ESTIMATING THE UNCERTAINTY IN DIAMETER GROWTH MODEL PREDICTIONS AND ITS EFFECTS ON THE UNCERTAINTY OF ANNUAL INVENTORY ESTIMATES[1]

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Abstract—Uncertainty in diameter growth predictions is attributed to three general sources: measurement error or sampling variability in predictor variables, parameter covariances, and residual or unexplained variation around model expectations. Using measurement error and sampling variability distributions obtained from the literature and Monte Carlo simulation methods, the uncertainty in 10-year diameter growth model predictions is estimated as are its effects on annual basal area estimates obtained using an annual inventory system. The results indicate that although annual diameter growth is difficult to predict precisely, the effects of the uncertainty in the growth predictions are greatly attenuated when diameter estimates are aggregated to estimate plot basal area and mean basal area over all plots.

INTRODUCTION

The Forest Inventory and Analysis (FIA) program of the USDA Forest Service has initiated an annual forest inventory system featuring measurement of a proportion of plots each year (McRoberts 1999). One approach to obtaining annual inventory estimates with this system is to use growth models to update to the current year data for plots measured in previous years and then base estimates on the data for all plots. If the updating procedure is sufficiently unbiased and precise, this approach provides nearly the same precision as using all plots but without the adverse effects of using out-of-date information. With this estimation approach in mind, a prototype set of individual tree, diameter at breast height (DBH) (1.37 m aboveground) growth models has been constructed and calibrated for use in updating FIA plot information.

The data used to calibrate the models were taken from measurements of forested Minnesota FIA plots for the 1977 (Spencer 1982) and 1990 periodic inventories (Miles et al 1995). Only trees alive and measured in both inventories were used. For each tree, average annual DBH growth was used as a surrogate for annual growth and was calculated as the ratio of the difference in DBH measurements for the two inventories and the number of growing seasons between measurements. Predictor variables were average DBH for the measurement interval, initial crown ratio (CR), initial crown class (CC), average plot basal area (BA), average plot basal area in trees larger than the subject tree (BAL), and physiographic class (PC). BA and BAL represent the sum of cross-sectional areas of live tree boles at breast height, and, unless otherwise noted, references to both BA and BAL are assumed to have been scaled to a per unit area basis.

The DBH growth models consist of the product of two components, an average component corresponding to regional average DBH growth with respect to DBH and a modifier component that adjusts DBH growth predictions in accordance with local plot and tree conditions. The average component is based on a 2-parameter gamma function with a constant multiplier and uses DBH as the predictor variable, while the modifier component consists of the product of exponential factors of which each incorporates a single predictor variable. Each factor in the modifier product expresses a multiplicative effect on growth predictions in terms of departures from regional or ecosystem averages for a single predictor variable. The general form of the DBH growth model is

$$E(\triangle DBH) = Ave(DBH)*Mod(CR,CC,BA,BAL,PC) \quad [3a]$$

where E(.) denotes statistical expectation, ΔDBH is annual DBH growth,

$$Ave(DBH) = ß_1\ DBHß_2\ exp(ß_3\ DBH) \quad [3b]$$

and

$$\begin{aligned} &Mod(CR,CC,BA,BAL,PC) \\ &\quad = exp[ß_4(CR-C_4)+ß_5(CC-C_5)+ß_6(BAL-C_6) \\ &\quad + ß_7(BA-C_7)+ß_8(PC-C_8)+ß_9(PC-C_8)^2], \end{aligned} \quad [3c]$$

where the ßs are parameters to be estimated and the Cs are constants representing regional or ecosystem averages for the corresponding predictor variables. Using iteratively reweighted least squares techniques, the model was fit separately for individual species. If a parameter was not statistically significantly different than zero, its estimate was fixed at zero. Lessard and others (submitted) provide details of the fitting procedure and verification and validation of the models.

THE ANNUALIZED INVENTORY DATABASE

An annualized database of plot and tree variables was constructed to evaluate the models. The database included measurements from forested FIA plots for both the 1977 (Spencer 1982) and the 1990 (Miles and others 1995) USDA Forest Service periodic inventories of Minnesota. Plots included in the 1977 inventory were actually measured between 1974 and 1978, while plots included in the 1990 inventory were actually measured between 1986 and 1991. Because additional investigations were necessary to

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estimate the uncertainty in model parameters estimated for each species, the data were further restricted to plots that included only the 15 most common tree species in Minnesota, i.e., if the species of any tree on a plot was not among the 15 most common species, the data for that plot was excluded from the database. The resulting database included data for 38,156 trees on 1,951 plots.

Plots measured for the 1977 and 1990 Minnesota inventories consisted of 10 subplots of which each is described as a variable radius plot due to the use of point sampling techniques. With these techniques, trees are selected with probability proportional to cross-sectional area rather than proportional to the frequency of occurrence in the population (Myers and Beers 1971). With point sampling, the number of trees in the population represented by a sample tree, termed the tree factor, varies by tree and is calculated as a scaling constant divided by the tree DBH. Tree factors are used to expand the measurements of sample trees to per unit area estimates.

Based on observations of individual trees with DBHs of at least 12.7 cm, an 11-year database was constructed that consisted of annual values for all model predictor variables and annual status with respect to survival, ongrowth, mortality, and harvest for each tree. Construction of the database required distributing total growth between inventories over varying numbers of years for individual trees in each of four categories: (1) survivor trees that were alive and measured at both inventories; (2) ongrowth trees that attained the 12.7-cm minimum DBH between inventories; (3) mortality trees that died between inventories due to causes other than harvest; and (4) harvest trees that were removed between inventories. For survivor trees, average annual DBH growth was calculated by dividing the total growth between inventories by the number of growing seasons between measurements. Measured DBH for the 1977 inventory was assigned to year 0, and DBHs for the 10 subsequent years were calculated by adding the average annual growth to the previous year's DBH. Because ongrowth trees were measured only in the 1990 inventory, DBH measurements for these trees were assigned to year 11, and DBHs for previous years back to year 0 were sequentially calculated by subtracting from current DBHs predictions of annual DBH growth obtained from the DBH growth models. Ongrowth status for these trees was designated in the year the tree attained the 12.7-cm minimum DBH. For mortality trees, a year of mortality between 1 and 10 was randomly selected from a uniform distribution and assigned to the tree independently of years of mortality assigned for other trees on the same plot. For harvest trees, a year of harvest between 1 and 10 was randomly selected from a uniform distribution and assigned to all trees harvested on the sample plot. For both mortality and harvest trees, DBHs measured in the 1977 inventory were assigned to year 0, and DBHs for subsequent years were calculated by adding previous year's DBHs and predictions of annual DBH growth obtained from the DBH growth models.

Calculation of unbiased estimates of change in BA (ΔBA) is difficult using data from variable radius plots (Van Deusen and others 1986). For these analyses, tree factors corresponding to year 0 were calculated for all trees and then held constant for the succeeding 10-year interval. Thus, annual database values of BA and BAL were calculated using the database of annual tree DBHs and the constant tree factors.

Although the procedures used to construct the annual database create somewhat greater uniformity in annual DBH growth, ongrowth, mortality, and harvest than would be observed, they represent a reasonable alternative. First, most other alternatives for distributing annual DBH growth or survivor trees would require either annual remeasurement or destructive sampling of all trees. Second, in the absence of precise knowledge of annual patterns of ongrowth, mortality, and harvest, uniform distributions represent overall long-term patterns that are reasonable for 10-year intervals. Finally, the impact on estimates of uncertainty, the primary entity of interest for this study, is expected to be minimal.

THE SIMULATION PROCEDURES

Monte Carlo simulations were used to obtain estimates of uncertainty for model parameter estimates; 10-year ΔDBH and DBH predictions, plot BA estimates, and mean plot BA estimates; and annual inventory estimates of mean plot BA. Before the simulations could be implemented, uncertainty had to be quantified for three components: tree- and plot-level predictor variables, residual variability, and parameter estimates. In all situations, uncertainty in model predictor variables was assumed to be non-negligible.

Uncertainty in Predictor Variables

Values of predictor variables are based on FIA field crew measurements and are subject to uncertainty. The tree-level predictor variables, DBH, CR, and CC, correspond to the measurement of a single physical entity, while the plot-level variables, BA, BAL, and PC, are sample estimates. Distributions for measurement errors for the tree-level predictor variables were obtained from the literature. McRoberts and others (1994) reported the results of a study in which 9-10 FIA field crews independently measured the same plots. They estimated a curve for describing the standard deviation of DBH measurements as a function of mean DBH. They also reported that distributions of ocular estimates of CR as percentages in the 0-1 range often deviated “0.3 around the median crew estimate. Nichols and others (1991) reported that when crews returned to plots later in the same growing season to obtain second ocular estimates of CC, 80 percent of estimates were unchanged while the remaining 20 percent were evenly distributed in the two adjacent classes. Uncertainty in BA and BAL estimates was simulated by using DBH measurements incorporating simulated DBH measurement error to calculate BA for each plot and BAL for each tree on each plot. Finally, because of the non-uniformity of plot soil, topographic, and vegetation conditions, PC is also subject to uncertainty due to sampling variability. However, because no empirical estimates of the sampling variability for PC are available, an arbitrary assumption was made that the coefficient of variation for PC is 10 percent.

Residual Variability

Estimates of residual variability were obtained as by-products of calibrating the models. Residuals were assumed to follow a Gaussian distribution with zero mean but with heterogeneous variances. The standard deviations of the distributions of residuals were found to be adequately described as follows:

$$E[\ln(\hat{\sigma}_{res})] = \alpha_1+\alpha_2\ln(\hat{\Delta}DBH), \qquad [4]$$

where E(.) denotes statistical expectation of the quantity between the parentheses, $\hat{\sigma}_{res}$, is the sample estimate of σ_{res}, and ΔDBH is predicted diameter growth from the models.

Uncertainty in Model Parameter Estimates

Model parameter covariances reflect uncertainty in the parameter estimates and must be included as a component of total uncertainty whenever model predictions are involved. When the models are relatively simple (eg., linear) and the uncertainty in predictor variables is negligible, parameter covariance estimates may be easily obtained using analytical methods. However, when the models are complex, nonlinear, and rely on predictors variables whose uncertainty cannot be assumed to be negligible, then Monte Carlo simulations are appropriate, if not also necessary, for reliably estimating these covariances. Failure to incorporate the uncertainty in the predictor variables results in underestimates of parameter covariances and, therefore, in underestimates of model prediction uncertainty. Using the distributions of uncertainty for the predictor variables and residual variation as previously described, distributions of model parameter estimates were obtained using a 4-step Monte Carlo procedure:

1. Simulated ΔDBH observations were obtained as the sums of two components: ΔDBH predictions obtained from the models using the parameter estimates obtained by calibrating the models to the observed data, and residuals randomly selected from a Gaussian distribution with zero mean and standard deviations obtained using equation [4] and the ΔDBH predictions.
2. Simulated values for predictor variables were obtained as sums of two components: observed values of the variables and either measurement error for DBH, CR, and CC, or sampling variability for PC obtained by randomly selecting values from the distributions previously described; using the simulated DBH observations, BA was calculated for each plot and BAL was calculated for each tree on each plot.
3. Model parameter estimates were obtained by fitting the models to the simulated ΔDBH observations obtained from Step 1 using the simulated values of the predictor variables obtained from Step 2; the resulting parameter estimates were recorded.
4. Distributions of model parameter estimates were obtained via 250 repetitions of Steps 1–3.

Uncertainty Estimation

Estimates of the uncertainty in ΔDBH and DBH predictions and in derived BA variables were based on Monte Carlo simulations. The essence of the simulation procedures, explained in detail below, is to initialize plot and tree conditions using the annualized database of values, add random variability where appropriate to mimic uncertainty, use the models to predict annual DBH growth, record estimates at fixed time intervals, and repeat the process a large number of times to create a distribution of estimates.

Two approaches to evaluating uncertainty were used. The ACCUMULATING approach produces DBH predictions for each of 10 consecutive years by sequentially predicting ΔDBH using the models and adding the prediction to previous year's DBH to obtain current year's DBH. Annual estimates of plot BA, mean plot BA, and the standard error of mean plot BA are obtained and are designated the MODEL10 estimates. Uncertainty in estimates obtained with this approach represent the accumulated uncertainty in DBH predictions over the 10-year prediction interval.

The second approach is designated the ANNUAL approach and is intended to mimic the annual inventory system being implemented by the FIA program of the USDA Forest Service. The sampling design for this system features an equal probability grid of field plots which has been systematically divided into five interpenetrating, non-overlapping panels. Each year the plots in a single panel are measured with panels selected on a 5-year rotating basis. To mimic the annual inventory procedures, the plots included for these analyses were ordered with respect to their plot numbers and distributed among five equal-sized panels by systematically assigning every fifth plot to the same panel. Because FIA plot numbers had been assigned sequentially on the basis of the geographic locations of the plots, the panel assignments approximated the systematic, interpenetrating feature of the annual inventory sampling design. Annual inventory estimates of mean plot BA and the standard error of mean plot BA were obtained using three methods: (10 the SAMPLE20 estimates were based on measurements for the current year's 20-percent panel of plots; (2) the MOVING estimates were based on the most recent measurements for all plots; and (3) the UPDATE estimates were based on measurements for the current year's 20-percent panel of plots and updated information obtained using the growth models for the four panels of plots measured in previous years.

Estimates of the uncertainty in ΔDBH and DBH predictions and estimates of plot BA for the ACCUMULATING approach and in estimates of mean plot BA and the standard error of mean plot BA with both approaches were obtained using a 4-step Monte Carlo procedure:

1. Year 0:
   a. Measurement of all plots was simulated by adding the year 0 values of DBH, CR, CC, and PC from the annualized database and simulated measurement errors and sampling variability obtained by randomly selecting values from the distributions previously described; simulated DBH observations were recorded for each tree.

   b. Simulated values of BA and BAL were obtained from the simulated DBH observations by calculating BA for each plot and BAL for each tree on each plot; plot BA, mean plot BA, and the standard error of mean plot BA were calculated and recorded.

   c. A set of model parameter estimates for each species was randomly selected from the distributions previously constructed.

2. Subsequent years:
   a. ACCUMULATING approach. Simulated observations of ΔDBH for all trees were obtained as the sums of previous year's DBHs, predicted ΔDBHs, and residuals randomly selected from Gaussian distributions with zero mean and standard deviations obtained using [4] and predicted ΔDBHs; the simulated observation of DBH and the difference between current and previous years' simulated DBH observations were recorded for each tree.

   b. ANNUAL approach.
      (i) For panels selected for measurement, field measurement was simulated for all plots by replacing values for each tree with values from the annualized database for the appropriate year and adding measurement errors and sampling variability randomly selected from the appropriate distributions.

      (ii) For panels not selected for remeasurement, an updated value for DBH for each tree was obtained as the sum of previous year's DBH, predicted ΔDBH, and a residual randomly selected from a Gaussian distribution with zero mean and standard deviation obtained from [4] and predicted ΔDBH.

   c. For each of the four estimation methods, BA was calculated for each plot, BAL was calculated for each tree on each plot, and mean plot BA and the standard error of mean plot BA were calculated; plot BA, mean plot BA, and the standard error of mean plot BA were recorded for all four methods.

3. Step 2 was repeated 10 times to obtain predictions and estimates for all four methods for years 1-11.

4. Steps 1-3 were repeated 250 times to obtain distributions of DBH and ΔDBH predictions, plot BA estimates, and estimates of mean plot BA and the standard error of mean plot BA for each method for each year.

ANALYSES

Standards of Comparison

The standards of comparison for evaluating bias and the contribution of uncertainty in model predictions to the uncertainty in estimates of mean plot BA were the annual estimates of mean plot BA and the standard errors of mean plot BA obtained from the annualized database values. For comparison purposes, these estimates represent a current year sample of the entire geographic area under consideration and are regarded as being without measurement error. Estimates based on these values use 100 percent of the sample plots and are designated the SAMPLE100 estimates. Because the DBH values on which the SAMPLE100 estimates are based are regarded as having no uncertainty, any uncertainty in the SAMPLE100 estimates is due simply to sampling variability of trees on plots and BA estimates among plots.

ACCUMULATING Approach

Uncertainty in ΔDBH and DBH predictions for individual trees, estimates of plot BA, and MODEL10 estimates of mean plot BA was quantified using the distributions of simulated estimates. Bias in the MODEL10 estimates of mean plot BA and the standard error of mean plot BA is evaluated by comparing these estimates to the comparable SAMPLE100 estimates. Differences between the medians of the distributions of MODEL10 estimates of the standard error of mean plot BA and the SAMPLE100 estimates quantify the effects of uncertainty in model predictions of DBH on the uncertainty of mean plot BA.

ANNUAL Approach

Bias and uncertainty in the annual inventory estimates of mean plot BA and estimates of the standard error of mean plot BA were evaluated using the medians of the distributions of simulated estimates. Comparisons of median estimates of mean plot BA for the SAMPLE20, MOVING, and UPDATE methods to the annual SAMPLE100 estimates of mean plot BA provide the bias check. Comparisons of the medians of distributions of estimates of the standard error of mean plot BA for the UPDATE method to the SAMPLE100 estimates reveals the effects of uncertainty in model predictions on annual inventory estimates of mean plot BA.

RESULTS

The adequacy of the 250 simulations was checked by evaluating the stability of coefficients of variation for the annual MODEL10 estimates of plot BA. For all plots, these coefficients of variation had stabilized by 100-150 simulations and were virtually unchanged for the final 50 simulations.

ACCUMULATING Predictions and Estimates

Histograms of coefficients of variation for 10-year DBH and 10-year DBH predictions indicate that although the median coefficient of variation for ΔDBH was relatively large, approximately 0.20, the median for DBH was small, approximately 0.02 (fig. 1). Thus, 10-year DBH may be predicted quite precisely, even though 10-year ΔDBH is difficult to predict precisely. This result is attributed to two factors: first, as a component of 10-year DBH predictions, ΔDBH is relatively small compared to the other component, initial DBH; and second, the larger component, initial DBH, has little uncertainty, because DBH measurement error is small.

Bias in the MODEL10 estimates of mean plot BA was evaluated by comparing the medians of the distributions of the MODEL10 estimates of mean plot BA to the SAMPLE100 estimates (table 1). The Wilcoxon Signed Ranks test (Conover 1980) detected no statistically significant differences (α=0.05) between the MODEL10 medians and the SAMPLE100 estimates. This result is consistent with observations that the medians of the MODEL10 estimates are in close proximity to the SAMPLE100 estimates and that they fall within a 2-standard error confidence interval around the SAMPLE100 estimates (fig. 2).

The medians of the distributions of the MODEL10 estimates of the standard error of mean plot BA were only slightly larger than the SAMPLE100 estimates. This result suggests that uncertainty in model predictions of ΔDBH has only a slight negative impact on the uncertainty in estimates of mean plot BA (table 1).

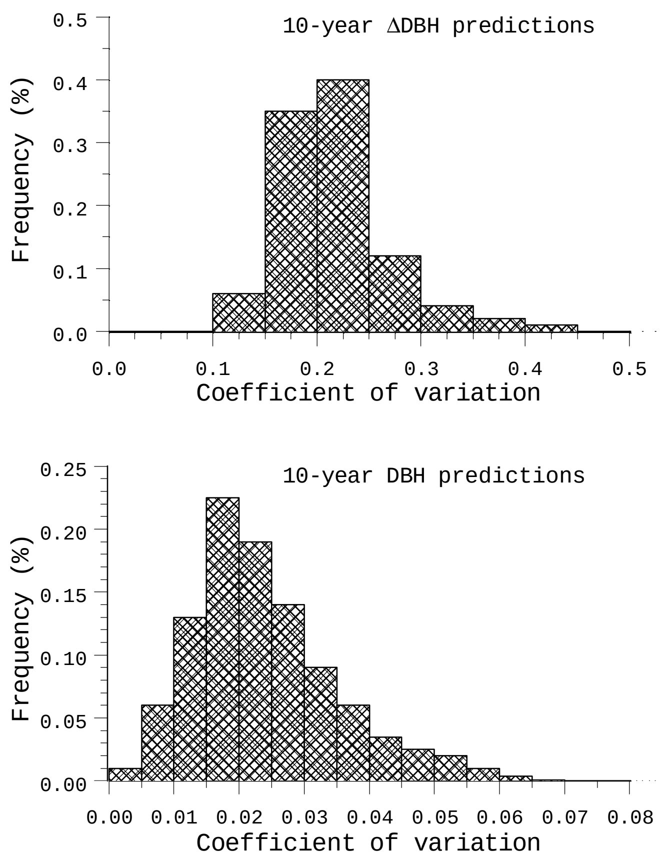

Figure 1—Distributions of simulated △DBH and DBH predictions.

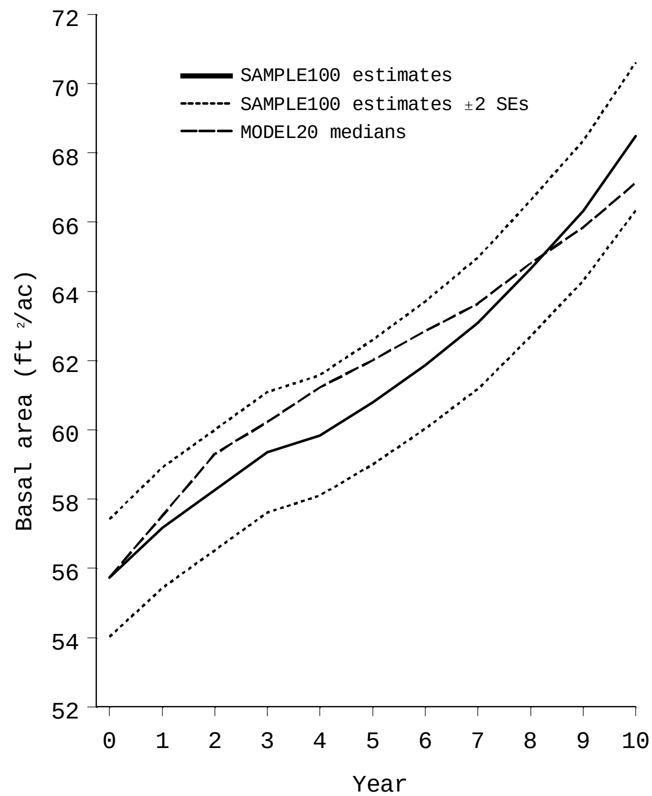

Figure 2—-Annual BA means obtained from DBH predictions.

ANNUAL Approach

Bias in the annual inventory estimates of mean plot BA was evaluated by comparing the medians of the distributions of the SAMPLE20, MOVING, and UPDATE estimates to the SAMPLE100 estimates (fig. 3, table 1). The medians of the SAMPLE20 estimates deviated considerably from the SAMPLE100 estimates due to the SAMPLE20 small sample size, while the medians of the MOVING estimates exhibited consistent bias due to the trend in the SAMPLE100 estimates. The medians of the distributions of the UPDATE estimates track the SAMPLE100 estimates quite closely, a result confirmed by the failure of the Wilcoxon Signed Ranks test to detect statistically significant differences (α=0.05).

The medians of the distributions of the UPDATE estimates of the standard error of mean plot BA were only slightly larger than the SAMPLE100 estimates, again indicating that uncertainty in model predictions of DBH has only a slight negative impact on the uncertainty of annual inventory estimates of mean plot BA.

Table 1—Mean plot basal area estimates

	SAMPLE100		MODEL10		SAMPLE20		MOVING		UPDATE	
Year	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE
	ft²/ac									
0	55.72	0.85	55.74	0.86	55.72	0.85	55.72	0.85	55.72	0.85
1	57.17	0.87	57.51	0.87	60.12	2.02	56.22	0.86	57.79	0.88
2	58.26	0.87	58.87	0.88	59.57	1.97	57.37	0.86	59.20	0.88
3	59.35	0.87	60.23	0.88	57.04	1.94	57.59	0.87	60.61	0.88
4	59.84	0.87	61.23	0.89	61.01	1.92	58.50	0.87	61.08	0.89
5	60.80	0.90	62.01	0.91	59.45	1.85	59.44	0.87	61.98	0.91
6	61.87	0.92	62.85	0.93	64.33	2.16	60.28	0.89	62.87	0.93
7	63.08	0.95	63.65	0.95	63.04	2.16	60.97	0.90	63.97	0.96
8	64.66	0.98	64.81	0.97	63.14	2.24	62.19	0.93	65.45	0.99
9	66.33	1.01	65.85	1.00	68.92	2.26	63.78	0.96	66.94	1.02
10	68.48	1.07	67.14	1.07	67.60	2.24	65.50	0.99	68.93	1.07

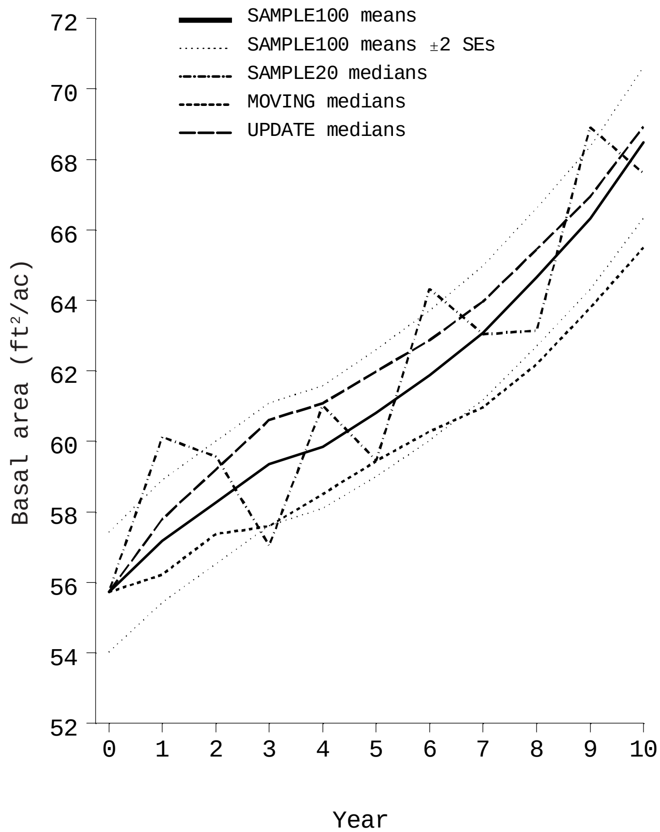

Figure 3—Annual BA means obtained from annual inventory system.

CONCLUSIONS

Two conclusions emerge from this study. First, even though ΔDBH is relatively difficult to predict precisely, 10-year predictions of DBH were quite precise. This conclusion is partially attributed to the observation that 10-year ΔDBH is generally a relatively small component of 10-year DBH. The second conclusion is that the uncertainty associated with model-based updating technique had only a slight negative impact on the uncertainty of 10-year estimates of plot BA, and 10-year and annual inventory estimates of mean plot BA. For the mean plot BA estimates, this conclusion is partially attributed to the observation that DBH prediction uncertainty is relatively small compared to natural variability among estimates of plot BA. Acknowledgment is made, however, that a complete updating system also requires techniques for predicting the survival, regeneration, and removal of trees, components that are not considered in this study. Nevertheless, the study demonstrates that sufficiently unbiased and precise updates of DBH may be obtained.

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