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Effects of Measurement Errors on Individual Tree Stem Volume Estimates for the Austrian National Forest Inventory

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National forest inventories typically estimate individual tree volumes using models that rely on measurements of predictor variables such as tree height and diameter, both of which are subject to measurement error. The aim of this study was to quantify the impacts of these measurement errors on the uncertainty of the model-based tree stem volume estimates. The impacts were investigated using two approaches: the law of propagation of error and Monte Carlo simulation. Estimates of total uncertainty also included variability associated with the model itself. Results for both approaches indicate that the relative standard deviation over plots of the volume estimates for all tree species is approximately 11%. A partition of the total uncertainty by sources indicates that error in measurement of the upper diameter makes the greatest contribution. Thus, the greatest potential for improvement in the precision of overall estimates lies in increasing the accuracy of upper diameter measurements. Although the uncertainty of individual tree stem volume estimates may be considered negligible for nationwide assessments of growing stock volume, it is relevant for small-scale and plot-level estimates used as training or accuracy assessment data for remote sensing applications that rely on emerging technologies such as airborne laser scanning.

Keywords: stem volume, uncertainty, measurement error, error propagation, Monte Carlo error simulation

Measurement errors play an important role in the assessment of uncertainties in forest inventory estimates, but they are not the only source of error. Cunia (1965) distinguishes among sampling variability, model error, and measurement error. For this study, errors from the first source were omitted because only plot-level estimates were considered. Several studies on error analysis refer to Cunia's classification, and some recognize classification or grouping error as an additional source of uncertainty (Gertner 1990, Gertner and Köhl 1992, Phillips et al. 2000). Other references classify error sources differently (e.g., Canavan and Hann 2004).

National forest inventories (NFIs) often consider only sampling variability. This study investigated the effects of individual tree measurement errors, which tend to be neglected due to the large sample sizes characteristic of nationwide inventories. However, they may have a considerable impact on single-tree volume estimates.

For estimating the stem volume of a tree, NFIs assess variables in the field (Tomppo et al. 2010) that are subject to measurement errors. When these variables are used as input to volume models, their errors affect stem volume estimates. The Austrian NFI uses a set of volume models that relies on up to four input variables to estimate the stem volume of a tree (Gabler and Schadauer 2008). The volume models were developed by Braun (1969), Pollanschütz (1974), and Schieler (1988). During the measurement process, observations for the four variables were rounded to the next full unit (Table 1).

Although many studies have focused on the assessment of measurement errors (e.g., Skovsgaard et al. 1998, Kaufmann and Schwyzer 2001, Barker et al. 2002, Elzinga et al. 2005, Kalliovirta et al. 2004, Kitahara et al. 2009), considerably less attention has been paid to the effects of these errors on volume estimates. On the other hand, Breidenbach et al. (2014) quantified the model-related variability of Norway spruce (*Picea abies* [L.] Karst.) biomass stock and change estimates using a Monte Carlo simulation technique. Özçelik et al. (2010) examined estimates of tree bole volume using artificial neural network models for four species. Clark et al. (1991) compared different taper models, input variables, and input variable estimates used to estimate tree stem volume. Their main interest was not in biomass or volume of the whole tree stem but in merchantable timber, and, therefore, they studied only the portions of the bole with diameters greater than a specified threshold, usually 10.2, 17.8, or 22.9 cm (4, 7, or 9 in.). Li

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Table 1. Input variables for the volume models.

Variable	Description	Tool	Unit
dbh	Diameter at breast height (1.3 m)	Calliper, tape	mm
h	Tree height	Vertex	dm
hk	Height to the living crown base	Vertex	dm
*d*03*h*	Diameter at 3/10 of the tree height	Mirror relascope	mm

and Weiskittel (2010) also juxtaposed several taper models, some of which were taken from Clark et al. (1991), but they estimated the volume of the whole tree stem. None of these four studies took measurement errors into account.

Williams et al. (1999) compared two laser dendrometers and investigated the influence of their measurement errors on taper models. Gertner (1990) assessed the impact of measurement errors on stand volume estimates when a nonlinear volume model was used to estimate the volume of individual trees. Westfall and Patterson (2007) analyzed measurement variation from quality assurance data and evaluated the effects on volume change estimates for 682 inventory plots. McRoberts et al. (1994) estimated the effects of variation in measurements on 20-year predictions of basal area and basal area growth using Monte Carlo simulations. Mowrer and Frayer (1986) analyzed variance propagation in growth and yield projections over five 10-year periods using Monte Carlo simulations and error propagation. Williams and Schreuder (2000) investigated the upper limit for tree height measurement error beyond which inclusion of tree height as a model input variable impairs rather than improves model estimates.

Kangas (1996) investigated the effects of measurement errors on volume estimates for 8,516 Scots pine sample trees but with different volume models. The focus was on the effects of estimated rather than measured input variables and possible corrections for resulting biases. Schmid-Haas and Winzeler (1981) compared several types of volume models for spruce trees with regard to measurement cost and total variability, including the effects of measurement errors.

Ståhl (1992) examined the standard errors (SEs) of compartment-wise stand volume estimates using multiple assessment methods: pure ocular estimation, a relascope method, a circular plot method, aerial photo interpretation, aerial photo interpretation combined with field control, and data acquisition from satellite images.

Thus, measurement errors have been investigated thoroughly for many instruments and under various conditions, and their effects are often well known. Much effort has also been committed to constructing volume models for tree stems, but less relevant information is available for the Austrian NFI on the connection between measurement errors and volume model prediction uncertainty. Because the variety of methods for estimating tree stem volume is huge and because different countries usually use different measurement instruments, input variables, and methods, the results from one country may provide little understanding of the situation in another country.

Information about the quality of volume estimates is relevant not only for stem volume estimation but also for aboveground carbon estimation under the United Nations Framework Convention on Climate Change and the Kyoto Protocol, where, according to the Intergovernmental Panel on Climate Change (2006) guidelines, the uncertainty must be reported. The results are also important for training and validating remote sensing-based algorithms and products based on laser scanning data, satellite imagery, or digital aerial photographs. Because these methods use plot data for training and validation purposes, the uncertainty for each plot estimate is important (Holmgren 2004). These plot data are also used to estimate growing stock and aboveground biomass efficiently for large areas in Austria (Hollaus et al. 2009) and in many other regions around the world (Næsset 2007). Undoubtedly, they will continue to play an important role in NFIs long into the future.

Table 2. Relative standard deviations (SDs) of the tree measurements.

Variable	Average SD for conifers	Average SD for broadleaved trees
	..............(%)	....
dbh	1.1	1.2
h	3.3	6.6
hk	19.5	22.2
*d*03*h*	6.1	8.3

The objective of this study was to quantify the effects on individual tree stem volume estimates of measurement errors for input variables for volume models used by the Austrian NFI. To this end, the effect of measurement errors for *dbh*, *h*, *hk*, and *d*03*h* (Table 1) are investigated using two different approaches: first, we simulate the errors in a Monte Carlo simulation and study their impact on the uncertainty of stem volume estimates, and second, we use the law of error propagation.

Data and Methods

Data

Standard Deviations of the Measurements from a Control Survey

Berger et al. (2012) studied measurement errors for the Austrian NFI using multiple data sets to assess the magnitude of the measurement errors and the dependence of the measurement errors on the measurement conditions. Data from a control survey were best suited to represent measurement errors made during normal fieldwork. The control survey was conducted as part of the quality assurance program of the Austrian NFI. Approximately 5% of the sample plots were revisited a short time after the original survey, and the variables noted in Table 2, among others, were reassessed. The control survey measurements were assumed to be taken under the same conditions as the original measurements and to be of equal quality. Table 2 shows the standard deviations (SDs) of the four measurements used for this study as reported by Berger et al. (2012).

The presentation in Table 2 suggests that SDs depend linearly on measurement values and thus provide a simple model for the SD of the measurement

$$\widehat{SD}_i = i \cdot SD_{i,r}, \tag{1}$$

where i denotes the measurements *dbh*, *h*, *hk*, and *d*03*h* and $SD_{i,\,r}$ denotes the relative SD from Table 2. However, a closer evaluation revealed that linearity is merely an approximation. To construct measurement error models that can adapt to the data more flexibly, nonlinear regressions were conducted and evaluated using the same scheme for all variables documented by Hosmer and Lemeshow (1989) and described here for *dbh*. First, the data were sorted in ascending order with respect to the mean of the two measurements,

$$dbh^* = \frac{dbh_1 + dbh_2}{2}, \tag{2}$$

where dbh_1 is the original measurement and dbh_2 is the measurement from the control survey. Then, the first n (usually 25) trees were grouped into one class, the next n trees into the next class, and so on except for the last class, which was combined with the previous class if the last class contained fewer than n trees. For each class i, the mean of the measurements $\overline{dbh_i}$ and the SD_i of the measurement differences, $Diff_{dbh}$, were estimated as

$$\overline{dbh_i} = \frac{1}{n} \cdot \sum_{j=1}^{n} dbh_j^*, \tag{3}$$

$$Diff_{dbh} = dbh_1 - dbh_2, \tag{4}$$

and

$$SD_i = \frac{1}{n-1} \cdot \sqrt{\sum_{j=1}^{n} \left(Diff_{dbh,j} - \overline{Diff_{dbh,j}} \right)^2}. \tag{5}$$

Finally, the nonlinear regression model,

$$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 \cdot (1 - e^{\hat{\beta}_3 \cdot X}) \tag{6}$$

was fit to these data, where X is the class mean and $\hat{Y}$ is the SD. After several analyses, $\hat{\beta}$ was set to 0 for all models (Table 3), because the curves with and without $\hat{\beta}_1$ were so similar that no statistically significant differences in the quality of fit of the model to the data were detected. Thus, omitting the intercept should have no relevant impact on the overall result. In addition, negative values for $\hat{\beta}_1$ could produce negative values for the SD for small measurement values and large values of $\hat{\beta}_1$ would lead to unreasonably large errors for small values.

To obtain the SD of the measurement, $\hat{Y}$ must be divided by $\sqrt{2}$ because $\hat{Y}$ is the SD of the difference of two random variables assumed to be identically and normally distributed. Thus, the model for the SD of dbh is

$$\widehat{SD}_{dbh} = \frac{1}{\sqrt{2}} \cdot \hat{\beta}_2 \cdot (1 - e^{\hat{\beta}_3 \cdot dbh}). \tag{7}$$

The same model form, but with different parameter estimates, was used for all the input variables because this kind of curve can adapt well to the data points. For h, the data were separated into conifers and broadleaved trees because the resulting models were quite different (see also Table 2). However, the data were not similarly separated for dbh or hk because the curves were very similar or for $d03h$ because not enough data were available to construct separate models. Both models 1 and 7 were used in the analyses.

The "Model Stems" ("Funktionsstämme")

A large-scale survey was conducted in the 1950s to construct volume models for the Austrian NFI. More than 17,000 trees were felled, and many measurements were taken on each tree including dbh, h, hk, $d03h$, and diameters at heights of 0.5 m (1.64 ft), 1.5 m, 2.5 m, and so on up to the top. All diameter measurements were taken with a caliper, and all heights were measured with a tape, so they are considered to be precise and accurate.

One of the goals was to capture as much of the variability in the Austrian forest as possible. Thus, trees were sampled at different locations with different management types, species composition, and age structures. The proportions of these trees per species represent approximately their occurrence in the Austrian forest. For a more detailed description, see Braun (1974) and Gabler and Schadauer (2008).

The tree stem volume models currently used by the Austrian NFI were constructed using this data set (hence its name) and were used for volume estimation for this study (see next section).

Simulation and Estimation of Stem Volume Variability

To assess the effects of measurement errors on stem volume estimates, two different approaches were tested, both of which are based on the volume models used by the Austrian NFI (Braun 1969, Pollanschütz 1974, Schieler 1988). The magnitude of measurement errors is estimated from the measurement error models introduced in the section SDs of the Measurements from a Control Survey. For the first approach, which used Monte Carlo error simulations, the effects of randomly altered values of the explanatory variables on volume estimates were investigated (see *Stimulation of Measurement Errors* below). The second approach, which was based on the law of error propagation, used a first-order Taylor series approximation and thus the first derivatives of the model expression for volume with respect to the explanatory variables (see *Error Propagation* below).

Stem Volume Models

The Austrian NFI uses multiple models to estimate the stem volume of a sample tree. The particular model used depends on the species and size of the tree. All the models have the same basic form,

$$V = \frac{\pi}{4} \cdot dbh^2 \cdot h \cdot \hat{f}, \tag{8}$$

which is based on the volume of a cylinder with diameter dbh and height h multiplied by a form factor function, $\hat{f}$. Five different types of $\hat{f}$ are currently used. For small trees (50 mm $\leq dbh <$ 105 mm, 1.97 in. $\leq dbh <$ 4.13 in.),

$$\hat{f} = c_0 + c_1 \cdot [\ln(dbh)]^2 + c_2 \cdot \frac{1}{h} + c_3 \cdot \frac{1}{dbh} + c_4 \cdot \frac{1}{dbh^2} + c_5 \cdot \frac{1}{dbh \cdot h} + c_6 \cdot \frac{1}{dbh^2 \cdot h}. \tag{9}$$

This function relies on the variables dbh and h, whereas the functions for trees with $dbh \geq$105 mm also use $d03h$ for all tree species and hk for broadleaved trees

➤ for conifers, except silver fir:

$$\hat{f} = c_0 + c_1 \cdot \frac{d03h}{dbh} + c_2 \cdot \frac{h}{dbh} + c_3 \cdot \frac{1}{dbh} \tag{10}$$

➤ for silver fir:

$$\hat{f} = c_0 + c_1 \cdot \frac{d03h^2}{dbh^2} + c_2 \cdot \frac{h}{dbh} + c_3 \cdot \frac{1}{dbh} \tag{11}$$

➤ for broadleaved trees, except oak:

$$\hat{f} = c_0 + c_1 \cdot \frac{d03h}{dbh} + c_2 \cdot \frac{hk}{h} + c_3 \cdot \frac{h}{dbh^2} \tag{12}$$

➤ for oak:

$$\hat{f} = c_0 + c_1 \cdot \frac{d03h^2}{dbh^2} + c_2 \cdot \frac{hk}{h} + c_3 \cdot \frac{h}{dbh^2} \tag{13}$$

The c_i are the coefficients and depend on the tree species. They were estimated for all tree species and sizes by Braun (1969), Pollanschütz (1974), and Schieler (1988), and are available in Gabler and Schadauer (2008).

The basis for these calculations was data from the model stems described above. Using Huber's formula, the tree stem was approximated by cylinders 1 m (3.28 ft) in height with diameters as noted in this section. The only exception was the top cylinder whose height was adapted to accommodate the total tree height. Any misrepresentation of the stem by cylinders and any caliper measurement errors were considered negligible and ignored, and the sum of the volumes of the cylinders was assumed to represent the true tree volume without error. Finally, the c_i were estimated using ordinary linear regression analyses.

Variability of the Estimates of Stem Volume Due to the Volume Models

There are several sources of uncertainty due to the volume models such as residual uncertainty around the predicted regression curves and failure of the model to fit the data. However, we do not distinguish between these sources and only regard the pooled uncertainty, which is referred to as model uncertainty. For the purpose of evaluating the variability of estimated stem volumes due to the volume models, the volume of each tree of the model stem data was estimated using two methods

Method 1: As the sum of 1-m high cylinders yielding the "true" stem volume

Method 2: Using volume model 8 and $\hat{f}$ as in Equations 9–13

The relative error of the volume estimate obtained with the volume model was estimated for each tree as

$$e_{V_2,r} = \frac{V_2 - V_1}{V_1}, \tag{14}$$

where the subscript for V indicates whether method 1 or method 2 was used to obtain the stem volume V_i and the r in the subscript refers to the *relative* error. From $e_{V_{2,r}}$, the relative SD of the volume model $SD_{Vf,r}$ was computed.

To check the volume models for biases, the absolute ($V_2 - V_1$) and the relative ($e_{V_{2,r}}$) volume errors were examined by calculating the means of the errors and by producing histograms of the errors (Figure 3A and B).

Simulation of Measurement Errors

The simulations were based on the Monte Carlo concept. To quantify the variability in estimated stem volumes, including the effects of measurement errors, the volume of each tree was estimated using a third method

Method 3: Volume model 8 and $\hat{f}$ as in Equations 9–13 with randomly modified input values according to models 1 and 7 for the SDs of the measurements.

For Method 3, the input values were modified to simulate measurement errors as follows

1. Every input value (*dbh*, *h*, *hk*, and *d03h*) was multiplied by a number randomly generated from a normal distribution $N(1, SD_{i,r}^2)$ with $SD_{i,r}$ as in Equation 1.
2. To every input value, a randomly generated number from a normal distribution $N(0, \widehat{SD}_i^2)$ was added with the $\widehat{SD}_i$ as the SDs obtained from Equation 7.

Negative values were not an issue because the SDs were sufficiently small. After the random modification of the input values, the SD of the volume estimates was estimated from the errors $e_{V_{3,r}}$ as above with

$$e_{V_3,r} = \frac{V_3 - V_1}{V_1}, \tag{15}$$

where the subscript for V again indicates whether method 1 or method 3 was used to obtain the stem volume V_i. This procedure was replicated 15 times for the entire data set, and the mean was taken as the final result. Fifteen replications were considered appropriate and sufficient because there was little variation among estimates from the individual replications and because each replication included calculation of volumes for more than 17,000 tree stems. The calculations were implemented in GNU Octave (2012).

To evaluate the importance of the magnitudes of measurement errors and the potential for improvement, additional sensitivity analyses were conducted by adapting method 3. For one input variable at a time, the SD was halved, whereas the SDs for the other variables remained the same. In addition, the SD of volume estimates was estimated, assuming half SDs for all measurements.

Error Propagation

The law of error propagation, which is based on a first-order Taylor series approximation, states that if $V = V(x_i)$ is a function of the input variables x_i, the SD of V SD_{Vm} can be estimated from the SDs SD_{x_i} of the variables x_i by the following equation (Taylor 1997, p. 79)

$$SD_{Vm} = \sqrt{\sum_i \left(\frac{\partial V}{\partial x_i} SD_{x_i}\right)^2}, \tag{16}$$

where $\partial V/\partial x_i$ are the first derivatives of the expression for V, assuming the measurement errors (which are the cause of the SD_{x_i}) are pairwise independent from each other. Because of this independence, the covariances are zero and are not included in Equation 16.

The input variables x_i for the volume and form factor functions, Equations 8 to 13, are *dbh*, *h*, *d*03*h*, and *hk*. The measurement procedures are independent of each other, except for the procedure to assess *d*03*h*, which depends on *h*. However, the lack of correlation between measurement errors for *d*03*h* and *h* (see Appendix 1) allows us to use the law of error propagation as presented in Equation 16.

The volume models and their first derivatives can be found in Appendix 2. Inserting the derivatives into Equation 16 delivers the SD of the stem volume estimate SD_{Vm} for every tree. From SD_{Vm} we obtain the relative SD as

$$SD_{Vm,r} = \frac{SD_{Vm}}{V}. \tag{17}$$

The calculations were made using the model stem data. Note that $SD_{Vm,r}$ takes only the measurement errors into account. The variability due to the volume model is included in a second step. Thus, the true volume, $\bar{V}$, consists of the estimated volume V plus two independent error terms, the first as a result of incorrect measurements, e_m, and the second due to the shortcomings of the volume model, e_f,

$$\bar{V} = V + e_m + e_f, \tag{18}$$

where $E(e_m) = E(e_f) = 0$ (V unbiased), $\mathrm{Var}(e_m) = SD_{Vm}^2$ and $\mathrm{Var}(e_f) = (V \cdot SD_{Vf,r})^2$. Thus, to obtain the total variance of the single-tree volume estimate, the variance caused by the measurement errors and the variance of the volume model can simply be added (Cunia 1965, 1987, Schmid et al. 1971, Gertner 1990)

$$\mathrm{Var}(\bar{V}) = \mathrm{Var}(e_m) + \mathrm{Var}(e_f). \tag{19}$$

The relative SD for the volume estimate is

$$SD_{\bar{V},r} = \sqrt{SD_{Vm,r}^2 + SD_{Vf,r}^2}. \tag{20}$$

As in the section *Simulation of Measurement Errors*, the total SD was also estimated using half the SDs of the measurements. Similar analyses were conducted using data split into conifers and broadleaved groups.

As stated in Equation 19, the total variance is the sum of multiple variances and can be expressed by squaring and expanding Equation 16 and inserting it into Equation 19 as

$$\mathrm{Var}(\bar{V}) = \left(\frac{\partial V}{\partial dbh} SD_{dbh}\right)^2 + \left(\frac{\partial V}{\partial h} SD_h\right)^2 + \left(\frac{\partial V}{\partial hk} SD_{hk}\right)^2 + \left(\frac{\partial V}{\partial d03h} SD_{d03h}\right)^2 + \mathrm{Var}(e_f), \tag{21}$$

where the terms with SD_{hk} and SD_{d03h} enter the expression when they are also used in the respective volume model (Schmid-Haas and Winzeler 1981). For each tree, the relative contribution, RC, of each component to the total variance can be estimated as

$$RC_i = \frac{\left(\frac{\partial V}{\partial i} SD_i\right)^2}{\mathrm{Var}(\bar{V})}, \tag{22}$$

where i denotes dbh, h, hk, and $d03h$ and

$$RC_f = \frac{\mathrm{Var}(e_f)}{\mathrm{Var}(\bar{V})}, \tag{23}$$

which facilitates comparison of the separate contributions. The trees were stratified into conifers with $dbh \geq 105$ mm, broadleaved trees with $dbh \geq 105$ mm, and trees with $50 \text{ mm} \leq dbh < 105$ mm. The final results are the means $\overline{RC_{dbh}}$, $\overline{RC_h}$, $\overline{RC_{hk}}$, $\overline{RC_{d03h}}$, and $\overline{RC_f}$ over all trees of the respective stratum. In addition, for these analyses, the model stem data were used.

Results

Measurement Error Models

The measurement error model, Equation 7, has two parameters, and it can easily adapt to different point clouds. Table 3 presents the estimates, $\hat{\beta}_i$, and Figure 1A and B show the fits of the models to the data for h, as well as the linear error models as dotted lines for comparison.

Table 3. Estimates of parameters for the nonlinear measurement error models.

	No. of measurements	Class size n	No. of classes	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
dbh	4,411	25	176	0	6.9654	−0.0033
h conifers	1,134	25	45	0	9.8495	−0.0234
h broadleaved trees	336	25	13	0	18.3986	−0.0115
hk	1,292	25	51	0	24.754	−0.0266
d03h	180	15	12	0	22.3525	−0.0104

Stem Volume Uncertainty

Table 4 presents the results for the simulations using the linear measurement error models. The SD due to the volume model $SD_{Vf,r}$ was 5.6% for all species, 8.2% for broadleaved trees, and 4.9% for conifers. Braun (1969) originally estimated $SD_{Vf,r}$ as between 4.33 and 7.65% for single tree species, but not all tree species were included in his analyses.

Simulating the measurement errors for the input variables with the linear error models resulted in SDs of the volume estimates between 10.9 and 11.2%. The final result, the average of 15 replications, is 11.1%, which is almost twice as large as the SD caused by the shortcomings of the volume model.

Applying the equations for error propagation, Equations 16 and 17, yielded a relative SD $SD_{Vm,r}$ of 9.4%. By adding the variance of the volume model to the variance obtained from the error propagation (Equation 20), we obtain the total relative SD per tree,

$$SD_{\bar{V},r} = \sqrt{SD_{Vm,r}^2 + SD_{Vf,r}^2} = \sqrt{0.094^2 + 0.056^2} \approx 0.1094 \approx 10.9\%,$$

which compares very well to the results of the error simulations.

Using the nonlinear error models instead of the linear ones resulted in similar but slightly greater values, except for broadleaved trees. The main difference is that broadleaved trees and conifers now share the same error models except for h, and, therefore, the results for broadleaved trees and conifers are more similar. The results are presented in Table 4.

Sensitivity Analysis of the Volume Models to Measurement Errors

As mentioned above, additional analyses were conducted to quantify the effects of measurement errors on volume estimates. Obviously reducing the SD of the measurement error from any source reduces the total uncertainty, but there are large differences among the input variables. First, the linear measurement error model was used. Only very small changes resulted from halving the SD of dbh (a drop from 11.1 to 11.0%) and hk (an imperceptible drop due to rounding). Improving the measurement of h made a small difference (10.5%), but of all analyses using a modified SD of only one input variable, lowering the SD of $d03h$ yielded the best result by far (8.2%). To produce a guideline, the simulations were also conducted with half SDs for all variables. This yielded a SD of 7.5% for the volume estimates.

The SDs were modified in the same manner for the analyses using the law of error propagation. The results followed the same pattern but were slightly smaller for every case.

Finally, the trees were split into broadleaved trees and conifer groups, and the analyses were conducted for the two groups separately. Although uncertainties were greater for broadleaved trees, the

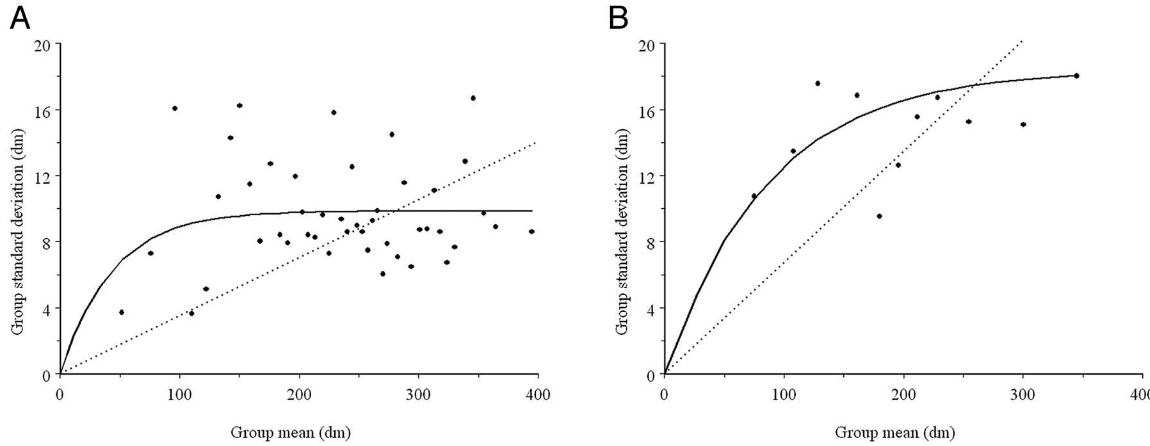

Figure 1. A. The linear model and the nonlinear model for the SD of *h* for conifers. B. The linear model and the nonlinear model for the SD of *h* for broadleaved trees.

Table 4. SDs of the tree stem volume per tree, linear, and nonlinear error models.

Data	Simulation: all trees	Error propagation: All trees	Broadleaved	Coniferous
	(%)			
Linear error model				
$SD_{Vf,\,r}$		5.6	8.2	4.9
$SD_{Vm,\,r}$		9.4	11.2	9.1
$SD_{V3,\,r}/SD_{\bar{V},\,r}$	**11.1**	**10.9**	**13.9**	**10.3**
Nonlinear error models				
$SD_{Vf,\,r}$		5.6	8.2	4.9
$SD_{Vm,\,r}$		9.6	9.9	9.6
$SD_{V3,\,r}/SD_{\bar{V},\,r}$	**11.4**	**11.2**	**12.9**	**10.8**

Table 5. Sensitivity analysis with modified SDs, linear, and nonlinear error models.

Data	Simulation: all trees	Error propagation: All trees	Broadleaved	Coniferous
Linear error models				
$SD_{V3,\,r}/SD_{\bar{V},\,r}$	**11.1**	**10.9**	**13.9**	**10.3**
Half SD of *dbh*	11.0	10.9	13.8	10.3
Half SD of *h*	10.5	10.3	12.6	9.8
Half SD of *hk*	11.1	10.9	13.8	
Half SD of *d*03*h*	8.2	8.2	11.6	7.4
Half SD of all	7.3	7.3	9.9	6.7
Nonlinear error models				
$SD_{V3,\,r}/SD_{\bar{V},\,r}$	**11.4**	**11.2**	**12.9**	**10.8**
Half SD of *dbh*	11.4	11.1	12.8	10.7
Half SD of *h*	10.8	10.4	11.6	10.2
Half SD of *hk*	11.4	11.1	12.9	
Half SD of *d*03*h*	8.4	8.3	11.1	7.6
Half SD of all	7.5	7.4	9.6	6.9

basic structure of the results remained the same. Again, the application of the nonlinear error models resulted in similar values. The complete results are presented in Table 5.

Partition of the Total Variance of the Volume Estimates

Even though the volume model for silver fir differs slightly from the model for the other conifers, the results were similar. The same holds for oak and the other broadleaved trees, but there are large differences between conifers and broadleaved trees with regard to both volume models and results. Thus, the data were separated with respect to the input variables used for the volume models into three tree groups: broadleaved trees with $dbh \geq 105$ mm, coniferous trees with $dbh \geq 105$ mm, and trees with 50 mm $\leq dbh <$ 105 mm. The results are presented in Table 6 and illustrated in Figure 2A and B.

Table 6. The relative contributions to the total variance.

Data error model	*dbh* ≥105 mm: Coniferous trees, Linear	Nonlinear	Broadleaved trees, Linear	Nonlinear	50 mm ≤ *dbh* < 105 mm: all species, Linear	Nonlinear
	(%)					
$\overline{RC}_{hk}$			1.3	0.7		
$\overline{RC}_{d03h}$	67.8	67.2	45.2	37.0		
$\overline{RC}_{h}$	10.8	9.9	19.9	19.4	14.0	31.7
$\overline{RC}_{dbh}$	0.2	0.2	0.5	0.5	3.1	4.2
$\overline{RC}_{f}$	21.2	22.6	33.1	42.5	82.9	64.1

Results obtained using the linear and the nonlinear error models differed somewhat. When the linear error models were used, the uncertainty in *d*03*h* (where available) always turned out to be the greatest contributor to the total variance followed by the uncertainty in the model itself, then in *h*, in *hk* (where available), and finally in *dbh*. The proportions of the total variance due to the model and to *h* are greater for broadleaved trees than for conifers and consequently the share of *d*03*h* is less. This result is attributed to the better approximation by the volume models for conifers (Table 4) and the much greater SD for *h* for broadleaved trees, whereas the SD for *d*03*h* is only slightly greater for broadleaved trees (Table 2).

Qualitatively, the results are the same as those reported in "Sensitivity Analysis of the Volume Models to Measurement Errors," except that in Table 5 the model variability is not analyzed. It shows more clearly that *hk* has a greater effect than *dbh* in Table 6, but they are of similar and small magnitude. In Table 5, some small differences are masked by rounding.

Because *d*03*h* is not assessed on trees with $dbh <$105 mm, the volume model uses less information and consequently provides a less precise estimate. In addition, because of the absence of *d*03*h*, the biggest relative contributor for the other strata is not available. Thus, the relative proportion of the overall variance contributed by the

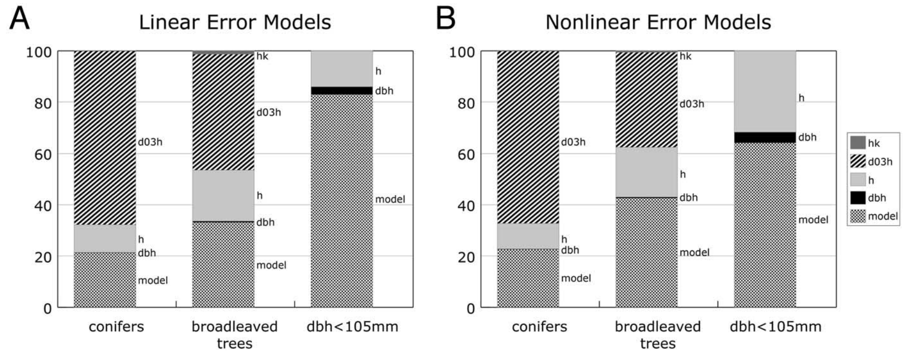

Figure 2. A. Contributions to the overall variance in percent, linear error models. B. Contributions to the total variance in percent, nonlinear error models.

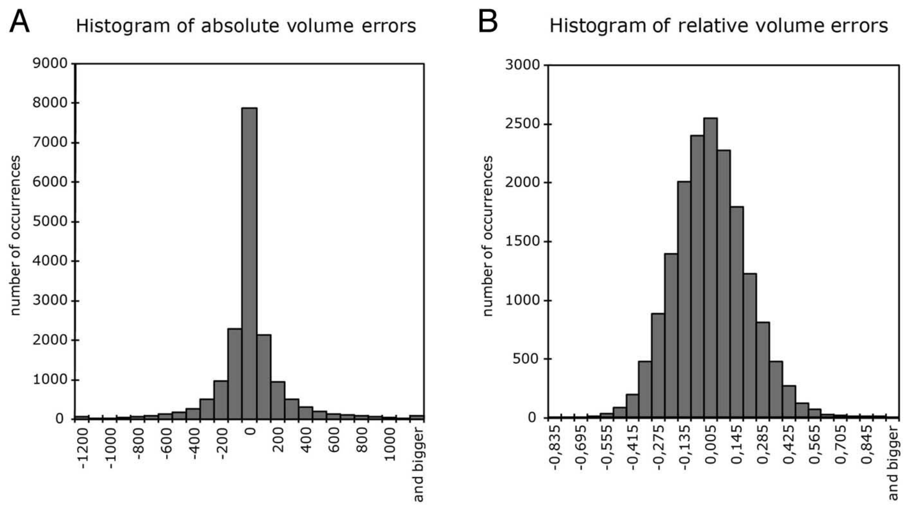

Figure 3. A. Histogram of the absolute errors in dm^3 of the volume estimates. B. Histogram of the relative errors of the volume estimates.

volume model must be considerably greater, in this case more than 80%.

The nonlinear error model for *d*03*h* is the same for broadleaved trees and conifers. Because the SD of *d*03*h* for broadleaved trees is greater than that for conifers when the linear error model is used, the measurement errors for *d*03*h* for broadleaved trees from the nonlinear model are a little smaller. Thus, the effects of the measurement errors for *d*03*h* for broadleaved trees are smaller.

The second main difference is that the nonlinear error models for *h* predicted much larger measurement errors for small trees (Figure 1A and B). Thus, the relative contribution of *h* measurements to the total variance is more than twice as great (31.7%) as that with the linear error models, and consequently the relative contribution of the volume model is 64.1% instead of 82.9%.

Deviations of the Mean

The mean of the relative errors was less than 1% for all analyses. The histograms in Figure 3A and B show the distributions of the absolute (dm^3) and relative errors. They demonstrate well that the errors are almost symmetrically distributed and that the mean is close to 0.

The shapes of the two histograms are so different, because trees of all sizes (50 mm $\leq dbh <$ 985 mm) are included in the data set. When absolute values are considered, all small trees fall into the classes of small error. Only large trees can fall into the other classes, and only those with average or big relative errors actually do so.

Discussion and Conclusion

Table 4 reveals that the SD of the volume estimates cannot drop below 5.6% without constructing new and better volume models, because the 5.6% is due to the shortcomings of the current volume models. With these models, the threshold of 5.6% can be reached only with perfect measurements without error. Factoring in the measurement errors increases this SD to approximately 11%.

Ståhl (1992) estimated relative SDs between 12.1 and 26.4% (between 12.1 and 15.1% for the methods relevant for this article) for compartment-wise stand volume estimates, depending on the method. The greater values are probably due to the fact that not all trees were assessed.

Li and Weiskittel (2010) compared several different taper models for estimating tree stem volume for balsam fir (*Abies balsamea* [L.] Mill.), red spruce (*Picea rubens* [Sarg.]), and white pine (*Pinus strobus* [L.]). Depending on the species, model, and input variables, they obtained relative root mean squared errors between 7.5 and 24.0%. Extracting the biases from these values resulted in relative SDs from 6.0 to 21.3% without including measurement errors. This result shows that the volume models of the Austrian NFI perform well compared with the analyzed taper models. Much smaller SDs of

2.8 to 8.6% depending on tree species and model were obtained by Clark et al. (1991), but they applied the taper models only to the part of the bole with diameter greater than 10.2, 17.8, or 22.9 cm (4, 7, or 9 in.), and the top of the tree stem was ignored. The top of the tree accounts for a large part of the total uncertainty of tree stem volume estimates because there are more irregularities in this part of the stem (Altherr 1960). Thus, the results of Clark et al. (1991) are consistent with the results obtained during this study.

Özçelik et al. (2010) investigated a very different approach. They estimated tree bole volume using artificial neural network models for four species, and their analyses resulted in percent root mean squared errors ranging from 4.1 to 16.0%, depending on species, model, and data set, mostly without significant bias.

Our sensitivity analysis showed (Table 5) that the greatest improvement in the SD of the estimates was achieved by enhancing the measurement of *d*03*h*. For our analyses using the law of error propagation, the total SD decreased from 10.9 to 8.2% (and from 11.2 to 8.3% for nonlinear error models), which is a relatively large amount considering that the other measurement errors remained unchanged and that the theoretical minimum for the current models is 5.6%.

Less potential for improvement was found for *h* (decreased to 10.3%/10.4%), and for *dbh* there was very little change (no decrease visible due to rounding/decrease to 11.1%). This result can probably be attributed to the rather precise measurements for these two variables (Table 2) and the greater effects of *d*03*h* on the form factor function $\hat{f}$.

The measurement of *hk*, on the other hand, is the most imprecise of the four measurements because of the difficulty in defining the base of the crown (Berger et al. 2012). However, the impact of halving its SD was even a little less than that for improving the measurement of *dbh*. There are two reasons for this result. First, *hk* enters into the volume models only for broadleaved trees, and second and more importantly, the coefficients c_2 in the respective volume models (see Appendix 2) have small values. Even for broadleaved trees alone, doubling the precision of the measurement of *hk* does not really change the overall result.

Of course, the best outcome was achieved by halving all SDs. This resulted in a decrease to 7.3% (7.4% with the nonlinear model), which is not very much less than the 8.2% (8.3%) obtained by improving only the measurement of *d*03*h*. To summarize, if the Austrian NFI wishes to enhance estimates of tree stem volume by improving the measurements, the focus should be on *d*03*h* for which new measuring instruments are available.

Gertner and Köhl (1992) came to the same conclusion that the error in the measurement of upper diameter had the greatest effect on the volume estimate followed by that of *h* and that the error of *dbh* had no relevant effect. However, Gertner and Köhl (1992) used different volume models, and *hk* was not used as an input variable. The upper diameter was D_7, which is the diameter at a height of 7 m, independent of the tree height. Therefore, the results can only be compared qualitatively.

Schmid-Haas and Winzeler (1981, p. 244) found that "The random measuring error in $d_{1.3}$ [*dbh*] induces a negligible volume variance" and that "the measuring error in the upper diameter accounts for the largest volume variance component," a result that is confirmed by the results of this study. For their study, only spruce trees were used, and the results for the volume model with *d*03*h* are very similar to the ones presented in Table 6 and in Figure 2A and B for coniferous trees.

Özçelik et al. (2010) stated that "The greatest limiting factor, at present, is the development of accurate and affordable instruments for measuring upper-stem diameters" when referring to the centroid method (Wood et al. 1990). The same is true for the volume models of the Austrian NFI.

Clark et al. (1991) obtained the most accurate results when including an upper diameter (in this case at the height of 5.27 m [17.3 ft]) in the volume models. These results underscore once more the importance and the relevance of upper diameter measurements.

Analyses for only coniferous and for only broadleaved trees demonstrated the well-known fact that volume estimates for conifers are more precise. They also showed that the proportional effects of the input variables are approximately the same for broadleaved trees, for coniferous trees, and for all trees combined, which was expected because the volume models are of the same general type.

The total uncertainty of the volume estimate per tree was estimated using two different measurement error models and both repeated simulations and the law of error propagation. Even though the measurement error models and consequently the contribution of the single components to the total variance differed notably at some points, the overall results for all methods were very similar, ranging from 10.9 to 11.4%. The results of the simulations were slightly greater than the results of the law of error propagation for both types of error models. These differences can be caused by minor effects that were not accommodated such as small correlations between the explanatory variables or simply by differences in the estimation process.

To be on the safe side, 11.4% should be used for future analyses, which results in a 95% confidence interval of ±22.3% at the single-tree level for all species combined. For a nationwide inventory, this level of uncertainty can be inconsequential because of the large sample and can be minimal relative to the most important source of uncertainty by far, sampling variability (Gertner and Köhl 1992). Breidenbach et al. (2014) also state that in all their simulations of biomass stock and change models, the largest part of the overall variability was caused by sampling-related variability. McRoberts and Westfall (2014) examined the effects of uncertainties of volume models for individual trees on large area estimates. They came to the conclusion that the model prediction uncertainties played only a minor role in the large area estimates and that the best way to improve those estimates was focusing on reducing the effects of sampling variability. Ståhl et al. (2014) investigated the sources of uncertainty for nationwide estimates such as biomass stock. They did not focus on individual tree stem volume uncertainty which is reasonable according to our findings.

However, results for small-scale estimates could be impaired. This uncertainty also must be accommodated when terrestrial data are used for training or validating algorithms that predict volume estimates from airborne measurements. For example, Næsset (2007) described an airborne laser scanning experiment for stand volume estimation. Plots with areas of 1,000 m^2 (0.25 acres) were divided into 250 m^2 (2,691 ft^2) quadrants with minimum stem counts as small as four trees per quadrant. The results are presented as SDs of the differences between results from laser scanning data and from ground truth data (10.8 and 12.8% for the two examined strata). However, the estimates from ground truth data are not "true" values, and thus they can inflate the total variance and introduce a bias. The uncertainty estimate reported for this study enables us to partition the different sources contributing to the total variation and thus to assess the real SD of the estimates from laser scanning data more accurately.

Because the mean of the relative errors was always less than 1%, which is very small compared with the SDs of the volume estimates, no relevant deviation of the mean was found in this study. Kangas (1996) found statistically significant biases only in volume estimates with input variables predicted from models, which was not part of this study. In practice, even these biases may be seen as negligible for single-tree estimates.

The facts that the inclusion of an upper diameter significantly improves the volume models (Pollanschütz 1974, Schmid-Haas and Winzeler 1981), that the upper diameter measurement is expensive and has a relatively high SD, and that the upper diameter accounts for the biggest share of the variance result in a dilemma. The large SD mitigates the advantages of the volume model to a certain extent. As mentioned above, in a nationwide survey, other error sources account for the largest part of the uncertainty and make the inclusion of an upper diameter irrelevant, although surveys for small strata will still benefit from it. Further research on when an upper diameter should be included and how the measurements can be improved, for example, by using better instruments, is advised.

Literature Cited

ALTHERR, E. 1960. Die Genauigkeit verschiedener Verfahren der Sektionierung in absoluter und relativer Schaftlänge. *Allgem. Forst. Jagdzeit.* 131:226–237.

BARKER, J.R., M. BOLLMAN, P.L. RINGOLD, J. SACKINGER, AND S.P. CLINE. 2002. Evaluation of metric precision for a riparian forest survey. *Environ. Monit. Assess.* 75:51–72.

BERGER, A., T. GSCHWANTNER, K. GABLER, AND K. SCHADAUER. 2012. Analysis of tree measurement errors in the Austrian National Forest Inventory. *Aust. J. For. Sci.* 129(3):149–177.

BRAUN, R. 1969. *Österreichische Forstinventur, Methodik der Auswertung und Standardfehler-Berechnung*. Mitteilungen der Forstlichen Bundesversuchsanstalt Wien No. 84. Österreichischer Agrarverlag, Wien, Austria. 60 p.

BRAUN, R. 1974. The methodological development of Austrian forest inventories. In *100 Jahre Forstliche Bundesversuchsanstalt*. Forstliche Bundesversuchsanstalt, Vienna, Austria. 359 p.

BREIDENBACH, J., C. ANTÓN-FERNÁNDEZ, H. PETERSSON, R.E. MCROBERTS, AND R. ASTRUP. 2014. Quantifying the model-related variability of biomass stock and change estimates in the Norwegian National Forest Inventory. *For. Sci.* In press.

CANAVAN, S.J., AND D.W. HANN. 2004. The two-stage method for measurement error characterization. *For. Sci.* 50:743–756.

CLARK, A. III, R.A. SOUTER, AND B.E. SCHLAEGEL. 1991. *Stem profile equations for southern tree species*. USDA For. Serv., Res. Pap. SE-282, Asheville, NC. 113 p.

CUNIA, T. 1965. Some theory on reliability of volume estimates in a forest inventory sample. *For. Sci.* 11:115–128.

CUNIA, T. 1987. On the error of continuous forest inventory estimates. *Can. J. For. Res.* 17:436–441.

ELZINGA, C., R.C. SHEARER, AND G. ELZINGA. 2005. Observer variation in tree diameter measurements. *West. J. Appl. For.* 20:134–137.

GABLER, K., AND K. SCHADAUER. 2008. Methods of the Austrian Forest Inventory 2000/02—Origins, approaches, design, sampling, data models, evaluation and calculation of standard error. BFW-Berichte, No. 142, Vienna, Austria. 121 p.

GERTNER, G.Z. 1990. The sensitivity of measurement error in stand volume estimation. *Can. J. For. Res.* 20:800–804.

GERTNER, G., AND M. KÖHL. 1992. An assessment of some nonsampling errors in a national survey using an error budget. *For. Sci.* 38:525–538.

GNU OCTAVE. (2012). *GNU Octave 3.2.4*. Available online at www.gnu.org/software/octave; last accessed July 18, 2011.

GROSENBAUGH, L.R. 1967. Tree form: Definition, Interpolation, Extrapolation. In *Wood measurement, conference proceedings*, Buckingham, F. (ed.). University of Toronto Faculty of Forestry, Toronto, ON, Canada. 222 p.

HOLLAUS, M., W. DORIGO, W. WAGNER, K. SCHADAUER, B. HÖFLE, AND B. MAIER. 2009. Operational wide-area stem volume estimation based on airborne laser scanning and national forest inventory data. *Int. J. Remote Sens.* 30(19):5159–5175.

HOLMGREN, J. 2004. Prediction of tree height, basal area and stem volume in forest stands using airborne laser scanning. *Scand. J. For. Res.* 19:543–553.

HOSMER, D.W., AND S. LEMESHOW. 1989. *Applied logistic regression.* John Wiley & Sons, New York. 307 p.

INTERGOVERNMENTAL PANEL ON CLIMATE CHANGE. 2006. *2006 IPCC guidelines for national greenhouse gas inventories, prepared by the National Greenhouse Gas Inventories Programme,* Eggleston, H.S., L. Buendia, K. Miwa, T. Ngara, and K. Tanabe (eds.). Institute for Global Environmental Strategies, Japan.

KALLIOVIRTA, J., J. LAASASENAHO, AND A. KANGAS. 2004. Evaluation of the laser-relascope. *For. Ecol. Manage.* 204:181–194.

KANGAS, A. 1996. On the bias and variance in tree volume predictions due to model and measurement errors. *Scand. J. For. Res.* 11:281–290.

KAUFMANN, E., AND A. SCHWYZER. 2001. Control survey of the terrestrial inventory. In *Swiss National Forest Inventory: Methods and models of the second assessment*, Brassel, P., and H. Lischke. (eds.). Swiss Federal Research Institute WSL, Birmensdorf, Switzerland. 336 p.

KITAHARA, F., N. MIZOUE, AND S. YOSHIDA. 2009. Evaluation of data quality in Japanese National Forest Inventory. *Environ. Monit. Assess.* 159:331–340.

LI, R., AND A.R. WEISKITTEL. 2010. Comparison of model forms for estimating stem taper and volume in the primary conifer species of the North American Acadian Region. *Ann. For. Sci.* 67:302.

MCROBERTS, R.E., J.T. HAHN, G.J. HEFTY, AND J.R. VAN CLEVE. 1994. Variation in forest inventory field measurements. *Can. J. For. Res.* 24:1766–1770.

MCROBERTS, R.E., AND J. WESTFALL. 2014. The effects of uncertainty in model predictions of individual tree volume on large area volume estimates. *For. Sci.* In press.

MOWRER, H.T., AND W.E. FRAYER. 1986. Variance propagation in growth and yield projections. *Can. J. For. Res.* 16:1196–1200.

NÆSSET, E. 2007. Airborne laser scanning as a method in operational forest inventory: Status of accuracy assessments accomplished in Scandinavia. *Scand. J. For. Res.* 22:433–442.

ÖZÇELIK, R., M.J. DIAMANTOPOULOU, J.R. BROOKS, AND H.V. WIANT JR. 2010. Estimating tree bole volume using artificial neural network models for four species in Turkey. *J. Environ. Manage.* 91:743–753.

PHILLIPS, D.L., S.L. BROWN, P.E. SCHROEDER, AND R.A. BIRDSEY. 2000. Toward error analysis of large-scale forest carbon budgets. *Global Ecol. Biogeogr.* 9:305–313.

POLLANSCHÜTZ, J. 1974. Formzahlfunktionen der Hauptbaumarten Österreichs. *Allgem. Forstzeit.* 85:341–343.

SCHIELER, K. 1988. *Methodische Fragen in Zusammenhang mit der Österreichischen Forstinventur.* Diploma thesis, University of Natural Resources and Life Sciences, Vienna, Austria. 99 p.

SCHMID, P., P. ROIKI-JOKELA, P. MINGARD, AND M. ZOBEIRY. 1971. The optimal determination of the volume of standing trees. P. 33–54 in *Beiträge zur Zuwachsforschung, 15th IUFRO World Congress.* Österreichische Agrarverlag, Vienna, Austria.

SCHMID-HAAS, P., AND K. WINZELER. 1981. Efficient determination of volume and volume growth. P. 231–257 in *Proc. of Forest Resource Inventory, Growth Models, Management Planning, and Remote Sensing,* *17th IUFRO World Congress*. International Union of Forestry Research Organizations, Niigata, Japan.

SKOVSGAARD, J.P., V.K. JOHANNSEN, AND J.K. VANCLAY. 1998. Accuracy and precision of two laser dendrometers. *Forestry* 71:131–139.

STÅHL, G. 1992. *A study on the quality of compartment-wise forest data acquired by subjective inventory methods.* Dept. of Forest Mensuration and Management, Rep. 24, Swedish University of Agricultural Sciences, Umeå, Sweden. (In Swedish with English summary.)

STÅHL, G., J. HEIKKINEN, H. PETERSSON, J. REPOLA, AND S. HOLM. 2014. Sample based estimation of greenhouse gas emissions from forests—A new approach to account for both sampling and model errors. *For. Sci.* In press.

TAYLOR, J.R. 1997. *An introduction to error analysis: The study of uncertainties in physical measurements.* University Science Books, Sausalito, CA. 327 p.

TOMPPO, E., T. GSCHWANTNER, M. LAWRENCE, AND R.E. MCROBERTS (EDS.) 2010. *National forest inventories: Pathways for common reporting.* Springer, Berlin, Germany. 612 p.

WESTFALL, J.A., AND P.L. PATTERSON. 2007. Measurement variability error for estimates of volume change. *Can. J. For. Res.* 37:2201–2210.

WILLIAMS, M.S., K.L. CORMIER, R.G. BRIGGS, AND D.L. MARTINEZ. 1999. Evaluation of the Barr & Stroud FP15 and Criterion 400 laser dendrometers for measuring upper stem diameters and heights. *For. Sci.* 45:53–61.

WILLIAMS, M.S., AND H.T. SCHREUDER. 2000. Guidelines for choosing volume equations in the presence of measurement error in height. *Can. J. For. Res.* 30:306–310.

WOOD, G.B., H.V. WIANT JR., R.J. LOY, AND J.A. MILES. 1990. Centroid sampling: A variant of importance sampling for estimating the volume of sample trees of radiata pine. *For. Ecol. Manage.* 36:233–243.

Appendix 1: The Relationship Between *d*03*h* and *h*

Methods: Dependence of *d*03*h* on the Error of *h*

Because the stem diameter of a tree generally decreases with increasing height, an error in measuring h would be expected to have an effect on the measurement of $d03h$. For example, if the h measurement is erroneously large, $d03h$ will be measured at a higher point on the stem and, therefore, will generally be smaller than if measured at the correct height and vice versa. To analyze this effect, data from the control survey of NFI plots (described above in SDs of the Measurements from a Control Survey) were used. First, the absolute differences of the measurements of $d03h$ and h were calculated for each tree as

$$Diff_h = h_1 - h_2 \tag{24}$$

$$Diff_{d03h} = d03h_1 - d03h_2 \tag{25}$$

where h_1 and $d03h_1$ refer to the measurements obtained during the regular NFI survey and h_2 and $d03h_2$ refer to measurements obtained during the control survey. Next, the relative difference (relative to the mean of the two measurements) was calculated as

$$Diff_{hr} = \frac{Diff_h}{\frac{h_1 + h_2}{2}} = \frac{2Diff_h}{h_1 + h_2}; \tag{26}$$

and similar calculations were made for $d03h$. Then, a correlation analysis was conducted to assess the degree to which differences in the measurements of h could predict differences in the measurements of $d03h$.

Results: The (Non)dependence of Errors of *d*03*h* on the Errors of *h*

The relationship between measurement errors for $d03h$ and for h was analyzed with data from the control survey. Relative errors for $d03h$ and h were graphed against each other, and a regression analysis was performed. The regression analysis returned a significant slope of the estimated regression line ($P = 0.2\%$), and a correlation coefficient of 22% when the complete data set was used. However, a closer evaluation revealed that these results were only due to two outliers. After removal of only two data points (~1% of 177) corresponding to the most extreme values, P increased to 63%, the correlation coefficient decreased to 3.7%, and the estimated regression line became almost horizontal. The analysis of absolute values of measurement differences yielded very similar results. Thus, $d03h$ and h are related, but the relationship is barely visible, and it has no relevant effect because the correlation is not statistically significantly different from 0.

Discussion: Independence of *d*03*h* from *h*

The intuitive assumption that the measurement errors for $d03h$ and h would compensate for each other was not confirmed. Instead, they can be considered statistically independent for which there are two major reasons

1. A measurement error for h translates into a much smaller error (only 30%) of the position of $d03h$. If h deviates by 1 m, the position of $d03h$ will change only by 30 cm. Thus, when the relatively high precision for the measurement of h is considered, the result is a small variation in the position of $d03h$.
2. At about 30% of its height, a tree stem is almost cylindrical (Grosenbaugh 1967, p. 4). Therefore, moving up or down along the stem does not change the diameter to a great extent. Instead, local factors such as bases and proliferation of branches as well as the measurement error with the relascope have a much greater effect on the total error of $d03h$.

Of course, the results obtained are only approximate because of measurement error terms occurring both in the denominator and the numerator of Equation 26. However, the correlation is so low that it is still safe to say that no correlation term needs to be included in the formula for the error propagation (Equation 16).

Appendix 2: Volume Models and Their First Derivatives

Conifers except silver fir:

$$V = \frac{\pi}{4} \cdot dbh^2 \cdot h \cdot \left(c_0 + c_1 \cdot \frac{d03h}{dbh} + c_2 \cdot \frac{h}{dbh} + c_3 \cdot \frac{1}{dbh} \right) \tag{27}$$

$$\frac{\partial V}{\partial dbh} = \frac{\pi}{4} \cdot h \cdot (2 \cdot c_0 \cdot dbh + c_1 \cdot d03h + c_2 \cdot h + c_3) \tag{28}$$

$$\frac{\partial V}{\partial d03h} = \frac{\pi}{4} \cdot c_1 \cdot dbh \cdot h \tag{29}$$

$$\frac{\partial V}{\partial h} = \frac{\pi}{4} \cdot dbh \cdot (c_0 \cdot dbh + c_1 \cdot d03h + 2 \cdot c_2 \cdot h + c_3) \tag{30}$$

Silver fir:

$$V = \frac{\pi}{4} \cdot dbh^2 \cdot h \cdot \left(c_0 + c_1 \cdot \frac{d03h^2}{dbh^2} + c_2 \cdot \frac{h}{dbh} + c_3 \cdot \frac{1}{dbh} \right) \tag{31}$$

$$\frac{\partial V}{\partial dbh} = \frac{\pi}{4} \cdot h \cdot (2 \cdot c_0 \cdot dbh + c_2 \cdot h + c_3) \tag{32}$$

$$\frac{\partial V}{\partial d03h} = \frac{\pi}{4} \cdot 2 \cdot c_1 \cdot d03h \cdot h \tag{33}$$

$$\frac{\partial V}{\partial h} = \frac{\pi}{4} \cdot (c_0 \cdot dbh^2 + c_1 \cdot d03h^2 + 2 \cdot c_2 \cdot dbh \cdot h + c_3 \cdot dbh) \tag{34}$$

Broadleaved trees except oak:

$$V = \frac{\pi}{4} \cdot dbh^2 \cdot h \cdot \left(c_0 + c_1 \cdot \frac{d03h}{dbh} + c_2 \cdot \frac{hk}{h} + c_3 \cdot \frac{h}{dbh^2} \right) \tag{35}$$

$$\frac{\partial V}{\partial dbh} = \frac{\pi}{4} \cdot (2 \cdot c_0 \cdot dbh \cdot h + c_1 \cdot d03h \cdot h + 2 \cdot c_2 \cdot dbh \cdot hk) \tag{36}$$

$$\frac{\partial V}{\partial d03h} = \frac{\pi}{4} \cdot c_1 \cdot dbh \cdot h \tag{37}$$

$$\frac{\partial V}{\partial h} = \frac{\pi}{4} \cdot (c_0 \cdot dbh^2 + c_1 \cdot dbh \cdot d03h + 2 \cdot c_3 \cdot h) \tag{38}$$

$$\frac{\partial V}{\partial hk} = \frac{\pi}{4} \cdot c_2 \cdot dbh^2 \tag{39}$$

Oak:

$$V = \frac{\pi}{4} \cdot dbh^2 \cdot h \cdot \left(c_0 + c_1 \cdot \frac{d03h^2}{dbh^2} + c_2 \cdot \frac{hk}{h} + c_3 \cdot \frac{h}{dbh^2} \right) \tag{40}$$

$$\frac{\partial V}{\partial dbh} = \frac{\pi}{4} \cdot dbh \cdot (2 \cdot c_0 \cdot h + 2 \cdot c_2 \cdot hk) \tag{41}$$

$$\frac{\partial V}{\partial d03h} = \frac{\pi}{4} \cdot 2 \cdot c_1 \cdot d03h \cdot h \tag{42}$$

$$\frac{\partial V}{\partial h} = \frac{\pi}{4} \cdot (c_0 \cdot dbh^2 + c_1 \cdot d03h^2 + 2 \cdot c_3 \cdot h) \tag{43}$$

$$\frac{\partial V}{\partial hk} = \frac{\pi}{4} \cdot c_2 \cdot dbh^2 \tag{44}$$

Small trees (no *d*03*h* and *hk*):

$$V = \frac{\pi}{4} \cdot dbh^2 \cdot h \cdot \left(c_0 + c_1 \cdot (\ln(dbh))^2 + c_2 \cdot \frac{1}{h} + c_3 \cdot \frac{1}{dbh} \right. \\ \left. + c_4 \cdot \frac{1}{dbh^2} + c_5 \cdot \frac{1}{dbh \cdot h} + c_6 \cdot \frac{1}{dbh^2 \cdot h} \right) \tag{45}$$

$$\frac{\partial V}{\partial dbh} = \frac{\pi}{4} \cdot (2 \cdot h \cdot dbh \cdot (c_0 + c_1 \cdot \ln(dbh) \cdot (1 + \ln(dbh))) \\ + 2 \cdot c_2 \cdot dbh + c_3 \cdot h + c_5) \tag{46}$$

$$\frac{\partial V}{\partial h} = \frac{\pi}{4} \cdot (dbh^2 \cdot (c_0 + c_1 \cdot (\ln(dbh))^2) + c_3 \cdot dbh + c_4) \tag{47}$$