
Allocating Resources to Large Wildland Fires: A Model with Stochastic Production Rates

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ABSTRACT. Wildland fires that grow out of the initial attack phase are responsible for most of the damage and burned area. We model the allocation of fire suppression resources (ground crews, engines, bulldozers, and airdrops) to these large fires. The fireline at a given future time is partitioned into homogeneous segments on the basis of fuel type, available resources, risk, and other factors. Each is assigned a utility value corresponding to the importance of holding the segment. For a given resource allocation, the probability of holding the segment is modeled in terms of the (random) width of fireline built. The task is then to find the allocation that maximizes the expected total utility. With certain restrictions, it proves possible to formulate the optimization as a linear programming problem.

Use of the model is demonstrated with a case study of a large fire representative of conditions on the Los Padres National Forest in southern California. One feature is that different assumptions about the *uncertainty* in the predictions of constructed fireline widths can lead to differences in the optimal resource allocations. Thus, if one inappropriately took the uncertainty to be zero (the deterministic case), the resulting allocation may well not be the optimal one. This illustrates the potential advantage of probabilistic modeling over the previous deterministic approach. *FOR. SCI.* 38(4):842-853

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A NUMBER OF COMPUTER-BASED MODELS HAVE BEEN DEVELOPED to simulate initial attack operations against wildland fires of limited size (Green et al. 1983, Mees 1985). These models generally assume an elliptical shape for the fire boundary, with the highest spread rate along the major axis. One assumption common to all such models is that of uniform and time-invariant local conditions along the fire boundary; another is that the width of fireline constructible by given firefighting resources is known deterministically. The models are used as a component of large-scale economic models designed to evaluate the effectiveness of present and future fire budgets and resource allocations, on both a forestwide and a regional basis (Bratten 1981, U.S.D.A. 1983).

Although much fewer numerically, the fires that have grown beyond the initial attack phase are of great importance because they account for most of the damage and burned area (Strauss et al. 1989). Despite this fact, little work on modeling the attack on such fires has been done. One reason is the complexity of the task; for example, spatial variations in fireline conditions must now be taken into account. Our optimization modeling for these large fires is related to earlier work (Bratten 1970). That approach differs from ours in various ways, however, in-

cluding (a) the deterministic modeling of the constructed fireline width and (b) the need for ad hoc tables giving the chances that various fireline widths will hold against the fire. We will see that the deterministic approach, though simpler, can lead to suboptimal allocation of resources.

In broad outline, our procedure is as follows:

1. For a specified time-point in the future, a perimeter for possible fire containment is mapped out. The fire management officer, or other user, partitions this into a number of homogeneous segments of the fireline, on the basis of similarity (on the basis of fire behavior or of importance of holding the fire), or because of restrictions on feasible attack possibilities. The user assigns each segment a subjective, relative utility value, indicating its importance (or, equivalently, the penalty associated with failure to hold the segment).
2. The resources allocated consist of a specified number of (indivisible) units of ground crews, bulldozers, engines, and areal retardant drops. We define a strategy to be an allocation of a unit to just one segment. There are many constraints on the class of possible strategies. An obvious example is that a segment cannot be assigned an engine when there is no access for such a unit.
3. We developed a model specifying the probability that a given allocation to a given segment results in a constructed fireline that holds against the fire. The chance of holding depends on factors such as flame length, and also on the width of constructed fireline. The latter is taken to be probabilistic; specifically, it is modeled as a gamma-distributed random variable, with mean depending on the quantity of resources allocated (as in a deterministic model) but with an empirically determined variance.
4. The expected total utility for a strategy is the sum over segments of the product of utilities and holding probabilities. An optimal strategy is one that maximizes this.
5. The number of possible strategies may be too large for direct enumeration. By setting up dummy variables for whether a particular collection of units is applied to a particular segment, however, and by carefully eliminating many unsuitable possibilities, it proves feasible to formulate the optimization as an integer linear program.
6. The output of the procedure is the optimal allocation of resources by segment, together with the achieved utility values (total and by segment). The process can then be repeated with different assumptions or parameters to see how the strategy and utilities change. It can also be helpful to repeat with different target times. The earliest feasible control line location may prove not to be the best.

The whole procedure, which is available for PC users in fairly friendly form, is in experimental use with forestry personnel in the Fremont, Ochoco, and Los Padres National Forests. The necessary input for the model is at present too demanding for the method to be practicable for resource allocation in large, ongoing fires. It can be used as a training tool, however, in the study of previous large fires, and ultimately as a component in the economic evaluation of large fires. This paper describes the method and theory in detail and describes an application with a representative large fire from the Los Padres National Forest. The fire to be simulated is located in an area that fire personnel are familiar with, having encountered large fires under similar conditions and with similar suppression capability.

THEORY AND METHOD

In this section we describe the method in more detail, with the necessary definitions and theory. To simplify the exposition a number of technical complications, which may not be of interest to the general reader, are relegated to footnotes.

Suppose that a fire is reported during the morning hours and has resisted initial

attack. The user selects one or more target times, such as 3 PM, and related fireline location for possible containment of the fire. That is, the user locates on the map potential control lines that the fire will not have reached before 3 PM, and divides the perimeter into segments. An example of how this might look in practice will be given in the next section. Let the utility of holding the i th fireline segment be U_i , specified by the user, and let the probability of its holding (or containment) be H_i . The H_i of course, depends on the particular resource allocation; our modeling for this will be explained shortly.

The optimal strategy is that which maximizes the expected total utility

$$ETU = \sum U_i H_i \quad (1)$$

The process might be repeated for a different target time, say 6 PM, with a different set of segments, utilities, and holding probabilities. By comparing the maximized $ETUs$, the relative merits of the two target times may be informally compared.

Next, the user determines the number of available units within each of the four categories of ground crews, bulldozers, engines, and air retardant drops. In each category the units are assumed (1) to be indivisible, (2) to contribute an equal amount of time on the fireline, and (3) to work at equal average rates under similar conditions.¹ With these restrictions, a strategy is equivalent to a partition of the n_k units of category k ($k = 1, 2, 3, 4$) into nonnegative integers n_{ik} , with $\sum n_{ik} \leq n_k$, where i indexes the segments; that is, a strategy is a specification of how many units of each category will be allocated to each segment.

A major issue is the modeling of the holding probabilities H_i to which we now turn. The following two aspects must be considered:

1. The probability density $f(x)$ of the random variable x , the width of the fireline that can be constructed at a given segment with given resources;
2. The conditional probability $p_i(x)$ that segment i holds, given that a width x is constructed. Then the required (marginal) holding probability for segment i is given by

$$H_i = \int_0^{\infty} f(x) p_i(x) dx \quad (2)$$

To model (1), the width of the constructed fireline segment, we actually need to work with the area of land cleared by the resource units. The constructed fireline may be taken to be rectangular, so that x is simply the cleared area divided by the (fixed) length of the segment; thus the probability distribution for x is simply determined from that for the area.² The expected value of the area cleared is computed separately for each of the four categories (ground crews, bulldozers, etc.) and summed.³ The standard deviation of the area cleared is given by

$$SD(\text{area cleared}) = \sqrt{\sum \sigma_k^2} \quad (3)$$

¹ Actually, the four categories are further divided into resource pools by the user of the model; for simplicity we will limit our discussions to the four basic categories. The units within each pool must again be indivisible and work at the same rate and for the same length of time.

² The variable area cleared by a resource unit is converted into a variable width x given a fixed length for each segment. Wildland fire personnel work with variable length produced given an assumed fixed width. For computational ease, we used additive variable widths applied to the entire segment length.

³ For all resources, the fuel type for the segment is classified into one of the four resistance to control classes (light fuels, such as grass; medium brush, such as light to medium chamise; heavy brush, such as medium oak; heaviest brush, such as dense chamise) given by Lindquist (1970). In each category, Lindquist tabulates the average production rate (square yards cleared per hour) for different

where σ_k is the standard deviation of the area cleared by category k ($k = 1, 2, 3, 4$).⁴ By taking the SD in (3) to be zero (rather unrealistically), we arrive at the conventional deterministic modeling of production rates, and this often provides a useful comparison. An example of how the optimal strategy may depend on assumptions about the SD will be given later.

An appropriate model for the probability distribution of the width x is the gamma density (Hogg and Craig 1978, p. 103):

$$f(x) = e^{-x/\beta} x^{\alpha-1} / \beta^\alpha \Gamma(\alpha) \quad (4)$$

where α , β are parameters, Γ is the gamma function, and the width x is always positive. The mean and variance are $\alpha\beta$ and $\alpha\beta^2$, respectively. By equating these to the mean width and associated SD for each combination of resources against a given segment, and one can solve for the required values of α and β . This completes step (a).

Step (b), the holding probability $p_i(x)$ given a constructed width x , is more difficult to determine empirically since data is hard to come by. Our current choice is as follows:

1. Select a lower bound a , say, for the holding probability when the width x is small; a is often, though not always, taken to be 0.
2. Select an upper bound c , say, to the holding probability for large width x .
3. Determine a lower bound m , say, to the widths that give any appreciable benefit; that is, the holding probability stays at a for all $x \leq m$.
4. It is reasonable to suppose that $p_i(x)$ approaches its upper bound c asymptotically as x becomes large. The simplest formulation consistent with these assumptions is

$$p_i(x) = \begin{cases} a, & \text{if } x \leq m \\ a + (c - a)(1 - \exp\{-(x - m)/b\}), & \text{if } x > m \end{cases} \quad (5)$$

The scale parameter b governs the rate at which the holding probability changes with x . The lower and upper bounds for the probability of holding the segment are dependent on the segment flame length.^{5,6} The fire management officer (FMO)

numbers of total work hours, based on observation of actual work crews. Not surprisingly, the mean hourly production rate tends to decrease as the total work time increases. In our procedure, we computed the total areas cleared for each number of total hours worked and fitted a quadratic regression curve for each of the four cases. We used the fitted curves to compute an expected cleared area for a given number of crew-hours' work.

⁴ The standard deviations of the areas cleared are difficult to determine empirically. We think it advisable, when using the model, to repeat with several different assumptions about the standard deviations, this gives an idea of how sensitive the results are to the assumed uncertainty of areas cleared. According to the variability figures implicit in Lindquist's tables, it seems reasonable for ground crews to take the standard deviation of area cleared to be a constant multiple of the mean (the variability thus being proportional to the expected amount of work). The figure of 40% appears to be acceptable as a rough overall multiplying factor. Mean bulldozer production rates are a function of size, slope, fuel, and other variables. In general, existing bulldozer production rates suggest a standard deviation equals 20% of the mean (Phillips et al. 1988). Little is known about the variation in areal retardant drop effectiveness; a standard deviation equaling 40% of the mean was suggested.

⁵ The flame length, FL , for a given segment is used to define a and c as follows: $a = 0.0$ for $FL > 2$ ft, $a = 0.025$ for $1.5 \text{ ft} < FL \leq 2$ ft, $a = 0.05$ for $1 \text{ ft} < FL \leq 1.5$ ft, $a = 0.1$ for $FL \leq 1$ ft, and $c = 1$ for $FL \leq 8$ ft, $c = 0.9$ for $8 \text{ ft} < FL \leq 12$ ft, $c = 0.8$ for $12 \text{ ft} < FL \leq 16$ ft, $c = 0.7$ for $FL > 16$ ft.

⁶ Following suggestions made by field personnel, we set the parameter m equal to 0.6 of the target width for each segment. The probability of holding, $p_i(x)$ is then set at 0.7 for x equal to 0.8 of the width to be cleared for each segment. The parameter b is then solved using Equation (5).

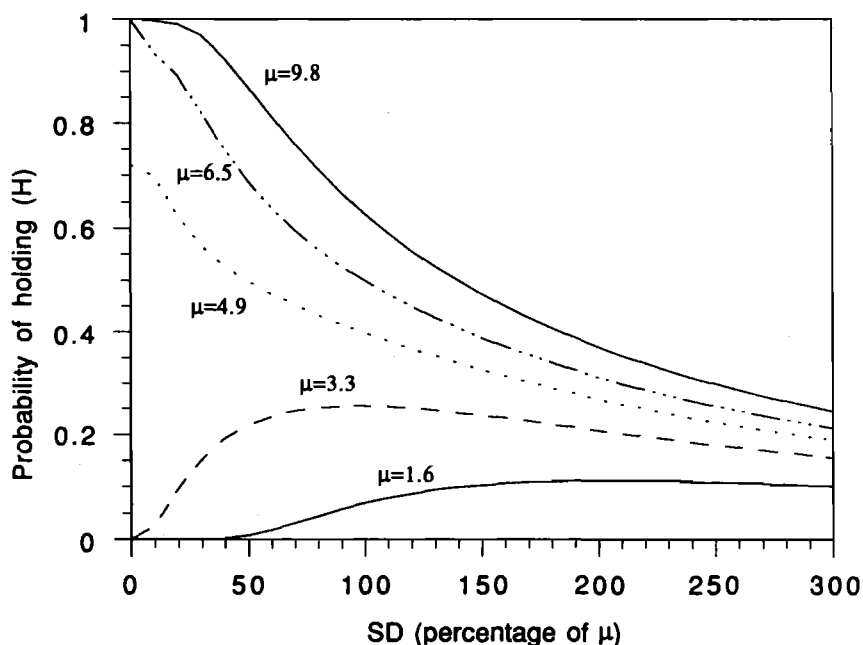


FIGURE 1. The curves are plots of H (the probability of holding a fire line segment against the SD (expressed as a percentage of μ), for various values of the expected widths μ . The target width is 6 ft, $m = 3.6$ ft, $a = 0$, $c = 1$, and $b = 1$.

selects a desired fireline width for each segment, the target width, which is also flame-length dependent and may hold the advancing fire. The parameter m is selected as a fraction of the target width. The user of the model has the option of resetting the parameters a , c , m , and b based on the expected flamelengths and needed target widths for each segment.

The integral (2), which determines the overall holding probability, is straightforward when (4) and (5) are assumed. The result is:

$$H_i = c - (c - a)G_\alpha(m/\beta) - (c - a)e^{m/b}[1 - G_\alpha(m\{1/b + 1/\beta\})]/(\beta/b + 1)^\alpha \quad (6)$$

where G_α is the incomplete gamma function (Shea 1988):

$$G_\alpha(t) = \int_0^t e^{-y} y^{\alpha-1} dy / \int_0^\infty e^{-y} y^{\alpha-1} dy.$$

Some illustration of how our modeling works may be helpful at this point. Figure 1 shows how the probability H_i of holding a segment [derived from Equation (6)] varies with the modeled standard deviation of width build. The SD is expressed as a fraction of the mean, as discussed above. For this segment,⁷ with an assumed SD of 0.0, a mean of 9.8 feet results in a holding probability $H_i = 1$, while

⁷ The target is taken to be 6 ft. The parameter m is then 0.6×6 ft; the parameters a , c are taken to be 0 and 1; b is determined as specified above, by setting $p_i(x) = 0.7$ when $x = 4.8$ ft ($= 0.8 \times 6$ ft).

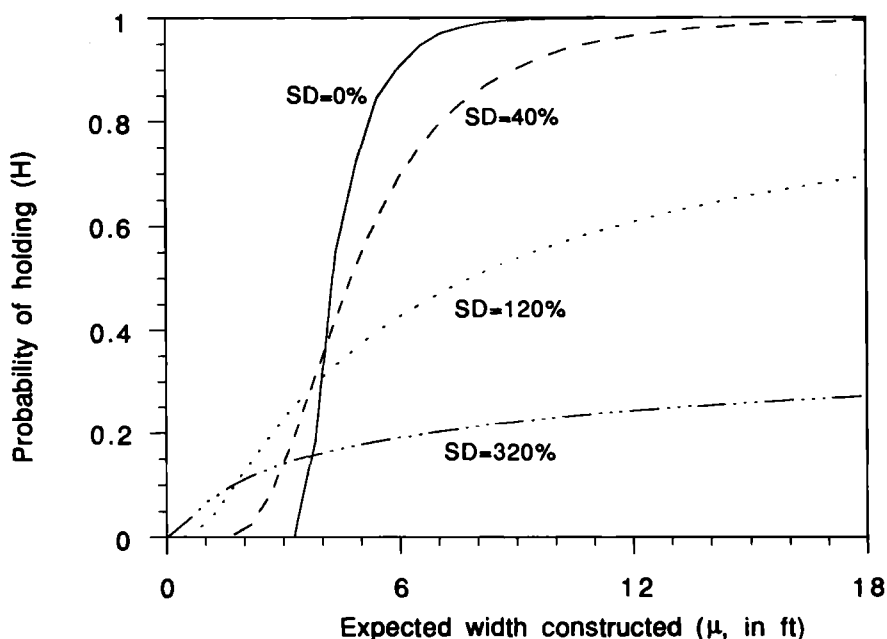


FIGURE 2. H plotted against μ , for selected SD s (percentage of μ). Again, the target width was 6 ft, $m = 3.6$ ft, $a = 0$, $c = 1$, and $b = 1$.

anything less than 3.3 ft would give $H_i = 0$. For large expected widths, increases in the SD naturally result in a decrease in holding probability; for small expected widths, an increase in SD may actually increase H_i slightly by offering a "ray of hope" that sufficient width may be built.

Figure 2 plots H_i against μ for various values of the SD , the latter again expressed as a fraction of μ . For the gamma distribution of our model it is, of course, quite possible for the SD to exceed the mean, and such large uncertainty value is, according to field personnel, quite realistic in some cases. Note in the figure that when the SD does exceed the mean, even a very large μ (representing a heavy investment of resources) results in a holding probability well short of 100%.

OPTIMIZATION

There remains the issue of how the optimization (over all possible allocation strategies) is to be performed. There are usually too many possible strategies to permit a direct search, and without some modification the problem does not appear to fall into any of the standard tractable classes.⁸ At some cost to the size

⁸ For example, a natural method of attack is to create dummy variables for allocation of a given resource to a segment and optimize over these. This proves to be unhelpful, however, as the problem is then both discrete and nonlinear. The nonlinearity is clear when one notices that the utility gained by applying resource unit j to segment i depends on what other units are applied to that segment. It may be of interest to note that the problem would be an integer linear program in this formulation if

of the task that can be handled, the problem can be cast as one of integer linear programming as follows.

Let $j = 1, 2, \dots$ index the feasible combinations of resources that could be applied to a one segment. For example, $j = 1$ might be "no resources used at all," $j = 2$ might be "one ground crew unit, two airdrop units, and one bulldozer," and so on. If we allocate combination j to segment i , we know the expected benefit c_{ij} ; it is given by $U_i H_{ij}$, where H_{ij} is the probability of holding the segment i given the combination j of resources. Now define the variables z_{ij} for the optimization by

$$\begin{aligned} z_{ij} &= 1 \text{ if we assign combination } j \text{ to segment } i, \\ &= 0 \text{ otherwise.} \end{aligned}$$

Then we need to maximize the objective function

$$\sum c_{ij} z_{ij}. \quad (7)$$

This is subject to the constraints

$$\sum_j z_{ij} = 1 \text{ for each segment } i, \quad (8)$$

and

$$\sum_{ij} A_{jk} z_{ij} \leq n_k \text{ for each category } k. \quad (9)$$

where A_{jk} is the number of units from resource category k in combination j . Constraint (8) says that every segment must be assigned exactly one resource combination, and (9) limits the number of resources used from each category.

This is an integer linear programming problem (STORM 1989, Chapter 5) for the variables z_{ij} . The method requires that for each segment i , a list of all feasible combinations be set up in advance, together with tables of computed A_{jk} 's and c_{ij} 's. In a typical problem solved on a PC, the number of potential combinations for any segment should not exceed about 100 if the linear programming is to be manageable. In the examples we have considered so far, it has been easy enough to limit the numbers of combinations of any section to 100, since many combinations are readily excluded.

The number of acceptable resource combinations against a given segment is limited by both user input and model constraints. The user is aware of fireline conditions and can limit both the number and kind of resources which are to be used against a segment. In addition, the user has the option of eliminating all resource combinations that produce an expected width outside a specified range width. For example, having a 10-ft target width to hold an approaching fire front with a 6-ft flame length, the user may select a minimum width of 8 ft and a maximum of 15 ft. The model also limits certain resource combinations, such as the ratios of bulldozers to crews and airdrops to all ground units.

The model consists of three programs to be run in the following sequence:

(1) $p_i(x)$ were strictly proportional to x and (2) the standard deviations of width constructed were all zero—the deterministic case.

1. A preprocessor that generates all feasible resource combinations against each segment and prepares the objective function and constraints.
2. An integer linear programming code generates the optimum solution.
3. A program to convert the output from step (2) into the output familiar to the user.

AN EXAMPLE

The Los Padres National Forest in southern California has been the site of many devastating large fires. Fire management personnel provided data for a typical July fire on the Ojai Ranger District, with a 1 PM start, temperatures in the 85–90° range, humidity 20–25%, and southwest winds of 8 mi/h. The area where the fire occurs is one that fire personnel are thoroughly familiar with.

The fire is assumed to escape initial attack, and two firelines for possible containment (Figure 3), corresponding to 3 PM and 6 PM, are selected by the FMO. As Figure 3 indicates, the two firelines share some common perimeter. We consider the 3 PM fireline first. The FMO decided on the seven-segment partition shown in the figure. The corresponding utility values are given in the second column of Table 1. The FMO also specified (1) the fire definition, including expected flame length for the segments and (2) for each resource category, the number of units and available work time before 3 PM. In our case the available units were 3 ground crews; 10 engines; 4 bulldozers; and 24 aerial retardant

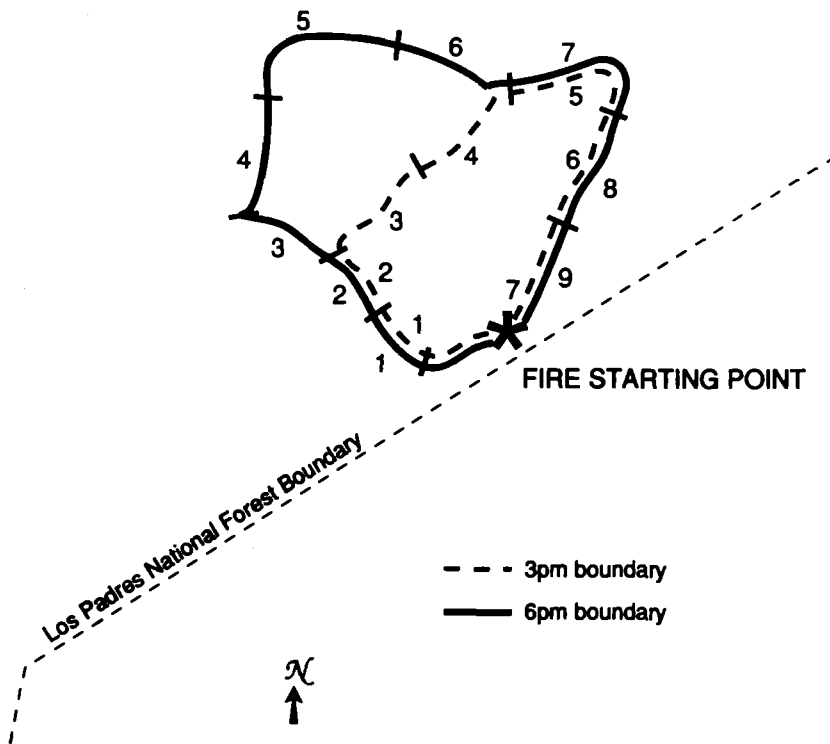


FIGURE 3. The projected 3 PM (7 segments) and 6 PM (9 segments) firelines for the Los Padres National Forest example. The scale is 1:24000. The lengths of the firelines are approximately 3.25 and 4.38 mi long.

TABLE 1.

The 3 PM fireline consists of seven segments.^aLos Padres National Forest
3 PM fireline

(1) Segment	(2) Assigned utility	(3) Expected utility for $SD = (0,0)$	(4) Expected utility for $SD = (0.4,0.2)$	(5) Expected utility for $SD = (1.0,1.0)$	(6) Expected utility for user allocation $SD = (0.4,0.2)$
1	9.0	9.0	8.9	6.3	5.4
2	5.0	5.0	4.5	2.9	2.8
3	3.0	0.0	0.0	0.0	0.0
4	4.0	4.0	3.9	2.7	0.4
5	10.0	8.5	6.4	3.8	0.4
6	6.0	0.0	0.0	0.0	0.0
7	8.0	8.0	7.2	4.9	8.0
Totals	45.0	34.5	30.9	20.6	17.0
Resources used		33	33	36	29

^a Listed are the assigned utilities, the expected utilities for $SD = (0,0)$, $SD = (0.4,0.2)$, $SD = (1.0,1.0)$, and the user allocation. The total number of resources is also given.

drops. We will subsequently denote such a set of resources by a string: (3,10,4,24).

We considered three choices for the uncertainty (as measured by the standard deviations) of production. One choice (as suggested from the production data itself) was: $SD = 40\%$ of mean for crews, engines, and airdrops; $SD = 20\%$ of mean for bulldozers. We denote this choice by (0.4,0.2). Our second choice was the deterministic model (0,0). The third choice, (1.0,1.0), is a plausible upper bound to the uncertainty in fireline production.

The optimal strategies for these three choices result in the expected utilities shown in Table 1, columns 3–5. The entries are the product of the assigned utility values U_i and the holding probabilities H_i ; they are the output of our integer linear program. Note that as the putative standard deviations increase, the optimized ETU decrease: from 33 (out of a possible 45) in the deterministic case to only 20.6 in the high-uncertainty case. The total suppression resources used in each case is shown in the bottom row of the table. Note that the complete set of available units (totaling 41) is never used. This results from the severe constraints on the acceptable combinations. For example, in the deterministic case, all resources were used except for eight airdrops; it turned out that these could not be used on other segments because of a lack of the necessary ground crews.

The difference in optimal allocation by segment for different choices of SD is an important feature of the optimization process. The allocations for the deterministic case, $SD = (0,0)$, and the extreme uncertainty case, $SD = (1,1)$ are shown in Table 2. The total allocation consisting of 33 units for all segments when $SD = (0,0)$ is (3,9,4,17) and for the 36 units when $SD = (1,1)$ it is (3,10,3,20). The total number of resources used for the 6 PM fireline does not vary substantially with SD

TABLE 2.

The 3 PM fireline allocations by segment for $SD = (0,0)$ and $SD = (1.0,1.0)$.^a

Los Padres National Forest
3 PM fireline

Segment	Allocations for $SD = (0,0)$	Allocations for $SD = (1.0,1.0)$
1	(0,1,2,1) ^a	(1,2,1,5)
2	(0,4,0,5)	(3,0,0,5)
3	(0,0,0,0)	(0,0,0,0)
4	(0,2,2,3)	(2,0,2,2)
5	(2,2,0,4)	(1,3,0,4)
6	(0,0,0,0)	(0,0,0,0)
7	(1,0,0,4)	(1,0,0,4)

^a Integers indicate the number of ground crews, engines, bulldozers, and aerial retardant drops.

because of the limitations on the resource combinations. Again, although the total number of resources does not vary with SD , the segments receive varying combinations with different SD s.

For comparison with our optimal results, the FMO was asked to give his preferred resource allocation. The expected utilities corresponding to his choice, calculated on the basis of $SD = (0.4,0.2)$, are shown in the last column of Table 1. His ETU (total expected utility) is only 12.9 out of a possible 45. The FMO had assigned too few resources to most segments, concentrating unduly on segment 7. This may reflect a lack of understanding of the influence of the SD s on the results, as well as a difficulty in dealing with segmented firelines as required by the model.

The 6 PM fireline has nine segments and an additional number of resources are available to it. This line is to be constructed under reduced temperatures, partial shading, and increased relative humidity. The estimated flame lengths and corresponding target widths required for each of the nine segments are therefore reduced. Whatever SD used, the achieved ETUs, shown in Table 3, are proportionally higher than for the 3 PM fireline (Table 1). This reflects the combined effects of lesser target widths and additional resources.

On comparing the two sets of results, the FMO opined that the 6 PM fire perimeter was to be preferred. There are a number of unused resources available for the 6 PM fireline, and the utility values can be further increased by modifying the bounds on the required target widths and modifying some of the constraints to allow additional allocations into the solution.

DISCUSSION

We have developed a method to incorporate uncertainty in fireline width produced into a probability of holding and expected value for a fireline segment. Fire agencies and current wildland fire research use deterministic models which are limited to small fires with uniform and time-invariant conditions along the fireline. The method described here provides a way to treat both large and small fires in terms of holding probabilities and expected values.

TABLE 3.

The 6 PM fireline consists of nine segments.^aLos Padres National Forest
6 PM fireline

(1) Segment	(2) Assigned utility	(3) Expected utility for $SD = (0,0)$	(4) Expected utility for $SD = (0.4,0.2)$	(5) Expected utility for $SD = (1.0,1.0)$	(6) Expected utility for user allocation $SD = (0.4,0.2)$
1	9.0	8.7	8.4	5.8	5.9
2	5.0	4.9	4.7	3.3	3.4
3	3.0	2.9	2.5	1.6	3.0
4	4.0	3.7	2.8	1.9	0.8
5	5.0	4.4	4.2	2.4	0.7
6	9.0	8.8	8.5	5.8	2.1
7	10.0	9.8	8.3	5.5	10.0
8	6.0	5.3	4.5	2.7	0.2
9	8.0	7.8	6.6	4.4	8.0
Totals	59.0	56.3	50.5	33.5	34.1
Resources used		52	50	51	64

^a Listed are the assigned utilities, the expected utilities for $SD = (0,0)$, $SD = (0.4,0.2)$, $SD = (1.0,1.0)$, and the user allocation. The total number of resources is also given.

Perhaps the most striking feature of the above analysis is the influence of uncertainty in production rates on the optimal resource allocation and its corresponding utility. This illustrates the potential advantage of probabilistic modeling over the previous deterministic approach.

A positive feature of our approach is the involvement of the user in formulation of the problem, and the insights gained from running the model with different target widths, standard deviations, and other parameters. Another attractive feature is the treatment of simultaneous fires drawing resources from the same resource categories. These can easily be handled in our framework; the only additional requirement is that the individual fires must be segmented with relative assigned utility values.

The production rates used in this model apply to limited geographic areas, in this case southern California. The corresponding probability of holding $p_i(x)$ has four parameters (a, c, m , and b) which must be applicable to the same geographic area. Wildland fire personnel responsible for USDA Forest Service large fires are required to estimate the holding probabilities (H_i) for all segments during extended attack operations. Estimates for these probabilities vary from person to person and are based on individual experience, with no uniform guidelines or procedures for their development. Agreement on production rates and the parameters (a, c, m , and b) by geographic area would provide consistency in computed estimates for the holding probabilities. The calculated holding probabilities provided by this model can then be made part of comprehensive economic wildland fire models.

We should note that our model is concerned with the optimal allocation of a

given set of resources. A quite different question is that of how many resource units to use on the fire, taking account of resource costs and the losses associated with failure. In the latter context, wildland fire managers may (and often do) behave risk aversely, deploying extensive resources in order to achieve near certainty of success. This should not be confused with the strategy of overallocating to one segment at the expense of the others. If the result is a relative small improvement in containment probability at one segment at heavy cost to the others, the strategy is not risk-averse at all: it is merely suboptimal. The use of a stochastic model, in conjunction with estimated uncertainties, may help avoid such undesirable extremes.

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