Forest-fire models

Fire Risk Models

Wildfire risk models are concerned with quantifying likelihood and potential impacts of fire events. Information on wildfire risk may be presented in the form of expected annual loss for such outcomes as total area burned; total suppression costs; or impacts to **ecosystem** services. Information may also be presented as the probability of an extreme event occurring in a given location and time. The latter is useful for wildfire suppression planning activities such as the allocation of initial attack resources among geographic regions. Fire occurrence models are important for estimating both expected loss and the likelihood of an extreme fire event. Statistical fire occurrence models assume fire ignitions are realizations of a spatial–temporal point process with (x_i, y_i, t_i) indicating the location (x, y) and time t of the ith fire. Next, space–time is divided into discrete voxels of, for example, 1 km by 1 km by 1 day and a binary random variables is defined by $Y = 1$ or 0 according to whether there has been a fire of a given size or not in the particular voxel. The following model is often used to estimate the probability of fire ignition or large fire occurrence given ignition:

$$\text{logit}\ \ \Pr[Y = 1|x, y, d] = s_1(x, y) + s_2(d)$$

with d being the day-in-year and s_1 and s_2 either parametric functions or nonparametric scatter plot smoothers such as locally weighted scatter plot smoothers or splines (*see* **Splines in nonparametric regression**). The above model is used to estimate long-term/historic fire occurrence probabilities as a function of location and day-in-year [1, 2]. Other explanatory variables, such as fire danger and fire weather indices [3, 4], are added to the above model to produce estimates of fire occurrence probabilities for a given date. These are then used for short-term/real-time forecasts of probabilities of fire risk [5, 6]. The risk of a fire burning a large area (more than H hectares) is estimated by

$$\text{Prob}\,[\text{fire size} > H] = \text{Prob [ignition]} \times \Pr\,[\text{fire size} > H|\text{ignition}]$$

In Figure 1, an estimate of the risk of losing more than 40 ha due to wildland fire is shown for the month of July and the month of October in Southern California. These estimates were obtained from historic fire occurrence and size data for the period 1994–2006. The highest risks are seen to be during July and in regions of the Los Padres and Cleveland National Forests. Some elevated risk levels seem to persist into October in the most southern California forest.

A second component of fire risk is the distribution of fire sizes. Distributions such as the Pareto, log-Pareto, and the extreme value functions are some of the models that have been used to fit large fire sizes [7–9]. For the period and location in the Southern California example above, the fire size distributions for fires greater than 40 ha observed during the months of July and October indicate a heavier tail distribution needed for the month of October (Figure 2).

Apparently, even though a greater number of large fires is expected in July (due to larger occurrence probabilities) the percentage of fires that will eventually burn over 1000 ha is larger during October (30%) than during July (16%). Hence the loss as measured by total area burned requires the estimation of both fire occurrence probabilities and fire size distributions with associated standard errors.

On a smaller spatial scale, managers may be concerned with the risk of losing ecosystem services like wildlife habitat, carbon, or clean water, due to wildfires. These risk calculations require estimates of burn probability and intensity. For small fires (e.g., 0.1–5 ha), burn probabilities are roughly equivalent to fire occurrence probabilities. However, for large fires, maps of fire occurrence are less informative because the fires spread over long distances and affect the burn probability over large areas. Large fire occurrence is generally a rare event and the historic fire records have been insufficient to estimate burn probabilities at a scale meaningful to managers concerned with wildfire risk. Consequently, a combination of large fire occurrence probabilities and fire spread models (see next section) are often used to estimate and map burn probabilities on large landscapes. One

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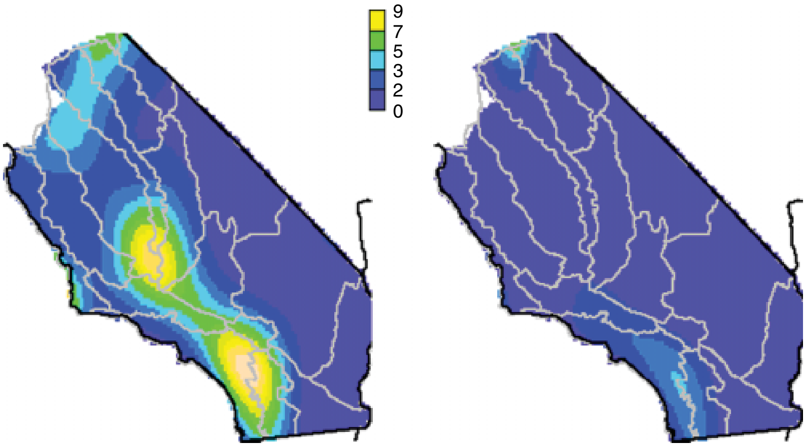

Figure 1 Risk (probability per 10 000) of a large fire occurring during the month of July (left panel) and October (right panel) estimated from historic wildland fire occurrence data in Southern California. Data provided by Thomas Rolinski, USFS Predictive Services.

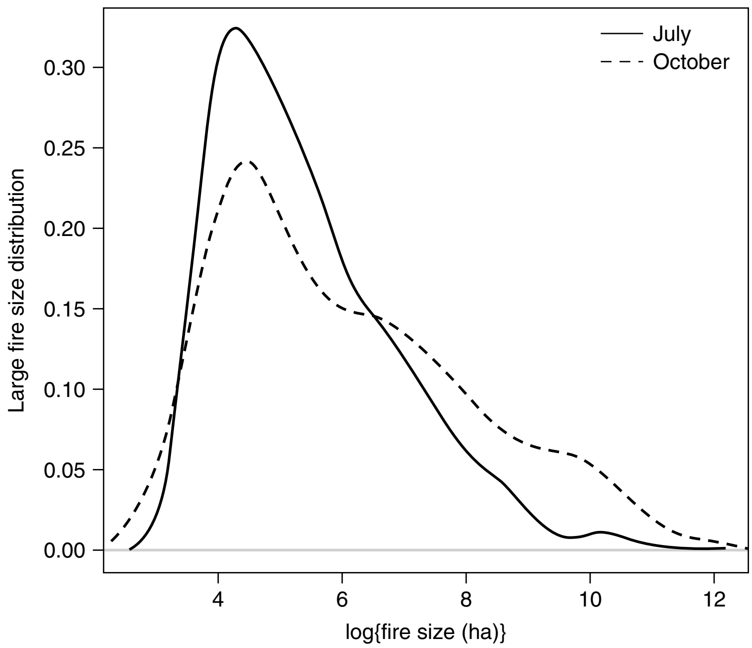

Figure 2 Empirical distribution of large fires in Southern California for the months of July and October obtained from historic fire occurrence data during 1994–2006. Data provided by Thomas Rolinski, USFS Predictive Services.

approach is to estimate the probability of a large fire ignition on a given day using historical relationships between weather and large fire occurrence. Monte Carlo methods (*see* **Simulation and Monte Carlo methods**) are then used to simulate the ignitions over the fire season, and each ignition is modeled using local information on fuel, weather, and topography [10, 11]. Fire containment can also be simulated as a function of weather during the fire [12]. By simulating many fires one can estimate a probability

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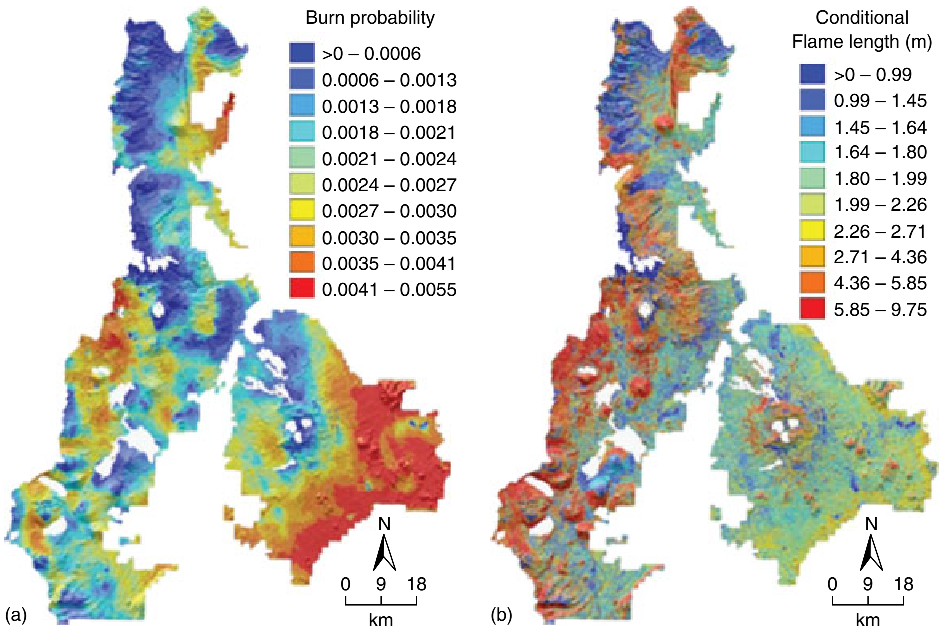

Figure 3 Map of burn probability (A), conditional flame length (B), for the 650 000-ha Deschutes National Forest in Oregon, United States. Burn probability is the chance of a pixel burning simulated given a random ignition and average weather conditions in the study area. Conditional flame length is the average flame length observed on a given pixel for all simulated fires. Simulations consisted of 50 000 wildfires that replicated historical fires within the area. Simulations performed at 90-m resolution with Randig, a command line version of FlamMap [17]. Figure derived from Ager *et al.* 2011. Courtesy of Hindawi Publishing Corporation.

that each point or pixel on the landscape encounters a fire. Fire intensity can also be obtained from the simulations, along with maps of fire sizes and other outputs (Figure 3). These maps are useful for wildfire risk assessments and fuel management planning [13, 14, 15] (Figure 4a). Fire progression maps from simulations and actual fires have also been used in generalized regression or **generalized additive models** [16] to explore causative factors.

Fire Spread Models

A wide range of approaches have been taken to model fire spread [19–21]. At one end of the spectrum are the detailed physical-based models that incorporate the combustion and emission, at the other end are empirical regression models estimating rate of spread of a fire [21]. One fire spread approach, sometimes referred to as *the forest-fire model*, assumes that the forest fuel bed can be tessellated by a regular grid with each cell having a probability of burning that depends on conditions in the cell and in surrounding cells [22]. A second approach assumes that each point on the fire front at a given time is an ignition point of a small fire that burns an elliptical region in the next increment of time [23]. However, the vast majority of fire spread models in use by practitioners and researchers employ Rothermel's one-dimensional empirical fire behavior models [24–29]. Fire simulators like Behave [30] link the Rothermel spread model for predicting surface and crown fire rates of spread, with VanWagner's [3, 26] or Scott's [31] crown fire transition and propagation models, and provide outputs of several fire behavior characteristics (e.g., rate of fire spread, fireline intensity, crown fire activity). Algorithms for simulating two-dimensional fire spread based on Huygen's principle of propagating waves [32] include FlamMap [17], FarSite [33], and the Canadian wildland simulation model Prometheus [34]. Fire growth models like FarSite and Prometheus simulate mechanistic fire spread as a vector wave front. FlamMap, FSIM and FSPRO [11, 35] use a minimum travel time

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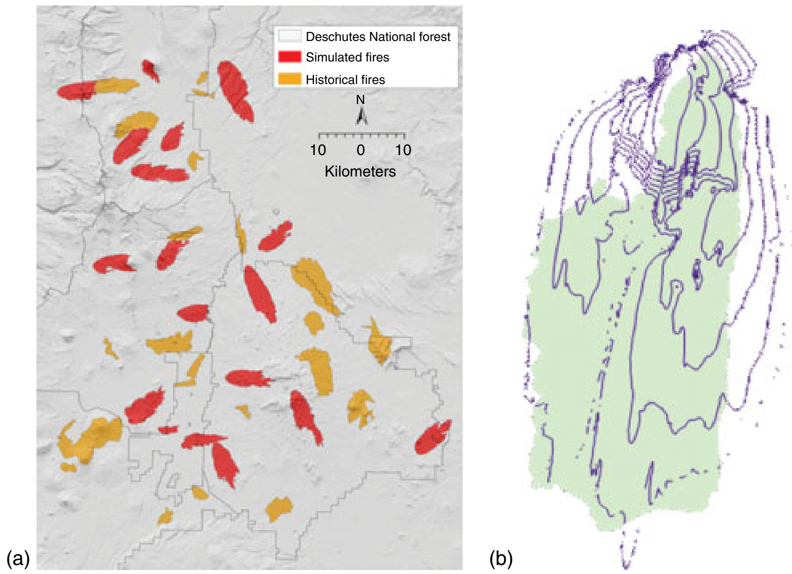

Figure 4 (a) Comparison of historical and modeled fire perimeters for the 650 000 ha Deschutes National Forest (gray outline) in Oregon, United States. Historical perimeters are shown for fires greater than 500 ha. Modeled fire perimeters are a sample from 50 000 simulations started at random ignition points. Simulation parameters for wind speed and direction were sampled from distributions based on historic wind patterns in the area. Figure derived from Ager *et al*. 2011. Courtesy of Hindawi Publishing Corporation. (b). Comparison of observed region burned (green) and simulated contours of fire spread for ca 2 h of fire growth. Simulations were based on a MTT fire spread algorithm in the FlamMap. Fire suppression activities on the backing and flanking portions of the fire led to overestimation of burned area by the simulation. FlamMap does not model fire containment. Input data for simulations provided by Nicole Vaillant.

(MTT) approach that offers a dramatic reduction in computation time [36]. Extensive application has demonstrated that the MTT algorithm is a reasonable predictor of fire spread and the replication of large fire boundaries on heterogeneous landscape (Figure 4, [11]). The MTT algorithm has made it feasible to run large numbers of simulations for very large landscapes and hence estimate burn probabilities (see above) over large areas (>2 million ha) in a matter of hours [14]. The MTT algorithm is also widely applied for strategic and tactical wildfire management planning operational wildfire problems throughout the United States [37, 38] and has been integrated into several aspects of fire planning on US federal lands (see [11, 37, 39]). In-depth discussions of these models and their limitations can be found in several recent papers [18, 40–44].

Fire Effects Models

Fire effects models are also important in the context of fuel management planning, and are used to examine the potential impacts of modeled fuel management activities and fire in terms of tree mortality, carbon, soil, and other ecosystem services [45–49]. Some important fire effects include tree injury and mortality, smoke production, and soil heating. Other, longer-term effects of interest to forest managers include fuel dynamics, erosion, air and water quality, and insect outbreaks. Fire effects models may be empirical, rule-based, or process-based [48]. Examples of process-based models include BlueSky [50] for predicting the smoke dispersion, FOFEM [51] for predicting soil heating underneath fires and ERMit [52] for predicting post-fire soil erosion.

Empirical and mechanistic models are also used to study fire effects. These are mostly regression models including, for example, **logistic regressions** for predicting tree mortality in postfire stands [53]. Mechanistic statistical models are based on physical principles driving model behavior. In these models fundamental principles are used to set down equations with physically meaningful parameters. Next

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a statistical model is employed to estimate model parameters. For example, the Fourier heat equation

$$\frac{\partial T(t,x)}{\partial t} = \frac{k}{c\rho}\frac{\partial^2 T(t,x)}{\partial x^2}$$

is used to study soil-temperature levels beneath fires. In the above differential equation, $T(t, x)$ is the soil temperature at depth x and time t, k is the thermal conductivity, c is the specific heat capacity, and ρ is the soil bulk density. Nonlinear regression models are employed to fit the solutions of the differential equation [54]. Another example is the use of Philip's equation for water infiltration [55], $I = St0.5 + \mathrm{A}t$, to study soil water repellency properties after a fire, where in the equation above I stands for infiltration, S is sorptivity, and A corresponds to the gravitational force [56].

In summary, forest-fire models have been developed for a wide range of management and research applications, and some specific models have become important components in decision support systems for tactical and strategic planning problems. Much of the current research concerns new models that represent the physics of combustion as part of fire spread models to improve upon existing methods. Fire models have become important tools for a range of purposes that contribute to reducing catastrophic ecological and human loss from wildfire events.

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(*See also* **Forestry**; **Wildfire risk analysis**; **Forest ecology**; **Forest carbon cycling**; **Point processes, temporal**; **Point processes, spatial–temporal**; **Point processes, spatial**; **Tessellations**; **Splines in nonparametric regression**; **Generalized extreme value distribution**; **Risk assessment, probabilistic**)

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