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SAMPLING ERROR IN TIMBER SURVEYS[1][2]

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INTRODUCTION

In cruising timber, the mean volume of any given area is usually estimated from the mean volume per unit area of a small percentage of the timberland taken as a sample. This estimated mean volume will inevitably differ to some extent from the true mean volume. With the elimination of such factors as bias in measurement of trees and of areas or use of inappropriate volume tables, the probable range of difference between true mean volume and estimated mean volume is dependent upon sampling error alone. If the plots or sampling units making up the sample meet the conditions of independent and random selection, the sample itself contains the information needed for estimating sampling error.

While it is of interest to know with what precision the estimated mean has been found, it is vitally more important from an economic standpoint to be able to predetermine what shall be a sufficient sample for an assumed allowable range of error. If the sampling error is not already known, it is necessary to take a preliminary sample of the area to gain this information. From this, it is possible to establish the intensity of the cruise that will produce results within the assumed allowable range of error of the mean. It is then necessary only to provide for occasional supplementary checks, as the cruise progresses, for adequacy of the work done.

In regular cruising practice a systematic arrangement of plots is used in which the sample is made up of contiguous plots forming equidistant strips, or of plots taken at regular intervals along equidistant lines. In most timber surveys, cover type and topographic maps are made in conjunction with the cruise, and for this purpose it is desirable that cruise lines be spaced equidistant in order that all parts of the area may be mapped to a satisfactory standard. As stand and type variation are generally greatest at right angles to contour lines, the cruise lines are usually run in that direction, and plots are spaced closer along lines than between lines. By this procedure type and contour lines are more readily located and mapped, and the more intensive sampling in the direction of greatest variation improves the accuracy of volume estimates.

This arbitrary spacing of plots presents difficulties, should a test of adequacy of sampling be attempted by treatment as a random sample in statistical analysis, since it violates the basic requirement that each possible plot in an area have an equal and independent

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chance of selection in sampling. Experiments in field crops have shown time and again that there is systematic variation in yield from one part of an area to another. Adjacent or neighboring plots tend on the average to be more alike than plots farther apart. Without previous knowledge of an area, it should be assumed that a population of plots is more or less heterogeneous, regardless of appearance of uniformity. The only method of assuring that the elements contributing to heterogeneity are represented in the sample in about their true proportion is to select at random the plots or parts of a sample contributing the estimate of sampling error. The plots indicated for cruising may, for example, be selected by drawing from thoroughly mixed numbers designating each possible plot location. After each draw, the numbered slip is returned and the numbers mixed before another draw. Repeats of plot numbers are rejected, as the objective is a fixed percentage of total area in the sample with each plot selected contributing the same amount of information per unit of area. An improvement over this method, assuring the same freedom of selection, is the use of Tippett's random sampling numbers (*11*).[4] The locating of such plots on the ground may be less convenient than with systematically spaced plots and may also make necessary the running of additional line for mapping. This is the sacrifice necessary for assurance of a valid estimate of error.

The fact that the sampling of timber stands, except for technique in collection of data, is essentially the same as any other problem in sampling in which soil heterogeneity is likely to be present, has not been fully recognized. Apparently advantage has not been taken of methods of testing for heterogeneity and, if indicated, of eliminating its effect on estimates of sampling error—methods first proposed and described by Fisher and Mackenzie (*3*) in 1923. Areas considered as of the same timber type, condition class, age class, and site quality have been treated as homogeneous populations. Various arbitrary plot spacings have been used. Mudgett and Gevorkiantz (*6*) in estimating the reliability of forest surveys in Wisconsin used plots at one-eighth mile intervals along parallel lines 1, 2, or 3 miles apart. Within the break-downs made in a type the errors of random sampling were based on total variation between systematically arranged plots. In the bottom-land hardwood forest survey, Schumacher and Bull (*9*) based their estimate of error on plots at one-eighth mile intervals along lines 3 miles apart. The plots were grouped according to forest condition, i. e., virgin, cut-over, second growth, etc., and the weighted mean of the standard deviations so obtained was used in estimating sampling error. In New England, Goodspeed (*4*) compared line-plot survey, using plots at 165-foot intervals along lines 330 feet apart, with continuous-strip survey along the same lines, taking each 165-foot length of strip as an independent observation. Since the strips and plots overlapped 44.5 percent, variation in sampling was actually confined to 55.5 percent of each sample. From statistical analysis of the data he concluded that the two methods, when applied with equal precision, gave essentially similar and equally reliable results. Preston (*7*) recommended statistical analysis of systematic line-plot cruises and suggested that timber types be mapped separately and prior to sampling. He pointed out the futility of adhering to a 5- or 10-percent, or any other fixed, preconceived intensity of cruise.

[4] Italic numbers in parentheses refer to Literature Cited, p. 735.

In Canada, Wright (*12*) used plots at 10-chain intervals along lines 10 chains apart. In strip survey his strips were run at half-mile intervals, and the tally kept separately by 2-chain segments. Seven estimates were made by taking the first 6 chains in each half mile of strip as a plot, the next 6 chains for a plot in the second sample, and so on to the seventh sample which was made up of plots 4 chains in length. The average of these seven sample estimates of variation was applied to the mean of all plots in calculating sampling error. Wright stated that the main requirement in the use of statistical methods in examining the accuracy of an estimate was to have a reliable figure for standard deviation. Robertson (*8*) favored small plots evenly distributed over the area as against a few hand-picked large plots, with analysis of data from time to time during the survey to insure the taking of sufficient plots to give an accuracy within the required limits for any given factor. The line-plot arrangement he found to be more accurate than the strip. Since in strip survey the plots are contiguous in one direction, he suggested that they represented conditions in that direction beyond their due proportion. Candy (*1*) stated that any method of survey for which it is possible to calculate the accuracy of the estimate obtained, is very much superior to methods in which the accuracy of estimate is doubtful and not at all calculable. He used both systematic strip and line plot, and concluded that only with line plots could adequacy of sampling be determined.

In the present study the true mean volume and the true variation between plots are known for an area of 5,760 acres. The distribution of plot volumes is known to approach that of the normal frequency distribution. Thus all of the essential statistical information regarding the population is at hand. The expected range of sample means from the true mean is calculated, taking into account the effect of such factors as size, shape, arrangement of plots, and intensity of sampling. From samples taken according to the specifications set up, the observed range of means from the true is compared with the expected range. Checks of theoretical with actual efficiency of sampling methods are readily made, since the timbered area studied is part of the Blacks Mountain experimental forest, in which the total timbered area of 9,078 acres has been completely inventoried and plot locations, cover types, and topography mapped. The usual assumption that systematically spaced plots do fulfill the requirements of independent and random selection is tested, both for strip and line-plot arrangements. Efficiency of different methods of cruising is determined on the basis of relative intensity of sampling required for a given accuracy of estimate. For cruises giving valid estimates of sampling error, the precision of sample estimates of population variance is given.

MATERIAL AND METHODS

The 5,760 acres of virgin timber on which the analyses are based is located in northeastern California, within the Lassen National Forest. The timber type is classed as pure pine, with more than 90 percent of the volume in ponderosa pine (*Pinus ponderosa* Dougl.) and Jeffrey pine (*P. jeffreyi* Oreg. Com.). The other timber species are white fir (*Abies concolor* Lindley and Gordon) and incense cedar (*Libocedrus decurrens* Torr.). The stand is all-aged, with young and overmature age classes well represented, but is understocked owing to light representation in the intervening age classes. With the exception of

small areas, the stand over the entire area appears uniform to the eye and not stratified according to any of the criteria used in mapping such stands. In other words, it appears to be a fairly homogeneous population.

The nine sections shown in figure 1 were selected for study because they were full sections and, with the exception of small areas, completely timbered. Board-foot volumes of trees 12 inches and more in diameter were totaled for the 2,304 individual 2.5-acre plots contained in the nine sections. These plots, rectangular in shape (2.5 by 10 chains), are designated as basic plots. In addition, because their

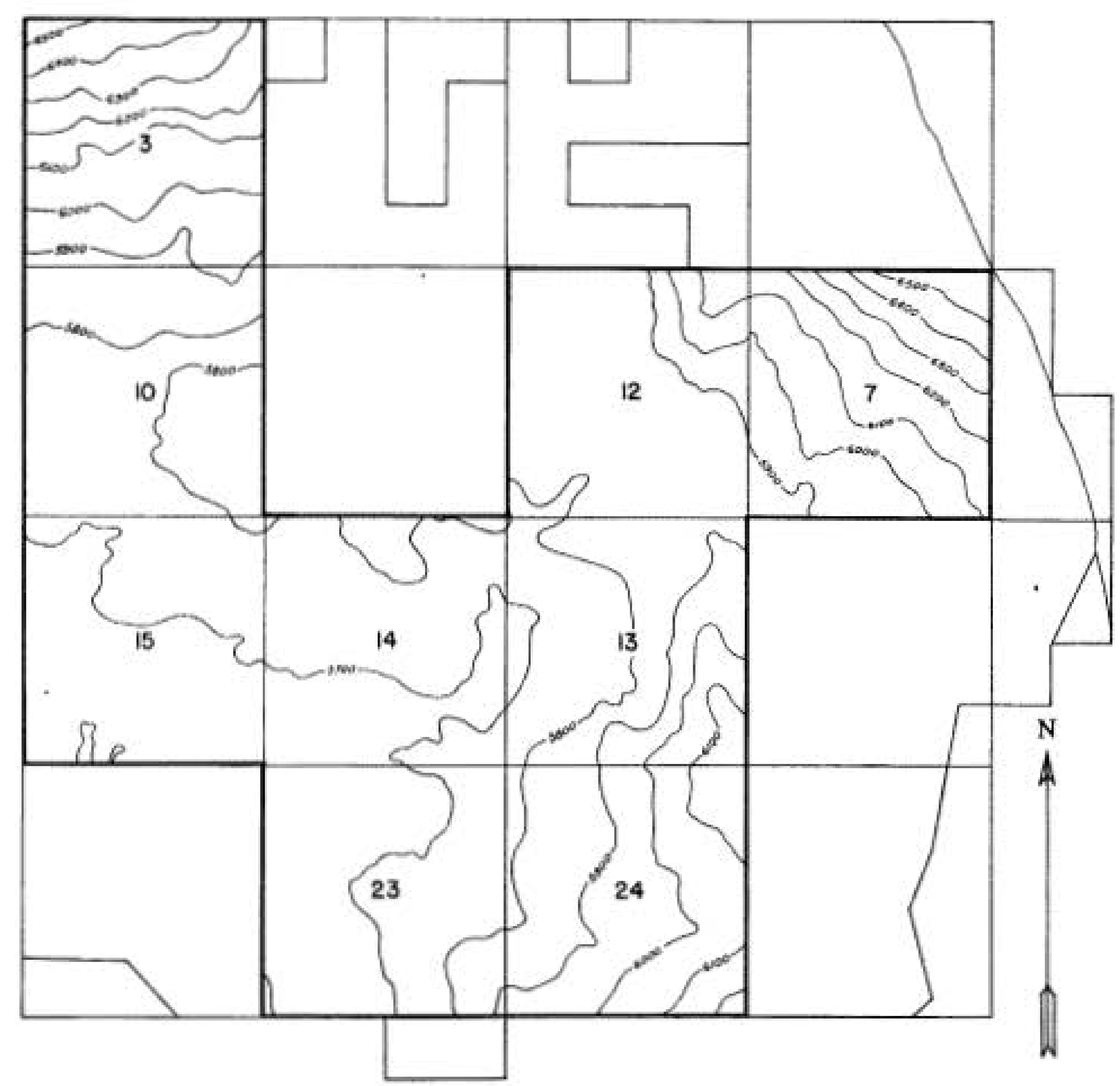

FIGURE 1.—Contour lines and section numbers of area selected for analysis from the Blacks Mountain Experimental Forest.

locations are mapped, adjacent plots may be combined into totals of 1,152 5-acre plots (5 by 10 chains), 576 10-acre plots (10 by 10 chains), or other shapes and sizes in multiples of the basic plot size and dimensions.

Some irregularity in size and shape of basic plots along section lines was unavoidable owing to variation of sections from exact mile squares. For these plots the volumes were proportionately reduced or increased to the 2.5-acre basis. The adjustments needed amounted to a negligible percent of the total. All basic plots were taken with the long axis in an east-west direction.

Subdivisions larger than plots are termed blocks, and are usually taken as regular Land Office subdivisions such as forties, quarter sections, half sections, or sections. Figure 2 shows the size and shape of plots and blocks mainly used. When a plot making up part of a sample is selected independently and at random it constitutes a random sampling unit. When plots are spaced at regular intervals along lines, and the lines selected independently and at random, the line-plot combination is the sampling unit. Likewise, where plots are contiguous end to end forming strips and the strips are selected independently and at random, the strip constitutes the sampling unit. The resulting arrangements may be termed random line plot and random strip, respectively. When the sampling units are taken at

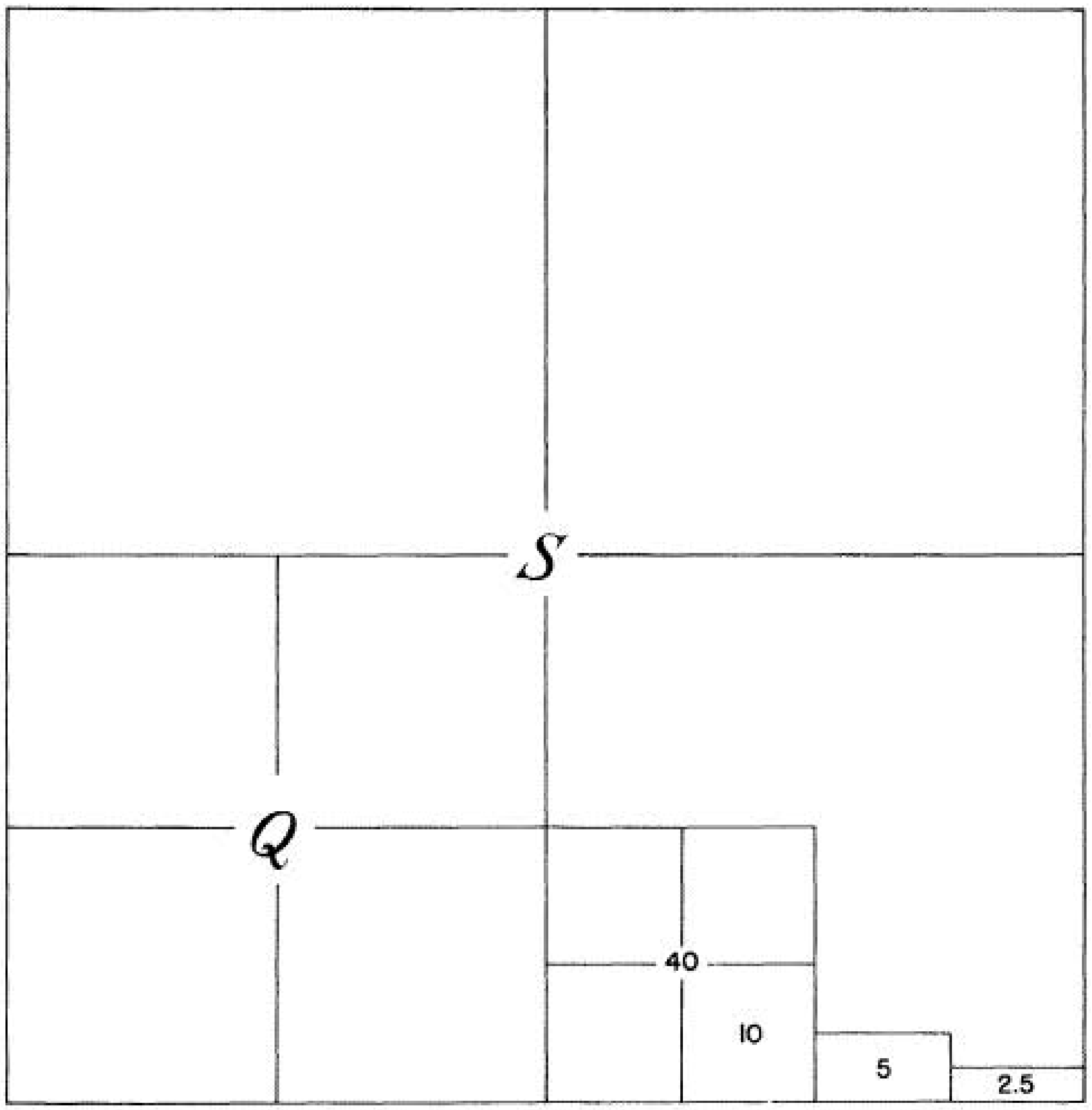

FIGURE 2.—Size and shape of plots and blocks: *S*, Section (640 acres); *Q*, quarter section (160 acres); smaller units designated by area in acres.

random from all possible in the population, the sampling is referred to simply as random, as contrasted to selection of equal numbers of units from each block, which is termed *"random within blocks."* Systematic strip and line-plot arrangements are representative samples in that each block is sampled to the same intensity, but are not random.

The total variation between random sampling units is apportioned into parts contributed by various known sources and the error variance segregated by the analysis of variance method of Fisher. Descriptions of this method and others used in the study are given by Fisher (*2*, *3*) and Snedecor (*10*), and an instructive application of the method by Immer (*5*).

RESULTS

SIZE AND SHAPE OF PLOT

The essential information concerning stand variability is given in table 1. The mean square ratios all exceed F at 0.01 as given by Snedecor (*10*), and therefore volume is correlated with place and the population is heterogeneous. In general, variation between plot

TABLE 1.—*Analysis of variance*

Source of variation	Sum of squares	Degrees of freedom	Mean square	F[1]	F at 0.01[2]
Sections from general mean	109, 693. 3526	8	13, 711. 6691	6. 39	3. 26
Quarter sections within sections	57, 937. 4032	27	2, 145. 8297	3. 48	1. 98
Forties within quarter sections	66, 569. 5344	108	616. 3846	1. 65	1. 11
Tens within forties	161, 109. 7722	432	372. 9393	1. 76	1. 08
Fives within tens	121, 940. 8201	576	211. 7028	1. 83	1. 04
Basic plots within fives	133, 337. 7395	1, 152	115. 7446	--------	--------
Total, basic plots from general mean	650, 588. 6220	2, 303	[3] 282. 4961	--------	--------

[1] $F=\frac{\text{larger mean square}}{\text{smaller mean square.}}$

[2] As given by Snedecor (see text). Values equal to or exceeding these indicate that differences are highly significant statistically.

[3] $\sqrt{282.4961}=16.81$=standard deviation of a basic plot from the population mean of 40.83 thousand feet board measure.

volumes tends to increase as the distance between plots is increased. For this reason, size and shape of plot will be an important factor in efficiency of sampling. Two plots taken side by side will in general include less of the stand variation within their combined area and more will remain between such pairs than will be the case with pairs of separated plots. Thus variation between 5-acre plots will be greater than variation between 5-acre sampling units each made up of two 2.5-acre plots spaced apart along a line. Likewise, owing to correlation between adjacent plots, variation between 5-acre plots, expressed in terms of single 2.5-acre plots taken as parts of 5's, will be greater than variation between 2.5-acre plots as random sampling units. Less stand variation per unit of area is included within 5-acre plots as sampling units than within 2.5-acre sampling units.

The mean square between 2.5-acre plots in table 2 is the total mean square from table 1. It is the error variance of randomly selected 2.5-acre plots. The corresponding value for 5-acre plots is obtained from table 1 by pooling sums of squares and degrees of freedom from 5's within 10's up to and including sections from general mean. The quotient of these two summations—449.3926—is the mean square of randomly selected 5-acre plots, expressed in terms of single 2.5-acre plots. This is considerably larger than 282.4961, and the effect on sampling efficiency is evident from the following formula for variance of a mean:

$$\sigma_M{}^2=\frac{\sigma^2}{n} \tag{1}$$

in which

$\sigma_M{}^2$=the variance of a sample mean, or the standard error squared
σ^2=the variance, or mean square, of the basic plots
n=the number of basic plots.

For a given standard error, n will need to be 449.3926/282.4961=1.59 times as great with 5-acre plots as with 2.5-acre plots. In random sampling from an unlimited population, 1.59 times as much land is needed in sample area with 5-acre plots as with 2.5-acre plots to assure the same precision.

Stated in another way, 282.4961/449.3926=62.86 percent, the efficiency of 5-acre plots as compared to 2.5-acre. To equal in precision the estimate of volume based on sampling with 5-acre plots, an area only 62.86 percent as large would be needed with the smaller plots. The advantage of 2.5-acre plots over 10-acre plots is still greater.

When plots and sampling units larger than the basic plots are used, the variance is then weighted according to the number of basic plots in the larger plot. Likewise n is in terms of basic plots. Obviously, the resulting standard error obtained is not affected. To keep mean squares directly comparable regardless of size, shape, or arrangement of plots used, they are expressed in terms of single basic plots throughout this article.

It is evident from table 2 that, among the 10-acre plots, the long, narrow shape is the most efficient. This is due partly to the presence of small nontimbered areas of such shape that several 10 by 10 and 5 by 20 plots contained little or no volume and consequently increased the average variation. The effect of placing of plots with respect to direction of greatest stand variation is illustrated in figure 1. Variation in timber stands is usually greatest at right angles to the contour lines. In collecting the present data, the longer plot axis always extended in an east-west direction. Thus, within sections having marked changes in elevation, sections 24 and 13 were cruised with plot length in the direction of greatest variation, section 7 at a 45-degree angle to greatest variation, and section 3 parallel to greatest variation. In this respect it would seem that sampling by sections about balances. Had length always been taken at right angles to the contours, the advantages of long, narrow plots would be greater, and possibly the 5 by 20 plots would appear as superior to the 10 by 10.

It should be emphasized that the results in table 2 are based on random sampling from a theoretically infinite parent population. In sampling methods discussed later, where corrections are made to the mean of the limited population and variation between blocks eliminated, these relative efficiencies will not be exactly true, although quite close.

TABLE 2.—*Relative efficiency of plots of varying size and shape in the use of land*

Plot size and shape	Mean square	Efficiency	Relative size of sample for a given precision of estimate
		Percent	
2.5-acre: 2.5 by 10 chains [1]	282.4961	100.00	1.00
5-acre: 5 by 10 chains	449.3926	62.86	1.59
10-acre:			
2.5 by 40 chains	[2] 566.0602	49.90	2.00
5 by 20 chains	[2] 711.8291	39.68	2.52
10 by 10 chains	687.4958	41.09	2.43

[1] 1 chain=66 feet.
[2] Obtained by separate calculation of population variance. All other mean squares obtained from table 1 directly by pooling degrees of freedom and sums of squares.

RANDOM PLOT AND RANDOM-WITHIN-BLOCK CRUISES

As previously stated, in sampling a heterogeneous population it is necessary to select the sampling units independently and at random to insure that the heterogeneous elements are represented in about their true proportion. Using 2.5-acre plots as sampling units, random sample estimates of variance will tend to approach 282.4961, the true variance. As the number of sample estimates is increased, their average will approach more closely to the true, and likewise, estimates from larger samples will be grouped closely about the true variance and closely in accord with the normal law of frequency of error. A valid sample estimate of variance may be substituted in formula (1), in which n is known and the standard error calculated. This tells us the range from the sample mean within which the chances are 2 in 3 that the true or population mean lies. By doubling the standard error obtained we have the range for which the chances are 19 in 20.

With intensive sampling, formula (1) gives appreciable overestimates of sampling error because it estimates the range from the theoretical infinite parent-population mean while here we are interested in range from a parent population limited to 2,304 2.5-acre plots. By substituting 2,304 as n in the formula we can get the estimated range of the population mean from the limited population mean. This is the irrevelant portion of total variance of sample means and is represented by $\frac{\sigma^2}{N}$ in the following formula:

$$\sigma_M{}^2=\frac{\sigma^2}{n}-\frac{\sigma^2}{N} \qquad (2)$$

in which

N=the number of 2.5-acre plots in the limited population (a constant=2,304).

In practice a single estimate of variance together with the sample mean are the two statistics supplied. With the present data the true variance and mean are known for the limited population, and by varying n in formula (2) the expected grouping of sample means about the true mean can be calculated for different intensities of sampling. Since σ^2 changes with varying size and shape of plot, the effect of these factors is also reflected in the expected grouping of sample means. The appropriate mean squares for substitution as σ^2 in formula (2) are given in table 3, the values for which are derived from table 1. The expected standard errors for random samples of varying intensity, made up of plots of varying size, are given in table 4. In all, the expected standard errors based on true variance are shown for seven such cruises by the random arrangement of plots. For each cruise, four independent samples were drawn to compare expected values with observed values. The 2,304 plots were numbered consecutively and the required number of plots for each cruise selected by use of Tippett's *(11)* random sampling numbers. The sample estimates of standard error and deviations from the true mean are shown in table 4 along with the corresponding true standard error.

TABLE 3.—*Variance of plots from the general mean and from block means*

Plot size and source of variation	Sum of squares	Degrees of freedom	Mean square [1]	Variance of plot means [2]	Standard deviation
2.5-acre:					
From general mean	650, 588. 6220	2, 303	282. 4961	282. 4961	16. 81
Within sections	540, 895. 2694	2, 295	235. 6842	235. 6842	15. 35
Within quarter sections	482, 957. 8662	2, 268	212. 9444	212. 9444	14. 59
Within forties	416, 388. 3318	2, 160	192. 7724	192. 7724	13. 88
5-acre:					
From general mean	517, 250. 8825	1, 151	449. 3926	224. 6963	14. 99
Within sections	407, 557. 5299	1, 143	356. 5683	178. 2842	13. 35
Within quarter sections	349, 620. 1267	1, 116	313. 2797	156. 6398	12. 52
10-acre:					
From general mean	395, 310. 0624	575	687. 4958	171. 8739	13. 11
Within sections	285, 616. 7098	567	503. 7332	125. 9333	11. 22
Within quarter sections	227, 679. 3066	540	421. 6283	105. 4071	10. 27

[1] Variance of a single basic (2.5 acre) plot.
[2] Variance of the mean of basic plots making up larger plots.

TABLE 4.—*Expected and observed deviations of sample means from the true mean volume* [1]

Plot and sample size	Row No.	Random sampling from total population			Random sampling within blocks [2]		
		Standard error based on population variance	Standard error based on sample estimate of population variance	Observed deviation of sample mean from the true mean	Standard error based on population variance	Standard error based on sample estimate of population variance	Observed deviation of sample mean from the true mean
		Percent of 40.83	*Percent of 40.83*	*Percent of 40.83*	*Percent of 40.83*	*Percent of 40.83*	*Percent of 40.83*
2.5-acre:							
3⅛-percent	1	4. 77	4. 85	−4. 38	4. 14	4. 36	+9. 18
	2		4. 95	+1. 89		4. 26	−. 12
	3		4. 87	+1. 10		4. 36	−3. 58
	4		4. 92	−3. 11		4. 38	+1. 86
6¼-percent	5	3. 32	3. 11	+4. 16	2. 88	2. 64	−. 83
	6		3. 16	−. 81		2. 82	+. 37
	7		3. 31	−1. 03		3. 60	+3. 87
	8		3. 40	−1. 47		2. 91	+3. 08
12½-percent	9	2. 27	2. 28	+1. 86	1. 87	1. 62	−2. 08
	10		2. 38	+1. 79		1. 69	+2. 91
	11		2. 23	−2. 89		1. 66	−1. 08
	12		2. 20	−. 42		1. 69	−1. 40
5-acre:							
6¼-percent	13	4. 18	3. 99	−3. 16	3. 50	4. 14	+3. 80
	14		4. 19	+9. 58		3. 92	−3. 92
	15		3. 53	+1. 08		3. 82	−3. 70
	16		4. 51	−1. 49		3. 13	−1. 10
12½-percent	17	2. 86	2. 86	+2. 47	2. 39	2. 60	+. 86
	18		2. 89	+1. 74		2. 57	+. 44
	19		2. 77	−5. 85		2. 30	−. 17
	20		2. 86	−2. 50		2. 67	+4. 56
10-acre:							
6¼-percent	21	5. 18	5. 29	−1. 10	4. 43	3. 50	−3. 38
	22		5. 12	−6. 81		4. 38	+1. 47
	23		5. 31	+. 51		4. 14	+1. 05
	24		4. 41	+3. 23		5. 93	+1. 79
12½-percent	25	3. 54	3. 37	+1. 59	2. 77	2. 62	+2. 28
	26		3. 39	−. 39		2. 67	−1. 69
	27		3. 67	+4. 58		2. 84	+3. 43
	28		3. 62	+. 34		2. 94	−4. 80

[1] The true mean volume of 2.5-acre plots is 40.83 thousand feet board measure.
[2] Equal numbers of plots selected at random from each block. Blocks are taken here as the smallest square unit of area, sampled equally and with a minimum of 2 plots.

By means of the analysis of variance, however, it may be possible to reduce these standard errors by changing the arrangement of plots to permit the breaking up of total sum of squares into parts contributed from known sources, i. e., into a part due to variation between blocks, and a part due to variation within blocks. Each block is sampled equally with the plots selected at random. To estimate the within mean square, a minimum of two plots to the block is required. For the present, blocks will be considered as the smallest square unit of area so sampled. Degrees of intensity of sampling will be introduced by variations in block and in plot sizes. This arrangement of plots will be contrasted with random selection of plots over the entire area, the latter selection being almost certain to result in an unequal sampling of blocks.

The number of degrees of freedom for the sum of squares due to variation between blocks will be one less than the number of blocks. For the squares due to variation within blocks, the number of degrees of freedom will be the number of sampling units minus the number of block means from which deviations are measured. An application of the F test shows in all cases that the between mean square is significantly the greater, a significant portion of variance irrelevant to sampling error being eliminated by using the within mean square as error variance instead of the total. This analysis is illustrated for quarter-section blocks with 2.5-acre plot size in table 5, using data from table 3 with the necessary additional computations.

TABLE 5.—*Analysis of variance for quarter-section blocks and 2.5-acre plots*

Source of variation	Sum of squares	Degrees of freedom	Mean square	F
Between quarter sections	167, 630. 7558	35	4, 789. 4502	22. 49
Within quarter sections	482, 957. 8662	2, 268	212. 9444	
Total	650, 588. 6220	2, 303	282. 4961	

This procedure is legitimate since mean square within blocks, treating each block as a population, is the same within the range of error of random sampling, regardless of differences in block means. The pooled estimates of mean square within all blocks is much more precise than the estimate from a single block, even when applied to a particular block, because of the larger number of degrees of freedom. The reduction in error variance gained by use of blocks of smaller sizes is made clear in table 3.

In making a 6¼-percent cruise with 2.5-acre plots as sampling units, by the random arrangement shown in figure 3, the error variance is 282.4961, and standard error 3.32 percent of the mean. By random within blocks, the 144 plots will average 4 to the quarter section, which is the block unit, as in figure 4 and table 5; error variance will be 212.9444; and the standard error will be reduced to 2.88 percent. All of the reductions in expected standard errors for random within blocks, as compared to random for total population in table 4, are due solely to differences in plot arrangement. For each standard error based on population variance, four estimates based on actual samples are given and the corresponding deviations of sample means from the true mean.

The effect on sampling error of size, shape, arrangement of plots, and intensity of sampling is reflected in the standard errors as calculated. If the effect of these has been correctly determined—population heterogeneity having been successfully overcome either by randomization alone, or by partial elimination followed by randomization—the deviations of sample means from the true mean

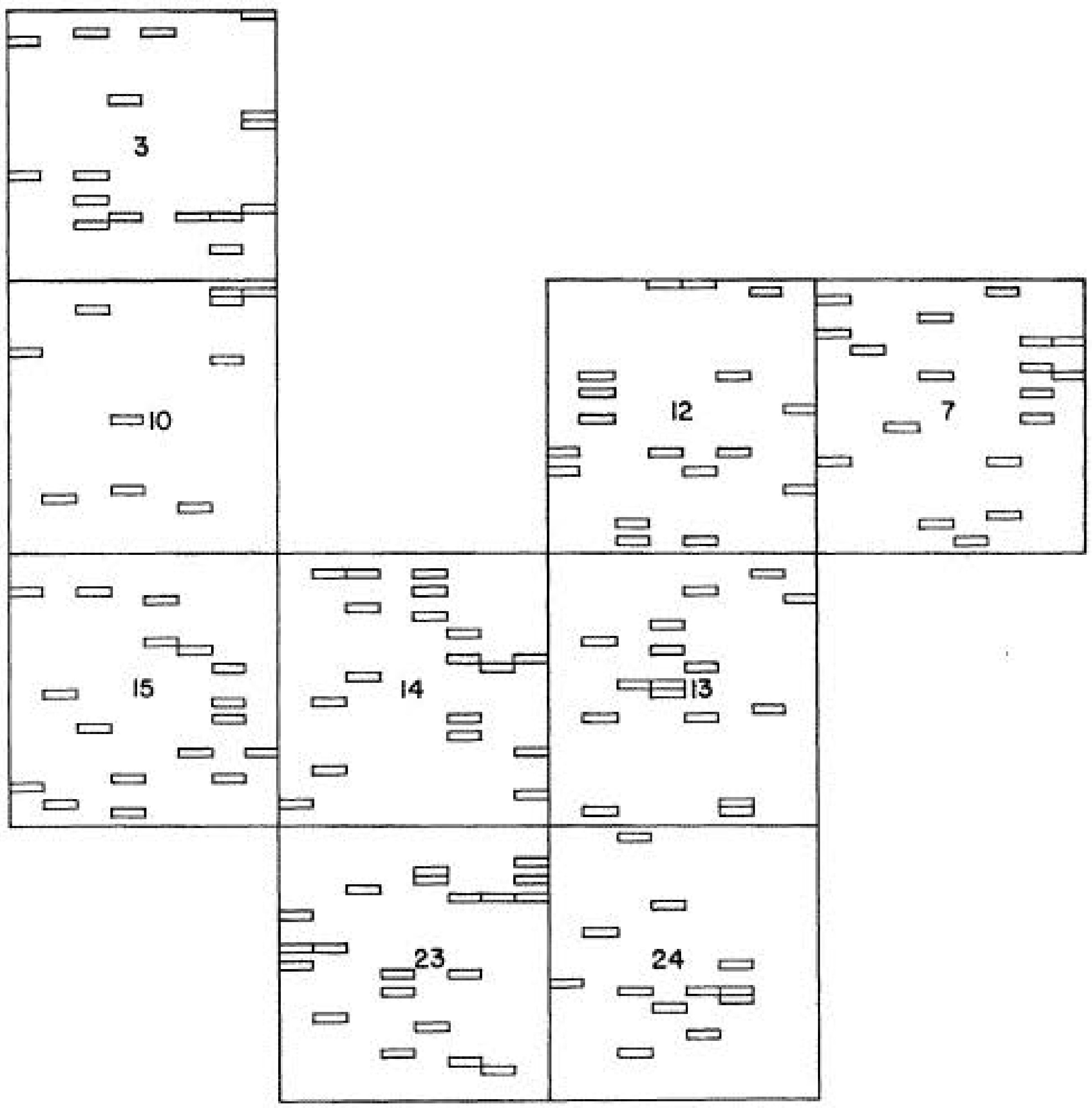

FIGURE 3.—Arrangement of 2.5-acre plots in a 6¼-percent random cruise of nine sections. Plots selected at random from the total population.

expressed in terms of standard units of the normal curve should not differ from a normal frequency distribution by more than may be attributed to error of random sampling alone. If the samples contain the information needed for assessing error due to sampling, the range of deviations in terms of standard units, arrived at from sample estimates of standard error, should likewise agree with expectation of normal grouping. That observed results agree with expected results is shown in table 6. The observed frequencies are within the range expected in 95 percent of such trials.

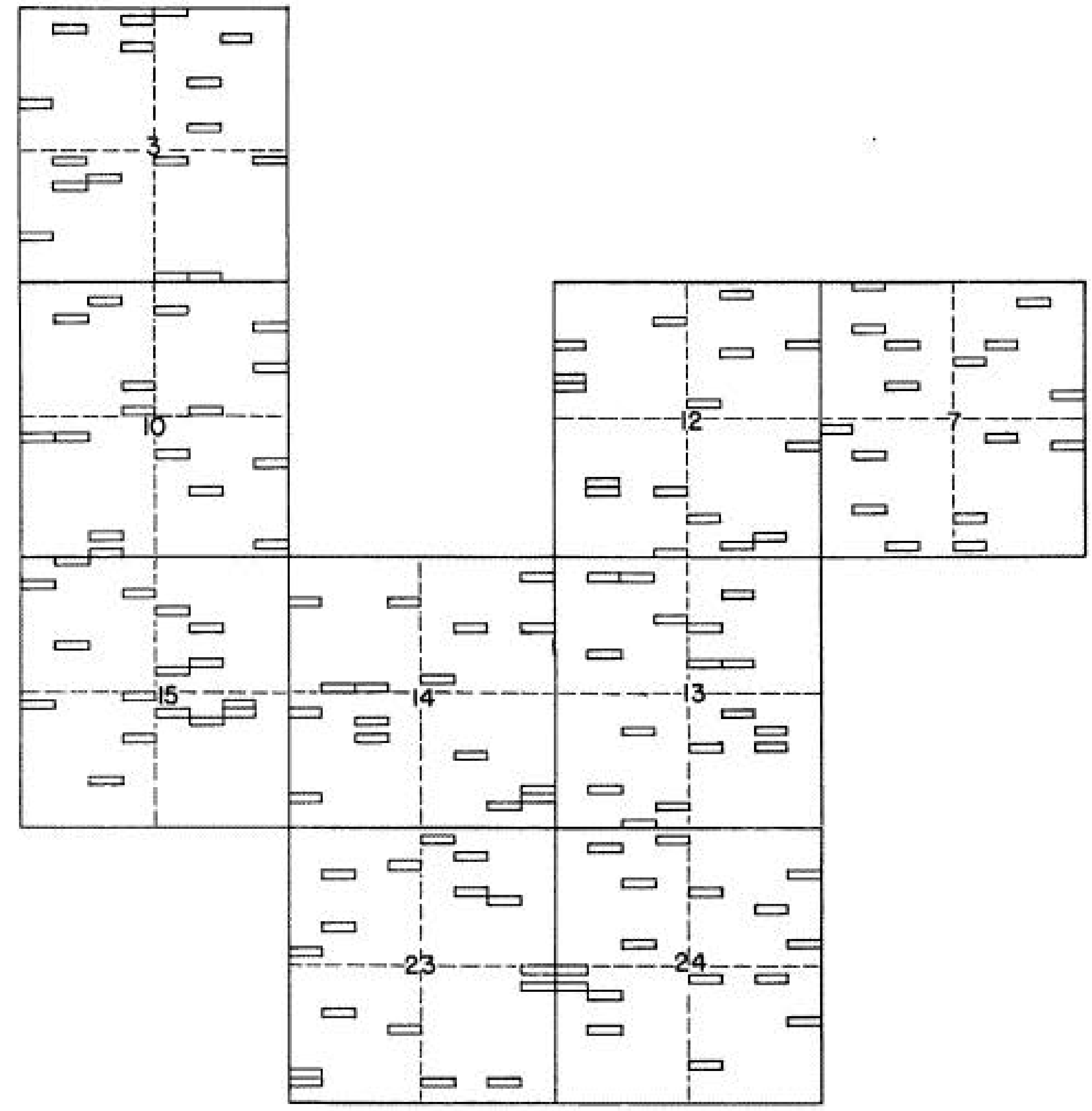

FIGURE 4.—Arrangement of 2.5-acre plots in a 6¼-percent random-within-block cruise of nine sections. Quarter section (40 by 40 chains) taken as the block unit.

TABLE 6.—*Expected and observed grouping of sample means about the true mean*

Range from zero difference in standard units of the normal curve	Expected frequency	Random, observed frequency		Random-within-block, observed frequency	
		Based on true standard error	Based on estimated standard error	Based on true standard error	Based on estimated standard error
Less than 0.253	5.6	7	6	5	4
0.253 to .524	5.6	6	7	6	7
.524 to .842	5.6	6	6	5	4
.842 to 1.282	5.6	5	4	7	8
1.282 and over	5.6	4	5	5	5
Total	28.0	28	28	28	28

STRIP CRUISES

Thus far the most efficient method of sampling indicated is the use of the random-within-block, or "stratified" arrangement of 2.5-acre plots, using blocks of such size that a minimum of two plots is taken within each. A disadvantage of this arrangement is that it presents some difficulty in locating plots on the ground and in mapping all parts of the area to the desired standard. Within some blocks the plots are clustered together leaving large areas which could not be mapped without running additional line for that purpose alone. The irregularity of line running to locate plots may be partially overcome by use of the strips taken at random within blocks, as shown in figure

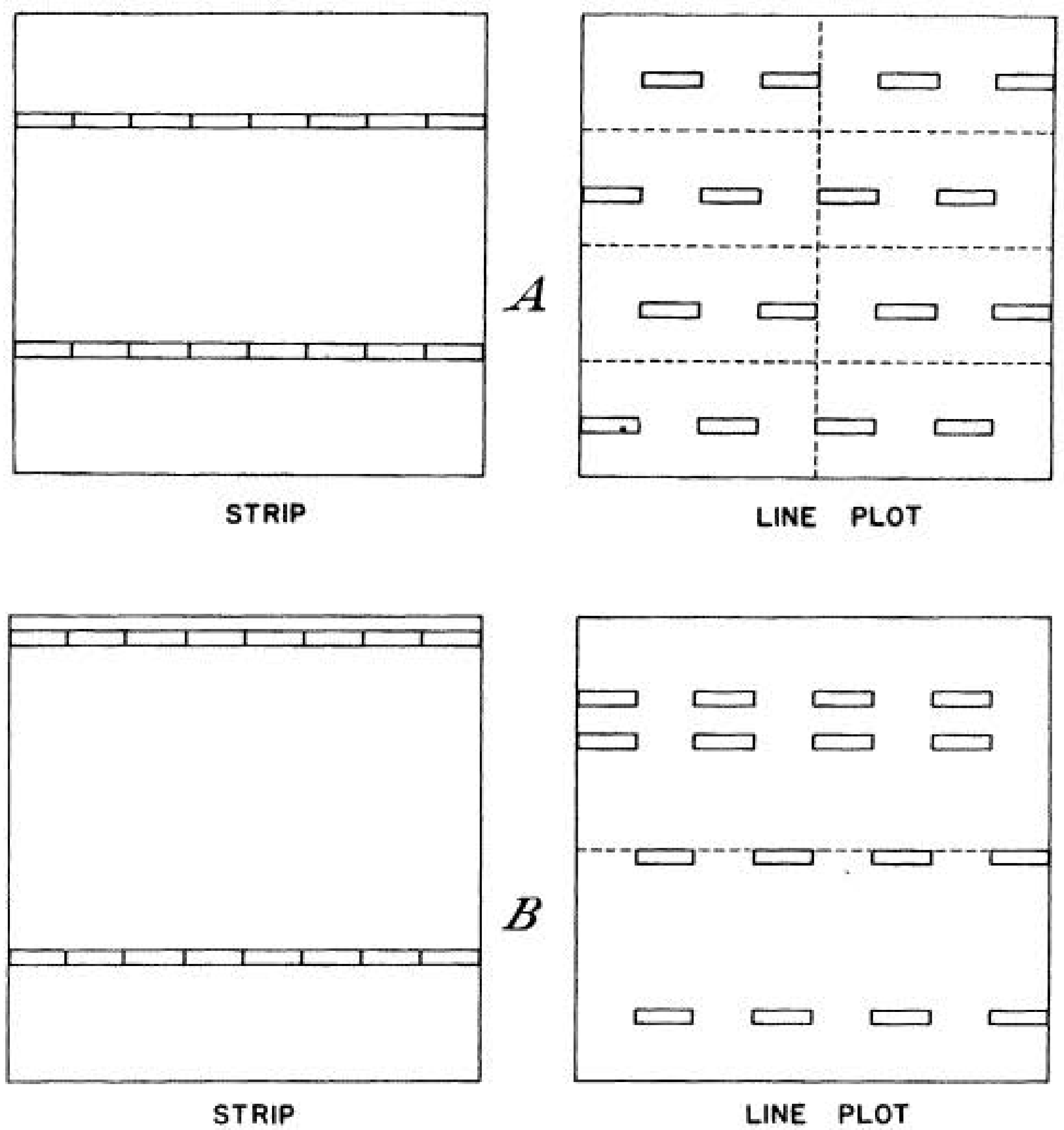

FIGURE 5.—Typical arrangements of plots in 6¼-percent strip and line-plot surveys: *A*, Systematic cruises; *B*, random-within-block cruises. Broken lines indicate block boundaries within sections.

5. Here the sampling unit is the strip, 2.5 by 80 chains, 2 of which are taken from the 32 possible in a section, the block unit. The mean square (in terms of a single 2.5-acre plot) between 28 such cruises is shown in table 7 as 385.5211. According to the previous reasoning, this differs by sampling error alone from mean square between strips within sections, which is estimated here as 425.1970. The F test fails to disprove this hypothesis.

TABLE 7.—*Analysis of variance based on strip cruises of 6¼-percent intensity*

Source of variation	Sum of squares	Degrees of freedom	Mean square	F [2]	Variance associated with blocks
RANDOM-WITHIN-BLOCK STRIP [1]					
Between cruises	10, 409. 0688	27	385. 5211	n. s.	
Sections within cruises	277, 894. 8542	224	1, 240. 6020	*2. 92*	407. 70
Strips within sections	107, 149. 6481	252	425. 1970		
Subtotal	385, 044. 5023	476	808. 9170		
Total, between strips	395, 453. 5711	503	786. 1900		
POPULATION VARIANCE [3]					
Between cruises			381. 2952		
Sections	109, 693. 3526	8	13, 711. 6691	*35. 96*	428. 49
Strips within sections	106, 381. 3677	279	381. 2952		
Total, between strips	216, 074. 7203	287	752. 8736		
SYSTEMATIC STRIP [4]					
Between cruises	2, 835. 2736	15	189. 0182	2. 21	
Sections within cruises	153, 411. 7346	128	1, 198. 5292	2. 87	390. 57
Strips within sections	60, 104. 5909	144	417. 3930		
Subtotal	213, 516. 3255	272	784. 9865		
Total, between strips	216, 351. 5991	287	753. 8383		

[1] Mean volume of 28 cruises=40.79.
[2] n. s.=nonsignificant; values in italics exceed F at 0.01.
[3] True mean=40.83.
[4] Mean volume of 16 cruises=40.83.

True population mean square of strips within sections is 381.2952. Whether or not the estimate obtained differs significantly from this may be tested by the chi-square distribution:

$$\text{Chi square} = \frac{\text{Sum of squares}}{\sigma^2} \tag{3}$$

For 252 degrees of freedom chi square is 209.5104 at $P=0.975$, and is 297.4360 at $P=0.025$. Substituting 381.2952 for σ^2, solving for each corresponding sum of squares, and dividing by 252 gives the range of mean square expected in 95 percent of such trials. The values obtained are 317.0052 and 450.0433. There is therefore no reason to suspect the sample estimate.

Since variation between sections is significantly greater than variation within sections, the use of total variance for error would give a serious overestimate, and not a valid one.

Systematic strip survey would seem to be quite comparable to strips taken at random within blocks as regards estimates of error mean square. Sixteen 6¼-percent cruises taken side by side include 100 percent of the area or the total population. Population variance of strip cruises is analyzed on this basis in the lower portion of table 7. Variation between such cruises is about one-half that between the random-within-blocks. This advantage in favor of the systematic cruise is surprisingly large, even considering that such a cruise is more representative in that each half section is sampled equally, whereas in the random the smallest block is the section.

The variation between cruises is less than would be expected in 95 percent of such trials if the sampling were random. The use of the F test is not legitimate, however, because the strips are equidistant. The analysis of population variance is made to find how the components of total variance differ in the systematic arrangement as compared to random. Obviously total population variance between strips is the same for both arrangements—the two values shown here are not absolutely the same owing to the use of short-cut methods of computation with the cruise estimates. The component parts do differ. Variation associated with sections is less by systematic than by random, and variation of strips within sections is correspondingly greater. Systematic sample estimates of mean squares do not tend to the same values as do random, and the basis for segregating error mean square is not known. In a single cruise the information for doing this is not provided. With the random cruise, the plot arrangement determines the one and only basis supplied by the data for estimating error mean square, and a single cruise will provide this.

With intensive cruising of fairly uniform stands of timber, such as this, the sacrifice in precision of volume estimate by use of random strip as compared to systematic strip is considerable. In some cases it might be feasible to cruise scattered blocks by random strip until sufficient degrees of freedom are available for a satisfactory estimate of error variance, and then cruise the remainder on the systematic basis; this should give reasonable certainty that the more representative sampling will keep the estimate within the allowable range of error. The advantage gained in mapping is also a consideration favoring this. While for some areas suitable maps are available prior to the cruise, map making in connection with sampling is still an important consideration in most cruising.

LINE-PLOT CRUISES

In the random line-plot arrangement in figure 5, the random sampling unit consists of four 2.5-acre plots spaced equidistant across the section. Two such units are taken at random from the 32 possible in a half section, which is the block unit. The true variance between such cruises is 221.8812 (table 8). In random sampling the appropriate estimate of error is provided by mean square of line plots within half sections, which tends to the true value. The corresponding estimates obtained from 56 such cruises are within a range attributable to random sampling error, and do not differ significantly from each other.

TABLE 8.—*Analysis of variance in line-plot cruises of 6¼-percent intensity*

Source of variation	Sum of squares	Degrees of freedom	Mean square	F [2]	Variance associated with blocks
RANDOM-WITHIN-BLOCK LINE PLOT [1]					
Between cruises	10, 109. 0880	55	183. 8016	n. s.	
Half sections within cruises	660, 240. 1175	952	693. 5295	*5. 05*	233. 18
Line plots within half sections	228, 976. 3700	1, 008	227. 1591		
Subtotal	889, 216. 4875	1, 960	453. 6819		
Total, between line plots	899, 325. 5755	2, 015	446. 3154		
POPULATION VARIANCE [3]					
Between cruises			221. 8812		
Half sections	128, 774. 5115	17	7, 574. 9713	*34. 14*	236. 72
Line plots within half sections	123, 809. 7229	558	221. 8812		
Total, between line plots	252, 584. 2344	575	439. 2769		
SYSTEMATIC LINE PLOT [4]					
Between cruises	3, 106. 6848	15	207. 1152	1. 21	
Half sections within cruises	177, 375. 0742	272	652. 1142	2. 60	200. 88
Line plots within half sections	72, 102. 4804	288	250. 3558		
Subtotal	249, 477. 5546	560	445. 4956		
Total, between line plots	252, 584. 2394	575	439. 2769		
Between cruises	3, 106. 6848	15	207. 1152	1. 06	
20 by 40 within cruises	394, 693. 3468	1, 136	347. 4413	1. 58	64. 10
Between plots within 20 by 40	252, 558. 0700	1, 152	219. 2344		
Subtotal	647, 251. 4168	2, 288	282. 8896		
Total, between plots	650, 358. 1016	2, 303	282. 3960		
POPULATION VARIANCE [5]					
Between cruises			221. 8812	*1. 10*	
20 by 40	202, 289. 9365	71	2, 849. 1540	*14. 18*	89. 04
Plots within 20 by 40	448, 298. 6856	2, 232	200. 8507		
Total between plots	650, 588. 6221	2, 303	282. 4961		

[1] Mean volume of 56 cruises = 40.70.
[2] n. s. = not significant; values in italics exceed F at 0.01.
[3] True mean = 40.83.
[4] Mean volume of 16 cruises = 40.83.
[5] True mean = 40.83.

A difficulty arises in connection with the systematic line-plot cruise shown in figure 5, in deciding upon the block unit, i. e., whether the arrangement is comparable to the random line plot or more comparable to random selection of two plots within blocks 20 by 40 chains in dimensions. In practice, with but one cruise supplying the total data, there is no way to determine by which means the most useful information will be obtained. With a population of 16 such cruises, it is possible to determine this for this particular area, but the results will not apply generally. When sampling units are selected independently and at random there is never any doubt as to the one valid basis for estimating error mean square.

If the systematic arrangement permits the same treatment as for the random line plot, the variance associated with blocks is less and mean square within is greater than the true for random sampling. As with the strip survey, such sample estimates tend to the true value for total variance, and variance between cruises is less than that of corresponding random cruises. The apportionment of total variance to sources within cruises is biased. The overestimate from one source is balanced by an underestimate from another source. This is still more pronounced when the 20 by 40 unit is taken as the block. The F test, if legitimate here, would show that both estimates of error mean square are in a range attributable to sampling. Of the two, mean square within 20 by 40's provides the more useful information. In other cases, where a single sample is taken, there would be no assurnace which if either would give a satisfactory estimate of error. The guess with strip survey was as logical as either choice here, and yet was shown to be of little use. In neither case is there any justification for use of total variance as error variance.

RELATIVE EFFICIENCY OF CRUISES

For a standard by which to measure relative efficiency of different cruises we may set up a range of 8.00 percent of the mean at $P=0.05$ for 3⅛-percent cruises. By doubling the intensity to 6¼ percent, we expect a reduction of error to 5.33 percent, and by doubling again to 12½ percent, an error of 4.00 percent. This follows from formula (1). The error mean square necessary to assure this precision is 199.9584 on a single 2.5-acre plot basis. Were the correction to limited population mean made, the efficiency would be greater, since the error mean square would then be reduced by multiplying by the proportion of population not in the sample—as is done by use of formula (2). This correction is not made in table 9 mean squares, since the relative efficiencies would not be changed greatly. As they are given, the effect of decrease of block size with increased intensity of cruise is segregatcd completely. The reciprocal of 199.9584, or invariance $=0.00500$, is taken as a unit of information per 2.5 acres and the different cruises rated on this basis. One-half unit indicates that sample size would need to be doubled to assure the same precision as that by a cruise supplying a full unit. If costs of cruising were the same per unit of area in the samples, use of the former method would double the cost.

TABLE 9.—*Relative amounts of information by cruises*

Sampling unit and arrangement	Mean square	Invariance [1]	Units of information per 2.5 acres [2]
3⅛-PERCENT SAMPLING			
2.5-acre, 2 plots within quarter section	212.9444	0.00470	0.94
2.5 by 80-chain strips, random	752.8736	.00133	.27
4 2.5-acre plots equidistant along 80-chain line, 2 lines within section	252.0121	.00397	.79
6¼-PERCENT SAMPLING			
2.5-acre, 2 plots within 20 by 40	200.8507	0.00498	1.00
2.5 by 80-chain strips, 2 within section	381.2952	.00262	.52
4 2 5-acre plots equidistant along 80-chain line, 2 lines within half section	221.8812	.00451	.90
5-acre, 2 plots within quarter section	313.2797	.00319	.64
10-acre, 2 plots within half section	477.6623	.00209	.42
12½-PERCENT SAMPLING			
2.5 acre, 2 plots within forty	192.7724	0.00519	1.04
2.5 by 80-chain strips, 2 within half section	323.3341	.00309	.62
4 2.5-acre plots equidistant along 80-chain line, 2 lines within 20 by 80	189.3106	.00528	1.06
5-acre, 2 plots within 20 by 40	291.6305	.00343	.69
10-acre, 2 plots within quarter section	421.6283	.00237	.47

[1] Invariance$=\frac{1}{\text{mean square}}$. Value of 0.00500=1 unit of information.

[2] Based on 1 unit for 8.00-percent error at 0.05 level with 3⅛-percent cruises, 5.66-percent error with 6¼-percent cruises, and 4.00-percent error with 12½-percent cruises. Correction for limited population not included.

The random-within-block 2.5-acre plot cruises meet the standard of precision set up for each of the three different intensities. The random-within-block line-plot cruises are not far behind in the lower intensities and are slightly the more efficient in the 12½-percent intensity. Because of advantages in locating plots on the ground along with satisfactory sampling efficiency, the random-within-block line-plot arrangement of 2.5-acre plots appears to be the best selection.

For a given sampling unit, the increase in information with increased intensity of cruise, as given in table 9, is due to the decrease in block size and the consequent elimination of more variation irrelevant to sampling error.

The 100-percent inventory of the population in this case makes it possible to compare the range of error of systematic cruises with that of similar random cruises. The systematic cruises cannot be treated as were the random in table 9, but for 6¼-percent cruises, both strip and line plot, the true variation between means has been found for this limited population. In both cases the mean squares are less than for similar random cruises. This finding, together with the advantage in mapping, makes it appear doubtful whether these arrangements should be completely discarded simply because single samples do not provide the information needed for estimating sampling error. There are good possibilities that their sampling error can be closely approximated. Where maps are not needed, or are already available, a random cruise can be made in the same time as a comparable systematic cruise. The cruiser must decide whether the added precision in estimating sampling error is offset by the loss in accuracy of volume estimate.

It has been previously explained[5] that the range of systematic sample means about the true corresponds satisfactorily with the range expected for random-within-block sampling in which square blocks are used. If a number of such blocks in a population were selected at random and cruised by a random arrangement, the pooled estimate of error variance so obtained would then be considered as satisfactorily applicable to the entire population sampled. The remaining blocks could then be cruised by a systematic arrangement, in which the same number, size, and shape of plots would be used. Discarding the restriction as to shape of blocks, blocks 40 by 20 chains (north-south by east-west dimensions) are suitable in the present study for a systematic strip arrangement as shown in figure 5. Random variation within such units tends to 195.5448. This compares very well with the mean square between cruises, 189.0182 in table 7. Obviously, however, this proposed method of approximating error variance could easily be carried beyond a reasonable point by the use of very short strip segments as plots and long, narrow blocks. Probably length of blocks should not exceed twice their width.

Where systematic line plots are taken, some such method of approximating error variance would appear quite safe. The mean square for error of random sampling with 20 by 40 blocks is 200.8507, and the mean square between systematic cruises is 207.1152. The justification for this approach to sampling error is apparent from inspection of the diagrams in figure 5. The systematic arrangements are more representative than the random. For line-plot sampling, each 20 by 20 is sampled by a plot; while in random sampling the plot locations may fall so that some 20 by 20's are not sampled at all and some are sampled with two plots.

F. Yates, chief statistician, Rothamsted Experimental Station, England, has suggested that the sampling error of systematic cruises in which a single sampling unit was taken in the center of each block, might be approximated by using error variance estimates from two randomly selected sampling units in blocks of the same size. Only sufficient random cruising would be done in a population to get an adequate estimate of variance within blocks, and the survey could be continued on the systematic basis. The gain in representativeness by the systematic arrangement as compared to random would then be reflected in reduction of error variance by use of blocks of just half the size needed with random cruises of equal intensity. With the 6¼-percent systematic strip cruise the mean square between the 16 cruises is 189.0182. The mean square between random strips within half sections is shown in the report previously cited as 323.3341. With only 15 degrees of freedom available for estimate, the difference is within a range attributable to error of sampling in an unlimited population. The value 189.0182 is true for the limited population only. The estimates from such cruises in other similar areas might well tend to something like 323.3341. There is no reason, for example, to expect that the strip arrangement should consistently give as close or closer estimates of volume than the line plot, where the spacing apart of plots would appear to sample a block better than by a single strip of contiguous plots. Also the advantage over random strip should not be much greater than can be attributed to more representative sampling alone. It is suggested that this method of approximating error of systematic strip survey be used in preference to the ones previously suggested in which strip segments were considered as plots.

[5] HASEL, A. A. ANALYSIS OF SAMPLING METHODS FOR VOLUME DETERMINATION IN A PONDEROSA PINE FOREST. Calif. Forest and Range Expt. Sta. Prog. Rept. June 1937. [Multilithed.]

With line-plot arrangements the results by the two methods are quite alike. Variance between line-plot sampling units within 20 by 80's is 189.3106. This differs little from 200.8507, the variance between plots within 20 by 40's, and neither differs significantly from 207.1152, which is based on 15 degrees of freedom.

It has been noted that doubling of intensity, other factors remaining constant, reduces a sampling error of 8.00 percent to one of 5.66 percent. Extremely close estimation of sampling error and small changes in intensity based on estimates of error variance are not important, unless it is realized at the same time that biased error from various sources may contribute much the greater part of total error of cruise estimate. Time saved by lowering intensity can be very profitably spent on reduction of biased error.

PRECISION OF ESTIMATES OF ERROR VARIANCE

The mean squares taken as error variance have been based upon true values for the population dealt with. They are the values to which sample estimates tend. The range within which sample estimates group themselves about the true is dependent solely upon the number of degrees of freedom available for their estimate. The range of 95 percent of such estimates is given for the various random cruises in table 10, derived by use of formula (3). Where the estimate is based on as few as 8 or 9 degrees of freedom a single sample may easily give a very coarse basis for error calculation—not close enough for much reliance. In this respect random plots have a marked advantage over line plots, which in turn are better than strip. By use of a much smaller and probably more efficient size of plot than 2.5 acres, however, the ordinary intensity of cruise in a few sections will give an adequate basis for estimate. Such tests as are given in table 10 should be made the basis for decision as to the number of blocks that should be cruised by a random arrangement before continuing with the systematic as suggested previously.

TABLE 10.—*Range of sample estimates of variance*

Sampling unit and arrangement	Degrees of freedom	Mean square	Range of 95 percent of sample estimates	
3⅛-PERCENT SAMPLING				
				Percent
2.5-acre, 2 plots within quarter section	36	212.9444	123.8059– 319.2763	(−42)–(+ 50)
2.5 by 80-chain strips, random	8	752.8736	203.8405–1648.7932	(−73)–(+119)
4 2.5-acre plots equidistant along 80-chain line, 2 lines within section	9	252.0121	74.4276– 532.1375	(−70)–(+111)
6¼-PERCENT SAMPLING				
2.5-acre, 2 plots within 20 by 40	72	200.8507	139.4796– 270.2647	(−31)–(+ 34)
2.5 by 80-chain strips, 2 within section	9	381.2952	112.6092– 805.1260	(−70)–(+111)
4 2.5-acre plots equidistant along 80-chain line, 2 lines within half section	18	221.8812	100.4136– 388.2921	(−55)–(+ 75)
5-acre, 2 plots within quarter section	36	313.2797	182.1408– 469.7107	(−42)–(+ 50)
10-acre, 2 plots within half section	18	477.6623	216.1724– 835.9090	(−55)–(+ 75)
12½-PERCENT SAMPLING				
2.5-acre, 2 plots within 40	144	192.7724	150.2018– 239.0980	(−22)–(+ 24)
2.5 by 80-chain strips, 2 within half section	18	323.3341	146.3266– 565.8347	(−55)–(+ 75)
4 2.5-acre plots equidistant along 80-chain line, 2 lines within 20 by 70	36	189.3106	110.0652– 283.8334	(−42)–(+ 50)
5-acre, 2 plots within 20 by 40	72	291.6305	202.5212– 392.4172	(−31)–(+ 34)
10-acre, 2 plots within quarter section	36	421.6283	245.1347– 632.1473	(−42)–(+ 50)

APPLICATION OF RESULTS

In cruising it is necessary to make compromises between what is theoretically correct and what is practically possible. Experienced cruisers would never consider using the random arrangement exclusively because it does not lend itself to map making, and the volume estimates are not as close as with the systematic arrangement. It is important, however, to have a valid and adequate estimate of error variance, and this is possible without a great initial sacrifice in time and money. Having that, it is possible to assure an estimate of the required precision so far as sampling error is concerned and by the cheapest method of cruise. Knowing sampling error, it is possible by later checks of estimate against actual cut to segregate error due to biased measurements, and direct efforts toward its reduction which are commensurate with its importance.

A timbered area to be cruised usually consists of several separate populations, which are segregated according to criteria used in mapping. In working out the method of cruise for a population, even though the stand appears uniform to the eye, heterogeneity of variation should be assumed. It follows then that size and shape of plot is an important consideration and tests should be made to find the kind of plot or sampling unit which includes the maximum of stand variation per unit of area. Obviously the minimum size of plot used in the present study is too large to give an indication of what minimum intensity of cruise is possible for a given range of error. As a rough guide until further studies are made, the minimum size with which plot volumes approach the normal type distribution is suggested, although the analysis of variance method does apply to definitely skewed distributions, which are not normal. At any rate the minimum size tested should not often contain zero volume. The time required to cruise these basic plots and multiples of basic plots should be recorded, as well as travel and mapping time per unit of distance between plots. It may prove more economical in practice to use more area in fewer large plots than the theoretical minimum indicated with more and smaller plots. By selecting the plots at random within blocks additional needed information on initial intensity of sampling is gained. Plot arrangement and intensity will require further tests unless previous experience in such stands is available. A more intensive cruise than is believed necessary should be made by use of the random plot or line-plot arrangement within blocks of the minimum size that will be considered. If the stand is not patchy it will be possible to combine adjacent block units into larger blocks, and by pooling within and between sums of squares and degrees of freedom, to estimate variance within blocks of different sizes.

Advantage should be taken of the usual procedure of running cruise lines and orienting plots for the purpose of sampling more intensively in the direction of greatest variation. The Land Office subdivisions are convenient block units, and within sections the cruise lines are generally run north-south or east-west depending on direction of variation. By taking blocks of such size and shape that variation within is kept low, and that between correspondingly high, the maximum of variation irrelevant to sampling error is eliminated. When sufficient degrees of freedom have been built up to provide estimates of variance within a predetermined allowable range, the units of information supplied per basic plot may be calculated for varying arrangements and block sizes. By knowing the average man-hours required to travel to and cruise a plot by each method, the cost per unit of information will give the most efficient method of sampling by a random arrangement. If the cruise is continued with a systematic arrangement of the same intensity of cruise, the estimate of error will be high. A closer approximation can be made by reducing the size of block by a half, so that a single systematically placed sampling unit is in the center of each. Knowing from the preliminary work the variance of random sampling within blocks of this size, this estimate of within variance, when divided by the total number of blocks, or number of systematically placed plots, will give the approximate sampling variance of the mean. By extracting the square root the standard error of the cruise mean is obtained.

The above-suggested procedure for starting a cruise would not need to be repeated in similar stands, except to check on or improve error-variance estimates by cruising randomly selected blocks by the random arrangement. It is intended to apply to intensive cruising of stands which appear uniform within blocks of 40 acres or more, as in the pure ponderosa pine type. A procedure for random sampling within types occurring in small irregular patches is difficult to work out, particularly if a type map is not available beforehand. The same is true of extensive cruising, as in the Forest Survey in parts of the country where types change often. If these scattered small type areas were brought together, however, there is little doubt that analysis of variance would show that they too are heterogeneous populations and that total variance based on systematically spaced plots is not a valid estimate of error variance. Such an estimate is useful only in that we can be sure that it is an overestimate.

Owing to the volume of timber inventory work in progress or in prospect, as in connection with preparation of timber-management plans and land-use studies, efforts to determine efficient methods of getting adequate samples in major timber types and for various stand conditions would be likely to yield results of considerable practical importance. Existing and proposed experimental forests are expected to represent fairly well the principal stands of timber within national forests. Complete inventories of these areas, with volumes recorded separately by sufficiently small units, would provide a good basis for working out sampling methods for the stands they represent.

CONCLUSIONS

The heterogeneous nature of stand variation within a 5,760-acre tract of the Blacks Mountain Experimental Forest is evidence that timber stands, even though they appear uniform to the eye, are similar to other soil crops in exhibiting systematic and yet partly disordered variation from point to point.

In a heterogeneous population, size and shape of plot are important factors in efficient sampling. A valid estimate of sampling error is possible only when the sampling units are selected independently and at random. By dividing the area into blocks of uniform size and shape, and selecting equal numbers and at least two random sampling units in each, a significant reduction in error variance is possible by Fisher's method of analysis of variance. The customary use of total variance as error variance and the statistical treatment of systematically spaced plots as random sampling units would be legitimate only if the population sampled were shown to be homogeneous. Such a condition seldom if ever exists. It is only by the use of random sampling that the elements contributing to heterogeneity are most likely to be represented in the sample in their true proportion.

Systematic cruises give closer estimates of volume than do similar random cruises and lend themselves better to map making. Since variation within separate blocks in a population varies within a range attributable to sampling error alone, and independently of the block means, only sufficient random cruising is suggested to assure an adequate estimate of error variance. If the remainder of the population is cruised by systematic spacing of sampling units, the estimate of error tends to be a little high. A closer approximation to sampling error is suggested if random variation within blocks of such size that they contain but one systematically placed sampling unit is used for error variance. In this way the gain in added representativeness of systematic sampling, as compared to random, is taken into account.

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