysis to indicate fire suppression difficulty (cumulative ERC and windspeed) demonstrated a high degree of variation, and extreme firefighting conditions were observed.

The intercept and slope parameters shown in Table 2 were used to compute the fractal dimension (D) of fire perimeters using Eqn 2.[B] Regression results were obtained for Forest Service Regions 3, 4, 5 and 6. Because an insufficient number of observations were available for Regions 1 and 2, we used the parameter estimates from pooling all regions to compute the fractal dimension for those regions. We note that the fractal dimension computed using all data was 1.19, which is nearly identical to the fractal dimension of 1.15 reported for wildfires in Canada (McAlpine and Wotton 1993), and smaller than the fractal dimension of 1.49 reported for wildland fires in Spain (Diaz-Delgado *et al.* 2004). Further, our results indicated that fire perimeters were quite smooth in Region 5 ($D = 1.05$) and Region 6 ($D = 1.08$) and were highly irregular in Region 3 ($D = 1.32$) and Region 4 (1.34). Future research could be directed to evaluate the degree to which various factors such as topography, fuel type and suppression technique contributed to these findings.

For the fixed-effects model estimated using all data (Table 3, Model 1), all of the parameter estimates (except dozers) for the purchased inputs had the expected sign and were significantly different from zero at the 0.10 level or better. In Model 2 (Table 3), which excludes extreme days, all parameter estimates on purchased inputs were significant at the 0.10 level or better, and the R^2 goodness-of-fit statistic is much larger than for Model 1. Apparently, extreme fire days induce statistical noise in the data and reduce the explanatory power of the production model.

The sum of the parameter estimates on the purchased inputs specified in a Cobb–Douglas production function (excluding management team type, which is measured as a dummy variable) provides an estimate of the degree of homogeneity (returns to scale) of the production function. For the fixed-effects model estimated using all of the data, the degree of homogeneity is ~2.0. This means that if all purchased inputs are increased by 100%, output increases by 200% and indicates increasing returns to scale in the suppression of large wildfires. Alternatively, this result may indicate that as fire suppression resources are accumulated on large fires, fire managers learn how to more effectively deploy those resources to contain the fire. Further, we note that the degree of homogeneity estimated for days in which extreme fire behaviour are excluded (Model 2) substantially exceeds the degree observed in Model 1 (~2.3 *v.* 2.0), presumably because overall productivity drops on extreme fire days. Although this is the result we would anticipate, we note that it is largely due to the relatively large contribution of dozer productivity on non-extreme fire days.

The parameter estimates on the variables reflecting the difficulties posed by daily conditions had the anticipated (negative) sign in Models 1 and 2, although only the parameter estimate on the ERC variable in Model 1 was statistically significant. In Model 1, the results can be interpreted to mean that a 1% increase in daily ERC will decrease daily fireline constructed by ~2.2%. In turn, this suggests that sustained high levels of ERC (particularly if combined with high levels of maximum wind speed) could have substantial negative effects on fireline productivity. Further, the lack of statistical significance of either variable in Model 2 suggests that these variables

Table 3. Parameter estimates for the fixed-effects Cobb–Douglas fireline production model
The dependent variable is the logarithm of daily fireline constructed, measured in metres. Standard errors reported in parentheses. Probabilities are significant at **, <0.05; ***, <0.01. IMT, incident management team

Variable	Model 1 Fixed-effects (all data)	Model 2 Fixed-effects (non-extreme days)
ln(crews)	0.709** (0.324)	0.782*** (0.235)
ln(dozers)	0.036 (0.292)	0.488** (0.215)
ln(engines)	0.550** (0.283)	0.411** (0.207)
ln(helicopters)	0.690** (0.338)	0.595** (0.249)
ln(erc_cumulative)	–2.148** (1.096)	−0.287 (0.796)
ln(max windspeed)	−0.663 (0.618)	−0.304 (0.454)
Type 1 IMT	1.944*** (0.530)	1.763*** (0.385)
Adjusted R^2	0.25	0.48
n	650	626

are collinear with extreme fire days (and therefore insignificant when extreme fire days are excluded from the analysis).[H]

The use of a type I incident management team on large wildfires substantially enhanced fireline productivity. For each day that a type I team was used on a fire in Model 1, daily fireline productivity increased by ~1.9%. Although it is not clear exactly how this result was achieved, it may simply reflect management team dynamics during a large wildland fire.[I] When a fire blows up and escapes initial attack, a type 1 team may be called in to engage the fire at a time when it is necessary to construct a substantial amount of fireline. When containment appears certain, the fire may be turned over to other teams to begin mop-up and rehabilitation activities. Clearly, further research is required to gain a clearer understanding of the role that management plays in wildfire suppression productivity and efficiency.

The results of the random parameter models provide further insight into the effect of firefighting conditions on the productivity of purchased inputs (Table 4). In all models, the homogeneity of the production function was greater than unity, which is consistent with the results of the fixed effects model. The variables reflecting the difficulties posed by daily conditions were not significantly different from zero in three of the four models, suggesting that they were collinear with other sources of heterogeneity not included in the model specifications.[J] The log-likelihood statistics for the random parameter models indicate that including all sources of heterogeneity improves the statistical fit over models that include only daily heterogeneity (comparing Model 4 *v*. Model 3 and Model 6 *v*. Model 5).

The mean parameter estimates on handcrew, engine and helicopter productivity were statistically significant and relatively constant across all of the random parameter models. The mean parameter estimates on dozers were only statistically different from zero in the random parameter models when we excluded extreme fire days (similar to the results of the fixed-effects model).

To gain some idea of the levels of maximum productivity revealed in these data, we evaluated the distribution of the input elasticities using parameter estimates for mean productivity as well as the scale parameter estimates. In Model 5 (non-extreme days and including daily heterogeneity in fire suppression difficulty), the mean and scale parameter estimates were significantly different from zero for helicopters and dozers. Plugging these parameter estimates into Eqn 11, we calculate that input elasticities for the best 5% of fire suppression days were 1.31 for helicopters and 2.28 for dozers. Relative to Model 2 (which likewise excludes extreme fire days, and for which estimated mean elasticities were 0.60 and 0.49 for helicopters and dozers), these elasticity estimates indicate that the most productive days of fireline construction for these suppression resources vary from approximately twice to nearly five times greater than average days of fireline construction. This result is consistent with those of prior research suggesting that fireline production rates are stochastic and highly variable (Lindquist 1970; Haven *et al.* 1982; Fried and Gilless 1989; Hirsch and Martell 1996).

The statistical significance of the scale factors on dozers and helicopters remains when factors that vary across fires are included in the model specification (Model 6). This result implies that the variation in purchased input productivity reported in Model 5 can be attributed to heterogeneity in factors varying both within and across fires. The parameter estimates on several factors representing heterogeneity across fires were statistically significant at the 0.05 level, suggesting the importance of topography, fuel type and population at risk on fireline productivity for large wildland fires. In particular, we note how the standard deviation of elevation (a proxy for terrain roughness), timber fuel type, and population living and working within the final fire perimeter (a proxy for intensity of point protection effort) are all significant factors in Model 6.

The Cobb–Douglas parameter estimates were used to compute the average daily productivity (measured in metres) of inputs affecting fireline production (Table 5) and then compared with standard fireline productivity estimates (Broyles 2011;

[H]Subsequent analysis using a logit model specification indicated that the probability of experiencing an extreme fire suppression day was positively correlated with ERC. The removal of extreme fire days in Model 2 thereby reduces the effect of ERC on fireline productivity.

[I]An alternative hypothesis is that Type 1 teams are more likely to burn out fuels leading to natural fuel breaks, thereby increasing fireline productivity.

[J]In Model 5, which excludes extreme fire days, the parameter estimate on the cumulative ERC variable was positive and significant.

Table 4. Parameter estimates for random parameters Cobb–Douglas fireline production models
The dependent variable is the logarithm of daily fireline constructed, measured in metres. Standard errors reported in parentheses. Probabilities are significant at *, <0.10; **, <0.05; ***, <0.01. IMT, incident management team

Variable	Model 3 Random parameters (all data) with daily heterogeneity	Model 4 Random parameters (all data) with all heterogeneity	Model 5 Random parameters (non-extreme days) with daily heterogeneity	Model 6 Random parameters (non-extreme days) with all heterogeneity
Means for random parameters				
constant	2.866 (3.020)	7.686** (3.743)	−1.390 (2.123)	2.466 (2.512)
ln(crews)	0.850*** (0.244)	0.753*** (0.264)	0.712*** (0.128)	0.777*** (0.152)
ln(dozers)	0.046 (0.217)	0.233 (0.259)	0.502*** (0.181)	0.533*** (0.189)
ln(engines)	0.428** (0.183)	0.364* (0.194)	0.486*** (0.140)	0.324** (0.146)
ln(helicopters)	0.410* (0.220)	0.561** (0.231)	0.430*** (0.156)	0.508*** (0.166)
ln(erc_cumulative)	−0.240 (0.629)	−0.398 (0.735)	1.028** (0.468)	0.754 (0.514)
ln(max windspeed)	0.260 (0.472)	−0.213 (0.619)	−0.041 (0.302)	−0.103 (0.385)
Type 1 IMT	1.205*** (0.447)	1.894*** (0.641)	1.137*** (0.333)	1.545*** (0.423)
Scale parameters				
constant	0.870*** (0.162)	0.139 (0.149)	0.854*** (0.118)	0.130 (0.114)
ln(crews)	<0.001 (0.067)	0.004 (0.065)	0.005 (0.049)	0.046 (0.050)
ln(dozers)	0.019 (0.118)	0.117 (0.078)	0.906*** (0.108)	0.505*** (0.010)
ln(engines)	0.078 (0.058)	0.074 (0.056)	0.066 (0.042)	0.004 (0.044)
ln(helicopters)	0.190* (0.107)	0.406*** (0.116)	0.450*** (0.086)	0.444*** (0.092)
ln(erc_cumulative)	0.231*** (0.038)	0.024 (0.034)	0.379*** (0.030)	0.078*** (0.026)
ln(max windspeed)	0.284*** (0.057)	0.360*** (0.057)	0.131*** (0.040)	0.387*** (0.039)
Type 1 IMT	0.022 (0.340)	0.054 (0.330)	0.315 (0.252)	0.013 (0.249)
Non-random parameters				
s.d. elevation (1000 m)	–	−11.635*** (2.613)	–	−11.287*** (1.777)
mean elevation (1000 m)	–	0.269 (0.300)	–	0.291 (0.217)
timber model	–	−1.636*** (0.413)	–	−1.339*** (0.292)
population at risk	–	−0.033*** (0.008)	–	−0.035*** (0.006)
σ	3.683*** (0.064)	3.642*** (0.067)	2.527*** (0.052)	2.556*** (0.044)
n	650	650	626	626
log-likelihood	−1795.77	−1781.87	−1526.398	−1513.693

G. Broyles, pers. comm.).[K] Our estimates of engine and dozer productivity demonstrated the largest gap from the standard rates (representing ~14 and 18% of the standard rates). We suspect that these results are largely explained by the fact that engines and dozers are commonly used for point protection rather than for constructing fireline. Dozers are also used to construct contingency lines, which often do not contribute to the final fire perimeter. Handcrew productivity was ~35% of standard rates, on average. This result is likely due to a variety of factors that affect crew productivity, such as crew size (Broyles 2011), shift assignments (such as building contingency lines) and fire conditions under which it was unsafe to engage the fire. The highest rates of productivity, relative to standard rates, were estimated for helicopters (~93% of the standard).

Discussion and conclusion

The results of this study are largely exploratory in nature and have left unaddressed many issues related to a complete evaluation of on-the-ground wildland firefighting productivity and efficiency. However, given the ability of the econometric models to provide a set of robust parameter estimates obtained under a variety of model specifications, we conclude that the use of economic production models, coupled with econometric analysis of operational data, provides a useful framework for analysing the productivity of fire suppression efforts during extended attack episodes. Despite the data limitations we faced, which likely introduced random error into our measure of daily fireline constructed and reduced estimates of production parameter standard errors, we further conclude that standardised fireline production rates are typically greater than rates estimated utilising economic models and operational data recorded on extended attack episodes.

The advantage of using operational data to estimate productivity of firefighting resources is that these data reflect the challenges faced by decision-makers as they make actual day-to-day fire containment and resource protection decisions. Over lengthy episodes of extended attack, operational data provide the ability to identify transient factors that impede fireline

[K]To make a meaningful comparison, the productivity estimates in Broyles (2011) (which are reported in chains per hour) were adjusted to a daily rate assuming that crews work a 14-h shift per day, and that 33% of each shift (4.62 h) is dedicated to line construction (Broyles 2011). These rates were then adjusted to metres.

Table 5. Comparison of marginal productivity estimates from Model 2 Cobb-Douglas production function with standard productivity estimates

Standard productivity estimates provided by G. Broyles (2011; pers. comm.) for timber fuel types

Variable	Cobb–Douglas daily productivity (m day^{-1})	Standard productivity (m day^{-1})	Ratio
Handcrews	197.66	563.27[A]	0.35
Dozers	438.73	2414.02	0.18
Engines	60.00	422.45	0.14
Helicopters	448.57	482.80[B]	0.93

[A]Type 2 crew.
[B]Type 2 helicopter.

productivity (such as extreme fire days) as well as providing a framework for investigating the influence of management decisions on fire containment and the investment of suppression resources.

Of particular interest was the finding that fire suppression on large wildland fires demonstrated increasing returns to scale. Although this result suggests a reason why fire managers may obtain levels of resources that appear to be excessive when viewed *ex post* (i.e. that resource productivity increases with the scale of resource use) this result also may be explained by the possibility that resource use becomes more productive over the course of a fire as managers learn how to deploy resources more effectively. Further research is needed to evaluate the nuances of economies of scale in firefighting operations.

Inferences of substantial gaps between economic production function estimates of input productivity and standards based on field studies provide an indication of the effect of unlucky events, point protection, the construction of contingency line and other factors on contiguous fireline productivity over the duration of a fire. Efforts are currently underway to obtain more complete, spatially explicit, data from fire suppression operations during extended attack episodes. It is anticipated that these data will help remove or reduce the ambiguity implicit in our comparative analysis.

Our results indicated that helicopters had higher productivity rates than did other suppression resources relative to established production rates. One potential explanation for this may be that helicopters are one component of an overall aviation package that engages the fire. If helicopter use is correlated with other aviation use not measured in this study, helicopter productivity would be overstated in this study. Efforts to include the effect of fixed-wing aircraft on fireline productivity within our economic framework are currently being pursued. A better understanding of the influence of fixed-wing resources on fire suppression is critical given the considerable costs, and safety factors, associated with their operation.

Economic production models based on spatially explicit data acquired during real-time events would likely produce improved understanding of the relative effectiveness of resources in producing fireline, the amount of fireline that engages the final fire perimeter, and other activities that suppression resources engage in that do not result in fireline production. In addition to measuring productivity and efficiency, geospatial analyses may allow for enhanced evaluation of exposure of firefighters to fireline dangers. A formal system to collect and archive relevant data could provide opportunities to better understand the complexities of the fire management environment and methods to improve strategic assessment and resource acquisition strategies, and could facilitate after-action reviews for improved team learning and outside training opportunities.

Finally, we emphasise that a better understanding of where and when large fire suppression resources are most productive and efficient has the potential to improve wildfire management by reducing potential resource loss, unnecessary exposure to wildland firefighters and management costs. Hopefully, future collaborations between research and field personnel will be able to assist the attainment of these goals.

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References

Bhaduri B, Bright E, Coleman P, Urban M (2007) LandScan USA: a high resolution geospatial and temporal modeling approach for population distribution and dynamics. *GeoJournal* **69**, 103–117. doi:10.1007/S10708-007-9105-9

Broyles G (2011) Fireline production rates. USDA Forest Service, National Technology & Development Program, Fire Management Report 1151–1805. (San Dimas, CA)

Butry DT, Prestemon JP, Abt KL, Sutphen R (2010) Economic optimization of wildfire intervention activities. *International Journal of Wildland Fire* **19**, 659–672. doi:10.1071/WF09090

Calkin DE, Gebert KM, Jones G, Neilson RP (2005) Forest Service large fire area burned and suppression expenditure trends, 1970–2002. *Journal of Forestry* **103**(June), 179–183.

Calkin DE, Finney MA, Ager AA, Thompson MP, Gebert KG (2011) Progress towards and barriers to implementation of a risk framework for Federal wildland fire policy and decision making in the United States. *Forest Policy and Economics*. doi:10.1016/J.FORPOL.2011.02.007

Díaz-Delgado R, Lloret F, Pons X (2004) Spatial patterns of fire occurrence in Catalonia, NE, Spain. *Landscape Ecology* **19**, 731–745. doi:10.1007/S10980-005-0183-1

Donovan GH, Brown TC (2005) An alternative incentive structure for wildfire management on National Forest land. *Forest Science* **51**, 387–395.

Donovan GH, Brown TC (2007) Be careful what you wish for: the legacy of Smokey Bear. *Frontiers in Ecology and the Environment* **5**, 73–79. doi:10.1890/1540-9295(2007)5[73:BCWYWF]2.0.CO;2

Donovan GH, Rideout DB (2003) An integer programming model to optimize resource allocation for wildfire containment. *Forest Science* **49**, 331–335.

Douglas PH (1976) The Cobb–Douglas production function once again: its history, its testing, and some new empirical values. *The Journal of Political Economy* **84**, 903–915. doi:10.1086/260489

Finney MA (1998) FARSITE: Fire Area Simulator – Model development and evaluation. USDA Forest Service, Rocky Mountain Research Station, Research Paper RMRS-RP-4. (Fort Collins, CO).

Finney MA, Grenfell IC, McHugh CW (2009) Modeling containment of large wildfires using generalized linear mixed-model analysis. *Forest Science* **55**, 249–255.

Finney MA, Grenfell IC, McHugh CW, Seli RC, Tretheway D, Stratton RD, Brittain S (2011*a*) A method for ensemble wildland fire simulation. *Environmental Modeling and Assessement* **16**, 153–167. doi:10.1007/S10666-010-9241-3

Finney MA, McHugh CW, Grenfell IC, Riley KL, Short KC (2011*b*) A simulation of probabilistic wildfire risk components for the continental United States. *Stochastic Environmental Research and Risk Assessment* **25**, 973–1000. doi:10.1007/S00477-011-0462-Z

Fried JS, Gilless JK (1989) Expert opinion estimation of fireline production rates. *Forest Science* **33**, 870–877.

GAO (2007) Wildland fire management: a cohesive strategy and clear cost-containment goals are needed for federal agencies to manage wildland fire activities effectively. United States Government Accountability Office, Report GAO-07–1017T (Washington, DC)

Gouriéroux C, Monfort A (1996) 'Simulation-based econometric methods.' (Oxford University Press: New York)

Greene WH (2005) Reconsidering heterogeneity in panel data estimators of the stochastic frontier model. *Journal of Econometrics* **126**, 269–303. doi:10.1016/J.JECONOM.2004.05.003

Greene WH (2007) 'Limdep version 9, Econometric modeling guide, Volume 1.' (Econometric Software, Inc.,: Plainview, New York)

Hanson PD, Abell CA (1941) Determining desirable size of suppression crews for the National Forests of Northern California. *Fire Control Notes* **5**, 156–160.

Haven L, Hunter TP, Storey TG (1982) Production rates for crews using hand tools on firelines. USDA Forest Service, Pacific Southwest Forest and Range Experiment Station, General Technical Report PSW-62. (Berkeley, CA)

Hesseln H, Amacher GS, Deskins A (2010) Economic analysis of geospatial technologies for wildfire suppression. *International Journal of Wildland Fire* **19**, 468–477. doi:10.1071/WF08155

Hirsch KG, Martell DL (1996) A review of initial attack fire crew productivity and effectiveness. *International Journal of Wildland Fire* **6**, 199–215. doi:10.1071/WF9960199

Klein GA (1993) A recognition-primed decision (RPD) model of rapid decision-making. In 'Decision-making in Action: Models and Methods'. (Eds GA Klein, J Orasanu, R Calderwood, CE Zsambok) pp. 138–147. (Ablex Publishing Corporation: Norwood, NJ)

Leibenstein H (1966) Allocative efficiency vs. 'X-efficiency'. *The American Economic Review* **56**, 392–415.

Leibenstein H (1979) A branch of economics is missing: micro-micro theory. *Journal of Economic Literature* **17**, 477–502.

Lindquist JL (1970) Building firelines – how fast do crews work? *Fire Technology* **6**, 126–134. doi:10.1007/BF02588899

Lovejoy S (1982) Area-perimeter relation for rain and cloud areas. *Science* **216**, 185–187. doi:10.1126/SCIENCE.216.4542.185

MacGregor DG, González-Cabán A (2008) Decision modeling for analyzing fire action outcomes. USDA Forest Service, Pacific Southwest Research Station, Research Paper PSW-258. (Riverside, CA).

Mandelbrot BB (1967) How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science* **156**, 636–638. doi:10.1126/SCIENCE.156.3775.636

McAlpine RS, Wotton BM (1993) The use of fractal dimension to improve wildland fire perimeter predictions. *Canadian Journal of Forest Research* **23**, 1073–1077. doi:10.1139/X93-137

Mercer DE, Haight RG, Prestemon JP (2008) Analyzing trade-offs between fuels management, suppression, and damages from wildfire. In 'The Economics of Forest Disturbances: Wildfires, Storms, and Invasive Species'. (Eds TP Holmes, JP Prestemon, KL Abt) pp. 247–272. (Springer: Dordrecht, the Netherlands)

Oak Ridge National Laboratory (2008) Landscan USA: high resolution population distribution model. (Oak Ridge, TN)

Prestemon JP, Mercer DE, Pye JE (2008) Natural disturbance production functions. In 'The Economics of Forest Disturbances: Wildfires, Storms, and Invasive Species'. (Eds TP Holmes, JP Prestemon, KL Abt) pp. 35–58. (Springer: Dordrecht, the Netherlands)

Schmidt P, Sickles RC (1984) Production frontiers and panel data. *Journal of Business & Economic Statistics* **2**, 367–374.

Simon HA (1955) Rational decision making in business organizations. *The American Economic Review* **69**, 493–513.

Stevenson S (1951) 'Hot shot' crews. *Fire Control Notes* **12**, 29–31.

Taynor J, Klein GA, Thordsen ML (1990) Distributed decision making in wildland firefighting. US Army Research Institute for the Behavioral and Social Sciences, ARI Research Note 90–42. (Alexandria, VA)

Appendix. ICS-209 resource definitions

- Handcrew: 20 firefighters that have been organised and trained and are supervised principally for operational assignments on an incident.
- Engine: any ground vehicle providing specified levels of pumping, water and hose capacity but with less than the specified level of personnel: typically 3–5 firefighters per engine.
- Dozer: any tracked vehicle with a front-mounted blade used for exposing mineral soil.
- Type 1 Helicopter: 'heavy' helicopter. Greatest overall capacity due to power and size.
- Type 2 Helicopter: 'medium' helicopter. Moderate capacity due to power and size.
- Type 3 Helicopter: 'light' helicopter. Smallest capacity due to power and size.