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forest management

A Spatial Stochastic Programming Model for Timber and Core Area Management Under Risk of Fires

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Previous stochastic models in harvest scheduling seldom address explicit spatial management concerns under the influence of natural disturbances. We employ multistage stochastic programming models to explore the challenges and advantages of building spatial optimization models that account for the influences of random stand-replacing fires. Our exploratory test models simultaneously consider timber harvest and mature forest core area objectives. Random fire samples are built into the model, creating a sample average approximation (SAA) formulation of our stochastic programming problem. Each model run reports first-period harvesting decisions along with recourse decisions for subsequent time periods reflecting the influence of stochastic fires. In each test, we solve 30 independent, identically distributed (i.i.d.) replicate models and calculate the persistence of period one solutions. Harvest decisions with the highest persistence are selected as the solution for each stand in a given test case. We explore various sample sizes in our SAA models. Monte Carlo simulations of these solutions are then run by fixing first-period solutions and solving new i.i.d. replicates. Multiple comparison tests identify the best first-period solution. Results indicate that integrating the occurrence of stand-replacing fire into forest harvest scheduling models can improve the quality of long-term spatially explicit forest plans.

Keywords: sample average approximation, mixed integer programming, harvest scheduling, spatial optimization

Natural disturbances such as fire, wind, insects, and diseases interact with forest management activities across space and time. These disturbances can influence timber flow, economic return, forest age-class distribution, and forest spatial structure. A number of research studies have highlighted the importance of accounting for the influence of such stochastic disturbances in forest planning models (e.g., MacLean 1990). In many forests, fire is a major disturbance factor influencing timber supplies. Interest in planning harvest schedules that account for the risk of fire blossomed in the 1980s and has continued through today, as outlined below.

Stochastic programming models to develop harvest schedules that consider fire risk were introduced as early as 1980. Stochastic programming models are formulated presupposing a sequence of decisions and observations wherein a first stage contains decisions that must be taken "here and now" considering the available data. These data are presumed to include conditional probabilities for random events that could affect the outcomes of those decisions. Subsequent stages contain "wait and see" recourse decisions that can be used to take corrective measures or adjustment plans as random events preceding the second stage (and possibly later decision stages) are revealed (Birge and Louveaux 1997). Martell (1980) developed an aspatial stochastic programming model to optimize the harvest schedule for a single stand of trees with a given fire risk. His work demonstrated the value of fire management activities, finding that these activities often boosted timber production enough to more than pay for themselves. Gassmann (1989) developed an aspatial stochastic programming model with seven time periods to evaluate strata-based optimal forest wide harvesting in the presence of fire. He found in some cases that he could use one-point discretizations for the last three planning periods (aggregating periods four to seven into a single decision stage), thus reducing the model to four stages, without affecting the quality of first-period solutions. In other cases, all seven planning periods had to be modeled as individual decision stages. Gassmann suggested using penalty terms in the objective function instead of restricting timber flow through constraints because he found that timber flow constraints are difficult to enforce in a stochastic programming framework. His model suggested that

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uncertainty leads to smaller optimal harvest sizes. Similarly, Boychuck and Martell (1996) developed an aspatial multistage stochastic programming model with 10 discrete planning periods, the first four modeled as decision stages. They examined the impacts of timber flow limitations and fire severity and found that timber flow limitations in a stochastic environment force a much smaller optimal harvest. Boychuck and Martell suggest the use of buffer stock to allow a forest to produce a nondeclining timber supply, as they also found timber flow constraints difficult to enforce.

In other stochastic modeling work, Routledge (1980) incorporated the probability of a catastrophe and the expected salvage proportion in a stand into the classic Faustmann formulation for identifying optimal stand rotation, demonstrating the importance of including stochastic information in harvest scheduling models. More recent aspatial stochastic models have been used to examine balancing timber production with other goals. One such model was developed by Spring and Kennedy (2005), who used a stochastic dynamic programming (SDP) model to study the tradeoffs between producing timber and protecting endangered species under the threat of random fires. While their study did include effects of random fires, it did not analyze how the risk of fire would change trade-off decisions. Ferreira et al. (2011) developed an SDP model that assumed management of each stand would be implemented at the end of the stage, after fires occur. They found that higher wildfire risk tends to decrease rotation lengths.

Zhou and Buongiorno (2006) incorporated Markov chain models describing stand transitions of mixed loblolly pine-hardwood forests under the influence of natural disturbances into a stochastic optimization model to study the tradeoffs between landscape diversity and timber objectives. They found that both large-scale and small-scale disturbances can be beneficial to diversity, but achieving higher landscape diversity has a high timber production cost. Markov chain models have also been incorporated into a forest level goal programming simulation/optimization model using simulated annealing, which was used to search for the optimal harvest schedule for a forest subject to random wildfires (Campbell and Dewhurst 2007). When the objective was to return the landscape to a historical condition, Campbell and Dewhurst found that it could be difficult to meet timber objectives.

Because stochastic optimization modeling can be computationally expensive and slow, researchers have experimented with other methods to examine the impact of fire on timber harvests, although in many cases recourse decision opportunities are not modeled. Van Wagner (1983) used simulation to examine the weaknesses of forest level harvest schedules that fail to take fire into account. He analyzed the effect of fire on timber yield and found that the optimal yield is decreased by fire risk. The use of these decreased annual harvests led to an annual yield that was much less sensitive to fire events. Reed and Errico (1986) formulated an aspatial forest level linear programming harvest scheduling model with flow constraints to maximize the return from harvests under the influence of mean value fire disturbance rates. While this was initially formulated as a stochastic problem, the authors decided to solve only the mean value problem. The authors suggest that in large enough forests the mean value problem is a satisfactory simplification. Their results were similar to Van Wagner's (1983). Reed and Errico (1989) later enhanced their model by accommodating the possibility of partial salvage, multiple timber types, accessibility constraints, and variable recovery costs. Armstrong (2004) developed an aspatial stochastic simulation model to test deterministic annual allowable cut (AAC) solutions in Alberta, Canada. After determining an AAC using a linear programming model, Armstrong evaluated the results using Monte Carlo simulations by assuming the proportion of area burned in each period is random. The study compared the effects of fire on AAC under different harvest schedules in different types of forest. More recently, a deterministic aspatial model was built by Pasalodos-Tato et al. (2010) to evaluate the interaction between fire and timber management in stands of Maritime Pine (*Pinus pinaster*). Both the Armstrong (2004) and Pasalodos-Tato et al. (2010) results are consistent with previous work, finding that increased fire risk leads to a decreased optimal harvest.

In some recent work, researchers have investigated the potential effects that harvesting has on fire spread rates for future fires. Konoshima et al. (2008) tested a two stage spatially explicit stochastic dynamic programming model in a regularly shaped landscape with seven hexagonal stands. Random fire events were captured by probabilities of ignition and different weather scenarios. Under the assumption that harvesting a stand produces higher fire spread rates across the stand, rotation lengths are not necessarily shortened; given the higher spread rates a fire in a younger stand tended to threaten more stands than a fire in an older stand. In contrast, Acuna et al. (2011) built a spatial model that assumes fire does not spread through harvested stands. This model integrated fire ignition, fire spread, fire suppression, and fire protection values in a combined simulation and spatial timber model to develop harvest schedules that purposefully impact landscape flammability. The harvest scheduling component in the model is an extension of Reed and Errico's 1986 model. Acuna et al. (2010) found that they could increase harvest volumes when looking at harvesting as a fire mitigation strategy. Because they assumed fire does not spread through harvested areas, they could reduce the risk to high value stands by harvesting stands nearby.

As Boychuck and Martell (1996) summarized, the effects of fire disturbance on timber supplies over time appear to vary considerably depending on a number of factors. In multistand or multistrata harvest scheduling models with forest-level constraints such as nondeclining flow, a frequent outcome is that "attempting implementation of mean value problem solutions in a stochastic system leads to infeasibility with high probability" (e.g., see Pickens and Dress 1988, Hof et al. 1995). They observed, however, that mean value problem solutions generally provided fair approximations to stochastic programming problem first-period solutions. In systems where periodic replanning occurs, they suggest that mean value solutions may even be good approximations where allowances are made for fire risk by harvesting less than the solution indicates (i.e., by retaining a timber supply buffer stock). Boychuck and Martell (ibid) indicate that more complex stochastic programming methods "would be justified in areas with a tight timber supply, lacking sufficient overmature areas, having high and highly variable fire losses, and where harvest quantity declines are particularly unwanted."

Many forest planning problems include objectives besides allowable cut or financial returns from timber harvests. These nontimber objectives often are spatially explicit and many require spatial optimization methods to account for landscape patterns and arrangement effects (e.g., see Hof and Bevers 1998, 2002, Murray 2007). Forest planning problems with harvest area ("adjacency") constraints (e.g., Murray 1999, Weintraub and Murray 2007, McDill et al. 2002, Goycoolea et al. 2005), forest patch considerations (Tóth and McDill 2008) or habitat requirements for species that dwell in the interior ("core area") of mature forests (e.g., Öhman and Eriksson 1998, 2002, Öhman 2000, Hoganson et al. 2005, Wei and Hoganson 2007, Zhang et al. 2011) are typical examples. We note that, so far, few studies on the subject of forest planning under fire risk have been conducted with spatially explicit models. We anticipate that mean value approaches to modeling fire disturbance may be untenable in many spatially explicit stand- or cut block-based landscapewide harvest scheduling problems, motivating this research.

In this paper, we explore the use of a spatial multistage stochastic programming model to select optimal harvest schedules for timber and core area joint production under the influence of wildfires. Because core area production is affected by the arrangement of stand conditions over a landscape and the number of possible arrangements is large for even a small forest, we incorporate random stand-replacing fire events into our model using a sample average approximation (SAA) formulation (Kleywegt et al. 2001, Bevers 2007). This model is similar in many respects to the model III harvest scheduling formulation (Gunn 1991), which models forest growth and harvesting by balancing the area of forest entering and leaving each state (age class) in each stage (planning period). Boychuck and Martell (ibid.) developed their stochastic programming formulation from a model III framework by replacing mean values for fire disturbance rates with discrete two-point probability distributions of fire disturbance. In this study, we employ random stand-replacing fires that maintain stand boundaries to address not only fire effects on allowable cut but the additional spatial concern of retaining mature forest core area over time. We built and solved our models on a 64-bit workstation equipped with two quad-core processors and 32 GB of memory using IBM's ILOG-CPLEX Parallel v.12.2 solver. The models proved difficult to solve in many cases. Consequently, we conducted our experiments on simplified artificial forest problems to study model performance and the effects of fires on timber and core area production. We also resorted to using relatively small sample sizes and solving multiple independent, identically distributed (i.i.d.) model replications, followed by statistical selection methods to evaluate candidate solutions, as suggested for some problems by Kleywegt et al. (2001) and described below.

Methods

We first introduce a deterministic even-aged harvest scheduling formulation that is constructed to maintain the integrity of stand boundaries. We step through the development of our model in subsequent sections.

A Deterministic Timber Harvesting Formulation

We consider the stand as the smallest management unit so that decisions need to be made for an entire stand. For example, if clearcutting is the only management prescription to be applied, this model will assume we can either clearcut a whole stand or not cut it at all. Binary variables are used to track management decisions for each stand in each period. This model coordinates stand level decisions across time to maximize forest level timber-based economic returns.

$$Max \sum_i \sum_j \sum_k \sum_t \; v_{ijkt} X_{ijkt} - q \sum_{t \epsilon \{2,T\}} Q_t \qquad (1.1)$$

St.

$$\sum_k X_{ij_i1k1} = 1 \qquad \forall\, i \qquad (1.2)$$

$$\sum_{j'} \sum_{k \epsilon K_{ij'j}} X_{ij'k(t-1)} - \sum_k X_{ijkt} = 0 \qquad \forall i, j, t, \epsilon \{2, T\} \qquad (1.3)$$

$$Q_t \geq \sum_i \sum_j \sum_k c_{ijk} X_{ijk(t-1)} - \sum_i \sum_j \sum_k c_{ijk} X_{ijkt} \quad \forall t \epsilon \{2, T\} \qquad (1.4)$$

$$X_{ijkt} \; is \; binary \qquad \forall i, j, k, t \qquad (1.5)$$

X_{ijkt} denotes binary decision variables indicating, when set to 1, the selection of management option *k* for stand *i* at age class *j* during period *t*. *T* denotes the total number of planning periods across the entire planning horizon. $K_{ij'j}$ denotes the set of management options that can advance stand *i* from age class *class j'* to *j* across two consecutive periods. For example, without being harvested, a stand will grow into an older age class entering the next planning period; the age of harvested stands will be reset to age class one when entering the next planning period. Other management options such as thinning could also be formulated into the model. Stands not harvested and older than a defined maximum age are assumed to stay within the maximum age class. To maintain stand boundaries, which is necessary for tracking forest core area (explained later), this model does not merge stands into multistand strata with identical age classes and stand types as model III formulations and as Boychuck and Martell (ibid.) did. The term c_{ijk} denotes the timber yield in any time period from managing stand *i* following prescription *k* when this stand is at age class *j*. The discounted net revenue of managing stand *i* at age class *j* following prescription *k* in period *t* is denoted by v_{ijkt}. Both c_{ijk} and v_{ijkt} are zero in any time period for the "no cut" prescriptions.

The following example illustrates the transition of stand age classes between two consecutive periods in our model (Equation 1.3). In this example, we assume a stand can be in age class one (≤39 years old), two (40–79 years old), or three (80+ years), depending on what happened to the stand in earlier planning periods. Each period is set to 40 years to match the stand age classification in this example. We assume either a clearcut that happens in the middle of each planning period, or no cut, are the only two management options for this stand. A clearcut will reset the stand age back to zero right after the harvesting. The set of stand transition constraints between period *t*-1 and *t* for this stand can be written as

$$X_{age\_1,\,cut,\,t-1} + X_{age\_2,\,cut,\,t-1} + X_{age\_3,\,cut,\,t-1} = X_{age\_1,\,cut,\,t} + X_{age\_1,\,no\_cut,\,t}$$

$$X_{age\_1,\,no\_cut,\,t-1} = X_{age\_2,\,cut,\,t} + X_{age\_2,\,no\_cut,\,t}$$

$$X_{age\_2,\,no\_cut,\,t-1} + X_{age\_3,\,no\_cut,\,t-1} = X_{age\_3,\,cut,\,t} + X_{age\_3,\,no\_cut,\,t}$$

The objective function maximizes the total discounted return of managing a forest for *T* periods minus a penalty on any declines in timber production between the (*T*-1) pairs of consecutive periods. Penalties or constraints on declines in timber production commonly are used to secure a sustainable flow of timber supply to local timber markets and to prevent overharvesting in earlier time periods when discount rates have less effect (Nautiyal and Pearse 1967). We employed this penalty instead of a nondeclining yield constraint (similar to Gassmann 1989), using a positive coefficient *q* as the penalty for each unit of timber decline between each pair of consecutive periods. Constraint 1.2 requires that exactly one stand management activity, either clearcut or no cut, is selected for each stand at period

Table 1. Three-period example illustrating the sampling notation used in our sample-based stochastic programming model.

Samples set, index of each sample, and the simplified denotations		
Period 1 sample set $\tilde{N}_1$	Period 2 sample set $\tilde{N}_2$	Period 3 sample set $\tilde{N}_3$
$(1)_1$	$(1,1)_2$ denoted as $(1)_2$	$(1,1,1)_3$, denoted as $(1)_3$
		$(1,1,2)_3$, denoted as $(2)_3$
	$(1,2)_2$ denoted as $(2)_2$	$(1,2,3)_3$, denoted as $(3)_3$
		$(1,2,4)_3$, denoted as $(4)_3$
$(2)_1$	$(2,3)_2$ denoted as $(3)_2$	$(2,3,5)_3$, denoted as $(5)_3$
		$(2,3,6)_3$, denoted as $(6)_3$
	$(2,4)_2$ denoted as $(4)_2$	$(2,4,7)_3$, denoted as $(7)_3$
		$(2,4,8)_3$, denoted as $(8)_3$

We assume random fires can occur in each of the three periods. Suppose two random sample states are drawn for a stand in period one and two more samples are drawn for each resulting state in subsequent periods. In this example, we would have two sample forest states at the end of period one, four at the end of period two, and eight at the end of period three for this one forest stand. $\tilde{N}_t$ denotes the set of randomly generated stand sample states during period t. We index sample states from 1 to n at the end of period t using the abbreviation $(1)_t$, $(2)_t$,......$(n)_t$ for simplicity. Each stand state at the end of the three-period planning horizon reflects a sampled succession pathway up to the end of period three. For example, the state indexed by $(5)_3$ represents a sampled potential forest succession pathway of $(2)_1(3)_2(5)_3$ for the stand.

one. In Equation 1.2, j_{i1} denotes the current age class of stand i. Constraint 1.3 links the harvesting options between two consecutive periods for each stand. Constraint 1.4 defines a set of bookkeeping variables Q_t to track declines in timber production from periods (t-1) to t. This formulation is used as a base for further enhancement. In the next step, we add new decision variables and constraints, creating a spatially explicit stochastic programming formulation to model the influence of random stand-replacing fires and recourse stand-harvesting decisions.

Adding Random Fires With a Fixed Sequence Between Harvesting and Fires

In the example formulation below, we first build random samples of stand-replacing fires into our model based on the assumption that fires always occur after management activities within each planning period. This assumption will be relaxed in the section that follows. Another simplification in this work is that we do not explicitly model fire spread from one stand to another.

In the formulation below, N denotes a constant number of i.i.d. landscapewide samples of stand-replacing fires generated as outcomes for each existing stand sample state in each time period (see Table 1). The beginning of each time period is a decision stage in our model. Either a clearcut or a no cut decision is made for each stand sample state at the start of the period. Logical "branches" in the constraints model implementation of the decision for that stand sample state in that stage followed by N stand-replacing fire sample outcomes (either burned or not burned). Thus, decisions after the first period are recourse decisions, unique to the state of the stand at that point in time in that sample path, and nonanticipativity is imposed implicitly by presenting each decision with multiple random outcomes (Birge and Louveaux 1997).

We use $\tilde{N}_t$ to denote the set of randomly generated stand sample states during period t. $\tilde{N}_0$ includes only the initial stand state. Using the example in Table 1, if N is set to two, $\tilde{N}_1$ would include two sampled states in period one for each stand. Branching from each period one state creates two new states for each stand at period two; therefore, $\tilde{N}_2$ would include four sample states. $\tilde{N}_3$ would include eight states. By tracing from each of these eight sampled states back to the current stand state, we can identify a sample forest stand succession pathway across three periods (Table 1). We use $(n)_t$ to index each random state in the set $\tilde{N}_t$ in period t. This SAA formulation is given below

$$Max\ \sum_{t\epsilon\{1,T\}}\frac{1}{N^{t-1}}\sum_{(n)_{t-1}}\sum_i\sum_j\sum_k v_{ijkt}X_{ijk(n)_{t-1}}$$

$$-\ q\sum_{t\epsilon\{1,T-1\}}\frac{1}{N^t}\sum_{(n)_t}Q_{(n)_t} \quad (2.1)$$

St.

$$\sum_k X_{ij_{i1}k(n)_o} = 1 \qquad \forall i \quad (2.2)$$

$$D_{ij'}h(n)_t = 0 \qquad \forall i, j', h, t \in \{1, T\}, (n)_t \epsilon \tilde{N}^{\circ\circ}_{ij'ht} \quad (2.3)$$

$$\sum_j\sum_{k\epsilon K_{ijj'}}X_{ijk(n)_{t-1}} - \sum_h D_{ij'h(n)_t} = 0$$

$$\forall i, j', t \in \{1, T\}, (n)_t \notin \tilde{N}^{\circ\circ}_{ij'ht} \quad (2.4)$$

$$\sum_{j'}\sum_{h\epsilon H_{ij'j}}D_{ij'h(n)_t} - \sum_k X_{ijk(n)_t} = 0$$

$$\forall i, j, t\epsilon\{1, T-1\}, (n)_t \in \tilde{N}_t \quad (2.5)$$

$$Q_{(n)_t} \geq \sum_i\sum_j\sum_k c_{ijk}X_{ijk(n)_{t-1}} - \sum_i\sum_j\sum_k c_{ijk}X_{ijk(n)_t}$$

$$\forall t \in \{1, T-1\}, (n)_t \in \tilde{N}_t \quad (2.6)$$

$X_{ijk(n)_{t'}}$ $D_{ij'h(n)_t}$ are binary variables. $H_{ij'j}$ is the set of fire disturbances under which a stand i transits from age class j' to j. h is the index of the disturbances in set $H_{ij'j}$. For the simplified cases in this study, each set can hold two disturbance types: "stand-replacing fire" and "no-fire." Other types of fire disturbances such as stand-damaging fire could also be modeled. $D_{ij'h(n)_t}$ is a binary variable used to track the actual occurrence of fire disturbance type h in stand i for sample paths where it is in age class j' in time period t, based on the random sample $(n)_t$. $D_{ij'h(n)_t} = 1$ indicates that fire disturbance type h would occur in sample $(n)_t$, as explained below.

Period one forest management decisions are denoted by $X_{ij_{i1}}k(n)_0$ with $(n)_0$ indexing the current forest state. Note that the fourth subscript of $X_{ij_{i1}}k(n)_0$ is $(n)_0$ instead of $(n)_1$. Harvest decisions at period one will be made before knowing the occurrence of any fires, enforcing nonanticipativity. Similarly, at the start of each future period t, the fourth subscript of recourse decision variable $X_{ijk(n)_{t-1}}$ is $(n)_{t-1}$ instead of $(n)_t$ indicating one recourse decision for each disturbance sample from the preceding time period, enforcing that recourse decisions must also be made before knowing the occurrence of any subsequent fires. In Constraint 2.3, $\tilde{N}^{\circ\circ}_{ij'ht}$ denotes the period t sample set in which fire type h does not occur in stand i if this stand is in age class j'. For each sample $(n)_t$ in set $\tilde{N}^{\circ\circ}_{ij'ht}$, the value of corresponding $D_{ij'h(n)_t}$ will be set to zero exogenously by using Constraint 2.3 following the results of random draws; values of the remaining $D_{ij'h(n)_t}$ variables will be determined by the model through Equation 2.4, as illustrated (Figure 1). In Constraint 2.4, the term $(n)_{t-1}$ is used to index the direct precedent parent sample of $(n)_t$. Using the Table 1 example, $(1)_2$ is the direct parent sample of both $(1)_3$ and $(2)_3$. However, $(1)_2$ is not the direct parent of the other samples at period three ($t = 3$).

Figure 1 demonstrates how we can simulate fire disturbance in a

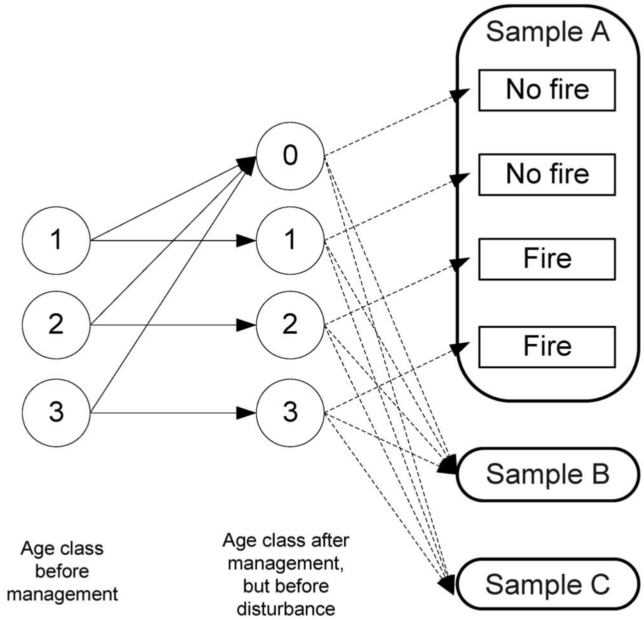

Figure 1. A stand could be in age class one, two, or three at the beginning of each period. We draw three random samples *A, B,* and *C* in this example to reflect the fire occurrences in this stand. The number in each circle represents stand age class; arrows represent the possible stand age class changes in the same planning period. Clearcuts reset stand age to zero; no cut does not change stand age class in the same period. Sample *A* represents a random draw that (1) stand in age class zero and no fire; (2) stand in age class one and no fire; (3) stand in age class two and fire happens; and (4) stand in age class three and fire happens. Different samples *B* and *C* could reflect different combination between fire occurrences and stand age class after management.

stand in a planning period to build Constraints 2.3 and 2.4. In this example, we assume a stand could be in one of three age classes at the start of each period: age class one, two, or three. Harvesting and no cut are the only two available management options for each stand. For this exploratory study, we assume the probability of fire in each stand depends on its age class. See Reed (1994) for a method of estimating the historic probability of stand-replacing fires. An example (denoted as sample *A*) was built based on random draws to reflect the possible fire occurrence in this stand. Constraints 2.3 and 2.4 can then be designed to reflect the fire occurrence as a function of stand condition (age class) in sample *A*. For simplicity we omit the subscripts of the stand and the sample index (sample A) from each decision variable. We use this example to demonstrate how to set up the transitions between *Xs* and *Ds*. Without harvesting, this stand will stay in the same age class in the same period (reflected by Equations SA.1 to SA.3). Harvesting would reset the stand age to zero (Equation SA.4). Equations SA.1 to SA.4 enumerate all possible age class transitions for this stand within the same planning period. After randomly drawing fires in this sample, we observe that some of these transition paths may not be possible. The example below depicts a case where a fire will occur if and only if the stand is in age class two or three and not harvested in this time period. We set the corresponding *Ds* to zero through Equations SA.5 to SA.8 to block other paths.

$$X_{age\_1,\,no\_cut} = D_{age\_1,\,no\_fire} + D_{age\_1,\,fire} \tag{SA.1}$$

$$X_{age\_2,\,no\_cut} = D_{age\_2,\,no\_fire} + D_{age\_2,\,fire} \tag{SA.2}$$

$$X_{age\_3,\,no\_cut} = D_{age\_3,\,no\_fire} + D_{age\_3,\,fire} \tag{SA.3}$$

$$X_{age\_1,\,cut} + X_{age\_2,\,cut} + X_{age\_3,\,cut} = D_{age\_0,\,no\_fire} + D_{age\_0,\,fire} \tag{SA.4}$$

$$D_{age\_0,\,fire} = 0 \tag{SA.5}$$

$$D_{age\_1,\,fire} = 0 \tag{SA.6}$$

$$D_{age\_2,\,no\_fire} = 0 \tag{SA.7}$$

$$D_{age\_3,\,no\_fire} = 0 \tag{SA.8}$$

For a different sample, e.g., sample B in Figure 1, the same set of X variables will be connected to another set of D variables that reflect fire sample B. Constraint 2.5 advances each stand into an older age class as it moves into the next planning period. If a stand is already in the oldest age class modeled, it will stay in this oldest age class. Constraint 2.6 tracks the declining timber flow in each sample between every pair of consecutive periods. In Constraint 2.6, $(n)_{t-1}$ is used to index the direct parent sample of $(n)_t$ as we described for Constraint 2.4. We use many samples to represent random fires and each sample has its own discounted return and associated timber flow. The new objective function (2.1) again has two components: the discounted return from managing the forest and the penalty on average timber production declining from any two consecutive periods. Discounted returns and timber flow penalties from each sample are weighted by one over the appropriate sample size N^{t-1} or N^t, creating a SAA model that maximizes the resulting estimate of expected net return.

Modeling Random Sequences Between Management and Fire

Stand-replacing fires in reality can happen before or after management activities in each period. For example, harvesting may liquidate the timber value before it can be destroyed by a stand-replacing fire; however, a stand-replacing fire might also make the harvesting impractical if the fire happens first. The assumption that harvesting always precludes fire may overestimate the allowable cut. In this study, we tested a more general formulation that assumes the sequence between harvesting and fire within a time period is also random. Following this assumption, we built an enhanced SAA formulation

$$Max\ \sum_{t\,\in\,\{1,\,T\}}\frac{1}{N^t}\sum_{(n)_t}\sum_i\sum_j\sum_k v_{ijkt}W_{ijk(n)_t} - q\sum_{t\,\in\,\{2,\,T\}}\frac{1}{N^t}\sum_{(n)_t}Q_{(n)_t} \tag{3.1}$$

subject to

$$\sum_k X_{ij_{i1}k(n)_0} = 1 \qquad \forall i \tag{3.2}$$

$$W_{ijk(n)_t} = 0 \qquad \forall i, j, k = 1, t \in \{1, T\}, (n)_t \in \mathring{N}_{ijkt} \tag{3.3}$$

$$D_{ij'h(n)_t} = 0 \qquad \forall i, j', h, t \in \{1, T\}, (n)_t \in \mathring{\mathring{N}}_{ij'ht} \tag{3.4}$$

$$X_{ijk(n)_{t-1}} - W_{ijk(n)_t} = 0 \quad \forall i, j, k, t \in \{1, T\}, (n)_t \notin \mathring{N}_{ijkt} \tag{3.5}$$

Table 2. Management decisions and fire occurrence can interact within a stand during each planning period. Randomly drawn samples model these interactions. In each sample, $W_{ijk(n)_t}$ tracks whether a clearcut could be implemented as a consequence of the within-period timing of fire occurrence. This table lists six possible scenarios of fire occurrence and forest stand management (mgmt) in a given time period. The action associated with the appropriate value of $W_{ijk(n)_t}$ is shown for each scenario.

Sampled fire occurrence	Model selected mgmt $X_{ijk(n)_{t-1}}$	Sampled sequence between mgmt and fire	Implemented mgmt $W_{ijk(n)_t}$	Actual fire occurrence
No	no cut	irrelevant*	no cut	No
No	clearcut	irrelevant*	clearcut	No
Yes	no cut	irrelevant*	no cut	Yes
Yes	clearcut	mgmt, fire	clearcut	No
Yes	no cut	irrelevant*	no cut	Yes
Yes	clearcut	fire, mgmt	no cut	Yes

*The sequence between fire and management does not matter in a given period if there is no fire, or if clearcut is not selected to manage this stand.

$$\sum_j \sum_{k\epsilon K_{ijj'}} W_{ijk(n)_t} - \sum_{h\epsilon H_{ij'}} D_{ij'h(n)_t} = 0 \quad \forall i, j', t \epsilon \{1,T\},(n)_t \notin \bar{N}^{\circ}_{ij'ht} \quad (3.6)$$

$$\sum_{j'} \sum_{h\epsilon H_{ij'j}} D_{ij'h(n)_t} - \sum_k X_{ijk(n)_t} = 0 \quad \forall i, j, t \epsilon \{0, T-1\},(n)_t \epsilon \bar{N}_t \quad (3.7)$$

$$Q_{(n)_t} \geq \sum_i \sum_j \sum_k c_{ijk} W_{ijk(n)_{t-1}} - \sum_i \sum_j \sum_k c_{ijk} W_{ijk(n)_t} \quad \forall (n)_t \epsilon \bar{N}_t, t \epsilon \{2,T\} \quad (3.8)$$

$$X_{ijk(n)_t}, W_{ijk(n)_t}, D_{ij'h(n)_t} \text{ are binary variables}$$

In this enhanced model, we still assume each stand could be harvested ($k = 1$) or not ($k = 0$) in each period. The model uses $X_{ijk(n)_{t-1}}$ to track whether at the start of period t harvesting is planned for each stand i at age class j. This decision applies for all subsequent branches of the sample path. Subsequent sample fire events in the same time period could preclude the implementation of scheduled cuts in some stands. We define a set of new variables $W_{ijk(n)_t}$ to track whether a scheduled cut ($k = 1$) would be implemented for stand i in period t in sample $(n)_t$. We use two random draws to create samples of fire occurrence for each stand at each existing forest state. The first draw indicates the occurrence of fire. If a fire occurs, the second random draw determines the sequence between this fire and any planned harvesting. $\bar{N}^{\circ}_{ijkt}$ denotes the subset of samples for which harvesting ($k = 1$), if selected, is precluded by a fire in stand i at age class j during period t. In this case, $W_{ij(k=1)(n)_t}$ is set to zero exogenously through Equation 3.3; otherwise, the value of $W_{ijk(n)'_t}$ is determined endogenously by the model-selected management actions $X_{ijk(n)_{t-1}}$ through Equation 3.5. Also note that the fourth subscript of $W_{ijk(n)_t}$ is used to enumerate items in each corresponding set from $\bar{N}_1$ to $\bar{N}_T$, which is different from the sets indexed by the fourth subscript of $X_{ijk(n)_t}$ that are from $\bar{N}_0$ to $\bar{N}_{T-1}$, again enforcing that decisions need to be made first at each stage prior to the realization of any random fire events (nonanticipativity). Table 2 gives an example enumerating the possible interactions between planned management decisions and fire occurrences in each stand. Equation 3.6 maintains the age balances for each stand within a planning period. When moving into the next planning period, Equation 3.7 increases the age class of every stand by one. Objective function (3.1) maximizes the estimated expected value of total discounted revenue only from the implemented harvesting decisions (reflected by the value of $W_{ijk(n)_t}$) minus the expected penalties for declines in timber harvest from one period to the next. Equation 3.8 tracks the declining timber flow across each sample between every pair of consecutive periods. Subtle differences also exist between Equations 2.6 and 3.8. Equation 2.6 uses decision variable $X_{ijk(n)_t}$ to calculate the timber production from each sample; Equation 3.8 instead uses the decision variable $W_{ijk(n)_t}$ to achieve the same type of bookkeeping. This is because a scheduled harvesting decision applies for all subsequent branches of the same path, but harvesting may not be actually implemented if a stand is first destroyed by a fire (i.e., see the last row of Table 2).

Modeling Forest Core Area

Preserving interior mature forest some distance from edge effects (core area) has been one of the important spatial forest management concerns in landscape forest planning (Zipper 1993, Baskent and Jordan 1995, Fischer and Church 2003). Mature forest core is the area of mature forest protected by a buffer area from edge effects of surrounding habitats. Core area has been integrated into forest planning through dynamic programming (Hoganson et al. 2005), mixed integer programming (Wei and Hoganson 2007), and heuristic models (Ohman and Eriksson 1998, 2002, Ohman 2000). Core area can be modeled by tracking the states of many predefined influence zones across time (Hoganson et al. 2005). While core area has been modeled in this fashion before, it has not previously been modeled in the presence of wildfire to our knowledge.

The concept of influence zones can be used to account for how forest management could preserve mature forest core area across a planning horizon. Influence zones are delineated through a separate geographic information system (GIS) process (Hoganson et al. 2005). Each influence zone can be considered as a portion of the forest where a unique set of stands influence whether the area in the zone will produce core area of mature forest in a given time period. An example forest of four stands (A, B, C, and D; Figure 2A) (Wei and Hoganson 2006) illustrates this concept and the associated modeling method. By buffering outward from the boundaries of each stand for a predefined distance, a set of influence zones {A, B, C, AB, BC, BD, CD, and BCD} can be identified (Figure 2B). Using influence zone AB as an example, the ages of both stand A and B must satisfy the age requirement of mature forest for this influence zone to be classified as mature forest core area. It is clear that implementing the concept of influence zone requires the model to maintain the stand boundaries. The probability of each zone becoming core area depends on the joint probability of all stands forming the zone to meet the age requirement of mature forest. Modeling joint

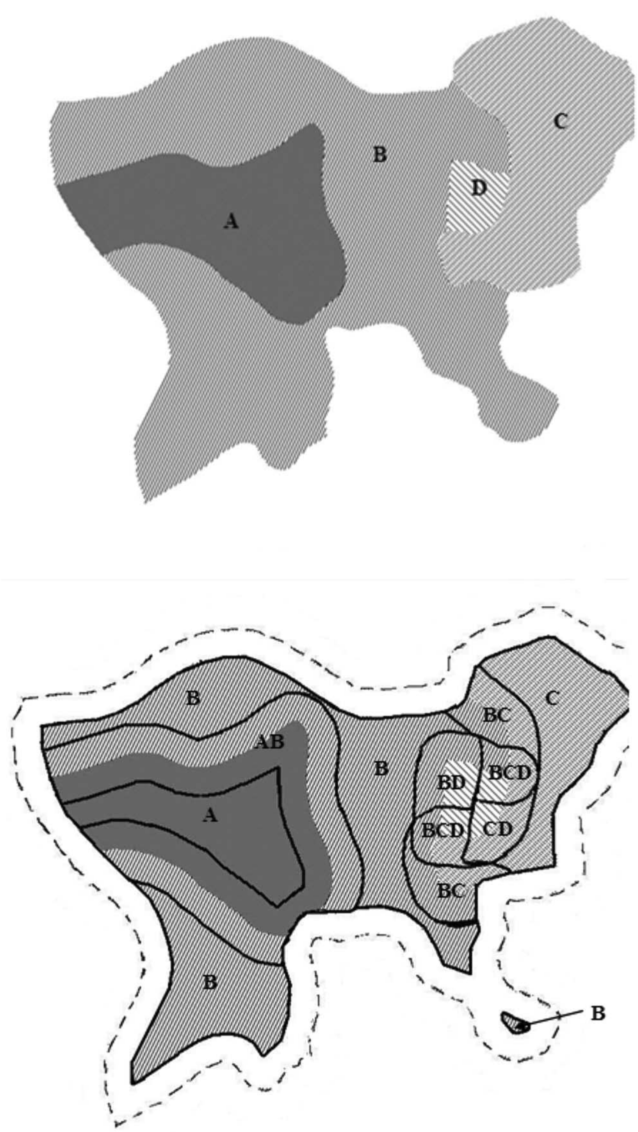

Figure 2. A forest is composed of four stands A, B, C, and D. Buffering the boundary of each stand creates eight influence zones. Whether each influence zone is core area at the end of each planning period depends on the age class of every stand in the zone.

probability directly is challenging due to potential nonlinearity. This makes the sampling-based approach more appealing.

The influence zone concept has been modeled in a deterministic context, in which harvesting decisions determine whether each influence zone would become core area at the end of each period. We build one constraint for every stand in each influence zone I_z during each planning period t. In this model, core area is tallied at the end of each planning period t.

$$Y_{zt} - \sum_{j \epsilon j''} \sum_{k \epsilon K'} X_{ijkt} \leq 0 \qquad \forall z,\, i \epsilon I_z,\, t \qquad (4.1)$$

The term z indexes every influence zone within a forest. This constraint is written in terms of the prescription options for each stand that could meet core area requirements. J'' denotes the set of stand age classes that satisfy the requirement of core area. Two conditions are required for any influence zone z to be core area at the end of period t: (1) all stands i associated with the influence zone z need to satisfy the age requirement for mature forest core area; and (2) prescriptions k ϵ K′ maintain the ages of all stands within influence zone z. When both conditions are satisfied, Y_{zt} will be set to one to indicate that influence zone z contributes to forest core area at the end of period t. The total core area maintained at the end of each period could be tracked in the objective function or through bookkeeping constraints.

For influence zone AB in the above example, stands A and B can be harvested ($k = 1$) or not ($k = 0$) during period t. If we assume stands in age class two or three could contribute to mature forest core area, two constraints can be used to track whether influence zone AB will be core area at the end of period t.

$$Y_{AB,t} - X_{A,2,0,t} - X_{A,3,0,t} \leq 0$$

$$Y_{AB,t} - X_{B,2,0,t} - X_{B,3,0,t} \leq 0$$

According to these two constraints, influence zone AB will be mature forest core area only if one of the variables $X_{A,2,0,t}$ and $X_{A,3,0,t}$, and one of the variables $X_{B,2,0,t}$ and $X_{B,3,0,t}$ are set to one (no harvesting for both stands).

Random fire disturbances could also influence core area production. For example, a stand-replacing fire might destroy the overstory trees within a stand and reset the stand age to zero. This would destroy core area within this stand and also stands for which this stand provides buffer. To account for these effects, we model the impact of fire on core area for each influence zone z at the end of each period t. Management decisions interact with fires to determine stand age at the end of each planning period. We substitute Equations 4.1 by the new set of Constraints 4.2.

$$Y_{z(n)_t} - \sum_{j \epsilon J''} \sum_{h \epsilon H'} D_{ijh(n)_t} \leq 0$$

$$\forall z,\, i \epsilon I_z, t \epsilon \{1, T\}, (n)_t \epsilon \tilde{N}_t \qquad (4.2)$$

Variable $Y_{z(n)_t}$ tracks whether influence zone z at the end of period t would contribute core area according to the sample indexed by $(n)_t$. H' denotes the subset of disturbances in stand i that could maintain the stand age and satisfy core area requirements. For example, when there are only two types of disturbances, stand-replacing fire or no fire, H' would only include the no fire option. Because management $X_{ijk(n)_t}$ will influence $D_{ijh(n)_t}$ through Equations 3.5 and 3.6, Equation 4.2 can be used to replace Equation 4.1 in the stochastic model.

Fire occurrence is simulated in each randomly generated sample. The SAA model tracks the amount of core area produced in each sample at the end of each planning period. For this study, we chose to use average minimum core area as our performance measure. To address this, a *Max(Min())* approach (e.g., Bevers 2007) is used. This is accomplished as follows

(1) For each sampled forest succession pathway, calculate the minimum core area produced from period one to T.

(2) Build a bookkeeping constraint to average the minima core area calculated in step 1 across all sampled succession pathways.

(3) Value the average minimum core area calculated from step 2 in the objective function.

Based on the above discussion, we built an integrated model to incorporate the impact of fire into a spatially explicit harvest-scheduling model with core area concerns. By including core area estimation, our SAA stochastic programming model addresses a much more complex landscape spatial optimization problem. This new model includes Constraints 3.2 to 3.7 to model fire occurrence and stand age class succession. Constraint 3.8 is included to track timber production declines between consecutive periods. Constraint 4.2 is included to track whether each influence zone produces forest core area at the end of each period given fire disturbances and management actions. Objective function (5.1) below is used to maximize the total weighted average return from timber, the average minimum core area along all sampled forest succession pathways, and a penalty for average timber production declines between any two consecutive periods across all samples. Bookkeeping Constraints 5.2, 5.3, and 5.4 below are used to support the *Max(Min())* model format. By using spatially explicit constraints to track core area preservation, this model maximizes the estimated expected total joint benefits from timber harvesting and core area conservation in a forest over a given planning horizon.

$$Max \sum_{t\in\{1,T\}}\frac{1}{N^t}\sum_{(n)_t}\sum_i\sum_j\sum_k v_{ijkt}W_{ijk(n)_t} - q\sum_{t\in\{2,T\}}\frac{1}{N^t}\sum_{(n)_t}Q_{(n)_t} + q'V_a \quad (5.1)$$

subject to

$$V_{(n)_t} = \sum_{z\in Z} o_z Y_{z(n)_t} \quad \forall t \in \{1,T\},(n)_t \in \breve{N}_t \quad (5.2)$$

$$\mathring{V}_{(n)_T} \leq V_{(n)_t} \quad \forall (n)_T,\ t \in \{1,T\},(n)_t \in P_{(n)_T} \quad (5.3)$$

$$V_a = \frac{1}{N^T}\sum_{(n)_T}\mathring{V}_{(n)_T} \quad (5.4)$$

The area of influence zone z is denoted as o_z. Constraint 5.2 calculates $V_{(n)_t}$, which is the total core area produced in each random sample at the end of planning period t. $\mathring{V}_{(n)_T}$ is the minimum total core area preserved in the forest along each sampled forest succession pathway. The term $P_{(n)_T}$ denotes the set of samples along the succession pathway ending with $(n)_T$. Using the example in Table 1, $P_{(5)_3}$ represents a sample set $\{(2)_1, (3)_2, (5)_3\}$. Equation 5.4 calculates V_a that denotes the average $\mathring{V}_{(n)_T}$ across all sampled pathways.

Selecting the First-Period Solution

A primary purpose for using a multistage stochastic harvest scheduling model is to identify first-period harvests that perform well over a range of plausible future conditions (Hoganson and Rose 1987). Harvesting decisions for the first period need to be implemented immediately and period one decisions may also have longer-term impacts on future timber production and spatial forest structure. The quality of the first-period decisions can be used to evaluate overall model performance.

In our SAA model, randomly generated fires are used to estimate the expected consequences of stand-replacing fires and recourse management actions. Replicate models built on different independent i.i.d. random fire samples sometimes suggest different period one harvest schedules. Larger sample sizes can be used to better inform the model about future fires up to a point. However, more samples also increase model complexity and make the model more difficult to solve. An alternative approach is to build multiple SAA models using different sets of i.i.d. fire samples (see Kleywegt et al. 2001). Solutions from the *R* different models can be used to estimate the "persistence" (Bertsimas et al. 2007) of first-period harvest decisions. For example, if more than half of the *R* models choose to harvest stand α in the first period, we might select this stand for harvesting. Note, however, that using persistence calculations stand-by-stand does not necessarily account for stand combinations that might be important, reflecting a heuristic design that could lead to suboptimal solutions (Bevers 2007).

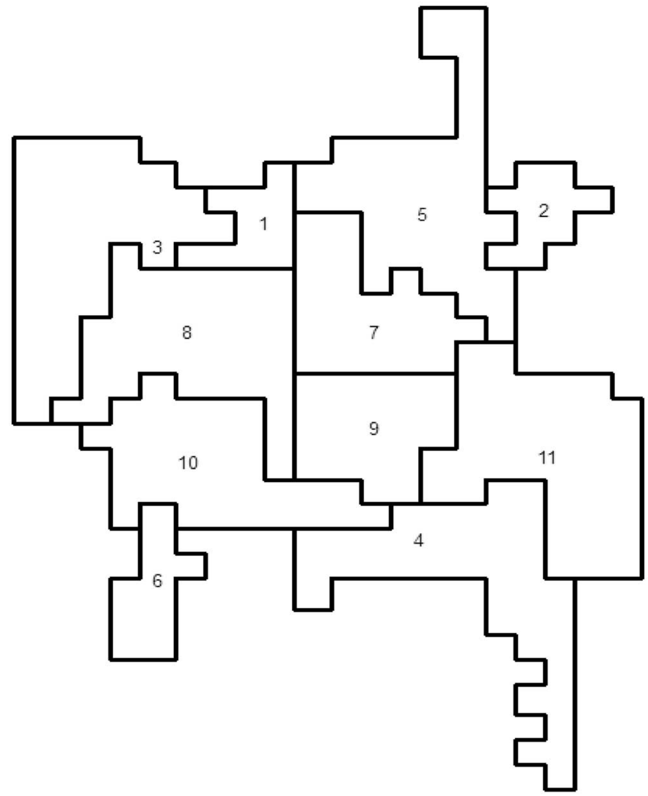

Figure 3. A forest composed of 11 stands used as the test site for our SAA model.

To better understand the quality of the first-period solutions from test cases employing various sample sizes, we created and ran multiple replicate models for each test case with each model built on a different i.i.d. set of randomly drawn fires, as described above. We then selected period one harvest decisions for each stand based on persistence measures. Next, we evaluated the quality of all resulting period one solutions using numerous Monte Carlo simulations generated by fixing first-period harvest decisions and resolving our models with i.i.d. samples. Results from these simulations were then compared using Tukey's multiple comparison method (see Goldsman and Nelson 1998). We used the results of these statistical tests to select first-period harvests. This approach is similar to one suggested by Kleywegt et al. 2001 except for our use of persistence measures.

Test Cases and Results

We built a computer-simulated forest with eleven stands as the test site (Figure 3). We divided a 120-year planning horizon into three 40-year planning periods. At the beginning of each period, a stand is in either age-class one (≤39 years), two (40–79 years), or three (80+). At the beginning of the planning horizon, stands 1, 4, 7, and 10 are assumed in age class one; stands 2, 5, 8, and 11 are in age class two; and stands 3, 6, and 9 are in age class three. Stands older than 40 years (in either age class two or three) are considered

Table 3. Thirty repetitions of the stochastic programming model runs are used to calculate the persistence of clearcut decisions for each stand in period one by using sample size *N* from 1 to 15. This test case assumes high fire risk for each stand of the forest. No penalty is imposed for declines in timber production. Bold numbers represent the decision to clearcut the corresponding stand based on solution persistence.

StandID	1	2	3	4	5	6	7	8	9	10	11
Core area price = $500/acre											
$N = 15$	0%	**100%**	**100%**	0%	**100%**	**100%**	0%	**100%**	**100%**	0%	**100%**
$N = 10$	0%	**100%**	**100%**	0%	**97%**	**100%**	0%	**100%**	**100%**	0%	**100%**
$N = 6$	0%	**100%**	**100%**	0%	**93%**	**100%**	0%	**77%**	**100%**	0%	**100%**
$N = 5$	0%	**93%**	**97%**	3%	**87%**	**100%**	0%	**67%**	**97%**	0%	**100%**
$N = 4$	0%	**97%**	**97%**	10%	**63%**	**100%**	3%	**63%**	**100%**	0%	**100%**
$N = 3$	0%	**83%**	**83%**	10%	**77%**	**93%**	0%	**53%**	**90%**	0%	**90%**
$N = 2$	3%	**80%**	**80%**	20%	43%	**93%**	7%	**63%**	**73%**	20%	**77%**
$N = 1$	23%	**73%**	**57%**	37%	**60%**	**77%**	13%	43%	47%	13%	**63%**
Core area price = $1000/acre											
$N = 15$*	0%	43%	**63%**	0%	20%	**100%**	0%	0%	**73%**	0%	**87%**
$N = 10$	3%	**83%**	**63%**	0%	47%	**100%**	0%	3%	43%	0%	**63%**
$N = 6$	0%	**60%**	**73%**	0%	30%	**100%**	0%	0%	**73%**	0%	**77%**

*The relative gap between the best solution found and the best possible solution is set to 5% to prevent the model from running out of memory. All other runs used a relative gap of 1%.

mature forest for core area calculations. Areas of mature forest 50 m away from the boundary of a young forest (<40 years old) or the boundary of the forest are counted as interior forest habitat, or core area. Using this assumption about 8,300 influence zones are identified to model core area.

Two management alternatives are assumed available for each stand in each planning period: no cut or clearcut. Clearcuts reset the stand age back to zero in the period of harvest. We assume clearcutting a stand in age class one generates no financial returns. Stand-replacing fire is assumed to occur randomly in each stand in each period following known probability distributions (defined below). Stand-replacing fire sets the stand age back to zero in the period the fire occurs. The sequence between fire and clearcut is also random.

Estimating the probability of stand-replacing fires (usually crown fires) is a complex research problem. Different conclusions have been drawn for different forest types. In the tests that follow, we assumed that the flammability of forest increases with age due to fuel accumulation (see for example Turner and William 1994, Minnich 1983, Despain and Sellers 1977). Two levels of hypothetical fire probability were tested. In low fire probability tests, we set the chance of fire at 0.1 for age class one stands, 0.2 for age class two stands, and 0.3 for age class three stands in each 40-year planning period. In high fire probability tests, we set fire probabilities for stands in age class one, two, and three to 0.2, 0.4, and 0.6, respectively, in each 40-year planning period. The chances of fire occurring before or after harvesting were set to 0.5 each. We assume stand-replacing fire and clearcutting are mutually exclusive in each stand in the same period. Random fire occurrences are reflected in the SAA model by setting the value of disturbance variables (D) and decision variables (W) as described in Equations 3.3 and 3.4.

Solution Persistence Under High Fire Risk Assumptions

Tests under the assumption of high fire probability show that persistence of first-period harvests improves with increasing sample size N. Under an assumed core area price of $500/acre, increasing sample size N from 1 to 15 caused decisions for stands 8 and 9 to switch from no cut to clearcut (Table 3). When N is set to 15, all SAA model runs consistently support a period one decision for every stand in the forest: harvesting stands 2, 3, 5, 6, 8, 9, and 11 and doing nothing to the other stands 1, 4, 7, and 10.

The tradeoffs between producing timber and maintaining forest core area also become clearer as sample size N increases. We first tested the assumption of $500/acre core area price with no penalty for timber production declines. When N is set to six, 30 model runs suggest maintaining an average of two acres of core area across the planning horizon (Table 4). By increasing N to 10, most model runs found no benefit in maintaining any core area (Table 4) and suggest all stands at age class two and three should be harvested during the first period to maximize expected returns (Table 3). The benefits of using larger sample sizes are also reflected by the decreased standard deviations of the average minimum core area, the timber productivity of each period, and the objective function value across the 30 tested SAA model runs (Table 4).

We assumed stands in age classes two or three are mature forest and can contribute to forest core area when they occur away from edges. However, older forest could be more susceptible to fires. Under this assumption, higher fire probability increases the cost of maintaining core area. Test results show, when core area price is set at $500/acre under the high fire probability assumption, no core area should be maintained across the planning horizon (Table 4). With the core area price doubled to $1,000/acre, SAA model runs suggest maintaining about 15 acres of core area (Table 4) by delaying harvests of stands above age class two (Table 3) during the first 40-year planning period.

Comparing the Quality of First-Period Solutions

Forest management occurs across space and time. Management decisions for period one need to be carried out without knowing future fire conditions with certainty. Decisions for later periods, on the other hand, can be adjusted based on the actual management activities and fire occurrences in earlier periods (Savage et al. 2010). A good period one solution should help facilitate adjustment of future forest management activities. Different sample sets used by the SAA model may suggest different period one solutions. Changing sample size N also causes the model to select different period one solutions as described in Table 3. While we expect better solutions to be derived from models built on larger sample sizes, the problem of selecting a single period one solution from a set of differing solutions remains.

We first revised the SAA model to find the optimal period one solution under the assumption of no fire risk, $500/acre core area price, and no penalty for timber production declines. This leads to a

Table 4. A statistical summary of 30 runs based on different random draws. Each run is based on either a core area price of \$500/acre or \$1,000/acre. No penalty is imposed for declines in timber production. High fire risk is assumed for each stand in the forest.

	Average value across samples					STD across samples				
		Timber yield at three periods					Timber yield at three periods			
Sample size	Average min core (acres)	P1	P2 (cords)	P3	Obj. (\$)	Average min core (acres)	P1	P2 (cords)	P3	Obj. (\$)
Core area price = \$500/acre										
$N = 15$	0	4,621	1,846	3,886	63,758	0	165	97	135	2,131
$N = 10$	0	4,628	1,807	3,889	63,992	1	288	149	199	3,216
$N = 6$	2	4,449	1,657	3,622	64,705	4	491	326	537	3,104
$N = 5$	6	4,328	1,497	3,398	67,189	6	631	411	654	6,008
$N = 4$	7	3,992	1,556	3,305	65,623	6	638	465	680	5,524
$N = 3$	11	3,849	1,234	3,251	68,006	8	921	523	953	8,194
$N = 2$	14	3,419	1,218	3,100	67,059	8	928	607	871	8,860
$N = 1$	17	3,690	1,590	3,028	75,732	14	1,415	810	1,678	18,781
Core area price = \$1,000/acre										
$N = 15$*	15	2,428	517	1,822	78,224	2	463	187	337	4,071
$N = 10$	16	2,309	423	1,745	79,578	3	504	169	388	5,868
$N = 6$	17	2,474	519	1,862	82,680	3	593	257	484	5,901

* The relative gap between the best solution found and the best possible solution is set to 5% to prevent the model from running out of memory. All other runs used a relative gap of 1%.

deterministic period one decision. Results indicated that only stand 6 should be harvested in period one to maximize the total return from timber and core area. Delaying harvests is preferred in this case without the risk of fires. We compared this solution with the three other period one solutions listed in Table 3 at the same core area price with sample size $N = 1$, 2, or 3.

For this comparison, we reran the SAA model 300 times with i.i.d. fire occurrence samples. In each of the 300 runs, we hardcoded the selected period one solution, simulated one random forest succession pathway across the three 40-year planning periods, and allowed the model to make recourse decisions for periods two and three to adapt to the simulated fires. The objective function value, timber production, and amount of core area were reported and saved for each sample.

We used Tukey's multiple comparison method to compare performance of the four solutions. Results in Table 5 show all period one solutions selected by our SAA models are significantly better than the deterministic solution, producing higher average objective function values. Increasing sample size N in the SAA model also led to higher average objective function values across our 300 independent samples. However, the differences are not statistically significant at the 95% confident level.

Table 5. Multiple comparison of the performance of different period one solutions discovered from the deterministic approach (denoted by DET) and from stochastic programming with sample sizes $N = 1$, 2, or 3 (denoted by N1, N2, and N3).

	Differences in objective function (Obj) values (\$)		
Comparison	Average difference in 300 samples	Lower bound of 95% confidence interval	Upper bound of 95% confidence interval
Obj(N1)–Obj (DET)	23,803	20,526	27,080
Obj(N2)–Obj (DET)	24,995	21,718	28,272
Obj(N3)–Obj (DET)	25,845	22,568	29,122
Obj(N2)–Obj(N1)	1,192	−2,085	4,469
Obj(N3)–Obj(N1)	2,042	−1,235	5,319
Obj(N3)–Obj(N2)	850	−2,427	4,127

The objective function values from implementing different period one solutions (Obj(DET), Obj(N1), Obj(N2), and Obj(N3)) are compared through 300 new random samples. Each new sample represents a random sequence of fire events and their interaction with management activities across three 40-year planning periods. In the stochastic model, we assume \$500/acre core area price, no penalty for declining timber production, and high fire probability. The confidence interval around the average difference in objective function value is calculated. Tests show the solution by using $N = 3$ gives the highest objective function value.

Influence of the Declining Timber Flow Penalty and Fire Probability

We tested two variations of forest management assumptions regarding the penalty on timber declines and core area prices: (1) \$500/acre core area price with a penalty of \$90/cord for timber production declines between two consecutive periods; and (2) \$500/acre core area price and no penalty for timber declines. In these tests, we also assumed the chance of stand-replacing fire is low (as previously defined). We used two sample sizes to build the SAA model: $N = 6$ and $N = 10$. Increasing N from 6 to 10 created a much larger mixed integer programming (MIP) model. To prevent our computer from running out of memory while using the CPLEX solver, we set the maximum computing time for solving each SAA model to 1 hour and we allowed the solver to stop when a feasible integer solution within 5% of the best possible solution was found.

Results show that a \$90/cord penalty effectively prevents timber production from declining between two consecutive planning periods in this test case (Table 6). Applying this penalty lowers the overall harvest level substantially across the planning horizon, especially during the earlier periods. Lower harvesting levels also allow the model to increase the average minimum core area by more than 20% across the 120 years.

Fire probability played an important role in timber and core area production (compare results in Table 4 and Table 6). Under the assumption of \$500/acre core area price without the declining timber penalty, doubling the expected fire probability in each age class causes the model to increase timber harvest levels by 2.4 to 2.8 times during each of the three planning periods. Higher fire probability makes the conservation of forest core area too risky to be justified and decreases the overall objective function values by 29.2% from \$91,400 to \$64,705 ($N = 6$), or 30.2% from \$91,627 to \$63,992 ($N = 10$) in this test case. Changing fire probability assumptions also influences the solution persistence when this model is used to

Table 6. Model performance with and without a penalty for declines in timber production assuming a core area price of $500/acre. This test case also assumes low fire risk in each stand of the forest.

	Average across samples					STD across samples				
		Timber yield					Timber yield			
Sample size	Average min core (acres)	P1	P2 (cords)	P3	Obj. ($)	Average min core (acres)	P1	P2 (cords)	P3	Obj.($)
$90/cord penalty for timber decline										
*N10	57	20	224	1,002	85,473	5	75	145	231	6,958
N6	55	90	318	1,115	85,215	7	186	316	345	10,886
No penalty for timber decline										
N10	45	1,750	648	1,413	91,627	9	617	281	527	7,754
N6	46	1,672	675	1,544	91,400	10	720	347	602	7,643

* Computing time is set to 1 hour to prevent "out of memory" error. The average relative gap is 4.12% between the best solution found and the best possible solution for all 30 runs.

select the period one decision for each stand. The test cases show, when fire risk is assumed to be low, a decision to clearcut will be preferred at period one only in stand 2 (21 out of 30 runs selected clearcut), stand 5 (21 of 30 runs selected clearcut), and stand 6 (all runs selected clearcut) when timber production declining is not penalized. Under the high fire risk assumption, 7 out of the 11 stands are selected for clearcut at period one (Table 3).

Conclusion and Discussion

Stochastic disturbances such as fire, insect, disease, or wind can have a significant impact on long-term forest management. Ignoring the effects of these disturbances in forest plans can lead to suboptimal or incorrect decisions and conclusions. Spatial concerns such as core area, edge effects, adjacency, and patch connectivity pose additional challenges to building disturbance impacts in forest management models. Without spatial considerations, stochastic disturbances might be modeled adequately using mean probabilities of different scenarios implemented as average fractions of land being disturbed. Under a spatial context, using average disturbance rates seems less likely to be adequate for approximating system behavior and optimal management. Even in stochastic programming models it can be difficult to select adequate representative scenarios from the enormous number of possible spatial disturbance patterns and resulting forest spatial structures; sample-based methods might be required, like those used in this study. Spatial forest management models often use binary variables to track and form desired spatial structures such as core area or to prevent undesired spatial conditions such as violations of adjacency constraints or harvest block size restrictions. Both scenario- and sample-based stochastic integer programming approaches can support these binary model formulations.

Harvest decisions for later time periods can often be adjusted through recourse actions, whereas implementation of many first-period decisions must begin immediately. A primary reason for using multistage stochastic programming is to improve the quality of the first stage, or first period, decisions while taking recourse opportunities into account. This research demonstrates that integrating fire risk explicitly in a harvest scheduling model can lead to better first-period decisions than decisions made using deterministic models. Modeling the opportunity to adjust plans through recourse decisions that account for recent fires clearly improved our solutions.

We examined the benefit of using larger sample sizes with a number of test cases. In our models, larger sample sizes improved the persistence of the period-one solutions. Larger sample sizes also reduced the variation in solutions, as indicated by lowering the SD of objective function values across many replicate i.i.d. SAA model runs. However, increasing the sample size also increased model complexity and model size. This study solved relatively small 11-stand and three-period artificial harvest scheduling problems under generous simplifying assumptions such as 40-year planning periods and independent fire events in each stand. These assumptions may not be too unreasonable in forests with long harvest rotations and with dense road networks that support rapid fire response. However, these assumptions do not hold in many other places. We employed these simplifications to allow us to focus on the stochastic model formulation itself. A potential future study might include additional details such as fire spread from stand to stand.

Solving larger forest planning problems with more complicated forest and fire conditions might be possible on computers with more memory by developing more efficient modeling formulations or by employing other solution approaches such as decomposition methods. This model also defines binary variables $W_{ijk(n)_t}$ to add more flexibility in tracking whether a scheduled cut could be implemented before a fire. It would be an interesting future study to compare between modeling detailed fire-management sequences enabled by $W_{ijk(n)_t}$ and using smaller time intervals, e.g., every 10 years.

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