

Modeling the Spreading of Large-Scale Wildland Fires

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Abstract—The objective of the present study is twofold. First, the last developments and validation results of a hybrid model designed to simulate fire patterns in heterogeneous landscapes are presented. The model combines the features of a stochastic small-world network model with those of a deterministic semi-physical model of the interaction between burning and non-burning cells that strongly depends on local conditions of wind, topography, and vegetation. Radiation and convection from the flaming zone, and ambient radiative heat loss are considered in the preheating process of unburned cells. The model is applied to an Australian grassland fire experiment. Predictions compare favorably to experiments in terms of rate of spread, area and shape of the burn. Second, the sensitivity of model outcomes (here the rate of spread) to six input parameters is studied using a two-level full factorial design.

Keywords: wildfire, network model, radiation, convection, sensitivity analysis, validation, prescribed burning

Introduction

In the area of wildfires, there are generally two relevant scales: the macroscopic scale (the scale of the fire front) and the gigascopic scale (the aerial or satellite images). The models developed at the macro-scale are combustion models: Grishin 1997; Porterie and others 2000; Porterie and others 2007; Mell and others 2007 which are based, for the most part, on a two-phase description of the flow and have a great generality. Most of the basic phenomena that govern the emergence and spread of wildfire are generally considered at the macro-scale: gas transport, turbulence, thermal degradation of the material, turbulent combustion, radiation exchange between the gas and the solid medium. A detailed model (Larini and others 1998; Linn and others 1997) requires the numerical solution of the balance equations of mass, energy and momentum for each phase and the radiative transfer equation. These detailed models must be considered as models of knowledge for understanding the behavior of the fire, to test alternative hypotheses or enrich the models, but are not suitable for simulating the spread of fire at the large scale (gigascopic). At this scale, we deal with propagation models.

Unlike algorithms of expansion based on direct-contact or on only nearest neighbors such as Cellular automata or percolation (Albinet and others 1986; Beer and Enting

1990; Duarte 1997), the expansion algorithm used in this study expands the perimeter through considering long range effects between sites by taking advantage of the stochastic method of Monte Carlo to simulate radiative fluxes.

Recently, Adou and others (2010) used the Monte Carlo method but the quasi-physical macroscopic model doesn't incorporate convection effect and involves a circular neighborhood for each site which is a limiting procedure. The propagation model is based on elliptic expansion (Anderson and others 1982) but the correlations of ellipses parameters for propagation are not physically based.

The present model allows investigating 3D fire propagation on a landscape without making the assumptions of one-dimensional spread Catchpole and others 1989; Weber 1991 neither of quasi-steady propagation (Dupuy and Larini 1999). The vegetation and the flame are represented in three dimensions. The macroscopic model has emphasized the most possibly precise description of the phenomena that occur at the macroscopic scale, namely the preheating of the receptive fuel layer by the flame radiation and convection of hot gases, but also its radiative cooling to the surroundings. As we will see, this is based on the resolution of an unsteady energy balance equation including all eventual thermal processes. The rate of spread (ROS), the perimeter shape, the depth of the flaming front and the burnt area can be evaluated at each timestep.

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Nomenclature

V

Control volume

D	Site diameter
H	Stratum height, Height
WFF	Wet fine fuel
DFF	Dry Fine fuel
T	Temperature
q	Power per unit of volume of heat transfer
n	Number of sites
t	Time
i	Index of a site on fire
j	Index of an intact receptor site
FMC	Fuel moisture content
FMC ₀	Initial fuel moisture content
FPPC	Fuel pyrolysis products content
FPPC ₀	Initial Fuel pyrolysis products content
c _p	Specific heat capacity
L	Specific enthalpy change
a	Absorptivity
p	Elementary power carried by a quantum
n"	Number of quanta emitted by each square meter of the flame
p"	Emissive power of the flame
SNB	Statistical narrow-banded
MCM	Monte Carlo Method
SFM	Solid flame model
B	Stefan Boltzmann constant
$\dot{Q}$	Rate of heat release within the flame
H _{f0}	Flame height without wind
H _f	Flame height with wind
S	Surface area
Δh _c	Heat combustion of volatile gases
t _c	The residence time of the flame
S _b	The superior surface area of the site
$\vec{U}$	Local wind
U	Wind speed
L	Length
D	Diameter

d	Distance
H	Height of the stratum
h	Mean convection coefficient
k	Thermal conductivity
$\vec{n}$	The Normal to the terrain (ground)
Re	Reynolds number
Pr	Prandtl number
g	Gravitational acceleration
X	Sensibility Factor
M	Matrix of levels
Ros	Rate of spread of fire
m	Mass

Greek symbols

δ	Penetration length of radiation (optical length)
δ _{eff}	Effective penetration length of radiation
α	Volume fraction or compactness
σ	Surface area/volume ratio
ρ	Fuel particle density
ε	Emissivity
α _f	Tilt angle of the flame
X _R	Radiant fraction of the heat release within the flame
ν	Cinematic viscosity
κ	Total absorption coefficient of the flame.
β	Average extinction coefficient

Subscripts

k	Solid phase
ign	Ignition
WFF	Wet fine fuel
FDF	Fine dry fuel
fb	Fuel bed
∞	Ambient
str	Stratum
char	Char
f	Flame
ij	Between (i) and (j)
c	Combustion

The Propagation Model

The Network

The vegetation can be homogeneous and uniform or, in the opposite case, it can be sparse and randomly distributed on the landscape. In the first case, the network can be constructed on a square or hexagonal based-structure. In the second case, it is necessary to build a poly-disperse (sites with different sizes) amorphous network with a density that fits the real rate cover (figure 1).

The Macroscopic Model Of Combustion

The propagation of the flame front is driven by the combustion of thin particles, typically smaller than 6 mm (such as leaves, needles and twigs) (Luke and Mc Arthur 1977) . The thick elements are involved in burning in the back of the fire front. The physical problem is displayed on the figure 2. We assume that the control volume involved in the processes of thermal degradation and combustion is a cylinder with volume $V = \pi D^2 \delta$, where D is the diameter of the fuel site and δ ($\delta \leq H$) the mean free path of radiation through the fuel bed. Beyond δ , we assume that the medium no longer interacts with radiation. δ is connected to the surface area/volume ratio σ_k for fine fuel elements, and to the volume fraction of the solid phase α_k , by $\delta = 4/\sigma_k \alpha_k$ (De Mestre and others 1989; Butler 1993). The combustion model is based on solving the balance equations of mass and energy for a receptor site. It assumes that the receptor site receives radiation from the flame front and a convective flux of hot gases carried by the wind. However, it is cooled by radiation to the surroundings. We follow then the evolution of the temperature of this receptor site until it ignites. Hence, it contributes in turn to the fire spreading.

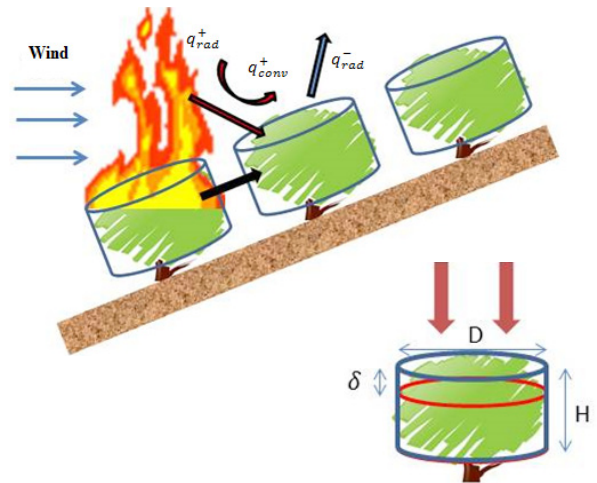


Figure 2—Heat transfers involved in the fire spreading and control volume of a site exposed to fire.

The combustion model assumes that the thermal degradation until ignition occurs according to four phases. The receptor site which is subject to the collective effect of sites constituting the fire front at time t is first heated to the temperature of vaporization of water, which is about 373K. It then dries at this constant temperature and inversely the fuel moisture content decreases until it becomes zero. Then, the dry fine fuel heats up to the temperature of pyrolysis. The last phase of pyrolysis is assumed to begin when the temperature of the dry fuel reaches the pyrolysis temperature. During this phase, if the quantity of pyrolysis products is sufficient, the site ignites. Thus, the energy conservation for a receptor site (j) exposed to N_{bc} burning sites at time t imposes:

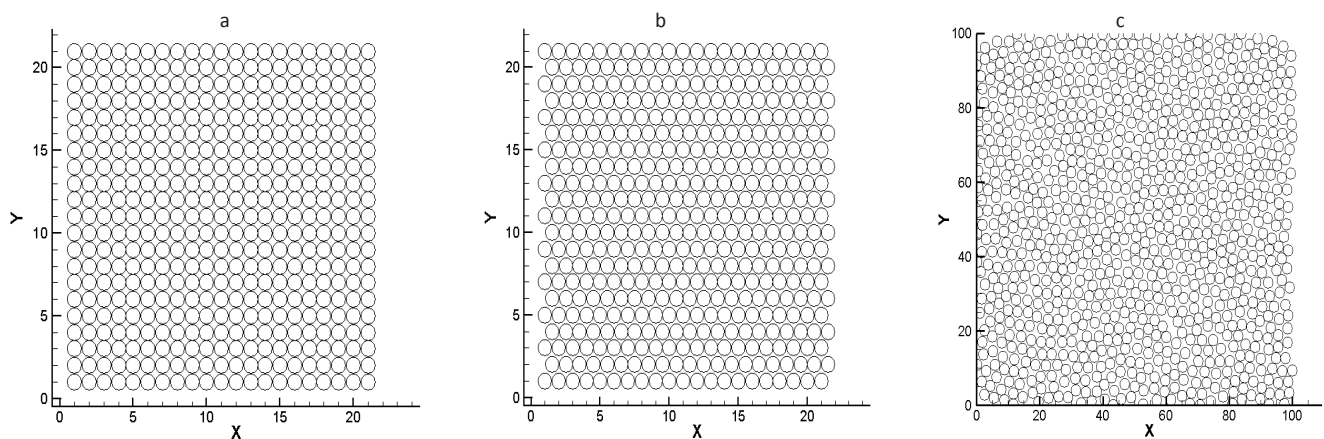


Figure 1—From left to right: examples of networks: a) Square based-structure network ,b) Hexagonal based-structure network and c) Amorphous network with a density of 70%.

$$\sum_{i=1}^{Nbc} [q_{rad}^+(i) + q_{conv}^+(i)]$$

$$= q_{rad}^-(j) + \left\{ \begin{array}{l} P_{WFF} C_{pWFF} \alpha_k \frac{dT(j)}{dt} \text{ for } T(j) < 373K \\ -P_{DFF} L_{vap} \alpha_k \frac{dFMC(j)}{dt} \text{ for } T(j) = 373K \\ P_{DFF} C_{pDFF} \alpha_k \frac{dT(j)}{dt} \text{ for } 373K < T(j) < T_{pyr} \\ -P_{DFF} L_{pyr} \alpha_k \frac{dFPPC(j)}{dt} \text{ for } T(j) = T_{pyr} \end{array} \right\}$$

where $q_{rad}^+(i)$ and $q_{conv}^+(i)$ correspond to the heat transfers due to flame radiation and due convection of hot gases carried by the wind. The term $q_{rad}^-(j)$ represents the radiative losses of site (j) to the surroundings. L_{vap} is the latent heat of vaporization of water, $FMC(j)$ the moisture content, defined as the mass of water divided by the mass of fine dry fuel, α_k is the volume fraction or compactness, $T(j)$ is the temperature of site (j), ρ and c_p are the density and the specific heat of the solid phase. DFF refers to dry fine fuel and WFF refers to wet fine fuel. L_{pyr} is the latent heat of pyrolysis and FPPC is the content of pyrolysis products of fine dry fuel. The initial FPPC is noted $FPPC_0$ and is defined by: $FPPC_0 = 1 - v_{char}$ where v_{char} is the initial content of char of dry fine fuel elements. Only the char remains if all the pyrolysis products are consumed. We assume moreover that the volume of fine fuel remains constant during the thermal degradation process. Each term of the energy balance equation is expressed below.

Radiation from the flame

The radiation received by the site (j) from the flame of site (i) is expressed as follows: $q_{rad}^+(j) = a_{fb} p_i N_{ij} / V_j$, where V_j is the control volume of site (j) and a_{fb} the absorptivity of the fuel bed.

In fact, a radiation model based on coupling the Monte Carlo method and the solid flame model is used. In the solid flame model, the flame is represented by a cylinder with only the surface that radiates. The Monte Carlo method is used to determine the number of quanta of energy launched from the site (i) on fire and which reached the receptor site (j). Each quantum carries an elementary power $p_i = P_i / n$ (in W), where n is the number of quanta emitted by each square meter of the flame and P_i is the emissive power of the flame attached to the site (i) on fire (see figure 3). The method is more detailed in Adou and others 2010. The radiation model is relatively simple but requires knowledge of the geometric properties of the flame and its emissive power. Moreover, the model takes into account not only the screening effect which manifests when another site on fire can exist between sites (i) and (j), but also the attenuation of the radiation by the atmospheric air layer. The average transmittance of the

air as a function of the thickness of the air layer crossed by radiation and of its relative humidity is determined using the Statistical Narrow banded Model of Malkmus (1967). The appendix A details how the attenuation is taken into account when using the Monte Carlo method.

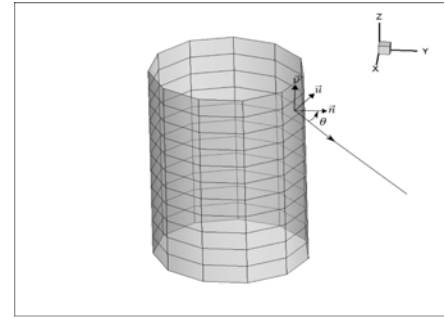


Figure 3—Energy beam leaving the flame mesh. The flame is assumed to have a cylindrical shape in the solid flame model.

Radiative losses

Radiative losses are a function of the emissivity of the fuel bed ϵ_{fm} , the Stefan-Boltzmann constant ($B = 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$) and the ambient temperature, T_∞ , as follows: $q_{rad}^-(j) = \epsilon_{fm} B (T_j^4 - T_\infty^4) / \delta$.

Convection of hot gases

The flame is not an impassable barrier and the wind, thanks to three-dimensional effects, can cross through it (Beer 1990). Thus, a site (j) located in the wake of a burning site (i) can be preheated by convection depending on the distance between the two sites, d_{ij} , according to a declining exponential law with a characteristic length approximately equal to three times the flame length (Koo and others 2005): $q_{conv}^+(i) = \frac{h}{\delta} (T_f - T_j) e^{-0.3 d_{ij} / L_f}$, where L_f and T_f are the length and the temperature of the flame, h is an average convection coefficient for a turbulent flow on a flat plate (Pagni and Peterson 1973) defined as: $h = 0.037 k Re^{0.8} Pr^{1/3} / d_{ij}$. The Reynolds number is defined by $Re = U d_{ij} / \nu$ based on

the tangential component of the local wind $\vec{U}$ to the terrain, $U = |\vec{n} \wedge (\vec{U} \wedge \vec{n})|$, where $\vec{n}$ is the normal to the terrain. The terms Pr , ν and k are respectively the Prandtl number, the cinematic viscosity, and the thermal conductivity of the air at the average temperature of $0.5(T_f + T_\infty)$.

The Input Parameters of the Propagation Model

The flame height in the presence of wind is evaluated by the correlation $H_f = H_{f0}(1 + 4 \frac{U^2}{gH_{f0}}) - 0.5$, where H_{f0} is the flame height without wind, deduced from the rate of heat released by combustion, $\dot{Q}$, and the source diameter, D , as $H_{f0} = 0.0148 \dot{Q}^{2/5} - 1.02 D$. The rate of heat released corresponds to the flaming combustion of volatile gases, such as: $\dot{Q} = (1 - \alpha_{char}) \frac{m_{DFF}''}{t_c} \Delta h_c S_b$ with $S_b = \pi D^2/4$.

As wind has little influence on flame length (Thomas and Pickard 1961; Thomas 1963; Nmira and others 2010), the flame length L_f is assumed to be equal to the luminous flame height with no wind. The emissive power per unit area of the flame is expressed as a function of the flame's area, $S_f = \pi D L_f$ and the radiated fraction to the exterior, χ_R , as: $P_f = \chi_R \dot{Q} / S_f$. We can then deduce the flame temperature assuming that the flame is a grey body with emissivity ϵ_f : $T_f = (P_f / \epsilon_f B)^{1/4}$ with $\epsilon_f = 1 - e^{-0.6 L_f}$.

Model Validation

Prescribed Burning

In the aim to validate the propagation model, I compared results with the prescribed burning conducted in Australia by the team of Cheney (Cheney and others 1993; Cheney and others 1998). The homogenous and continuous vegetation is mainly herbaceous (*Themeda Australis*) and covers a square plot of 200 m on each side. The wind direction is perpendicular to the left edge along which a line ignition of 175 m was performed.

The measured data are the following: $U = 4.6 \text{ m/s}$, $T_\infty = 303 \text{ K}$, $RH = 42\%$, $D = 3 \text{ m}$, $H = 0.51 \text{ m}$, $FMC_0 = 0.058$, $\chi_R = 0.3$, $t_c = 6 \text{ s}$, $\sigma k = 12240 \text{ m}^{-1}$, $m_{DFF}'' = 0.391 \text{ kg/m}^2$, $\alpha_{char} = 0.2$, $c_{PDFF} = 1480 \text{ J/kg/K}$, $P_{DFF} = 720 \text{ kg/m}^3$, $a_{fm} = 0.9$, $\epsilon_{fm} = 0.9$. On the figure 4, I have compared the fire contours predicted by the model (in red) and those measured after 56s (at left) and 86s (at right) of the ignition time (marked with square symbols). The comparison shows a very good agreement. After 86s, as mentioned by the authors, the comparison becomes impossible since the direction of wind has significantly changed.

In the example above, I should notice that the size of sites (strata) was conditioned by the flame height through the relation between the flame height and the size of a site as mentioned early, that can be considered as a means of calibration of the model.

I tested the radiative model (Adou and others 2010) and I realized that this model was insufficient and not able to correctly predict the fire behavior, at least in the case of this prescribed burn. It was necessary to introduce the convective contribution and the radiative losses which are non-negligible at the macroscopic scale of the fire front.

Real fire (Favone's Fire, Corsica 2009)

A second means of validation was conducted on a real fire that took place in Corsica in 2009. It is well documented in the work of Santoni and others 2011. This anthropogenic fire took place on July 8, 2009 near the village of Favone in Corsica. The coordinates of the ignition's point have been identified to $41^\circ 45' 57.08''$ in latitude, $09^\circ 23' 44.84''$ in longitude and 33 meters of altitude. The fire was detected very early, around 3:00 pm. Its spread was sustained by a Southwest wind with a magnitude of 18 km/h. A total area of about 30 ha was burned. The Favone fire presents an interesting case to study the effects of slope on fire behavior, since first the fire propagated uphill between points A and B, then downhill, between points B and C (figure 5).

The shape of the topography and the intensity of fire didn't allow a direct attack at the heading fire front. Four tanker planes worked first on the left flank to protect urban areas

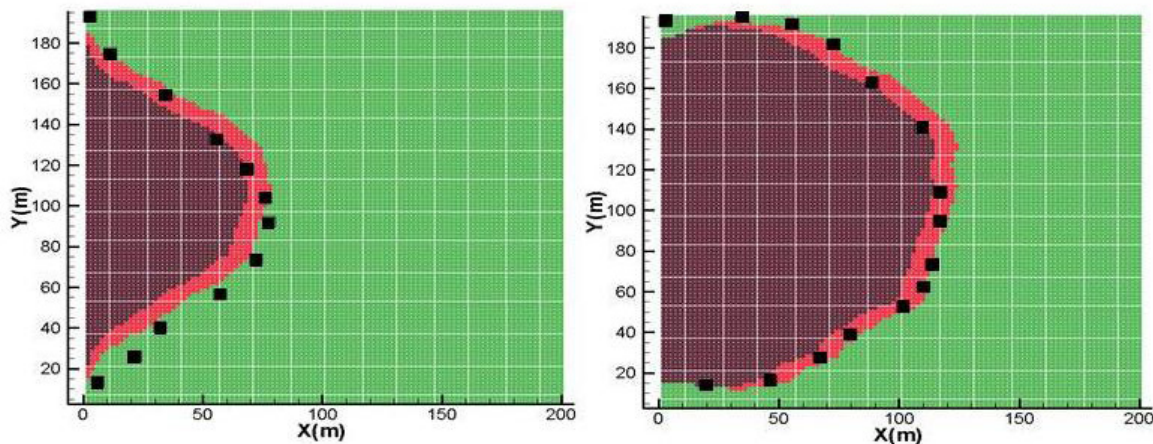


Figure 4—Fire contours predicted by the model (red line) and measured (■) after 56s (at left) and 86s (at right) of fire propagation.

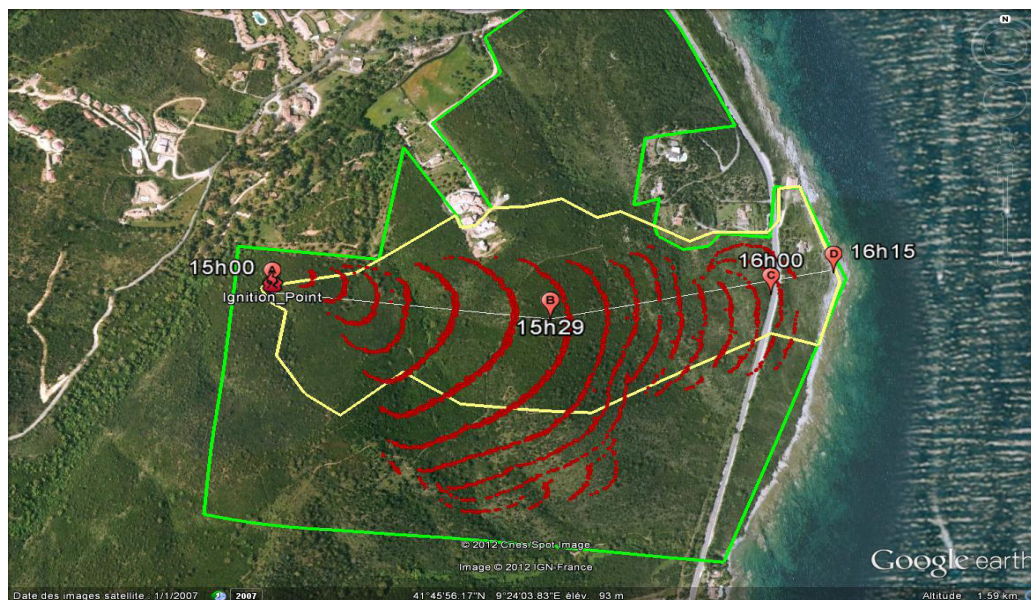


Figure 5—Fire contours (in red) simulated every 5 minutes. The yellow line delimits the area actually burned. The measured times of the fire front passage are indicated (Santoni and others 2011).

and allow firefighters to respond safely. They then attacked the right flank in the hope of pushing the fire towards the sea. The fire crossed the road at 4:00 pm, reached the sea at 4:15 pm and was finally under control at 7:00 pm. Two wind regimes dominate in this area. The first, most common, is an east wind due to the effect of the sea breeze, present all day on the east side of Corsica. The second is a west wind due to large-scale atmospheric circulation. It blows mostly at night and after 4:00 pm when the effect of the sea breeze fades. The predominant wind directions in the region are typically west and southwest. In the case of Favone's fire, wind speed was between 5 and 7 m/s and the wind direction varied between 240° and 270° . The air temperature ranged between 25°C and 27°C during the fire, and the relative humidity of the air ranged between 41 and 46 percent.

I compared the rate of spread and the burnt areas predicted by the model and those measured. The figure 5 shows the fronts of the simulated fire every five minutes. The yellow line marks the real area ravaged by fire.

The rate of spread observed (figure 5) in the rising edge between A and B is 16.8 m/min, although the ROS predicted by the model is 14 m/min. There is still an underestimation between points B and C when the fire goes down. The burned area is overestimated and presents a lateral expansion. The differences are mainly due to a change in the wind direction during the spread of fire, the fact that the intervention of the fighting means was not taken into account and so have not been modeled, or a bad estimation of some input parameters. That is why a sensitivity analysis was carried out.

Sensitivity Analysis

The aim of this sensitivity analysis is to study the effect of some input parameters of the model that are a priori the most influential on the rate of fire spread. The vegetation is represented by a square-based structure network of

sites distributed on a flat terrain of $100\text{m} \times 100\text{m}$ (the study area). Each site is a cylinder having a diameter of 2 m, and a height of 2.5 m. We have used here as input parameters of the fuel, the thermo-physical and the geometric properties of Kermes Oak (Drissi 2013). Six input parameters were retained for two reasons: the first is that quite large interval of values for each parameter is found in the literature and the second is that they present a big uncertainty when they are experimentally measured. These parameters are: the dry load of fine fuel elements m_{DFE}^n , the residence time of the flame t_c , the initial moisture content of the vegetal FMC_0 , the ignition temperature T_{ign} , the fraction radiated χ_R and the wind speed U .

The response is the rate of spread of fire between points A and B in the study area located respectively on the lines $Y = 20\text{m}$ and $Y = 80\text{m}$. Let's notice that a point's ignition was performed starting from the coordinates $X = 49\text{m}$ and $Y = 0\text{m}$ which corresponds to the middle of the bottom side of this domain. The sensitivity study is based on a two-level full factorial design. It consists in defining for each of the six retained parameter, a low level value and a high level value around a reference value. So $2^6 = 64$ simulations have been performed. The results are shown on a Pareto chart (figure 6): it describes how many of the selected factors and their interactions can affect the rate of fire spread. The factors with amplitude which exceeds the red line are the more significant parameters. According to this chart, the rate of spread is strongly influenced, in order, by: the wind speed, the load of dry fine fuel elements, the residence time of the flame, the fraction radiated, the initial moisture content and the ignition temperature. The second order effects are less significant like the interaction between the wind speed and the dry load. The third order terms have a negligible effect.

The negative or positive impact of the main parameters on the rate of spread can be evaluated through the chart of main effects. For example, as it can be seen on the chart,

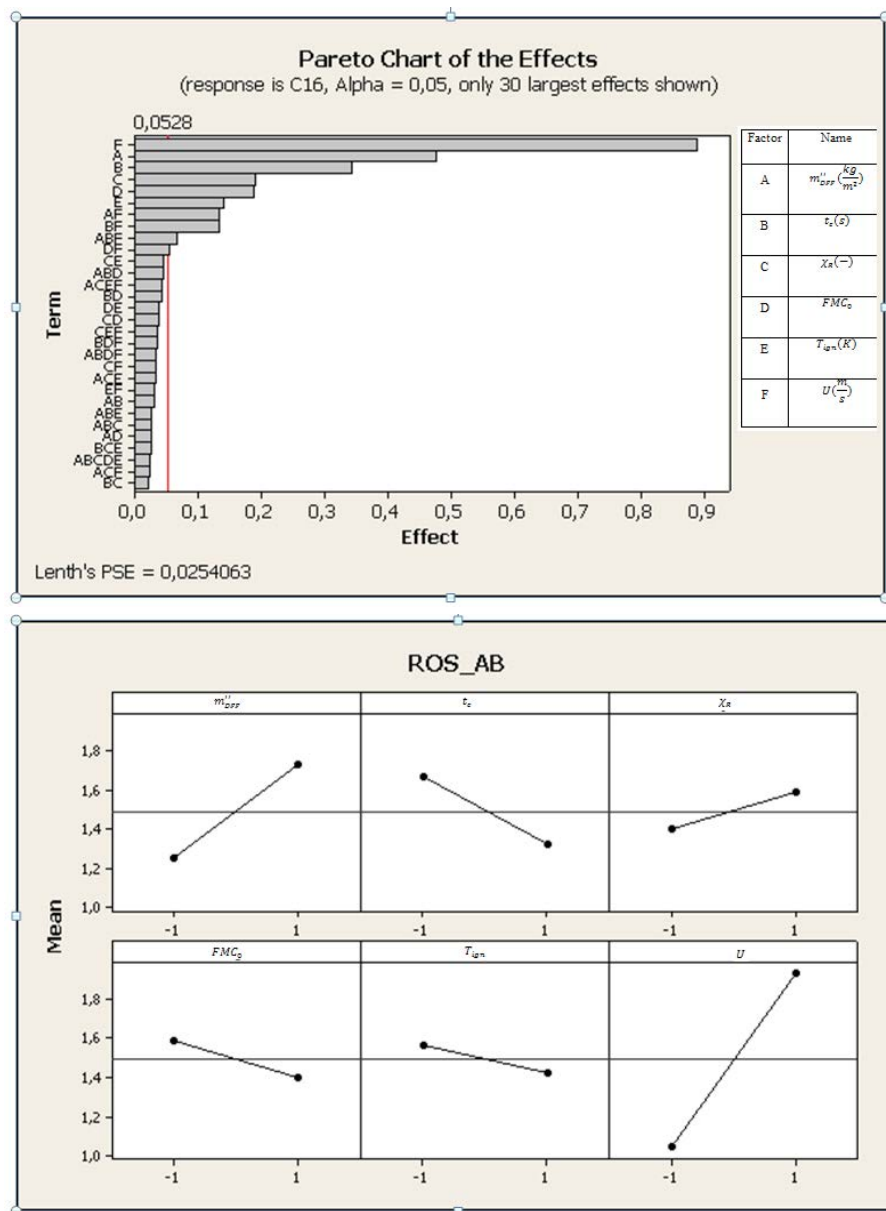


Figure 6—Pareto and main effects charts on the rate of fire spread.

when increasing the residence of flame time, the rate of spread decreases.

Conclusion

The hybrid model of propagation which combines the stochastic method of Monte Carlo and the semi-physical approach based on the resolution of the non-stationary energy equation was validated first on a prescribed burning that occurred in Australia and second on a real fire that took place in Corsica. There is an excellent fit in terms of rate of spread, fire contours and burned area. A full factorial plan of two levels allowed the classification and prioritization of the most influential parameters of the model as well as their interactions.

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