NONLINEAR PROGRAMMING MODELS TO OPTIMIZE UNEVEN-AGED SHORTLEAF PINE MANAGEMENT

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Abstract-Nonlinear programming models of uneven-aged shortleaf pine (*Pinus echinata* Mill.) management were developed to identify sustainable management regimes that optimize soil expectation value (SEV) or annual sawtimber yields. The models recognize three species groups (shortleaf pine and other softwoods, soft hardwoods and hard hardwoods) and 13 2-inch diameter-at-breast-height size classes. Reproduction, growth and mortality rates are a function of tree diameter, stand density and site productivity. The optimal economic and production regimes each involve a guiding maximum diameter for softwoods and periodic hardwood control, with the optimal maximum diameter a function of site productivity.

INTRODUCTION

Growing public demand for non-commodity forest values such as biological diversity, scenic beauty, recreational opportunities and wildlife habitat has lead to increased interest in uneven-aged management. Yet models to predict the effects of specific management regimes on stand structure, species composition, timber production, economic returns and sustainability are not readily available for many forest types. This remains true for shortleaf pine, despite its economic importance and wide distribution. To help address this situation, we developed mathematical programming models to identify sustainable management regimes that maximize economic returns or annual sawtimber production for uneven-aged shortleaf pine.

GROWTH MODEL

To estimate stand growth, a site- and density-dependent matrix transition model was developed using data from 1047 naturally regenerated, shortleaf pine re-measurement plots of the Southern Forest Inventory and Analysis (FIA) database (table 1, Hansen and others 1992). The average interval between inventories was 8.6 years. Observed upgrowth and mortality probabilities and ingrowth rates were converted to a one-year interval by exponential interpolation.

The model's structure follows Lin and others (1998). Trees are categorized into thirteen 2-inch diameter-at-breast height (DBH) size classes and three species groups: shortleaf pine and other softwoods, soft hardwoods and hard hardwoods. Size classes are denoted by their mid-point diameters and range from size class 2 to size class 26+, which contains all trees 25 inches DBH and larger. The model was calibrated on 838 plots (80 percent) chosen randomly from the 1047 available. The remaining 209 plots were used to test the accuracy of the model prior to re-estimating the parameters using data from all 1047 plots.

Table 1—Distribution of sample plots by state and inventory"

Inventory	State												
Year	AL	AL	AR	AR	LA	LA	MS	MS	OK	OK	TN	TX	TX
Current	'82	'90	'88	'95	'84	'91	'87	'94	'86	'93	'89	'86	'92
Previous	'72	'82	'78	'88	'74	'84	'77	'87	'76	'86	'80	'75	'86
Plots	108	47	192	174	34	21	85	42	82	85	14	a5	78

"Inventories may span more than one year.

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Table 2-Equations for annual ingrowth (trees/ac/yr)

Sp.[b]	Stand BA (ft^2/ac)	Site (ft^3/ac/yr)	Species BA (ft^2/ac)	Con-stant	R^2	dF
SW	**-0.597** **		0.41 **	41.9 **	0.12	1044
SH	-0.077 **	0.058 **	0.41 **	6.9 **	0.14	1043
HH	-0.091 **	0.059 *	0.13 *	12.7 **	0.02	1043

[a]Asterisks denote level of significance: *, 0.01; **, 0.0001.
[b]Species groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

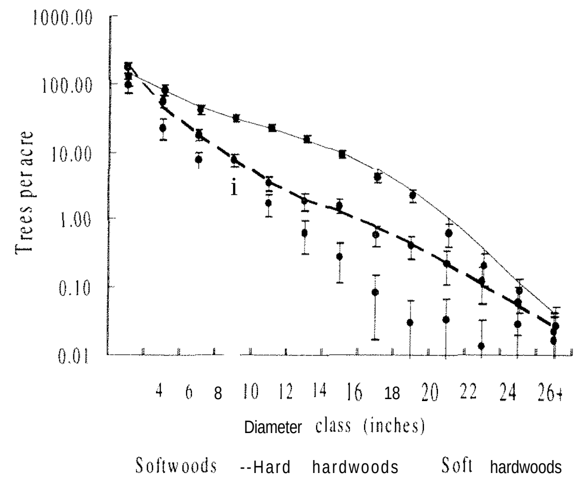

Figure l-Average observed (dots, with 95 percent confidence intervals) and predicted (lines) distributions of shortleaf pine and other softwood, soft hardwood and hard hardwood trees on 209 post-sample plots after an average 8.6-years growth.

Ingrowth Rates

Table 2 gives the parameter estimates for the final ingrowth equations. Ingrowth rates were inversely proportional to total stand basal area and directly proportional to the basal area of the given species group, presumably reflecting the presence of more seed-producing trees. Site productivity had a significant, positive effect on the ingrowth of the soft hardwoods and hard hardwoods but not the shortleaf pine and other softwoods.

Upgrowth Probabilities

The upgrowth probability equations' parameters are in table 3. As expected, upgrowth probabilities were inversely proportional to stand density, directly proportional to site productivity, and a quadratic function of tree diameter for all three species groups. Upgrowth probabilities were lowest at small diameters, peaked at intermediate diameters, and declined again at large diameters.

Mortality Probabilities

The parameter estimates for the mortality equations are in table 4. All three species groups exhibit the expected convex relationship between diameter and mortality. Mortality probabilities were highest at small diameters, reached their lowest levels at intermediate diameters, and increased again at large diameters. For the shortleaf pine and other softwoods group, mortality probabilities were also significantly higher at higher stand densities and on more productive sites.

Projection Accuracy

To test the accuracy of the growth model for projections as long as the interval between two FIA inventories, the initial model, developed with data from the 838 estimation plots, was used to predict the diameter frequency distributions of the 209 validation plots at the current inventory given their distribution at the previous inventory and any intervening harvest. Figure 1 shows how the predicted distributions compared with the observed distributions. For most species-diameter categories, the average of the predicted number of trees was within the 95 percent confidence interval of average observed number of trees, though there was a slight tendency for the model to over predict the number of large shortleaf pine and other softwood trees.

YIELD MODEL

Cubic-foot sawlog and pulpwood volumes of individual trees are estimated using equations fitted to the stem

Table 3-Equations for probability of transition between size classes in 1 year[a]

DF Spb	Stand BA (ft^2/ac)	Site (ft^3/ac/yr)	DBH (in.)	DBH² (in.^{2})	Constant	R^2	DF
SW	-0.00034 **	0.00021 **	0.01000 **	0.00034 **	0.02740 **	0.09	5702
SH	-0.00018 *	0.00020 **	0.00830 **	-0.00032 **	0.00170	0.08	1234
HH	-0.00020 **	0.00022 **	0.00750 **	-0.00022 **	0.00470	0.10	2854

[a]Asterisks denote level of significance: *, 0.01; **, 0.0001.
[b]Species groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 4-Equations for probability of mortality in 1 year[a]

Sp.[b]	Stand BA (ft²/ac)	Site (ft³/ac /yr)	DBH (in.)	DBH² (in.²)	DBH⁻¹ (l/in.)	Con- stant	R^2	dF
SW	0.0000560 **	0.00014 ***		0.000014 *	0.158 ***	-0.028 ***	0.22	5728
SH			-0.00470 ***	0.000160 ***		0.0360 ***	0.04	1244
HH			0.00085 **		0.092 ***	-0.007 ***	0.07	2878

[a]Asterisks denote level of significance: *, 0.01; **, 0.001; ***, 0.0001.
[b]Species groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 5—Equations for sawlog volume (cubic feet)

Sp.[b]	DBH² (in.*)	Sawlog (ft)	Con- stant	R^2	dF
SW	0.13	1.02	-36.1	0.94	299
SH	0.11	1.13	-23.6	0.96	229
HH	0.10	1.12	-31.6	0.96	225

[a]Fitted to the stem volume equations of Clark and Souter (1994). All coefficients are significant at the 0.0001 level.
[b]Species groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods: HH, hard hardwoods.

Table 6—Equations for pulpwood volume per tree (cubic feet)

Sp.[b]	Height (ft)	DBH² (in.²)	Con- stant	R^2	dF
SW	0.12	0.13	-36.1	0.97	47
SH	0.11	0.11	-23.6	0.97	80
HH	0.11	0.10	-31.6	0.97	80

[a]Fitted to the stem volume equations of Clark and Souter (1994). All coefficients are significant at the 0.0001 level.
[b]Species groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 7-Equations for top pulpwood volume per tree (cubic feet)

Sp.[b]	Height (ft)	Sawolg (ft)	DBH² (in.²)	Con- stant	R^2	dF
SW	0.067	-0.99	0.067	-21.5	0.89	298
SH	0.057	-1.06	0.057	-22.3	0.91	228
HH	0.056	-1.07	0.056	-17.0	0.91	224

[a]Fitted to the stem volume equations of Clark and Souter (1994). All coefficients are significant at the 0.0001 level.
[b]Species groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 8-Equations for total tree height (feet)

Sp.[b]	Stand BA (ft²/ac)	Site (ft³/ac /yr)	Site² (ft⁶/ac² /yr²)	DBH (in.)	DBH⁻¹ (l/in.)	Con- stant	R^2	dF
SW	0.090	0.42	-0.0010	1.04	-182	30.9	0.66	17815
SH	0.057	0.39	-0.0013		-274	56.4	0.55	1216
HH	0.071	0.44	-0.0013	0.75	-143	24.3	0.52	3654

[a]All coefficients are significant at the 0.0001 level.
[b]Species groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table S-Equations for sawlog length (feet)

Sp.[b]	Height (ft)	DBH (in.)	DBH⁻¹ (l/in.)	Con- stant	R^2	dF
SW	0.83	-2.2	-396	-36.1	0.75	11901
SH	0.49		-234	-32.6	0.44	291
HH	0.38	-1.9	-465	-31.6	0.37	1349

[a]All coefficients are significant at the 0.0001 level.
[b]Species groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 10—Stumpage prices[a]

	Species group Pulpwood ($/cord)	Species group Sawtimber ($/Mbf)
Softwoods	21.88	324[b]
Soft hardwoods	13.85	153[c]
Hard hardwoods	13.85	291[c]

Source: Timber Mart-South (Sept. 1999 - Aug. 2000).
[b]Scribner log rule; [c]Doyle log rule.

Table ll-Steady-state management regimes that maximize soil expectation value on low, medium and high productivity sites[a]. Trees harvested each cutting cycle are denoted by asterisks

Size	--Low site- · SW	SH	HH	-Medium site- · SW	SH	HH	-High site- · SW	SH	HH
2	269.5	22.8*	46.6*	239.2	34.8*	58.0*	218.5	73.5*	101.5*
4	112.9	0.4*	1.9*	110.1	2.0*	3.6*	113.8	9.4*	14.J
6	72.7	0.0*	0.1*	71.2	0.1*	0.2*	76.1	1.0*	1.7*
8	56.8			54.3			58.0	0.1*	0.2
10	49.4			45.6			31.4*		0.0*
12	18.6			18.5			11.3*		
14	3.7			4.1			2.8'		
16	0.4			0.5*			0.5		
18	0.0			0.0*			0.1'		
20									
22									
24									
26+									
Statistics[b]									
Cycle	8			6			9		
SEV	2093			2711			3430		
Saw	81			109			119		
H'_{tree}	41			41			35		

“Low site, shortleaf pine site index 67 feet at age 50; medium site, 102 feet; high site, 142 feet.
[b]Cycle, optimal cutting cycle (years): SEV, soil expectation value ($/acre); Saw, annual sawtimber production (ft^3/acre/year); H'_{tree}, percent of theoretical maximum tree diversity (pct), after harvest.

Table 12—Steady-state management regimes that maximize annual sawtimber production on low, medium and high productivity sites[a]. Trees harvested each cutting cycle are denoted by asterisks

Size	Low site SW	SH	HH	Medium site SW	SH	HH	High site SW	SH	HH
2	251.5	20.0*	46.3*	234.9	5.7*	10.4*	223.7	18.2	27.7*
4	94.9	0.8*	1.9*	101.5			111.3	0.3	0.5*
6	58.3	0.0*	0.1*	63.5			72.6		
8	44.2			47.8			54.6		
10	37.7			39.3			44.6		
12	34.7			34.7			8.2*		
14	14.8*			3.0”			0.4*		
16	3.4*								
18	0.5*								
20	0.0*								
22									
24									
26+									
Statistic[b]									
Cycle	7			1			2		
SEV	1595			486			2968		
Saw	88			111			133		
H'_{tree}	45			46			41		

“Low site, shortleaf pine site index 67 feet at age 50; medium site, 102 feet; high site, 142 feet.
[b]Cycle, optimal cutting cycle (years); SEV, soil expectation value ($/acre); Saw, annual sawtimber production (ft^3/acre/year); H'_{tree}, percent of theoretical maximum tree diversity (pct), after harvest.

volume tables of Clark and Souter (1994). Pulpwood is potentially available from poletimber trees (softwoods 5 to less than 9 inches DBH or hardwoods 5 to less than 11 inches DBH) and from the tops of sawtimber trees (softwoods 9 inches DBH and larger or hardwoods 11 inches DBH and larger). Pulpwood volumes of poletimber trees (table 5) are a linear function of tree height and diameter squared; whereas pulpwood volumes from the tops of sawtimber trees (table 6) are a linear function of tree height, sawlog length and diameter squared. Sawlog volumes (table 7) are a linear function of sawlog length and diameter squared.

Total heights of the average tree in each size class of a particular species group are estimated using equations based on more than 22,000 trees on the 1047 plots used to develop the growth model. Table 8 gives the empirical tree height equations. For a given size class, trees were taller in stands with more basal area and on more productive sites. Similarly, sawlog lengths are estimated using equations based on more than 13,000 trees from the same plots. The empirical sawlog length equations are in table 9. Sawlog length was a function of tree diameter and height.

OPTIMIZATION MODELS

Maximizing Soil Expectation Value

Knowing the maximum economic return that can be obtained from a particular site provides a useful measure for comparing the economic performance of alternative management regimes. The preferred measure of a management regime's economic performance, when applied to a stand of a given productivity, is the soil expectation value (SEV), the present value of all future harvests, net of all costs, including the opportunity cost of the growing

‘The optimization models presented in this paper have non-concave response surfaces, thereby necessitating the use of nonlinear programming techniques. Consequently, the optimal regimes they identify are locally, though not necessarily globally, optimal. To improve the likelihood of finding globally optimal solutions, each problem was solved 50 times, each time beginning with different initial values.

stock. Because SEV is highly influenced by a stand's initial structure and to ensure sustainability, only steady-state management regimes, those in which the stand returns to the same pre-harvest diameter distribution each cutting cycle, are considered here. Consequently, the model' to identify the sustainable, uneven-aged management regime that maximizes soil expectation value is:

$$\max_{\mathbf{y}_0, \mathbf{h}_0} SEV = \frac{\mathbf{s}\,\mathbf{h}_0 - F}{(1+r)^C - 1} - \mathbf{s}'(\mathbf{y}_0 - \mathbf{h}_0) \qquad (1)$$

subject to:

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{G}_0(\mathbf{y}_0 - \mathbf{h}_0) + \mathbf{I}_0 \\ \mathbf{y}_2 &= \mathbf{G}_1(\mathbf{y}_1) + \mathbf{I}_1 \\ \mathbf{y}_C &= \mathbf{G}_{C-1}(\mathbf{y}_{C-1}) + \mathbf{I}_{C-1} && (2) \\ \mathbf{y}_C &= \mathbf{y}_0 && (3) \\ \mathbf{y}_0 - \mathbf{h}_0 &\geq \mathbf{0} && (4) \\ \mathbf{h}_0 &\geq \mathbf{0} && (5) \end{aligned}$$

where C is the cutting cycle, $\mathbf{y}_t$ is a vector containing the number of trees per acre of species group *i* and size class j at the start of year *t*, $\mathbf{h}_0$ is a vector containing the number of live trees per acre of species group ***i*** and size class j harvested each cutting cycle, $\mathbf{G}_t$ is a matrix containing transition probabilities for year *t*, and $\mathbf{I}_t$ is a vector containing the ingrowth for year t (i.e., the number of trees entering the smallest size class of each species).

The stumpage values of individual trees, *s*, are obtained by multiplying their pulpwood (cords) and sawtimber (board-feet) volumes by their stumpage prices. The stumpage prices used in this analysis are 1999-2000 average prices, weighted by area, for the Southeastern United States (table 10, Timber Mart-South). Pulpwood cubic-foot volumes are converted to cords assuming 72 cubic feet per cord for softwoods and 79 cubic feet for hardwoods. Koch's conversion table (Koch 1972) is used to convert cubic-foot sawlog volumes to board-foot measures (Scribner log rule for softwoods and Doyle log rule for hardwoods). Costs not already reflected in the stumpage prices, *F*, such as administration and hardwood control, are assumed to total $80.00 per acre, while the real rate of interest, *r*, is set at 4 percent.

Equations (2) are the growth equations*. There is one equation for each year of the cutting cycle. Equation (3) is the steady-state constraint, which ensures sustainability by requiring the stand to return to the same pre-harvest distribution each cutting cycle. Equation (5) guarantees that the number of trees harvested from the stand does not exceed the number of trees present; whereas equations (4) and (5) together ensure that the number of trees in, and harvested from, each species-size category is nonnegative.

Maximizing Annual Sawtimber Production

While economic concerns may be a key concern of many forest landowners and managers, others are likely to be more interested in the volume of sawtimber that can be produced on a sustainable basis. The model to maximize annual sawtimber production is:

$$\max_{\mathbf{y}_0, \mathbf{h}_0} \text{Saw} = \frac{\mathbf{v}_s'\mathbf{h}_0}{C} \qquad (6)$$

subject to:
(2), (3), (4) and (5)

where $\mathbf{v}_s$ is a vector containing the cubic-foot sawtimber volumes of trees in each species-size category.

Measuring Tree Diversity

In addition to managing for economic returns and timber production, forest landowners are also increasingly interested in managing for biological diversity. Because the distribution of trees by species and size largely determines a stand's structure, and thus the ecological niches available to other organisms, tree diversity is a key component of a stand's overall diversity (Wilson 1974, Rice and others 1984). One of the most widely used and accepted diversity indices is Shannon's index (Pielou 1977, Magurran 1988). Here we define Shannon's index of tree diversity in terms of basal area, rather than number of individuals, to give added weight to larger trees:

$$H_{trees} = -\sum_{i=1}^{m} \frac{b_{ij}}{b+\varepsilon} \ln\left(\frac{b_{ij}+\varepsilon}{b+\varepsilon}\right) \qquad (7)$$

where b_{ij} is the residual basal area in species group i and size class *j*, *b* is the residual stand basal area and epsilon is a small, positive constant (0.001) used to avoid division by zero and natural logarithm of zero errors. As defined here, Shannon's index reaches its maximum value of 3.66 [ln(39)] when the residual basal area is distributed evenly among each of the thirty-nine species-size categories. It provides a useful measure for comparing the tree diversity of the optimal economic and sawtimber regimes.

RESULTS AND DISCUSSION

Table 11 gives the steady-state management regimes that maximize SEV on low (shortleaf pine site index 67 at age 50 years), medium (site index 102), and high productivity

[2]Because the parameters of the growth and ingrowth matrices are derived from regression equations which contain negative coefficients for residual stand basal area, it is possible for the predicted transition probabilities and ingrowth rates to be negative when the residual basal area is sufficiently high. To avoid such biologically infeasible predictions, the right hand side of each applicable regression equation, call it "z", was replaced by the expression "$[z + (z^2)^{1/2}]/2$. This expression returns the original value of "z" if it is positive and zero otherwise. This equation was also used, as needed, with regression equations for predicting sawtimber and pulpwood volumes.

(site index 142) sites. The optimal cutting cycles are 8, 6 and 11 years, respectively. In all three cases, the hardwoods are completely controlled at each harvest and the shortleaf pines and other softwoods are managed with a guiding maximum diameter of 11 inches DBH on low and medium sites and 9 inches DBH on high sites.

The optimal regimes give SEVs of $2,093, $2,711 and $3,430 per acre, while producing 81, 109 and 119 cubic feet of shortleaf pine and other softwood sawtimber per acre per year, respectively. The small diameters of softwoods and the absence of hardwoods in the residual stands result in relatively low Shannon indices of tree diversity of 41 percent of the theoretical maximum value on low and medium sites and 35 percent on high sites.

Sawtimber Production

Table 12 shows the optimal management regimes for producing sawtimber on low, medium and high productivity sites. The optimal cutting cycles are 7, 1 and 2 years, respectively. As was the case for the SEV-maximizing regimes, the optimal sawtimber regimes each involve complete hardwood control at each harvest and a guiding maximum diameter for shortleaf pine and other softwoods: 13 inches DBH on low sites and medium site and 11 inches DBH on high sites.

These regimes have annual shortleaf pine and other softwood sawtimber production rates of 88, 111 and 133 cubic feet per acre on low, medium and high sites, respectively. By leaving more large diameter softwoods in the residual stand than the SEV-maximizing regimes, Shannon's index of tree diversity improves to 45, 46, and 41 percent of its theoretical maximum on low, medium and high productivity sites, respectively. In contrast, SEV drops to $1595, $486 and $2968 per acre, respectively. This poorer economic performance is due, in part, to the shorter cutting cycles, which cause the fixed costs to be incurred more frequently.

CONCLUSION

Deciding how best to manage forestlands to meet specific objectives requires a clear understanding of what is possible on different sites. The nonlinear programming models presented here help define these limits for uneven-aged shortleaf pine by identifying sustainable steady-state management regimes that maximize either the soil expectation value or the average annual sawtimber production on low, medium and high productivity sites. In addition, the growth model developed for this study allows land managers to explore additional management strategies for meeting their own specific objectives.

Because tree growth, reproduction, and mortality are highly stochastic processes, our ability to model them accurately is limited. Therefore, the optimal regimes presented in this paper should be interpreted as tentative recommendations and not as proven strategies to be adopted unquestioningly. Likewise, simulation results obtained with the growth model should be interpreted as representing the expected average behavior of a number of similar stands, not as predicting the precise behavior of an individual stand.

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