An Expression for the Effect of Aspect, Slope, and Habitat Type on Tree Growth

Note by A. R. Stage

Abstract. An expression for describing the effect of aspect and slope on tree growth is illustrated by the relation of western white pine site index to aspect, slope, and habitat type. ***Forest Sci. 22: 457–460.***

Additional key words. Site, *Pinus monticola*, western white pine.

STUDIES of the effect of environmental factors on the growth of trees generally assume that the effect of aspect can be coded as a cosine function with the minimum shifted to the southwest quadrant and the maximum to the northeast quadrant. This assumption, first proposed by Gaiser (1951) with respect to site index, has been found to be an acceptable approximation in numerous later site index studies. However, for other measures of tree growth such as periodic diameter increment, the optimum aspect can be quite different. Indeed, in regression models, the aspect variable may have different effects depending on the choice of other factors to be included in the model. Beers and others (1966) have shown how the phase of the cosine function can be shifted to place the optimum at an arbitrary aspect. Searching techniques to locate the optimum for a given set of data as used by Hartung and Lloyd (1969) can be quite inefficient and depend for their success on a good initial guess of the location. Finally, the use of higher order sine or cosine functions to represent asymmetries was introduced by Carmean (1967).

The purpose of this note is to show how the foregoing developments can be combined into a single technique whereby the *a priori* assumption of a maximum in the northeast quadrant (45° azimuth) and a minimum in the southwest quadrant can be replaced by an empirically determined location of the optimum without repeated calculations of the regression fit. In addition, it is argued that expressions for the effect of aspect should always be considered as terms involving an interaction with slope. The simple symmetrical cosine curve assumption can also be modified to accommodate more asymmetrical forms of the response to aspect.

The effect of aspect proposed by Beers and others is stated mathematically as B cosine $(a\text{-}\theta)$ where B is the amplitude, a is the azimuth measured clockwise from north, and θ is the phase shift that generally has been assumed to be 45° (Trimble and Weitzman 1956). Suppose now that instead of assuming the optimum aspect *a priori* we wish to estimate the phase shift angle from the data at hand. This can be accomplished quite readily by introducing both the cosine of a and the sine of a as two independent variables in the multiple regression for predicting the response to aspect (e.g., equation (1) in Figure 1). The regression coefficients estimated by least squares for these two independent variables can then be used to determine the phase shift (θ) and the amplitude (B) from the relations given in Figure 1. The expressions for θ in degrees depend on the signs of b_1 and b_2 and the arc tan of the absolute value of the ratio: $|b_2/b_1|$. These equations are commonly found among the "formulae for reference" in appendices to many mathematical texts, but their application to growth analyses apparently is not widely recognized.

The simple variables, sine and cosine of azimuth, would suffice in the regression if all data were obtained from plots having about the same slope. However, plots on level ground supply no information on the effect of aspect on tree growth. Furthermore, it is reasonable to assume that the effect of aspect will increase on an adverse aspect up to the angle of slope that is perpendicular to the sun's rays. For greater slopes, the slight advantage of a decrease in incoming radiation is likely to be offset by decreasing soil depth.

For these reasons, I recommend that the variables to represent the combined effect of slope and aspect be defined as the tangent of slope (slope percent) times the sine and cosine, respectively, of the azimuth. In this way, plots on flat ground will have a zero value for these two variables, but plots on steep ground will have high weights for the sine and cosine of aspect. In addition, the tangent of slope

Principal Mensurationist, USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, Utah 84401, stationed in Moscow, Idaho, at the Forestry Sciences Laboratory, maintained in cooperation with the University of Idaho. Manuscript received April 15, 1976.

This file was created by scanning the printed publication.
Errors identified by the software have been corrected;
however, some errors may remain.

should also be included as a separate independent variable to overcome the symmetry of the sine and cosine functions. Otherwise, the steeper slopes on favorable aspects would be predicted to be more favorable than the flatter slopes. The additive variable for slope percent permits the model to describe adverse or negligible slope effects on the more favorable aspects and increasingly severe conditions with increasing slopes on the more unfavorable aspects.

Example of the application of this technique is taken from the study of the effect of environmental factors on the site index (*S.I.*) of western white pine (*Pinus monticola* Dougl.). The regression of site index on the factors of aspect, slope, and habitat type (Daubenmire and Daubenmire 1968) was calculated to be:

$$\ln(S.I.) = 0.08070s\cos(a) + 0.08423s\sin(a) - 0.12634s + hab$$

where

$\ln(S.I.)$ = logarithm of site index for western white pine

a = azimuth from north

s = slope percent $\div$ 100 = tan (slope angle)

hab = 4.16974 for *Abies grandis/Pachistima* habitat type; 4.15770 for *Tsuga/Pachistima* habitat type; 4.41374 for *Thuja/Pachistima* habitat type.

Substituting these regression coefficients in the expressions given in Figure 1, we find that the data indicate a phase shift of arc tan (0.08423/0.08070) = 46°. The amplitude of the cosine function with the phase shift is $\sqrt{0.08070^2 + 0.08423^2} = 0.11665$. Consequently, the simplified prediction equation is.

$$\ln(S.I.) = 0.11665s\cos(a\text{-}46°) - 0.12634s + hab.$$

Note that the regression coefficient for tan (slope angle) is –0.12634. It is opposite in sign and of almost the same magnitude as the amplitude of cos (a-θ) in the northeast quadrant. However, on the southwest facing slopes, the slope effect accentuates the adverse aspect effect. Figure 2 illustrates the solution of the final regression equation for two habitat types. The plotting of the *Abies grandis/Pachistima* habitat type is omitted because it would nearly coincide with the *Tsuga/Pachistima* habitat type.

In some cases, the symmetrical cosine function may not truly represent the effect of aspect on tree growth. For example, the function represented by the above coefficients for site index of western white pine is illustrated by the solid line in Figure 3. Suppose now that instead of this relationship an asymmetrical form, defined by the points represented by the filled circles, represents the true relationship. That is, the northeast and northwest aspects are equally favorable, and the

---

Relations for calculating amplitude (B) and phase shift (θ) to express the effect of aspect and slope on growth:

If:

$$G = b_0 + b_1s\cos(a) + b_2s\sin(a) + b_3s \quad (1)$$

where:

G = growth response

b_0 = constant term or sum of other predictor effects in the regression

a = azimuth in degrees from north

s = slope in percent $\div$ 100

then:

$$G = b_0 + Bs\cos(a\text{-}\theta) + b_3s \quad (2)$$

where:

$$B = \sqrt{b_1^2 + b_2^2}$$

and θ is given by:

b_2 \ b_1	positive	negative
positive	arctan $\lvert b_2/b_1 \rvert$	180° - arctan $\lvert b_2/b_1 \rvert$
negative	-arctan $\lvert b_2/b_1 \rvert$	180° + arctan $\lvert b_2/b_1 \rvert$

FIGURE 1. *Equations for calculating amplitude* (B) *and phase shift* (θ) *to express the effect of aspect on growth.*

most adverse sites are on southwest exposures. Such a relationship can be described by using additional sine and cosine terms for integer multiples of the azimuth. That is, their arguments would be two, three, or perhaps four times the azimuth. Such an approach will be familiar to those who have used Fourier analysis in engineering. Such an analysis was introduced to forest mensurationists by Anderson in 1937. When such analysis was applied to eight equally spaced data points from Figure 3, an expression of the following form was derived:

$$\ln(S.I.) = 0.0951s\cos(a) + 0.0675s\sin(a) - 0.0142s\cos(2a) + 0.0126s\sin(2a) - 0.1263s + hab.$$

Predictions using this equation are shown by the dashed line in Figure 3.

In the western white pine example, the location of the extreme sites coincides surprisingly well with the commonly postulated 45°! However, for basal area growth of individual trees of the same species on the same habitat types, the phase shift was calculated to be 201°, indicating an optimum on the southwestern aspects within the specified habitat type. In fact, similar analyses for western larch, Douglas-fir, grand fir, western redcedar, lodgepole pine, Engelmann spruce, and subalpine fir all showed southerly aspects to be superior for basal area growth. Only western hemlock, with an indicated optimum on the northeast aspects, coincided with the *a priori* expectation (personal communication, Robert A. Monserud and William R. Wykoff). With the analysis technique described in this note, these unexpected effects were uncovered using regular multiple regression calculations which are widely available.

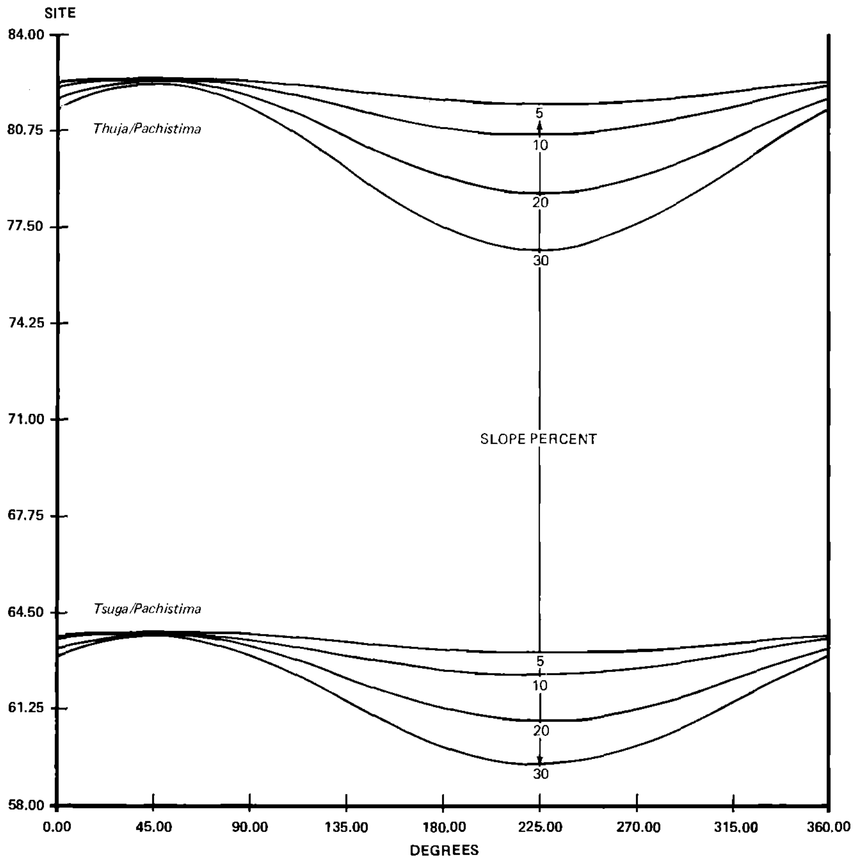

FIGURE 2. *Trends of western white pine site index with slope and aspect for two habitat types in northern Idaho.*

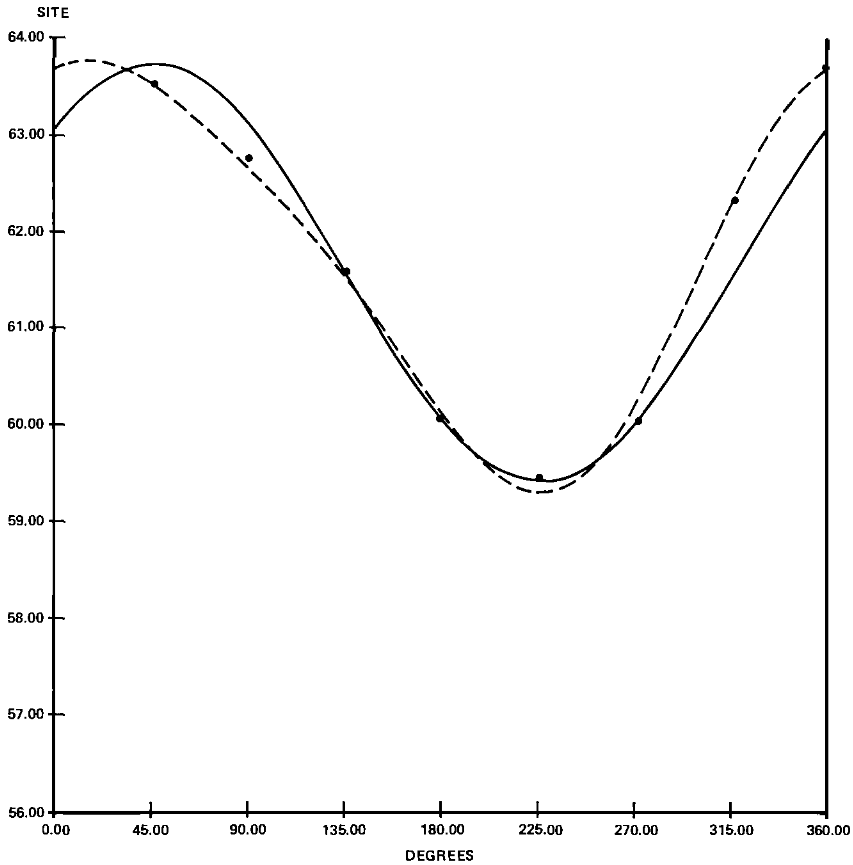

FIGURE 3. *Comparison of symmetrical effect of aspect (solid line) with asymmetrical effect (dashed line) obtained by adding sine and cosine of twice the azimuth as additional regression variables.*

Literature Cited

ANDERSON, ROBERT T. 1937. The application of Fourier's series in forest mensuration. J For 35:293–299.

BEERS, THOMAS W., PETER E. DRESS, and LEE C. WENSEL. 1966. Aspect transformation in site productivity research. J For 64:691–692.

CARMEAN, WILLARD H. 1967. Soil survey refinements for predicting black oak site quality in southeastern Ohio. Soil Sci Soc Am Proc 31:805–810.

DAUBENMIRE, R. R., and JEAN B. DAUBENMIRE. 1968. Forest vegetation of eastern Washington and northern Idaho. Wash Agric Exp Stn Tech Bull 60, 104 p.

GAISER, R. N. 1951. Relation between topography, soil characteristics, and the site index of white oak in southeastern Ohio. USDA Forest Serv, Cent States Forest Exp Stn Tech Pap 121, 12 p.

HARTUNG, ROBERT E., and WILLIAM J. LLOYD 1969. Influence of aspect on forests of the Clarksville soils in Dent County, Missouri. J For 67:178–182.

TRIMBLE, G. R., JR., and SIDNEY WEITZMAN 1956. Site index studies of upland oaks in the northern Appalachians. Forest Sci 2:162–173.