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NONTURBULENT DISPERSION PROCESSES IN COMPLEX TERRAIN

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Abstract—Mass divergence influences on plume dispersion modify classic Gaussian calculations by as much as a factor of two in complex terrain. The Gaussian plume was derived in flux form to include this process.

Prediction of dispersion from stationary point sources by means of Gaussian models has generally been less accurate in complex terrain than over level ground. Dispersion is frequently underestimated for a given stability (Hinds, 1970), when compared to the values given in Turner (1969). Dispersion estimates in complex terrain are improved either by altering the Pasquill–Gifford stability class (Leahey and Halitsky, 1972; Start *et al.*, 1975) or by using turbulence measurement to specify diffusion coefficients (MacCready *et al.*, 1974; Wooldridge and Lewis, 1975). Plume spread predictions using onsite turbulence measurements improve the Gaussian model and make it more applicable to complex terrain. However, it is difficult to generalize these results to areas where detailed measurements off turbulence have not been made. While turbulence is often greater in mountainous areas, persistent, nonturbulent processes also influence plume dispersion. Airflow patterns in complex terrain, such as the classical mountain and valley circulation, (Defant, 1951) are characterized by significantly large values of divergence and stretching deformation. One must have methods available to separate these bulk fluid motion influences from the influences of smaller scale turbulence. In this note, the development of the Gaussian plume model is restated in a form which considers nonturbulent dynamic influences on plume spread.

The solution obtained in this paper is illustrated for a highly simplified case in order to demonstrate how dispersion due to nonzero divergence fields enters the calculation. This solution is based on Robert's (1923) solution for constant eddy viscosity and mean wind speed along the plume axis. The mean cross axis flow is assumed to be negligibly small although derivatives of these cross flow components may be nonzero. An example illustrating the magnitude of errors in a typical mountain flow is provided following the derivation.

We consider the conservation of mass as

$$\frac{\partial \chi}{\partial t} + \nabla \cdot (\chi \mathbf{v}) = 0,$$

where molecular diffusion has been neglected and then apply the conventional approach to define an ensemble averaged velocity and close the turbulent concentration flux terms with a Fickian diffusion approximation. The conservation equation is then

$$\frac{\partial}{\partial t}\bar{\chi} + \bar{\mathbf{v}}\cdot\nabla\bar{\chi} + \bar{\chi}(\nabla\cdot\bar{\mathbf{v}}) = \nabla\cdot(\mathbf{K}\cdot\nabla\bar{\chi}), \qquad (1)$$

where

$$\bar{\mathbf{v}} \simeq \bar{u}, 0, 0$$

$$\delta = \nabla\cdot\bar{\mathbf{v}} = \frac{\partial\bar{u}}{\partial x} + \frac{\partial\bar{v}}{\partial y} + \frac{\partial\bar{w}}{\partial z}. \qquad (2)$$

A solution, following Robert's (1923) for $K_x = 0$, $K_y, K_z, \bar{u}$ – constant can be written

$$\bar{\chi} = \frac{Q}{\bar{u}\sqrt{2\pi}\sigma_y\sigma_z}\exp\left[-\frac{1}{2}\left(\frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2}\right)\right] \times \exp\left(-\frac{\delta x}{\bar{u}}\right), \qquad (3)$$

where it has been assumed that a Gaussian distribution in y and z for the concentration exists, and such a distribution is related to the eddy viscosity by

$$K_y = \frac{\bar{u}}{2x}\sigma_y^2 \qquad (4a)$$

$$K_z = \frac{\bar{u}}{2x}\sigma_z^2. \qquad (4b)$$

For simplicity we have restricted consideration to elevated plumes and concentrations near the center line. Equation (3) reduces to the conventional form when $\delta = 0$.

The assumptions involved in this approximation are consistent with the existence of regular, divergent mountain valley wind systems. These patterns tend to be rather fixed for defined periods of time and to exhibit a steady undirectional wind ($\bar{\mathbf{v}} = \bar{u}, 0, 0$) along the valley axis. The cross flow components are small while the gradients of the cross valley flow are large. These circumstances generate substantial divergence (Fosberg, 1969; Defant, 1951). While $\bar{v}$, $\bar{w}$ are approximately zero, the sign and magnitude of each component is spatially invariant along the plume axis because the thermal heating or cooling of mountain surfaces forces expansion or contraction of a unit mass of air. Divergence is therefore nonzero. The divergence, δ, like the mean wind, $\bar{u}$, is constant along the plume. The requirement that δ and $\bar{u}$ are constants is a major restriction of this analysis.

Inclusion of the divergence effect in complex terrain does not require additional assumption on the turbulent flow. Divergence and mean flow characteristics are based on ensemble averages. Mean flow along the plume is defined in the conventional fashion. Because this ensemble average velocity along the plume is defined as $\bar{\mathbf{v}} = \bar{u}, 0, 0$, a more general solution involving variable turbulent diffusion coefficients (Sutton, 1932; Bosanquet and Pearson, 1936; Pasquill, 1962) can be used to replace the assumption of constant K used in the above arguments. Because of this, the model can accommodate the usual Pasquill–Gifford dispersion coefficients. In fact, since modern concepts (Pasquill, 1962; Gifford, 1968) use the Gaussian distribution as an hypothesis on which to base a model containing coefficients determined empirically, the divergence corrected Gaussian may be interpreted in the same manner, i.e. an hypothesis that satisfies mass continuity.

In the following discussion, solutions from Turner's workbook (1969) have been used with the divergence term rather than the analytical solution obtained in this paper.

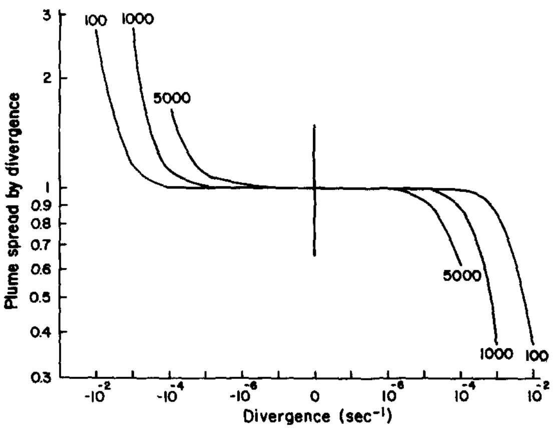

Fig. 1. Divergence influence on downwind plume concentrations. Line labels refer to time interval from release at the source.

The effect of divergence on dispersion can be estimated by comparing the concentration given by equation (3) with the classical solution such as given in Turner (1969). Since the classical equation is identical to (3) without the divergence exponential, the effect can be studied by forming a ratio of equation (3) to the classical solution:

$$\frac{\chi\bar{u}/Q}{\chi\bar{u}/Q_{\text{classical}}} = \exp(-\delta x/\bar{u}) = \exp(-\delta\Delta t),$$

where the characteristic time interval Δt over which conditions remain constant is used to replace $x/\bar{u}$. Figure 1 shows this ratio as a function of divergence and characteristic times. It can be seen that small divergences of small characteristic time intervals have an insignificant effect. However, when either the time interval or the divergence becomes large (long plumes or strong persistent local circulation patterns), it is possible to alter the concentration over the classical calculation. Flow convergence is seen to have a less pronounced effect than divergence on concentration.

To determine whether this influence is a mathematical refinement which has little practical significance, or is a real process which should be included in dispersion estimates, typical values of divergence must by established. Estimates of meso-scale divergence (Schaefer, 1973) showed maximum values of $10^{-4}\,\mathrm{s}^{-1}$ over moderate terrain, and values of $\pm 10^{-5}\,\mathrm{s}^{-1}$ over level ground. Small, toposcale circulations were found to have divergence values of $\pm 10^{-4}$ to $\pm 10^{-3}\,\mathrm{s}^{-1}$ with occasional extreme values of $-10^{-2}\,\mathrm{s}^{-1}$ (Fosberg *et al.*, 1976; Anderson, 1971).

While the increased turbulence intensities indicate that plume dilution is five or more times greater in complex terrain than on level ground, divergence influences can account for a factor of two change in concentration in intense local circulations.

The critical assumption mentioned earlier regarding the neglect of advection of the concentration away from the center line can be justified from an illustration. Consider Fig. 2, which represents a typical mountain valley circulation. A point source released at A would experience a divergence of $1.3 \times 10^{-3}\,\mathrm{s}^{-1}$ while the vertical velocity component is $7\,\mathrm{cm\,s}^{-1}$, which is small compared to $\bar{u}$ ($5\,\mathrm{m\,s}^{-1}$). Assume that the hypothetical source located at A emits $10^7\,\mu$g of pollutants per second with a total plume rise of 100 m. Using Turner (1969) and assuming class D stability yields a maximum ground level concentration of

$$\chi_{\max} = 16\,\mu\mathrm{g}^{-1}\,\mathrm{m}$$

at 3 km from the stack. The correction due to divergence would reduce this concentration maximum at 3 km to

$$\chi_{\max} = 16 \exp(1.3 \times 10^{-3} \times 600) = 7\,\mu\mathrm{g}^{-3}\,\mathrm{m}$$

which would appear to be a significant reduction.

The non-zero divergence imposes a limit on the distance or time over which the plume characteristics

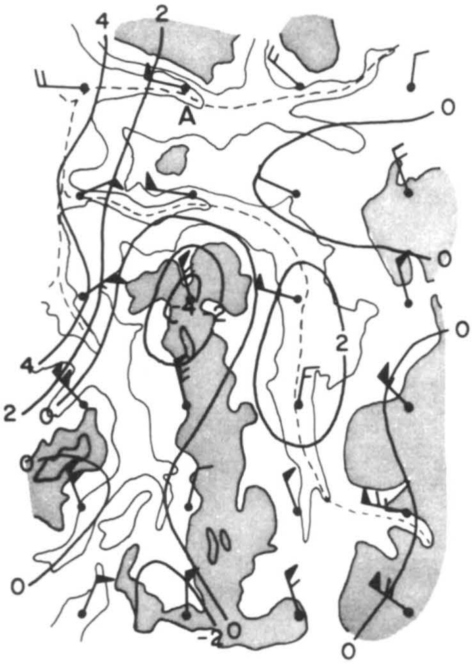

Fig. 2. Wind and divergence fields in a valley. Wind speed is indicated by (a) pennant, $5\ m\ s^{-1}$, (b) barb, $2\ m\ s^{-1}$, (c) half barb, $1\ m\ s^{-1}$. Divergence isopleths are in $10^{-3}\ s^{-1}$. Elev. contours are 1000 ft.

can be calculated. Concentration dispersed by divergence is accompanied by mass dispersal, and therefore the time interval or downwind distance is limited by the mass balance of the carrying air.

In conclusion, we suggest that a Gaussian dispersion model with a divergence correction can be very useful in mountain diffusion studies; however, caution must be exercised to ensure that conditions of strong divergence are accompanied by small lateral advections.

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