Remeasuring Tree Heights on Permanent Plots Using Rectangular Coordinates and One Angle Per Tree

Note by Robert L. Neal, Jr.

Abstract. Heights of permanent sample trees with tops visible from any point can be measured from that point with any clinometer, measuring one vertical angle per tree. Two horizontal angles and one additional vertical angle per observation point are necessary to orient the point to the plot. Permanently recorded coordinates and elevations of tree locations are used with the angles to calculate the instrument location and elevation, instrument-to-tree distance, and total tree height. The method is efficient only if a programmable calculator or a computer can be used for calculations. Procedures and formulas are suggested, including formulas for calculating the coordinates of tree locations in a plantation plot from tree identification numbers. Error relationships are discussed. Applications are suggested and one test is described. ***Forest Sci. 19:233–236.***

Additional key words. Spacing studies, stands, computers.

THE USUAL METHODS of periodically remeasuring tree heights on permanent sample plots become more difficult and expensive as the trees grow older. With the indirect method described here, the cost of height remeasurement can sometimes be reduced. Essentially, the method is the application of surveying techniques and electronic data processing to the remeasurement problem.

Heights of all permanent sample trees with tops visible from any point can be remeasured from that point with any clinometer with only one vertical angle measurement per tree. Two horizontal angles and one additional vertical angle must be measured at each observation point to relate the point to the plot. Observation points are temporary points selected for the convenience of the observer and can be different for each periodic remeasurement. No distance measurements are required for remeasurements, although they are needed in the initial measurement of the plot. The method is more efficient than conventional procedures only if electronic data-processing can be used for the necessary calculations. Computers and programmable calculators can handle the calculations with ease.

Though usable in virtually any stand, the method will probably provide the greatest savings in stands which have both severely limited lower-stem visibility and good upper-stem visibility. Many long-term studies either have these conditions throughout the life of the study, or include treatments which produce these conditions at one or more times in the course of the study. The method might be most useful in small, isolated, spacing studies, progeny-testing plantations, and open natural stands on brushy broken terrain. Even in ideal conditions for the method, a few trees usually can be remeasured more economically by conventional methods.

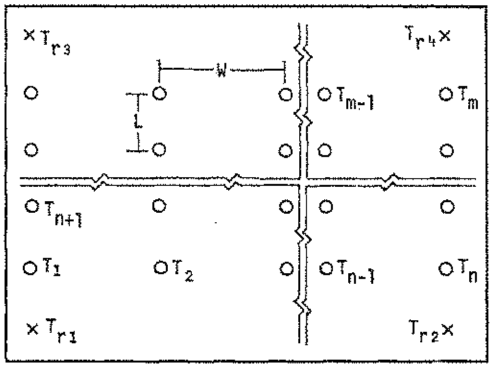
The one vertical angle per tree and distances calculated from the coordinates of tree and instrument locations are used to calculate tree height relative to instrument height. Instrument height relative to a datum plane is similarly calculated. Instrument height and the elevation of tree location relative to the datum plane are added algebraically to the calculated portion of tree height to obtain total height. Tree coordinates and elevation data are part of the permanent record for the plot and are obtained when the plot is established. Instrument location coordinates and elevation are calculated from the plot record information and the orienting angles measured at the instrument location.

The method has four distinct phases:

1. Coordinates and ground elevation for each tree on the plot are determined when the plot is established. This information becomes part of the permanent record for the plot. If the plot is a carefully laid out plantation, the amount of data to be entered to calculate the height of each remeasured tree can be reduced by calculating tree coordinates from tree numbers (Fig. 1). The formulas of Figure 1 can be used only if grid intersections are numbered consecutively and trees have the same number as the grid intersections. Because the coordinates of the tree locations are calculated from the tree numbers, only the program steps executing the formulas need be

---

The author is research forester, Pacific Southwest Forest and Range Exp. Stn., USDA Forest Service, Berkeley, Calif., stationed at Challenge, Calif. Manuscript received April 28, 1972.



T = any tree or dummy location, represented by a grid intersection; trees have the same numbers as intersections and all intersections must be numbered consecutively (from left to right from bottom to top in this sample)

T_1 = origin of the grid and a corner of the plantation; it may be any positive number and has the coordinates $P_1(x_1,y_1) = 0,0$

T_n = number of the tree at the end of the row begun with T_1, defining one corner of the plantation

T_m = number of the tree at the corner of the grid diagonally opposite T_1; it is the last tree in the plantation and defines a corner of the plantation

T_r = number of a dummy location used as a reference

L = distance between trees in columns

W = distance between trees in rows

N = T_n-T_1+1 = number of trees per row

Formulas for numbering of corner dummies:

$T_{r1} = T_1 - N$

$T_{r2} = T_1 - 2N + 1$

$T_{r3} = T_m + 1$

$T_{r4} = T_m + N$

Calculation of rectangular coordinates $P_i(x_i,y_i)$ of any tree (T_i), or dummy:

$$F_i = \frac{T_i - T_1}{N}$$

$[F_i]$ = integer part of F_i

$x_i = WN(|F_i - [F_i]|)$

$y_i = L[F_i]$

Note 1. Absolute value of F-[F] need be specified in the formula only if dummy locations are established as shown.

Note 2. See text for limitations on location of dummies.

FIGURE 1. *Procedure for calculating tree coordinates.*

stored in order for the computer or calculator to "know" the coordinates of all trees in the plantation. The rectangular layout procedures shown in Figure 1 can be adapted to any regular polygon pattern.

On a plot with random tree distribution, the coordinates of each tree location must be individually determined when the plot is established. The actual coordinates rather than tree number are then entered to calculate tree height. The use of polar coordinates is probably the easiest way to determine the tree coordinates, but polar coordinates must be converted to rectangular coordinates in order to use the formulas in Figures 2 and 3.

Tree-site elevations can usually be determined by conventional leveling methods. Occasionally slope may be uniform enough to be expressed as a formula, and elevation calculated from the tree-location coordinates.

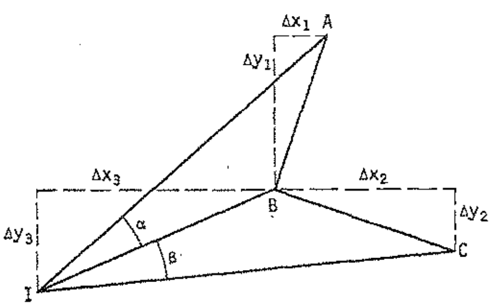
2. The coordinates of the observation point are determined by the use of two horizontal angles and the coordinates of three reference points in a "three-point problem" formula from surveying (Fig. 2). The three-point problem can be solved in several ways. The "analytic solution" used in Figure 2 was selected as the simplest of six methods on hand (other solutions may require less calculator room, however). No solution is possible if points *A*, *B*, *C*, and *I* all lie on the same circle. Three-point problem exposition and other methods of solution can be found in surveying texts and handbooks.

Points *A*, *B*, and *C* can be any points for which the coordinates are known or can be calculated. These points can be actual tree locations or dummy locations established and monumented specifically for minimum error in relating observation points to the plot. A sighting rod placed at a dummy can be positioned and plumbed to be free of the effects of tree or rod displacement and of tree lean possible at an actual tree location. Ideally, the dummies, or rods placed on them, should be visible over a wide area. Dummies should be established and monumented at the time the plot is established. Monuments for the dummies could also serve as benchmarks for the elevations on the plot.

Dummies located and numbered for direct use in the coordinate calculation formulas for plantations are shown in Figure 1. Spacing for the dummies is the same as plantation spacing. Other dummies with calculated coordinates can be located at both ends of all columns without affecting tree numbering. Trees cannot be numbered consecutively if dummies with calculated coordinates are located at the ends of rows. Note the out-ofsequence numbering of T_{r2}, which is necessary if the coordinates of T_{r2} are calculated.

3. Instrument height relative to the datum plane is calculated by the use of the vertical angle between horizontal and a known height at a known elevation and location (Fig. 3). One of the references used to calculate the coordinates of the instrument location is usually the most convenient known elevation and location, especially if one or more of the references are dummies. The rod used to provide a target for the horizontal angles needed to locate point I can also provide the known height. Alternatively, heights at known locations can be permanently established by tags placed at known heights on widely visible sample trees.

4. Tree height is calculated from the information developed in the preceding phases and the one measured vertical angle per tree



Given: $P_A(x_A,y_A)$, $P_B(x_B,y_B)$, $P_C(x_C,y_C)$, α, β

Calculation of $P_I(x_I,y_I)$

$$\Delta x_1 = x_A - x_B$$
$$\Delta x_2 = x_C - x_B$$
$$\Delta y_1 = y_A - y_B$$
$$\Delta y_2 = y_C - y_B$$
$$P = \Delta x_1 - \Delta y_1 \cot \alpha$$
$$Q = \Delta y_1 + \Delta x_1 \cot \alpha$$
$$R = \Delta x_2 + \Delta y_2 \cot \beta$$
$$S = \Delta y_2 - \Delta x_2 \cot \beta$$
$$K = \frac{P - R}{S - Q}$$
$$\Delta x_3 = \frac{KQ+P}{K^2+1} = \frac{KS+R}{K^2+1}$$
$$\Delta y_3 = K\Delta x_3$$
$$x_I = \Delta x_3 + x_B$$
$$y_I = \Delta y_3 + y_B$$

FIGURE 2. *Procedure for determining coordinates of the observation point.*

(Fig. 3). If percent or topographic units instead of degree are used to measure the vertical angle, the formula for total height in Figure 3 becomes

$$H = HI - E_i + (D/100)\,\theta,$$

and

$$H = HI - E_i + (D/66)\,\theta$$

respectively. The formula for HI is similarly changed.

Error relationships with the method are even more complex than those for conventional methods of indirect height measurement. Only three general observations about error relationships will be made here.

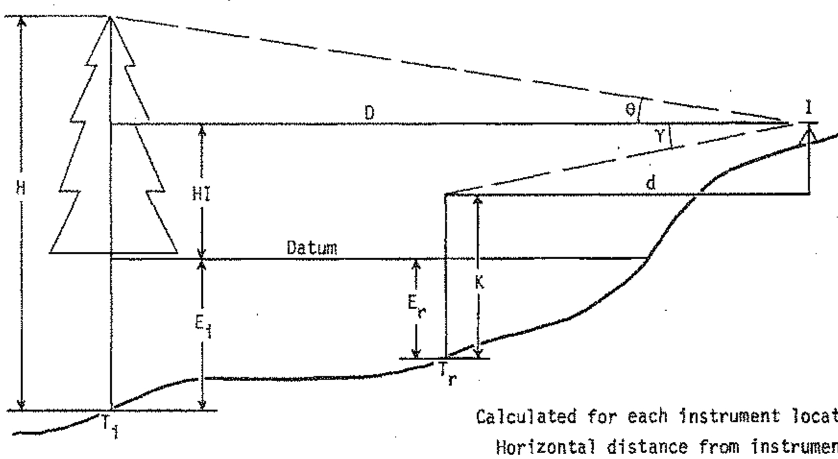
First, because the instrument-to-tree distance and direction are different for every tree, error in the calculated location of point I introduces different amounts of error into each height calculation. In the typical geometry of Figure 2, an error of about 15 minutes in the measurement of angles α or β if they are 30 degrees or larger would cause less than 1 percent error in the location of point I relative to the actual length of line BI. An error of about 3 minutes when α or β are as small as 5 degrees would also cause less than 1 percent error in the location of I. In the geometry of Figure 2, all sample trees would be behind lines BA and BC relative to point I. The relative error in instrument-to-tree distance would therefore be less than the error in the location of point I relative to the length of line BI. Some or all of these relationships would be different with other arrangements of points A, B, C, and I.

Second, once the observation point is selected, tree lean is ignored. Even small amounts of lean in the plane of observation can cause considerable error in height calculations. Error due to lean in any plane can be reduced by reducing the vertical observation angle, either by increasing the instrument-to-tree distance or by changing the elevation of the instrument, or by a combination of both. It may be preferable to measure some leaning trees by conventional methods if an instrument location with a satisfactorily low observation angle for trees leaning in the plane of observation is impossible.

Finally, instrument-to-sample tree distances are typically longer with this method than with conventional methods of height measurement. Therefore more precise measurements and more sensitive instruments may be necessary to achieve desired standards.

The method has been used in small progeny-testing plantations. Plots with as many as 50 trees each occupy a small part of a large clearing. Trees range from 20 to 40 feet in height with a few shorter trees. Lanes separate most of the plots, and the parts of the clearing not occupied by plantations have a head-high brush cover of medium density. Tree heights were measured with a transit by two men. One used the transit and took notes and the other used the rod and identified the trees. They easily measured all but an occasional stunted tree from one instrument location per plot. Distances from the instrument to the nearest tree in the plantation ranged from about 80 to 160 feet, depending on the height of the trees. Slopes averaged about 10 percent. One of the plots was measured from across an adjoining highway.

A variant of the method was tested briefly in virgin redwood with a rotary calculator and tables for calculations. The method was not developed further at that time because computation time exceeded the savings in field time.



Given:

$P_I(x_I, y_I)$, $P_r(x_r, y_r)$, $P_i(x_i, y_i)$

E = elevation relative to a horizontal zero datum (elevations above and below datum are signed + and - respectively, and are part of the permanent data for the plot).

K = known height at reference location T_r (T_r can be an actual tree location).

γ = vertical angle from instrument to K

θ = vertical angle from instrument to top of tree T_i

Note: γ and θ above and below horizontal are signed (+) and (-) respectively.

Calculated for each instrument location (I):

Horizontal distance from instrument to location T_r:

$$d = \sqrt{(x_I - x_r)^2 + (y_I - y_r)^2}$$

Height of instrument relative to datum:

$$HI = K + E_r - d \tan \gamma$$

Calculated for each tree (T):

Horizontal distance from instrument to location T_i:

$$D = \sqrt{(x_I - x_i)^2 + (y_I - y_i)^2}$$

Total height of tree T_i:

$$H = HI - E_i + D \tan \theta$$

FIGURE 3. *Procedure for calculating instrument-to-tree distance and tree height.*