INTERIM TAPER AND CUBIC-FOOT VOLUME EQUATIONS FOR YOUNG LONGLEAF PINE PLANTATIONS IN SOUTHWEST GEORGIA

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Abstract- Outside bark diameter measurements were taken at 0, 0.5, 2.0, 4.5, 6.0, 16.6 and 4 foot height intervals above 6 foot to a 2 inch dob top diameter on 42 longleaf pine trees selected from intensively managed longleaf pine (*Pinus palustris Mill.*) plantations in Dougherty and Worth Counties in southwest Georgia. Trees were sampled from unthinned, cutover stands in their 11th and 14th growing season that are currently part of an existing growth and yield study. Sample trees ranged from 2 to 7 inches in diameter and from 18 to 40 feet in total height. Parameters for a segmented polynomial taper and compatible cubic foot volume equation were simultaneously estimated using a seemingly unrelated nonlinear fitting procedure to volumes based on a generalized Newton formula and an overlapping bolt methodology. Resultant taper and volume functions were compared to published equations for longleaf plantations in the West Gulf physiographic region.

INTRODUCTION

Over the past decade there has been an increased interest in planting longleaf pine (*Pinus palustris Mill.*) in southwest Georgia. This interest is based on a historical, as well as an emotional relationship with this species, the existence of cost sharing programs, and the ability to consistently establish well stocked, uniform plantings that generally do not exhibit a “grass-stage”. The ability to establish these types of plantings is based on major advancements in seedling care, planting techniques, more intensive site preparation methods and inclusion of post planting herbaceous weed control.

Little is known regarding the growth and yield of longleaf plantations in the Southeast, especially for these more intensively managed plantations. Most of the published mensurational information on planted longleaf stands has been for cutover sites in the West Gulf physiographic region. Compatible taper and volume functions have been published for outside bark diameters (Baldwin and Polmer 1981) and inside bark diameters (Thomas and others 1995) for plantations in central Louisiana and east Texas. A total and merchantable cubic foot volume equation has also been developed from plantations in this same region by Baldwin and Saucier (1983). Whether these equations accurately model the taper and volume of trees in southwest Georgia has never been examined.

The purpose of this project was to develop compatible taper and cubic foot volume functions as part of a growth and yield study for unthinned longleaf pine plantations on cutover sites in southwest Georgia and to compare the resulting equations with those that have been developed for longleaf pine plantations in the West Gulf.

METHODS

Sample trees were selected during the summer of 2000 from three unthinned plantations in Dougherty and Worth Counties, Georgia that are part of an existing growth and yield study. Plantations were established on cutover stands that received mechanical as well as chemical site preparation. Plantations ranged in age from 12 to 14 years and were established on sandy loam soils using bare root seedlings. A description of these plantations is presented in table 1.

Approximately 15 sample trees were selected from the interior of each plantation from the area buffering existing permanent growth and yield plots. An attempt was made to stratify the sample by diameter class without leaving holes in the existing stand. Sample tree distribution by height and diameter class is displayed in table 2. Trees possessing multiple stems, broken tops, obvious cankers or crooked boles were not included in the sample. Each sample tree was felled at ground level and total tree height recorded to the nearest 0.1 foot. One inch sample disks were removed from the base, 0.5 foot, 2.0 feet, 4.5 feet, 6.0 feet and repeatedly along the stem at 4 foot intervals until reaching a 2 inch dob top diameter. An additional disk was also removed at 16.6 feet to represent Girard form class height. Diameter outside bark to the nearest 0.01 inch was measured for each disk using a diameter tape. The data set included 456 outside bark measurements on 42 sample trees. Cubic foot volume outside bark was calculated for each bolt utilizing an overlapping bolt method (Bailey, 1995) and a generalized Newton formula described by Wiant and others (1992).

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DATA ANALYSIS

The Max and Burkhart (1976) segmented polynomial taper function was selected as the first candidate taper model, which has the form:

$$\frac{d^2}{D^2} = \beta_1(Z_u - 1) + \beta_2(Z_u^{\,2} - 1) + \beta_3(\alpha_1 - Z_u)^2 I_1 + \beta_4(\alpha_2 - Z_u)^2 I_2 \quad (1)$$

Where d is diameter outside bark (in.) at some given height h (feet), D is diameter outside bark (in.) at breast height, Z_u is the ratio of the upper bolt height to total height, α_1 and α_2 represent the joint points estimated during the fitting procedure, and the ß's are model parameters. The I_i's are indicator variables and are defined as:

$$I_i = \begin{cases} 1, \ if\, Z_u \le \alpha_i \\ 0, \ if\, Z_u \succ \alpha_i \end{cases}$$

integration of the taper function over height results in the volume model:

$$V = kD^2H \left| \begin{array}{l} \frac{\beta_2}{3}(Z_u^3 - Z_l^3) + \frac{\beta_1}{2}(Z_u^2 - Z_l^2) - (\beta_1 + \beta_2)(Z_u - Z_l) \\ - \frac{\beta_3}{3}\left[(\alpha_1 - Z_u)^3 J_1 - (\alpha_1 - Z_l)^3 K_1\right] \\ - \frac{\beta_4}{3}\left[(\alpha_2 - Z_u)^3 J_2 - (\alpha_2 - Z_l)^3 K_2\right] \end{array} \right| \quad (2)$$

Where V is volume outside bark in ft^3, k is $\pi/576$, H is total height in feet, Z_u is the ratio of upper bolt height to H, Z_l is the ratio of lower bolt height to H, and the α's and ß's are as previously defined. The J_i's and K_i's are indicator variables and are defined as:

$$J_i = \begin{cases} 1, \ if\, Z_u \le \alpha_i \\ 0, \ if\, Z_u \succ \alpha_i \end{cases}$$

$$K_i = \begin{cases} , \ if\ Z_l \le \alpha_i \\ 0, \ if\ Z_l \succ \alpha_i \end{cases}$$

Traditional development of compatible taper and volume functions involves parameter estimation for the taper function, which is then integrated to provide volume. This approach will minimize the error associated with stem diameter estimation but does not ensure minimal error in volume estimation. In an attempt to simultaneously minimize the error associated with taper and volume, Equation (1) and Equation (2) were simultaneously fit as seemingly unrelated regressions (SUR) using SAS/ETS Model Procedure (SAS Institute Inc. 1993).

RESULTS

Statistics of fit and parameter estimates from the SUR fitting procedure for Equation (1) and Equation (2) are presented in tables 3 and 4, respectively.

Taper

The proposed taper function was compared to the equation published by Baldwin and Polmer (1981) for planted longleaf in the West Gulf region. Residuals for diameter outside bark at several relative height classes were compared using statistics similar to those applied by Parresol and others (1987). These included: (1) the Sum of squared relative residuals (SSRR); (2) Mean absolute residual (AbsD); (3) Bias (D); and (4) Standard deviation of residuals (Sd) (table 5). The Baldwin and Polmer (1981) model was superior only in relative height class 1 (relative height of 0.06 to 0.15) and to a lesser extent, in relative height class 2 (table 6). The superiority in this part of the stem is due to the fact that for the tree sizes evaluated in this study, relative height class 1 reflects the relative height at dbh and the Bennett and others (1978) model employed by Baldwin and Polmer (1981) constrains the model to equal dbh at 4.5 feet. A review of the residual plot for the Baldwin and Polmer model indicated an over estimation of stem diameter at the base of the tree, an under estimation of stem diameter between relative heights of 0.2 and 0.5, and an over estimation of stem diameter between relative heights of 0.5 and 0.9. Both models are constrained to a 0 inch top diameter at total tree height. No irregularities were detected from the residual plot for the proposed model.

Volume

The proposed compatible cubic foot volume function was compared to the total cubic foot volume estimates from the Baldwin and Polmer (1981) and Baldwin and Saucier (1983) models using their published parameter estimates. In terms of total stem cubic foot volume (ob), the Baldwin and Polmer equation was superior to the Baldwin and Saucier equation and the proposed equation was superior to the Baldwin and Polmer equation. The same statistics used to evaluate the differences in stem diameter

Table i-Description of sampled longleaf pine plantations

Plantation	Planting Spacing	Age	TPA	BA/AC	QMD (in)	DHT* (ft)
1	6*8	14	516	66.4	4.9	39.0
2	6*8	12	798	87.7	4.5	35.2
3	6*8	12	695	92.8	4.9	33.7

* Where DHT equals the average total height of dominant and codominant trees

Table 2-Distribution of felled longleaf pine sample trees by diameter and total height class

Dbh (in.)	20	25	30	35	40	Total
2	5	3				8
3		4	1			5
4		2	5	5		12
5			1	4	2	7
6				5	1	6
7				1	3	4
Total	5	9	7	15	6	42

Table 3-Nonlinear SUR summary of residual errors

Equation	DF Model	DF Error	SSE	MSE	R-Square	Adj R-Square
1	3	465	5.2521	0.0113	0.9579	0.9577
2	3	465	0.4426	0.000952	0.9684	0.9683

Table 4-Nonlinear SUR parameter estimates

Parameter	Estimate	Std Err	t value	P> |t|
B1	-3.0544	0.2902	-10.53	0.0001
B2	1.349745	0.1727	7.84	0.0001
B3	-1.36556	0.1662	-8.21	0.0001
B4	154.0197	22.8409	6.74	0.0001
Al	0.606008	0.0504	12.02	0.0001
A2	0.057371	0.00416	13.78	0.0001

Table 5—Statistics used to evaluate predicted diameters (ob) and total cubic foot volume (ob)

Sum of Squared Relative Residuals (SSRR)

$$\sum\left(\frac{y_i - \hat{y}_i}{y_i}\right)^2$$

Mean Absolute Residual (AbsD)

$$\frac{\sum ABS\left(y_i - \hat{y}_i\right)}{n}$$

Bias (D)

$$\frac{\sum\left(y_i - \hat{y}_i\right)}{n}$$

Standard Deviation of Residuals (Sd)

$$\left[\frac{\sum\left(y_i - \hat{y}_i\right)^2 - \frac{\left(\sum\left(y_i - \hat{y}_i\right)\right)^2}{n}}{n-1}\right]^{0.5}$$

Where: y_i represents either the observed diameter or volume (ob) and $\hat{y}_i$ represents either the predicted diameter or volume (ob).

estimates were applied to total cubic foot volume differences (table 7). The average residual (D) for the proposed model was 85 percent smaller than that for the Baldwin and Polmer equation. Differences between the standard deviation of the residuals were minute. A review of the residual plots indicated that the Baldwin and Saucier equation underestimated volume for all trees greater than 4 inches dbh. This bias increased directly with dbh with residuals ranging from -0.4 to 0.6 cubic foot. The Baldwin and Polmer equation underestimated volume for 78 percent of the trees with residuals ranging from -0.5 to 0.3 cubic foot. The proposed model was biased for trees in the 2 and 3 inch diameter class, however, this bias was small (< 0.1 cubic foot). Residuals ranged from -0.5 to 0.2 cubic foot.

Table 6-Statistics of fit for 10 relative height classes based on planted longleaf pine taper data

		Relative Height Class									
Model*	Statistic	0	1	2	3	4	5	6	7	8	9
1	SSRR	0.608	0.047	0.090	0.216	0.320	0.365	0.949	0.697	0.695	0.007
	AbsD	0.060	0.017	0.027	0.067	0.069	0.068	0.105	0.140	0.155	0.086
	D	-0.048	0.006	0.022	0.066	0.052	-0.007	-0.081	-0.132	-0.147	0.086
	Sd	0.339	0.101	0.122	0.168	0.231	0.287	0.330	0.228	0.207	
2	SSRR	0.313	0.084	0.071	0.058	0.131	0.230	0.518	0.257	0.234	0.002
	AbsD	0.043	0.026	0.031	0.031	0.043	0.050	0.080	0.072	0.085	0.039
	D	0.006	0.010	0.007	0.011	0.015	0.003	0.016	0.054	0.054	0.039
	Sd	0.249	0.136	0.128	0.156	0.188	0.236	0.275	0.175	0.177	

* (1) Baldwin & Polmer (2) Brooks and others

Table 7-Statistics to evaluate predicted total cubic foot volume for 42 longleaf pine trees

Statistics	Baldwin & Saucier (1983)	Baldwin & Polmer (1981)	Brooks and others
SSRR	**0.46073**	0.47951	**0.36356**
AbsD	0.12583	0.11511	0.09566
D	0.07009	**0.06385**	**0.00984**
Sd	0.16215	0.13627	0.13479

CONCLUSIONS

The objective of the study was to compare existing taper and cubic foot volume equations for planted longleaf pine in the West Gulf to an equation fit to 42 sample trees from plantations in southwest Georgia. It is not surprising that the proposed model had the smallest residuals since it was fit to the test data. How the existing equations predicted taper and/or volume compared to the proposed model was of primary interest. The Baldwin and Saucier (1983) volume equation possessed residual trends that would make it an unlikely candidate for use in these young plantations. The Baldwin and Polmer (1981) equations provided reasonable estimates of volume but was limited in its ability to accurately predict stem diameter. Further analysis is planned to fit the Bennett and others (1978) model and other nonlinear segmented polynomial models to this data set in an attempt to further reduce volume and taper estimation errors.

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