



Allometric equations for urban ash trees (*Fraxinus* spp.) in Oakville, Southern Ontario, Canada



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ABSTRACT

Tree growth equations are an important and common tool used to effectively assess the yield and determine management practices in forest plantations. Increasingly, they are being developed for urban forests, providing tools to assist urban forest managers with species selection, placement, and estimation of management costs and ecosystem services. This study describes the development of allometric equations for *Fraxinus americana* and *F. pennsylvanica* growing in Oakville, Canada. With data collected from 103 ash trees, five allometric models were tested to develop equations estimating diameter-at-breast-height (dbh), tree height, crown width and crown height, using age and dbh as explanatory variables. Mean annual growth rates are presented to demonstrate species growth performance and were not significantly different over the first 40 years of growth for the two species. Of all the tested random coefficient models for both species, the cubic with weight $1/x$ provided the best fit for estimating dbh from age. The best models for other parameters were the loglog for crown height from dbh, the quadratic for crown diameter from dbh, and the linear for tree height from dbh for *F. americana*. Model types showed more consistency for *F. pennsylvanica* with linear providing the best fit for crown diameter, crown height and tree height from dbh. The number of model types suggests the difficulty of fitting any single model to the vast array of conditions affecting plant growth in urban areas where management practices and environment can significantly influence tree size and growth. These models may be used to estimate the growth of ash tree populations in Oakville and communities with similar climate, soil, planting, and management environments.

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Introduction

Planting trees in urban settings must be a careful work that considers spatial and esthetic functions (Larsen and Kristoffersen, 2002; Urban Forest Innovations Inc. and Kenney, 2008). To better facilitate this, a systematic knowledge of the growth potential of individual species in relation to the local growing environment is required. Selecting the right tree for the right place to avoid damage to structures, costly tree maintenance, or even the removal of healthy-mature trees requires quantitative information on tree growth and mature size. Municipal and extension office publications, designed to educate the public and urban forest planners about how different tree species grow, provide information in qualitative, subjective terms such as slow-to-fast growth rate or small-to-large tree size. Because tree dimensions are not

exclusively determined by the species, but also local site and environmental conditions, these documents have limited applicability to many urban sites. Stoffberg et al. (2008) note the need for more precise information regarding the way tree species grow, and how such information may then be used to influence decisions on tree spacing and positioning in relation to man-made structures. In the case of urban forestry, however, the issue is less about precision than it is about the need for additional data which will lead to more precise growth estimates.

Growth equations, like those developed by Cao et al. (2002), exist for forest stands. Power laws used to investigate plant form (Niklas, 1994) and growth theories like those of Shinozaki and others (Chiba, 1990) have been extensively applied to quantitatively predict how forest tree dimensions change over time. However, numerous studies indicate that open grown trees grow and partition bole, branch, twig and leaf biomass differently compared to trees in managed stands (Wenger, 1984; Simonovic, 1991; Hahn, 1997; Zeng, 2003), strongly suggesting the need to develop models specific to open-grown trees. Municipal trees, particularly those growing along boulevards and roadways, tend to be open-grown.

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Table 1
Panel a lists the number random and arbitrarily-sampled trees for *Fraxinus americana* (FRAM) and *F. pennsylvanica* (FRPE). Panel b details the size range for measured trees and the final number of samples included (in brackets) in the analysis for each species, with age and dbh presenting higher sample numbers due to repeated sampling of tree cross-sections and 5-year intervals.

Species Code	Random Samples(<i>n</i>)	Arbitrary Samples (<i>n</i>)	Total (<i>n</i>)			
<i>Panel a</i>						
FRAM	17	10	27			
FRPE	38	38	76			
Species Code	No. of trees Sampled	Age (yr)	dbh (cm)	Height (m)	Crown Height (m)	Crown Diameter (m)
<i>Panel b</i>						
FRAM	27	5–52 [75]	1–48 [75]	3–20 [27]	1–18 [27]	1–14 [27]
FRPE	76	5–103 [277]	2–84 [277]	3–23 [73]	1–20 [72]	1–20 [74]

The development of equations to estimate dbh, height, crown diameter, and crown height for these species will enable arborists, researchers, and urban forest managers to more accurately forecast the effects of the trees in their immediate environment, the costs associated with tree maintenance, analyze alternative management scenarios, and determine the best management practices for sustainable urban forestry (Lukaszkiwicz and Kosmala, 2008; McPherson et al., 2000). This is especially important in an era where there is a need to “scrutinize expenditures often considered ‘nonessential’ such as the planting and management of municipal forests” (McPherson et al., 1999).

The field data measured to find correlations and develop allometric equations also form the basis for creating more realistic tree growth animations (Linsen et al., 2005; Brasch et al., 2007; Rudnick et al., 2007). Animated landscape planning and design applications offering users the option to “plant” specific tree species and observe them “growing” in situ would provide the user with an important tool for species selection, allowing planners, forest managers, arborists, and landscape architects to anticipate conflicts between mature trees and other infrastructure.

Several studies have been conducted to produce predictive models for trees growing in urban settings (e.g. in the United States: Peper et al., 2001a, 2001b; South Africa: Stoffberg et al., 2008; Denmark: Larsen and Kristoffersen, 2002; Poland: Lukaszkiwicz et al., 2005; Northeastern Italy: Semenzato et al., 2011), but equations have not been developed for urban species in Canada, more specifically for southern Ontario. Peper et al. (2001a,b) suggest that equations developed for species growing in one region cannot be used to model growth in another due to differences in environmental site conditions, tree maintenance practices, and the length of the growing season. However, the authors also suggest that the approach used to develop their models may be transferable to other regions.

The objective of this study was to develop allometric equations to estimate dbh from age, and total height, crown diameter, and crown height from dbh for populations of ash trees growing in a variety of urban environments in Oakville, Canada. Additionally, we wanted to compare the species to determine whether growth patterns differed.

Methods

Field measurements

The data were recorded in the town of Oakville, a municipality located in southern Ontario, Canada (43°27'N, 79°41'W), mainly characterized by clay and sandy loam soils. However, this parent material may or may not be consistently present in post development, urban sites. Like Toronto, Oakville is in plant hardiness

zone 6a with minimum temperatures averaging −23 °C to −21 °C (Agriculture and Agri-Food Canada 2000).

According to the most recent municipal street and active park tree inventory (completed in 2010), there are a total of 138,130 trees in Oakville (not including woodlots) of which 14,606 are ash. Green ash (*F. pennsylvanica*; species code FRPE) is the third most common species. Inventoried trees owned by the town are growing in medians, boulevards and yards in residential and commercial zones and public parks. Part of this database is made available to the public on the town's web site at <http://www.oakville.ca/residents/ash-tree-locator-map.html>. For this study, a total of 103 trees were selected for measurement, including 76 green ash and 27 white ash (*F. americana*; species code FRAM). From these, 38 green ash and 17 white ash were randomly selected. An additional 38 green ash and 10 white ash were arbitrarily selected (Table 1a). These were either scheduled for removal by the town's tree maintenance crews for safety reasons or removed due to high infestation with emerald ash borer (EAB). Although arbitrarily sampled, these trees were deemed representative of mature trees in each population by the forestry services manager for the town of Oakville.

The following quantitative and qualitative information were recorded for each of the sampled trees: dbh, tree height, crown height, crown diameter, tree-to-curb/street/sidewalk and sampled tree-to-closest tree distances, competition (or restriction) for space and sunlight, general condition, signs or symptoms of disease (in particular EAB), and any other factors that could contribute to tree stress and affect tree growth. Trees that were previously treated with the systemic insecticide TreeAzin to control Emerald Ash Borer (EAB) were excluded to prevent interfering with the treatment.

Tree height, crown diameter and crown height were measured using a TruPulse 200 model laser rangefinder to the closest 1.0 cm. Diameter-at-breast height was measured with a dbh tape to the closest 0.1 cm. Distances to infrastructure were measured with a measuring tape to the closest 1.0 cm. Competition for space and light (density) was recorded in qualitative terms, where competition for space was occurring if the distance between the flare of the trunk of the closest tree to the flare of the trunk of the sampled tree was less than at least half of the drip line of the sampled tree, and competition or restriction for sunlight was occurring if the branches of the neighbor tree(s) or man-made structures blocked any sunlight that the sampled tree could get otherwise. Foliage, crown and bole were assessed for general condition and recorded as good, fair or poor condition. The diseases were reported exclusively when the sampled tree displayed any visible signs or symptoms. Only the collected dendrometric data were used for this study while qualitative tree assessments were captured for potential use in developing planting plans addressing spacing, species competition, and health issues associated with various sites.

Coring and aging

Coring and cross-sectional analysis were required because Oakville did not have information on original planting dates or ages for these species. Tree cores provided a single tree age at the field-measured dbh, whereas cross-sections provided repeated measures of age-dbh data. These repeated measures were collected at 5-year increments for each cross-section analyzed.

The maximum dbh of 71 cm for the core-sampled trees was established based on the length of the increment borer's bit. The bit needed to be at least 2 cm longer than the tree's dbh to ensure that the pith would be included in the collected core; the minimum dbh of 20 cm was selected to avoid coring younger trees that would not provide as much tree growth information as older trees can and could potentially suffer a greater harm from coring. Following standard coring procedures, four cores were collected from each tree at breast height at approximately 90° intervals during the months of June and July of year 2010 (Grissino-Mayer, 2003). The depth of the drilling was half of the premeasured dbh, plus 1–2 cm to ensure collecting the pith or near the pith.

Collected cores were stored and mounted using procedures described in the literature (Orvis and Grissino-Mayer, 2002). Once samples were ready to be analyzed, WinDendro software (v. 2003b, Regent Instruments) was used to count and measure tree rings to within 0.001 mm. Seven samples that did not include tree piths were eliminated from analysis.

Cross-sections were collected by the maintenance crews from July to November, 2010, coming from trees ranging from 21 to 84 cm dbh as they were being removed throughout the town. The samples obtained from stumps at 8–15 cm above ground level were collected during January, 2012 from trees removed in November and December, 2011 in three areas of town with a high EAB density. For cross sections where piths were off-center, the longest and shortest paths were selected and measurements between rings averaged; otherwise, only one path was selected to count and measure ring widths.

There were a total of 58 repeated age and dbh measurements taken from the 10 arbitrarily-sampled white ash trees and 17 single age-dbh measurements taken from the 17 randomly-sampled trees for a total of 75 age-dbh samples. There were 239 repeated measures from 38 arbitrarily sampled green ash trees and 38 single measures from the 38 randomly sampled trees (Table 1b) for a total of 277 age-dbh samples.

Development of allometric equations

Prior to analysis, data points were plotted to examine potential outliers. There were no outliers for white ash, but several for green ash. These included 3 height, 4 crown height, and 2 crown diameter measurements that were data entry or measurement mistakes on removed trees. Table 1b shows the final sample numbers for each parameter. Following methods described by Martin and others (2012), those observations identified on residual plots that were greater than two units larger than the general spread of observations for that parameter were eliminated from analysis.

Previous research (Peper et al., 2001a,b; Stoffberg et al., 2008) indicated that the loglog equation, typically used to model growth in forest stands, best modeled tree size in urban settings as well. However, subsequent research for regional community tree guides in cities across the U.S. revealed that urban trees do not always follow the “norm” because growth may be impacted through management, particularly by changing height and crown dimensions through pruning. Various linear functions were tested for the tree guide species to determine best models (McPherson et al., 2000, 2010; Peper et al., 2007, 2009).

A blocking factor was not added in the models to account for the two sample types (randomly versus arbitrarily sampled trees) because the arbitrarily sampled trees were deemed representative of mature trees, and also because of the limited size of the sample. The purpose of this study is to produce reasonable growth estimates for tree planting and placement for two species under similar climate and management regimes. The randomly sampled trees tended to be older and grow larger than the non-randomly sampled trees. To estimate tree height, crown height and crown diameter from dbh, we tested the following models at four weights:

$$\text{Linear } y_i = a + bx_i + \frac{\varepsilon_i}{\sqrt{w_i}} \quad (1)$$

$$\text{Quadratic } y_i = a + bx_i + cx_i^2 + \frac{\varepsilon_i}{\sqrt{w_i}} \quad (2)$$

$$\text{Cubic } y_i = a + bx_i + cx_i^2 + dx_i^3 + \frac{\varepsilon_i}{\sqrt{w_i}} \quad (3)$$

$$\text{Loglog } \ln(y_i) = a + b \ln(\ln(x_i + 1)) + \frac{\varepsilon_i}{\sqrt{w_i}} \quad (4)$$

$$\text{Exponential } \ln(y_i) = a + bx_i + \frac{\varepsilon_i}{\sqrt{w_i}} \quad (5)$$

where y_i is the tree characteristic measured on tree i , a , b , c , and d , are constants to be estimated, x_i is the predictor variable for tree i , ε_i is the random error for tree i with $\varepsilon_i \sim N(0, \sigma^2)$, σ^2 , σ^2 is the variance of the random errors, and w_i is a known weight of one of the following forms:

$$\begin{aligned} w_i &= 1 \\ &= 1/\sqrt{x_i} \\ &= 1/x_i \\ &= x_i^2 \end{aligned}$$

Analysis was conducted using SAS 9.2 MIXED procedure (SAS 9.2 Cary, North Carolina, USA). Because of smaller sample sizes the second-order Akaike's information criterion (AIC_C) was used rather than AIC to compare and rank the models (Akaike, 1974). To obtain AIC_C values that would be comparable among all of the models, an additional analysis was performed for the loglog and exponential models. This additional analysis was only to obtain the comparable AIC_C values for those two models. The modified analyses concern the fitting of the following models:

$$\text{Loglog } \hat{y} \ln(y_i) = a^* + b^* \ln(\ln(x_i + 1)) + \frac{\varepsilon_i^*}{\sqrt{w_i}} \quad (6)$$

$$\text{Exponential } \hat{y} \ln(y_i) = a^* + b^* x_i + \frac{\varepsilon_i^*}{\sqrt{w_i}} \quad (7)$$

where $\hat{y}_i$ is the geometric mean of the y_i values. Multiplying by the geometric mean makes the AIC_C values comparable with the models where y_i is not transformed (Draper and Smith, 1998).

An (approximately) unbiased estimate for the loglog model

$$y_i = \exp \left(a + b \ln(\ln(x_i + 1)) + \frac{\varepsilon_i}{\sqrt{w_i}} \right) \quad (8)$$

that is found with

$$\hat{y} = \exp \left(\hat{a} + \hat{b} \ln(\ln(x + 1)) + \frac{\hat{\sigma}^2}{2w} \right) \quad (9)$$

$$\text{If } w = 1/\sqrt{x}, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b} \ln(\ln(x + 1)) + \frac{\partial^2 \sqrt{x}}{2} \right)$$

$$\text{If } w = 1/x, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b} \ln(\ln(x + 1)) + \frac{\partial^2 x}{2} \right)$$

$$\text{If } w = 1/x^2, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b} \ln(x+1) + \frac{\partial^2 x^2}{2} \right)$$

For the exponential model with

$$\ln \hat{y}|x = \hat{a} + \hat{b}x \quad (10)$$

we want an approximately unbiased estimate for

$$E(y) = E \left(\exp \left(a + bx + \frac{\varepsilon}{\sqrt{w}} \right) \right) \quad (11)$$

that is found with

$$\hat{y} = \exp \left(\hat{a} + \hat{b}x + \frac{\hat{\sigma}^2}{2w} \right) \quad (12)$$

$$\text{If } w = 1/\sqrt{x}, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b}x + \frac{\partial^2 \sqrt{x}}{2} \right)$$

$$\text{If } w = 1/x, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b}x + \frac{\partial^2 x}{2} \right)$$

$$\text{If } w = 1/x^2, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b}x + \frac{\partial^2 x^2}{2} \right)$$

The age-dbh data also included repeated measures on the arbitrarily removed (non-random), tree samples. We considered two potential statistical methods for addressing the repeated measures—auto-correlation or fitted models. If the trees were more densely sampled we could have applied auto-correlation, but age-dbh data were spread out with measurements taken every five years. We selected the random coefficients model because it would account for correlation over time. We fit five random coefficients models (Littell et al., 2006) at four weights, where one run assumed that the slopes and intercepts were the same for the “random” data and the “non-random data (repeated measures and single-tree measures data, respectively) and another assuming slopes and intercepts might be different:

$$\text{Linear } y_{ij} = (a + \alpha_i) + (b + \beta_i)x_{ij} + \frac{\varepsilon_{ij}}{\sqrt{w_i}} \quad (13)$$

$$\text{Quadratic } y_{ij} = (a + \alpha_i) + (b + \beta_i)x_{ij} + (c + \gamma_i)x_{ij}^2 + \frac{\varepsilon_{ij}}{\sqrt{w_{ij}}} \quad (14)$$

$$\text{Cubic } y_{ij} = (a + \alpha_i) + (b + \beta_i)x_{ij} + (c + \gamma_i)x_{ij}^2 + (d + \delta_i)x_{ij}^3 + \frac{\varepsilon_{ij}}{\sqrt{w_{ij}}} \quad (15)$$

$$\text{Loglog } \ln(y_{ij}) = (a + \alpha_i) + (b + \beta_i)\ln(\ln(x_{ij} + 1)) + \frac{\varepsilon_{ij}}{\sqrt{w_{ij}}} \quad (16)$$

$$\text{Exponential } \ln(y_{ij}) = (a + \alpha_i) + (b + \beta_j)x_{ij} + \frac{\varepsilon_{ij}}{\sqrt{w_{ij}}} \quad (17)$$

where y_{ij} is the tree characteristic measured on tree i on age x_{ij} , a , b , c , and d , are constants to be estimated, x_{ij} is the predictor variable for measurement j on tree i , ε_{ij} is the random error for measurement j on tree i with $\varepsilon_{ij} \sim N(0, \sigma^2)$, α_i , β_j , γ_i , and δ_i are random effects associated with each tree with variances σ_{α}^2 , σ_{β}^2 , σ_{γ}^2 , and σ_{δ}^2 , respectively (and corresponding covariances), σ^2 is the variance of

the random errors, and w_{ij} is a known weight of one of the following forms:

$$\begin{aligned} w_{ij} &= 1 \\ &= \frac{1}{\sqrt{x_{ij}}} \\ &= \frac{1}{x_{ij}} \\ &= \frac{1}{x_{ij}^2} \end{aligned}$$

As with the previous models, an additional analysis was performed for the loglog and exponential models. The following models were fitted:

$$\begin{aligned} \text{Loglog } \hat{y} \ln(y_{ij}) &= (a^* + \alpha^*) + (b^* + \beta^*) + \ln(\ln(x_{ij} + 1)) \\ &\quad + \frac{\varepsilon_{ij}^*}{\sqrt{w_{ij}}} \end{aligned} \quad (18)$$

$$\text{Exponential } \hat{y} \ln(y_{ij}) = (a^* + \alpha^*) + (b^* + \beta^*)x_{ij} + \frac{\varepsilon_{ij}^*}{\sqrt{w_{ij}}} \quad (19)$$

where $\hat{y}$ is the geometric mean of the y_{ij} values.

An (approximately) unbiased estimate for the mean for the loglog model

$$E(y|x) = E \left(\exp((a + b) \ln(\ln(x + 1))) + \frac{\varepsilon}{\sqrt{w}} \right) \quad (20)$$

$$\cong \exp(a + b \ln(\ln(x + 1)) + \frac{\sigma^2}{2w}) \quad (21)$$

is found with

$$\hat{y} = \exp \left(\hat{a} + \hat{b} \ln(\ln(x + 1)) + \frac{\partial^2}{2w} \right) \quad (22)$$

$$\text{If } w = 1/\sqrt{x}, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b} \ln(\ln(x + 1)) + \frac{\partial^2 \sqrt{x}}{2} \right)$$

$$\text{If } w = 1/x, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b} \ln(x + 1) + \frac{\partial^2 x}{2} \right)$$

$$\text{If } w = 1/x^2, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b} \ln(\ln(x + 1)) + \frac{\partial^2 x^2}{2} \right)$$

For the exponential model with

$$\ln \hat{y}|x = \hat{a} + \hat{b}x \quad (23)$$

we want an approximately unbiased estimate for

$$E(Y) = E \left(\exp \left(a + bx + \frac{\varepsilon}{\sqrt{w}} \right) \right) \quad (24)$$

and that is found with

$$\hat{y} = \exp \left(\hat{a} + \hat{b}x + \frac{\hat{\sigma}^2}{2w} \right) \quad (25)$$

$$\text{If } w = 1/\sqrt{x}, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b}x + \frac{\partial^2 \sqrt{x}}{2} \right)$$

$$\text{If } w = 1/x^2, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b}x + \frac{\partial^2 x^2}{2} \right)$$

Table 2The best models based on AICc for predicting y with coefficients and mean square error (MSE) for *Fraxinus americana* and *F. pennsylvanica*.

Species	Use x to predict y		Model	Weight	Estimated coefficients				MSE ^a
	x	y			a	b	c	d	
FRAM	Age	DBH ^b	Cubic	1/Age	1.3274	0.2480	0.0485	−0.0007	0.4176
FRAM	Age	DBH ^c	Cubic	1/Age	−33.201	6.3921	−0.1852	0.0017	0.4118
FRAM	Age	DBH ^d	Cubic	1/Age	1.3909	0.2902	0.0477	−0.0006	0.4118
FRAM	DBH	Crown diameter	Quadratic	1/DBH ²	0.7462	0.0701	0.0041		0.00001
FRAM	DBH	Crown height	LogLog	1/DBH ²	−0.7403	2.2646	0.0001		0.00005
FRAM	DBH	Tree height	Linear	1/DBH ²	2.1648	0.2743			0.00002
FRPE	Age	DBH ^b	Cubic	1/Age	−0.6385	0.8861	0.0163	−0.0002	0.4100
FRPE	Age	DBH ^c	Cubic	1/Age	−15.2205	2.9545	−0.0471	0.0002	0.4050
FRPE	Age	DBH ^d	Cubic	1/Age	−0.7775	0.8877	0.0149	−0.0001	0.4050
FRPE	DBH	Crown diameter	Linear	1/DBH ²	0.2483	0.2344			0.00002
FRPE	DBH	Crown height	Linear	1/DBH ²	0.2742	0.2434			0.00001
FRPE	DBH	Tree height	Linear	1/DBH ²	2.1213	0.2828			0.00003

^a For models predicting DBH from age, number listed under RMSE is the residual variance which must be multiplied by the square root of the age of the tree to obtain RMSE.

^b Combined data model.

^c Single-measurements model.

^d Multiple-measurements model.

$$\text{If } w = 1/x, \text{ then } \hat{y} = \exp \left(\hat{a} + \hat{b}x + \frac{\hat{c}x^2}{2} \right)$$

The models with the “best” fit as indicated by having the smallest AICc were selected to predict dbh from tree age and tree height, crown diameter and crown height from dbh for the two *Fraxinus* species. These were then used to “grow” the two species and compare their growth with one another. With the exception of age-dbh where repeated measures were taken from the same trees, these models represent a surrogate for growth based on measuring different trees represented across a range of dbh classes.

Results and discussion

Estimated coefficients and mean standard errors for all top models are given in Table 2. Final equation forms to use with the Table 2 coefficients are listed in Table 3. The random coefficients models for both species for age-dbh estimates are shown in Fig. 1a–d, with height, crown height and crown diameter models shown in Fig. 2. For all graphs, the selected models are fitted to the collected data points, and the estimated mean response (center line) and confidence interval for each model ($\alpha=0.05$) shown. Confidence intervals, rather than prediction intervals, are shown because our objective was to determine best allometric models for a population of trees rather than estimating individual tree size at a specific point in time. Prediction intervals would be much wider. Regardless, the results presented allow users to calculate either prediction or confidence intervals.

Age-DBH models

The top models for *F. americana* (FRAM) are cubic with 1/age as the weight. There was no significant difference between the

“assume equal slopes and intercepts” and the “assume potentially different slopes and intercepts” models with AICc weights of 0.33 and 0.40, respectively. No single-measurement data was collected for trees ranging from 3 to 20 cm dbh (approximately ages 7–24 in Fig. 1b). The fitted cubic single-measurement model demonstrates the problem of having a limited data set. The wide confidence intervals on each side of ages 24–37 suggest this model is less precise than the multiple-measurement model. It is also likely that the downward growth trend for the FRAM dbh is due to our limited sampling range associated with limited core lengths and lack of cross-sectional samples from larger trees rather than reflecting actual growth since Oakville's inventory lists 252 additional white ash ranging from 41 to over 100 cm dbh.

Comparing the combined data model in Fig. 2a with the multiple-measurement model in Fig. 1b differences are minor. In both cases, the bulk of measured data points lie within ages 5–40. At age 40 the multiple-measurement equation produces a dbh estimate only 1.7 cm larger than the combined data model in Fig. 1a. However, the confidence interval for the combined data model is slightly narrower than the multiple-measurement model, suggesting it may be the more precise.

For *F. pennsylvanica* (FRPE) the top models are again cubic, at the same 1/age weight as those for the FRAM. The AICc weights are 0.84 and 0.15 for different versus same intercepts and slopes, respectively, and there is more support for the “assume potentially different slopes and intercepts” model. Although growth may indeed slow significantly after age 40, as the single-measurement model in Fig. 1d shows, the bulk of the single-measurement data comes from a relatively small sample of trees ranging from 7 to 65 years old (3 to 71 cm dbh). The larger sample of single-measurement data for FRPE results in a more precise model than the FRAM single-measurement model, but the fitted slope remains steeper than the majority of the slopes for the multiple-measurement trees over the same age range. The confidence interval for the single-measurement model is also much wider after age 65 than for the multiple-measurement model, and there is a sudden growth rate reduction by age 40. As with FRAM, this growth reduction may be due to limited data collection, given that the city inventory lists 71 FRPE trees measuring over 100 cm dbh. Given more data with the multiple measures and the narrower confidence interval, the multiple-measurement model is the more precise of the two FRPE models shown in Fig. 1d. The combined data model (Fig. 1c) and multiple-measurement model (Fig. 1d) estimate nearly identical growth until age 40 where the latter produces mean dbh estimates that are increasingly larger with age compared

Table 3

The equation forms to use with models and coefficients listed in Table 2 to predict y from x (where x is age if predicting dbh or dbh if predicting height, crown height or crown diameter).

Model	Form
LogLog ($w_i=1/x$)	$\hat{y} = \exp \left(\hat{a} + \hat{b} \ln(\ln(x+1)) + \frac{\hat{c}x^2}{2} \right)$
LogLog ($w_i=1/x^2$)	$\hat{y} = \exp \left(\hat{a} + \hat{b} \ln(\ln(x+1)) + \frac{\hat{c}x^2}{2} \right)$
Linear	$\hat{y} = a + bx$
Quadratic	$\hat{y} = a + bx + cx^2$
Cubic	$\hat{y} = a + bx + cx^2 + dx^3$

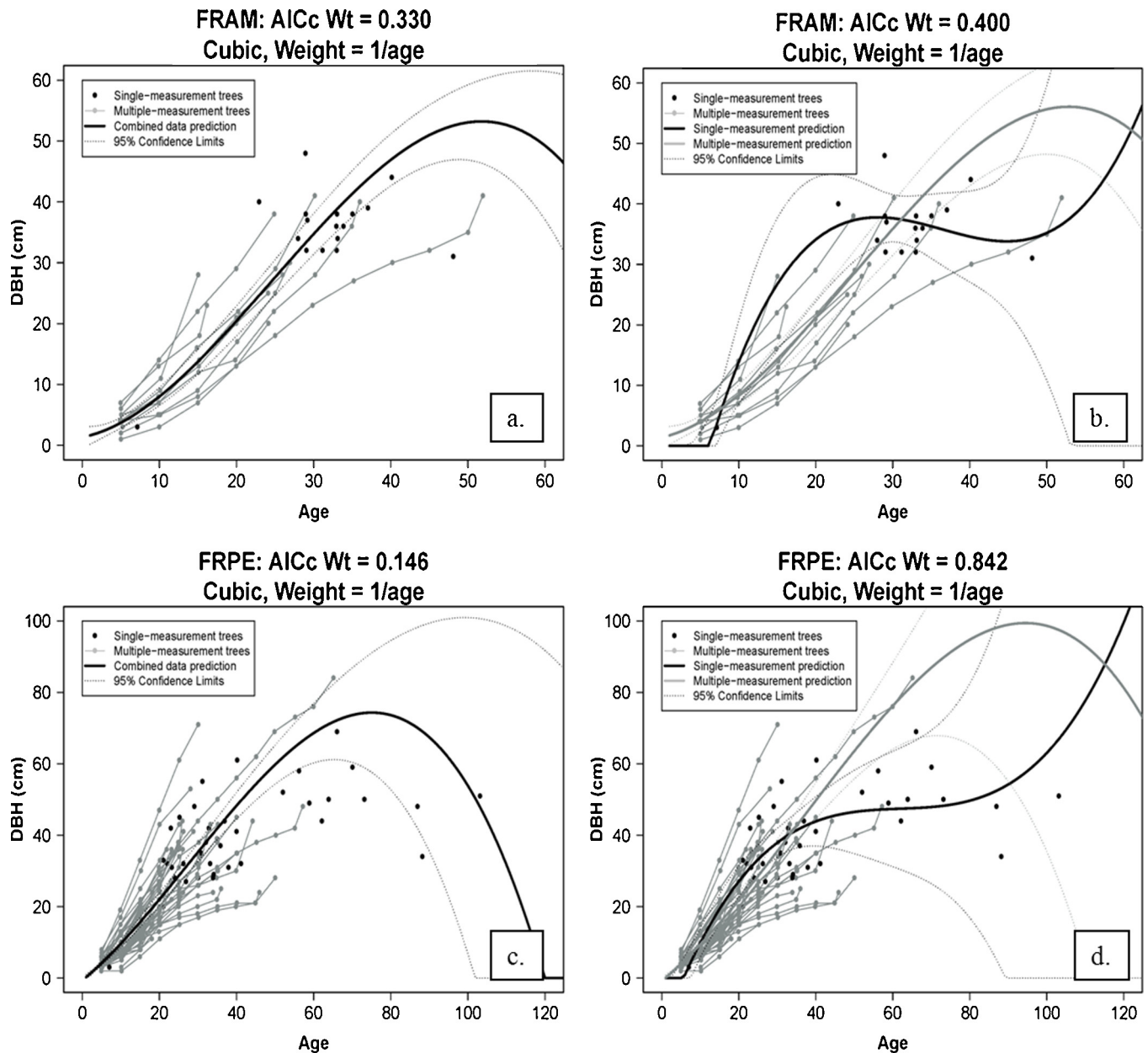


Fig. 1. (a)–(d) Top models for age-dbh models with dbh for those trees repeatedly measured connected by gray lines and single-sample trees shown with black circle symbols. Note slight difference between *Fraxinus americana* (FRAM) combined data prediction model (a) and multiple measurements model (b). Same models for *F. pennsylvanica* (FRPE) show more divergence after age 40.

to the combined data model (e.g., 8 cm greater by age 60). Given there is little difference between the combined data and multiple-measurement models over the first 40 years, and that the combined data model produces more conservative estimates after the age of 40, we suggest it may be the better model to apply when estimating ecosystem services associated with tree growth until more data for older trees can be collected.

Height, crown height, and crown diameter models

Fig. 2 shows the best models based on AICc fitted to the collected data points, with the estimated mean response (center line) and confidence interval for each model ($\alpha = 0.05$). The data represents both arbitrarily and randomly sampled trees with height, crown height, and crown diameter measured when the arbitrarily-sampled trees were removed.

Although fits are good with adjusted R^2 greater than 0.88 for all, the models for FRAM may not produce precise estimates for trees in the 5–21 cm dbh range because of the lack of measured data.

Green ash (FRPE), sampled across more size classes, shows little indication of diminishing height, crown height, or crown diameter dimensions at 80 cm dbh. The linear trend for all parameters may continue beyond the trees sampled since inventory lists numerous trees with larger dbh.

Limiting growth comparisons of both species to their shared data ranges from 20 to 40 years old. Table 4 shows negligible differences in mean size or growth through age 40 for bole diameter and total tree height. Trunk diameters grow about 1 cm annually through age 20, increasing to about 1.2 cm annually by age 30 and 1.6 cm by age 40. Estimated height, crown height and crown diameters are nearly identical through age 40 when some divergence appears to begin with green ash having greater crown height and

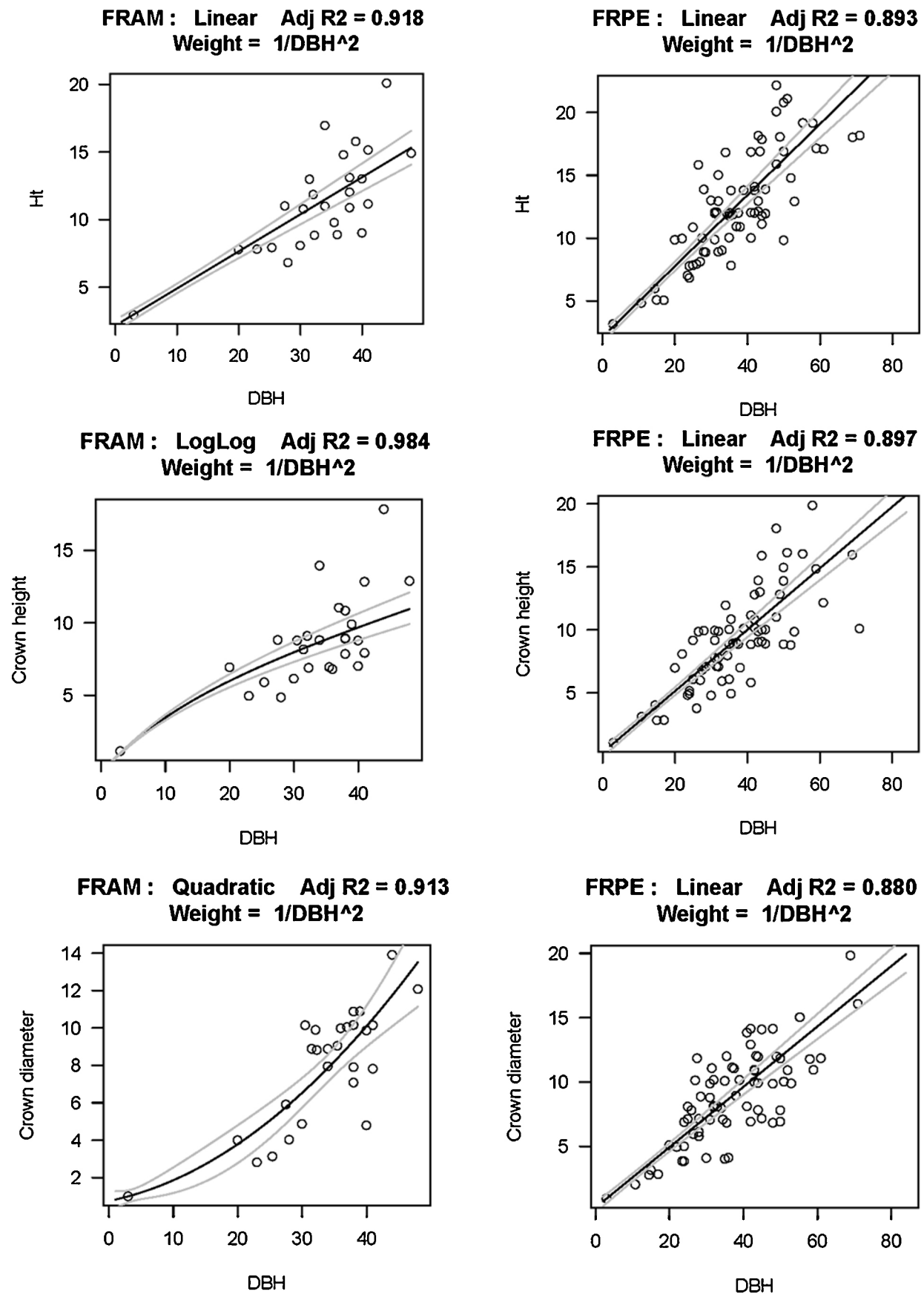


Fig. 2. Selected models for *Fraxinus americana* (FRAM) and *F. pennsylvanica* (FRPE), with the corresponding weight, and adjusted R^2 . Center lines represent predicted mean size (corrected mean after back transformation) along with upper and lower confidence intervals. DBH is in centimeters with other parameters reported in meters.

Table 4

Showing predicted size comparison at 10-yr intervals for both species.

	DBH (cm)			Height (m)			Crown height (m)			Crown diameter (m)		
Age	20	30	40	20	30	40	20	30	40	20	30	40
FRAM	21.9	36.1	47.9	8.2	12.1	15.3	6.3	8.8	10.3	4.2	8.6	13.5
FRPE	22.0	35.3	48.3	8.3	12.1	15.8	5.6	8.9	12.0	5.4	8.5	11.6

white ash greater crown diameter, less than 2 m difference in both cases. This may be due to genetic variation between the species or tree location. Street tree crown heights are influenced by setback distances, how far away they are planted from streets and major thoroughfares. Those closer to streets have crowns raised to allow traffic passage. Crown diameters may be influenced by planting distances and competition between tree crowns.

Conclusions

For both the green and white ash, all top models for age-dbh estimations were cubic, with 1/age weights, regardless of sample types. Based on test results, we recommend using the combined data model to estimate white ash dbh no further than age 40 due to thinness of data thereafter. Test findings support the different intercepts and slopes model for green ash; however, if using the models to estimate the values of ecosystem services into the future, we recommend applying the combined data model and extending estimates no further than age 60. Our reasoning for this is that although the combined data and multiple measurements models display similar estimated growth during their first 40 years, erring on the side of underestimating green ash growth after 40 may be preferable and the combined data model will provide more conservative estimates through age 60. Additional data are required for both species to determine whether these age-dbh models are adequate for older trees (past age 40) or if they will change significantly when the current data gaps are filled.

Similarly, measurement of additional trees in the 0–20 cm dbh range for both species, but particularly white ash, may require the fitting of new models to better estimate height, crown height and crown diameters of younger trees.

As mentioned previously, the inference level for this study was one city. Thus, it should not be assumed that the resultant models are transferable to climates and conditions differing significantly from the town of Oakville. Differences in growth rates across within and across regions may be associated with varied biotic and abiotic influences. In urban areas, it seems more likely that management differences (particularly pruning methods), extended drought or increased precipitation due to climate change, pest and disease infestation, soil types and compaction as well as soil pollutant loads, may change tree growth patterns, limiting future applications of these and other models. At this point, application of the models developed during this project should be limited to trees grown in similar conditions and locations as the sample trees.

The equations predicting dbh from age represent the only true growth models presented here since data points were derived by counting and measuring rings from cores and cross-sections for the same trees over time. The other models estimate size based on measurements of sample trees recorded at a single point in time. Adding a spatial or temporal stochastic structure to those predictive models by setting up and re-measuring permanent plots containing the trees that were cored for this project would likely improve future projections (Fox et al., 2001).

What perhaps is of more importance in this study is the range of model types required to best fit the collected data. Urban forest growth modeling remains an emerging science with little quantitative knowledge and data available. A single community's urban forest often grows across a vast range of environmental and

management influences. Based on the results of this study, and until more data is available, any assumption that tree size and growth modeling will mimic a single form appears faulty.

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