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Maximum Likelihood Estimation of the Parameters of a Bivariate Gaussian–Weibull Distribution from Machine Stress-Rated Data

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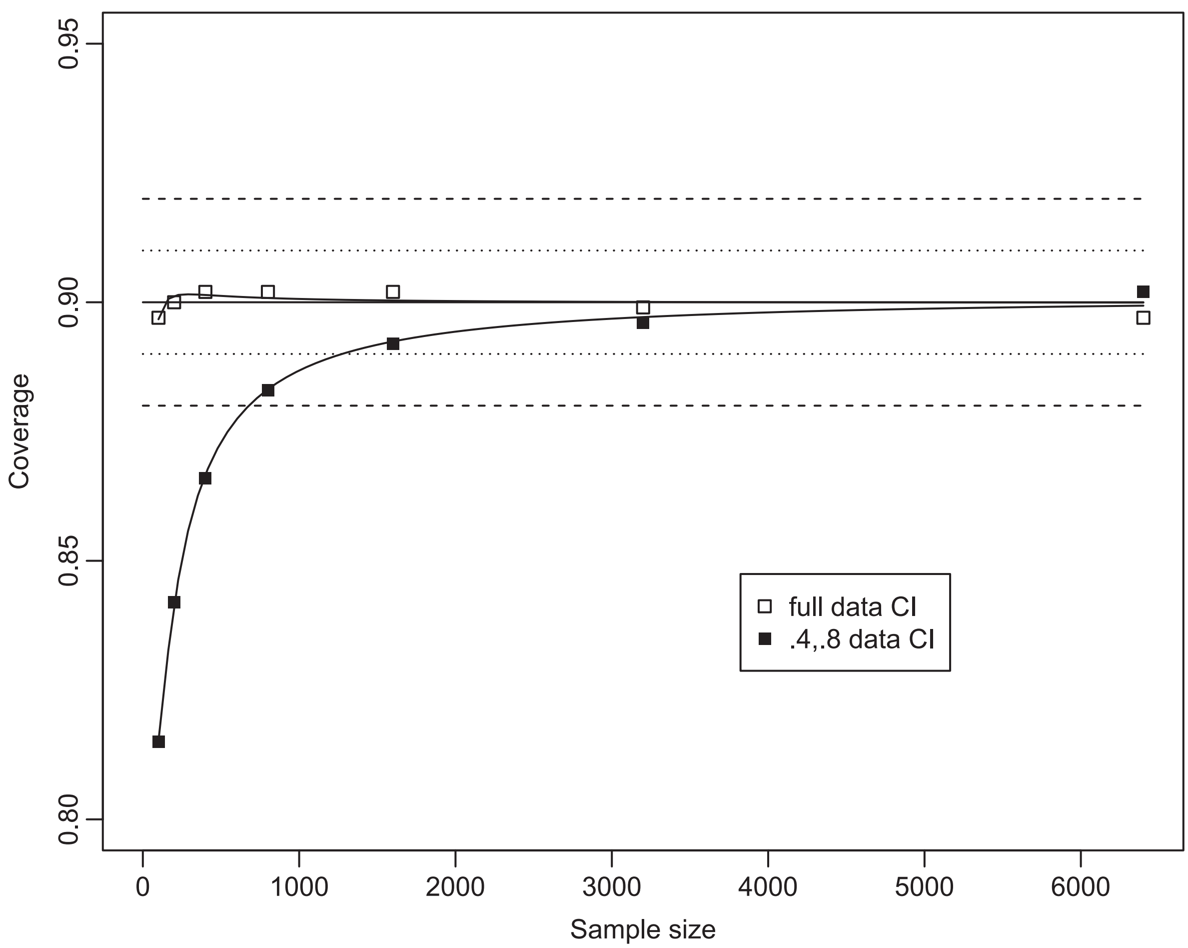

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Abstract

Two important wood properties are stiffness (modulus of elasticity, MOE) and bending strength (modulus of rupture, MOR). In the past, MOE has often been modeled as a Gaussian and MOR as a lognormal or a two- or three-parameter Weibull. It is well known that MOE and MOR are positively correlated. To model the simultaneous behavior of MOE and MOR for the purposes of wood system reliability calculations, a 2012 paper by Verrill, Evans, Kretschmann, and Hatfield introduced a bivariate Gaussian–Weibull distribution and the associated pseudo-truncated Weibull. In that paper, they obtained an asymptotically efficient estimator of the parameter vector of the bivariate Gaussian–Weibull. This estimator requires data from the full bivariate MOE,MOR distribution.

In practice, such data are often not available. Instead, in some cases "Machine Stress-Rated" (MSR) data are available. An MSR data set consists of MOE,MOR pairs, where a pair is accepted into the data set (a piece of lumber is accepted) if and only if the MOE value lies between predetermined lower and upper bounds. For such a data set, the asymptotically efficient methods appropriate for a full data set cannot be used. In this paper we present an approach that is effective for MSR data.

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Keywords: reliability, modulus of rupture, modulus of elasticity, normal distribution, Weibull distribution, bivariate Gaussian–Weibull distribution, bivariate normal–Weibull distribution, pseudo-truncated Weibull distribution, likelihood methods, machine stress-rated data, MSR data

Contents

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1 Introduction

Two important wood properties are stiffness (modulus of elasticity or MOE) and bending strength (modulus of rupture or MOR). In the past, MOE has often been modeled as a Gaussian and MOR as a lognormal or a two- or three-parameter Weibull (for example, ASTM 2010a, Evans and Green 1988, Green and Evans 1988).

Design engineers must ensure that the loads to which wood systems are subjected rarely exceed the systems' strengths. To this end, ASTM D 2915 (ASTM 2010a), and ASTM D 245 or ASTM D 1990 (ASTM 2010b,c) describe the manner in which "allowable properties" are assigned to populations of visually graded structural lumber. In essence, an allowable strength property is calculated by estimating a fifth percentile of a population (actually a 95% content, lower, 75% tolerance bound) and then dividing that value by "duration of load" (aging) and safety factors. The intent is that the population can be used only in applications in which the load does not exceed the allowable property. Of course there are stochastic issues associated with variable loads, uncertainty in estimation, and the division of a percentile with no consideration of population variability. Thus, this is not an ideal approach to ensuring reliability of wood systems. However, it is the currently codified approach.

To apply this approach, one must obtain estimates of the fifth percentiles of MOR distributions. Currently, one method for obtaining estimates involves fitting a two-parameter Weibull distribution to a sample of MORs. To obtain this fit, either a maximum likelihood approach or a linear regression approach based on order statistics is permitted under ASTM D 5457 (ASTM 2010d).

Unfortunately, these methods are often applied to populations that are not really distributed as two-parameter Weibulls. For example, in the United States, construction grade 2 by 4's are often classified into visual categories—select structural, number 1, number 2—or into machine stress-rated (MSR) grades. In the case of MSR grades, MOE boundaries are selected, MOE is measured nondestructively, and boards are placed into categories based upon the MOE bins into which the boards fall. Because MOE and MOR are correlated, bins with higher MOE boundaries also tend to contain board populations with higher MOR values. The fifth percentiles of these MOR populations are sometimes estimated by fitting Weibull distributions to these populations. Upon reflection, statisticians and reliability engineers recognize that this poses a problem. Even if the full population of lumber strengths were distributed as a Weibull, we would not expect that subpopulations formed by visual grades or MOE binning would continue to be distributed as Weibulls.

In fact, such a subpopulation is not distributed as a Weibull. Instead, if the full joint MOE–MOR population were distributed as a bivariate Gaussian–Weibull, the subpopulation would be distributed as a “pseudo-truncated Weibull” (PTW). Verrill et al. (2012a) obtained the distribution of a PTW and showed how to obtain estimates of its parameters by using asymptotically efficient methods to fit a bivariate Gaussian–Weibull to the full MOE–MOR distribution.

However, currently, full MOE–MOR data sets are generally not available. Instead, one might have, for example, a sample of number 2 data or a sample of MSR data.

In this paper, we use computer simulations to investigate the properties of maximum likelihood estimators of the parameters of a bivariate Gaussian–Weibull population (and the corresponding PTW) that are based on MSR-type data. We note that we do not provide a rigorous mathematical proof that our estimators are asymptotically efficient. However, our simulation results strongly suggest that our estimators do indeed have the desired optimality properties.

Our simulation results are described in Section 2. In Section 3 we describe a Web-based computer program that calculates estimates of the parameters of a bivariate Gaussian–Weibull distribution (and thus of a PTW distribution) from MSR-type data.

In the course of performing the computer simulations, we also found that if a full bivariate MOE–MOR population is truly bivariate Gaussian–Weibull, we can better characterize the properties of an associated MSR population and the corresponding PTW distribution (and thus probabilities of failure) by working with a sample of n observations from the full bivariate Gaussian–Weibull population than by working with n observations from the MSR population. This result might be counterintuitive to some readers. We discuss this result and try to make it more intuitive in Section 2.1.

As an aside, we remark that the bivariate Gaussian–Weibull distribution has uses other than as a generator of PTWs. For example, engineers who are interested in simulating the performance of wood systems must begin with a model for the joint stiffness/strength distribution of the members of the system. Provided that we are considering the *full* population, a Gaussian–Weibull is one possible model for this joint distribution.

Bivariate Gaussian–Weibull distributions first appeared in the literature in Verrill et al. (2012a). Gumbel (1960), Freund (1961), Marshall and Olkin (1967), Block and Basu (1974), Clayton (1978), Lee (1979), Hougaard (1986), Sarkar (1987), Lu and Bhattacharyya (1990), Patra and Dey (1999), Johnson et al. (1999), and others had previously investigated bivariate Weibulls.

We note that the bivariate Gaussian–Weibull distribution discussed in Verrill et al. (2012a,b; 2013; 2014) and the current paper is not the only possible bivariate distribution with Gaussian and Weibull marginals. In essence we begin with a “Gaussian copula”—a bivariate uniform distribution generated by starting with a bivariate normal distribution and then applying normal cumulative distribution functions to its marginals. However, there is a large literature on alternative copulas (multivariate distributions with uniform marginals). See, for example, Nelsen (1999) and Jaworski (2010). (Also see Wang et al. (2008) for an application of copulas to joint models of tree heights and diameters.) These alternatives would lead to alternative bivariate Gaussian–Weibulls. Ultimately, the test of the usefulness of our proposed version of a Gaussian–Weibull for a particular application will depend on the match between the theoretical distribution and data. Still, we believe that the ability to fit the version discussed in the current paper represents a useful step in the construction and evaluation of bivariate Gaussian–Weibull distributions.

2 Simulations of Gaussian–Weibull Fits

In Verrill et al. (2012a) we found that the joint probability density function of the proposed Gaussian–Weibull was

$$f(x, w; \mu, \sigma, \rho, \gamma, \beta) \equiv \gamma^{\beta}\beta w^{\beta-1}\exp\left(-(\gamma w)^{\beta}\right) \times \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma\sqrt{1-\rho^2}}\exp\left(-\left(\frac{x-\mu}{\sigma}-\rho y\right)^2/(2(1-\rho^2))\right) \tag{1}$$

where $x \in (-\infty, \infty)$ is the Gaussian value, $w > 0$ is the Weibull value, μ and σ are the mean and the standard deviation of the marginal Gaussian, ρ is the generating correlation, γ and β are the inverse scale and the shape of the marginal two-parameter Weibull (we assumed that $\beta > 1$ in our development),

$$y = \Phi^{-1}\left(1 - \exp\left(-(\gamma \times w)^{\beta}\right)\right) \tag{2}$$

and Φ denotes the N(0,1) cumulative distribution function. (In figures 1 – 9 of Verrill et al. (2012a), we provide contour plots of our version of a bivariate Gaussian–Weibull distribution for coefficients of variation equal to 0.35, 0.25, and 0.15, and generating correlations equal to 0.5, 0.7, and 0.9.)

In section 4 of the 2012a paper, we noted that the appropriate joint probablity density function of the corresponding truncated (on the Gaussian) bivariate Gaussian–Weibull distribution is given by

$$f_{\mathrm{tr}}(x, w; \mu, \sigma, \rho, \gamma, \beta) = f(x, w; \mu, \sigma, \rho, \gamma, \beta)/\left(\Phi((c_{\mathrm{u}}-\mu)/\sigma) - \Phi((c_{\mathrm{l}}-\mu)/\sigma)\right) \tag{3}$$

for x between c_{l} and c_{u}, and 0 elsewhere. In the MSR case, c_{l} and c_{u} are the (known) limits on the permitted MOE values.

In section 5 of the 2012a paper, we demonstrated that the univariate probablity density function of the PTW associated with this truncated (on the Gaussian) bivariate Gaussian–Weibull is

$$\begin{aligned} f_{\mathrm{PTW}}(w) &= \frac{d}{dw}F_{\mathrm{PTW}}(w) \qquad (4)\\ &= \gamma^{\beta}\beta w^{\beta-1}\exp\left(-(\gamma w)^{\beta}\right) \\ &\quad \times \left[\Phi\left((c_{\mathrm{u}}-\mu)/\left(\sigma\sqrt{1-\rho^2}\right) - \rho y/\sqrt{1-\rho^2}\right)\right. \\ &\quad \left. - \Phi\left((c_{\mathrm{l}}-\mu)/\left(\sigma\sqrt{1-\rho^2}\right) - \rho y/\sqrt{1-\rho^2}\right)\right] \div \left[\Phi((c_{\mathrm{u}}-\mu)/\sigma) - \Phi((c_{\mathrm{l}}-\mu)/\sigma)\right] \end{aligned}$$

where

$$y = \Phi^{-1}\left(1 - \exp\left(-(\gamma w)^{\beta}\right)\right)$$

Note that we can characterize a univariate PTW (obtain estimates of its parameters and thus, for example, its fifth percentile) by obtaining estimates of the (same) parameters of either the corresponding truncated (on the Gaussian) bivariate Gaussian–Weibull (density given by Equation 3) or the corresponding full bivariate Gaussian–Weibull (density given by Equation 1).

In the 2012a paper we also proved that

$$\sqrt{n}\left(\begin{pmatrix}\hat{\mu}\\ \hat{\sigma}\\ \hat{\rho}\\ \hat{\gamma}\\ \hat{\beta}\end{pmatrix} - \begin{pmatrix}\mu\\ \sigma\\ \rho\\ \gamma\\ \beta\end{pmatrix}\right) \xrightarrow{D} \mathrm{N}(\mathbf{0}, I(\boldsymbol{\theta})^{-1}) \tag{5}$$

where n is the sample size, $\boldsymbol{\theta} \equiv (\mu, \sigma, \rho, \gamma, \beta)^T$, $\hat{\mu}$ and $\hat{\sigma}$ are one-step Newton estimators based on the bivariate Gaussian–Weibull theory (that is, the gradient and Hessian used to calculate the Newton step correspond to the first and second partials of the full Gaussian–Weibull likelihood) that start at the standard univariate normal maximum likelihood estimators of the mean and standard deviation of a Gaussian, $\hat{\gamma}$ and $\hat{\beta}$ are one-step Newton estimators based on the bivariate Gaussian–Weibull theory that start at the standard univariate maximum likelihood estimators of 1/scale and shape for a Weibull, $\hat{\rho}$ is a one-step Newton estimator based on the bivariate Gaussian–Weibull theory that starts at the $\sqrt{n}$-consistent estimator of ρ introduced in appendix B of Verrill et al. (2012a), and the elements of $I(\boldsymbol{\theta})$ are listed in appendix A of Verrill et al. (2012b). The $\stackrel{D}{\to}$ notation here means "converges in distribution to."

Ideally we would like to be able to prove a result similar to (5) when we are working with MSR data rather than with full bivariate Gaussian–Weibull data. However, to prove (5) we showed (for example) that our initial parameter estimates were $\sqrt{n}$-consistent, we calculated the elements of the information matrix $I(\theta)$ exactly, we proved that $I(\theta)$ is positive definite, and we were able to establish various regularity conditions that were needed to invoke theorem 4.2 in chapter 6 of Lehmann (1983). The proof becomes much more difficult when we are working with truncated Gaussian and PTW data. Thus, in the current paper we numerically optimize the truncated (on the Gaussian) bivariate Gaussian–Weibull likelihood function appropriate for MSR data (Equation 3) and establish via simulations that

$$\sqrt{n}\left(\begin{pmatrix}\hat{\mu}\\ \hat{\sigma}\\ \hat{\rho}\\ \hat{\gamma}\\ \hat{\beta}\end{pmatrix} - \begin{pmatrix}\mu\\ \sigma\\ \rho\\ \gamma\\ \beta\end{pmatrix}\right) \approx \mathrm{N}(\mathbf{0}, I(\hat{\boldsymbol{\theta}})^{-1}) \tag{6}$$

where $\hat{\boldsymbol{\theta}} \equiv (\hat{\mu}, \hat{\sigma}, \hat{\rho}, \hat{\gamma}, \hat{\beta})^T$, $\hat{\mu}$, $\hat{\sigma}$, $\hat{\rho}$, $\hat{\gamma}$ and $\hat{\beta}$ are the results of the numerical optimization, and the j,kth element of $I(\hat{\boldsymbol{\theta}})$ is

$$-\sum_{i=1}^{n} \frac{\partial^2 \log\left(f_{\mathrm{tr}}(x_i, w_i; \boldsymbol{\theta})\right)}{\partial\theta_j \partial\theta_k}|_{\hat{\boldsymbol{\theta}}}/n \tag{7}$$

where f_{tr} is the density given by Equation (3), x_i, w_i is the ith Gaussian–Weibull pair, and n is the sample size. (That is, $I(\hat{\boldsymbol{\theta}})$ is the approximate information matrix.) The values of the second partials in Equation (7) are described in Appendix A.

We argue that the approximation in Equation (6) is justified by our simulations. In particular we show that as sample sizes increase, the biases in our parameter estimates tend to zero. Further, our simulations demonstrate that as sample sizes increase, the actual coverages of confidence intervals based on Equation (6) approach the nominal coverages.

In Subsection 2.1 we discuss a non-intuitive fact illustrated by these simulations. In Subsection 2.2 we discuss the "mechanics" of the simulation. In Subsection 2.3 we discuss the results of the simulation.

2.1 Estimating a pseudo-truncated population from non-truncated data — Intuition

An important conclusion of this paper is that if we want to estimate the parameters and the fifth percentile of a population of MSR lumber, it is better to fit a sample of n observations from the corresponding *full* bivariate population of MOE/MORs than a sample of n observations from the population of MSR MOE/MOR pairs. (This assumes that the full population actually is a bivariate Gaussian–Weibull.)

This might not be intuitively clear. We will demonstrate that the claim is correct via simulations, but it is also useful to develop some intuition. This can be done by considering a simple regression analogy.

Suppose that two variables, y and x, are related by the equation

$$y_i = a + b \times x_i + \epsilon_i$$

where a and b are constants and the ϵ_i are independent random errors centered at 0. We often assume that

$$\epsilon \sim \mathrm{N}(0, \sigma^2)$$

That is, the random errors are normally distributed and have variance σ^2.

Now suppose that, in practice, we are interested only in (x, y) pairs for x's in the interval $[40, 80]$ (in analogy to MSR data with the MOE lying between two bounds, c_{l} and c_{u}). To model the relationship between x and y, we could obtain n (x, y) pairs with the x values lying between 40 and 80, and do a regression fit to obtain estimates of a, b, and σ. *Or* we could recognize that even though we will eventually be working just with a population for which x lies between 40 and 80, in fact, the $y \approx a + b \times x$ relationship holds for x's beyond the [40,80] range and we know (if we have taken the necessary statistics course) that we can obtain our best estimate of b by taking half of our observations at the bottom of the range of x's for which $y \approx a + b \times x$ holds and half at the top of that range. That is, even though we will eventually be exclusively working with a *population* in which $40 \leq x \leq 80$, we can obtain a better b estimate to predict things about the x, y relationship for this population by taking a *sample* that has x values that lie *outside* the range appropriate for the population.

Similarly, in practice we might be interested in an MSR bivariate population for which c_{l} (the lower MOE cutoff) and c_{u} (the upper MOE cutoff) correspond to, for example, the 40th and 80th percentiles of the MOE population. In this paper, we refer to this as a .4,.8 MSR population (or a .4,.8 truncated [on the Gaussian] bivariate Gaussian–Weibull population). From Equation 4, we can see that to estimate the fifth percentile of the corresponding MOR (a PTW), we need estimates of the parameters μ, σ, ρ, γ, and β. (Of course, we also need the known c_{l} and c_{u} values.) We can use maximum likelihood techniques and Equation (3) to obtain estimates of these parameters and the covariance structure of the estimates (needed to obtain a lower confidence bound on the fifth percentile of the PTW MOR population) from a sample of the .4,.8 MSR population. However, alternatively we can use Equation (1) and maximum likelihood techniques to obtain estimates of μ, σ, ρ, γ, and β, and the covariance structure of the estimates from a sample of the *full* (untruncated) bivariate Gaussian–Weibull population.

Just as we can do better in the linear regression example in estimating the b needed to predict the relationship between x and y for x constrained to lie between 40 and 80 by considering x values that lie below 40 and above 80, we assert that we can do better in the MSR case by considering MOE values that lie below c_{l} and above c_{u}. (Of course, this *assumes* that the full population is distributed as a bivariate Gaussian–Weibull.) In particular, we do better by drawing a sample from the whole bivariate Gaussian–Weibull MOE,MOR distribution than by taking a sample of only MSR data.

We quantify the advantage of the full distribution sample in Subsection 2.3.

2.2 Simulation "mechanics"

Each of the 28 simulation runs (4 correlations, 7 sample sizes) involved 10,000 trials. In each of these simulations the generating μ was set at 100, the generating σ was set at 20 (for coefficients of variation equal to 0.20), the generating β was set at 5.7974 (for coefficients of variation equal to 0.20), and the generating γ was set at a value that would yield a Weibull median of 100 (given the β value).

In the optimizations that were performed by the programs on *full* bivariate Gaussian–Weibull data sets, no constraints were placed on the μ estimate, the ρ estimate was constrained to lie within the interval $[-1, 1]$, the σ and γ estimates were constrained to be non-negative, and the β estimate was constrained to lie within the interval $[1, 50.59]$ (that is, the coefficient of variation was constrained to lie between 1 and 0.025).

In the optimizations that were performed by the programs on *truncated* (on one or both ends of the Gaussian data) bivariate Gaussian–Weibull data sets, the μ estimate was constrained to vary from its initial estimate by no more than a factor of 2 (it could be halved or doubled), the σ estimate was constrained to vary from its initial estimate by no more than a factor of 8, the ρ estimate was constrained to lie within the interval $[-1, 1]$, and the γ and β values were constrained to be non-negative.

In Appendix D these constraints are discussed in slightly greater detail.

Listings of the computer programs that were used to perform these simulations can be found at `http://www1.fpl.fs.fed.us/sim_tr_gauss_weib.html` and `http://www1.fpl.fs.fed.us/sim_left_tr_gauss_weib.html`.

A single instance of a bivariate Gaussian–Weibull was generated as follows: Obtain independent N(0,1)'s, X_1, X_2, via a Gaussian random number generator. Set $X = \mu + \sigma X_1$ and $Y = \rho X_1 + \sqrt{1-\rho^2} X_2$. Then X is distributed as a $\mathrm{N}(\mu, \sigma^2)$, Y is distributed as a N(0,1), and their correlation is ρ. Now let $U = \Phi(Y)$. Then U is a Uniform(0,1) random variable that is correlated with X. Finally, let $W = (-\log(1-U))^{1/\beta}/\gamma$. Then W is distributed as a Weibull with shape parameter β and scale parameter $1/\gamma$, and together X and W have our joint bivariate Gaussian–Weibull distribution.

To simulate MSR data we added an additional step. p_l (lower truncation probability level) and p_u (upper truncation probability level) values were chosen to obtain full and MSR data. We considered the five cases listed in Table 1:

Data type	p_l	p_u
full	0.0	1.0
MSR	0.1	0.9
	0.2	0.8
	0.4	0.8
	0.4	1.0

Table 1: p_l and p_u values

The $p_\mathrm{l} = .4$, $p_\mathrm{u} = .8$ case was chosen to roughly represent Number 2 data. The $p_\mathrm{l} = .4$, $p_\mathrm{u} = 1.0$ case was chosen to roughly represent Number 2 and better data.

Given a particular p_l, p_u combination, we drew X,Y values until we had obtained the necessary sample size. In the course of the draw, we rejected X values that lay below $c_\mathrm{l} \equiv \mu + \sigma \times \Phi^{-1}(p_\mathrm{l})$ or above $c_\mathrm{u} \equiv \mu + \sigma \times \Phi^{-1}(p_\mathrm{u})$.

We considered two types of confidence intervals on the parameters.

1. Simulation-based (sim) confidence intervals:

$$\hat{\theta} \pm z_{1-\alpha/2} \times \sigma_{\text{sim}} \tag{8}$$

2. Asymptotic theory-based confidence intervals:

$$\hat{\theta} \pm z_{1-\alpha/2} \times \sigma_{\text{th}}/\sqrt{n} \tag{9}$$

Here, θ denotes one of the five parameters (μ, σ, ρ, γ, β). $z_{1-\alpha/2}$ denotes the appropriate N(0,1) quantile. (For example, for a 95% confidence interval, $\alpha = 0.05$ and $z_{1-\alpha/2} = 1.96$.) $\hat{\theta}$ denotes the maximum likelihood estimate of the parameter obtained by maximizing Equation (1) in the full case or Equation (3) in the truncated cases.

σ_{th} denotes the square root of the appropriate element of the estimated asymptotic covariance matrix (obtained from the inverse of the estimated information matrix — see (7)). σ_{sim} is obtained from the simulation. It is given by

$$\sigma_{\text{sim}} = \sqrt{\sum_{i=1}^{N} \left(\hat{\theta}_i - \bar{\hat{\theta}}\right)^2 /(N-1)} \tag{10}$$

where N is the number of trials, $\hat{\theta}_i$ is the estimate of the parameter in the ith simulation trial, and $\bar{\hat{\theta}}$ is the average of the N $\hat{\theta}_i$'s.

For a given correlation/sample size condition, the actual coverage associated with the sim (for example) type of confidence interval was the fraction of trials in which $\hat{\theta} \pm z_{1-\alpha/2} \times \sigma_{\text{sim}}$ included the true θ value.

2.3 Simulation results

For a 0.20 coefficient of variation (for both the Gaussian and Weibull marginals), generating correlations of 0.5, 0.6, 0.7, and 0.8, (the "generating correlations" are approximately equal to the observed correlations between the Gaussian random variable and the Weibull random variable—see table 1 of Verrill et al. (2012b)), and sample sizes of 100, 200, 400, 800, 1600, 3200, and 6400, we performed simulations that permitted us to produce the following tables:

1. Tables $2,\mu - 2,\beta$. These five tables contain simulation estimates of the percent biases and percent root mean squared errors (rmse) of the parameter estimates. They demonstrate that both the percent biases and the percent rmses of the parameter estimates decline to very small values (at least in the full bivariate Gaussian–Weibull case) as sample sizes increase. They also demonstrate that full data sets yield *much* better parameter estimates than do .4,.8 data sets.

2. Tables $3,\mu - 3,\beta$. These five tables present ratios of the rmses of parameter estimates obtained from full bivariate–Gaussian data sets with the rmses of parameter estimates obtained from .1,.9, .2,.8, .4,.8, and .4,1 data sets. The tables also present the inverses of these ratios squared. These latter values are estimates of the relative sample sizes needed to obtain the same quality of result. Thus, for example, for μ estimates, the rmses for full data sets of a given size tend to be roughly 1/5 or less of the rmses of .4,.8 data sets of the same size, and for a .4,.8 data set to yield estimates of the parameters as good as those obtained from a full data set, it would generally have to contain 25 or more times as many observations (for μ estimates).

3. Tables 4,75,μ – 4,99,β. These 20 tables provide the actual coverages (in 10,000) trials of nominal 75%, 90%, 95%, or 99% confidence intervals on the parameters for sample sizes 100, 200, 400, 800, 1600, 3200, and 6400. The larger sample sizes (e.g., 800, 1600, 3200, 6400) were included in the simulation because the more highly truncated data sets (e.g., a .4,.8 data set) sometimes require sets this large for actual coverages to closely approach nominal coverages.

   In designing an experiment in which one wants to obtain confidence intervals on bivariate Gaussian–Weibull parameters, one should consult Table 4 to choose an adequate sample size. However, our Web program provides some protection against inadequate sample sizes at the analysis stage. It provides nominal 75%, 90%, 95%, and 99% simulation and theoretical confidence intervals on the parameters. However, it also provides on-the-fly simulation estimates of the actual coverages of these confidence intervals. (Details on the manner in which on-the-fly simulation estimates of coverages are calculated are provided in point 7 of Appendix C.) If these simulation estimates of coverages significantly diverge from the nominal coverages, then this fact should be reported and the simulation estimates of coverages should be used rather than the nominal coverages.

   We note that it is important to draw a distinction between two types of "needed sample sizes." We have been talking about the n's needed to ensure that we can trust nominal confidence levels. That is, we want the sample size that ensures that a confidence interval constructed to cover the true value of a parameter at least 95% (for example) of the time really does cover the parameter at least 95% of the time. This is distinct from a separate sample size issue. The separate issue is whether the sample size is large enough to ensure that a confidence interval is narrow enough or that our ability to detect a difference (statistical power) is large enough. It is quite possible that the n needed to ensure that actual confidence levels are close to nominal confidence levels could be as low as 15 (in the full case — see Verrill et al. (2012b)), while the n needed to ensure that confidence interval widths are sufficiently small could be much higher than 15. These are two separate issues. We do not consider the second issue in this paper.

4. Table 5 presents results from a more careful analysis of a portion of the results presented in Table 4.

   For both full and .4,.8 data sets, a nominal 90% confidence level, and ρ values 0.5, 0.6, 0.7, and 0.8, we used least squares to fit the interpolating model

$$\text{actual coverage} - \text{nominal coverage} = a_1/\sqrt{n} + a_2/n + a_3/n^{3/2} \tag{11}$$

   to the data. Here n denotes the sample size. Two examples of the data and the associated fits to the data are given in Figures 1 and 2.

   Given these fits, we could calculate the n needed to ensure that actual coverages lay in, for example, [.88,.92] (the "Wide" case). In this case, if a curve approached the horizontal 0.92 line from above, then we would obtain the needed n by using a nonlinear equation solver to solve

$$0.92 = 0.90 + a_1/\sqrt{n} + a_2/n + a_3/n^{3/2}$$

   for n where a_1, a_2, and a_3 were obtained from the least squares fit of model (11). If a curve approached the horizontal 0.88 line from below (as do the full and .4,.8 curves in both Figures 1 and 2), then we would obtain the needed n by using a nonlinear equation solver to solve

$$0.88 = 0.90 + a_1/\sqrt{n} + a_2/n + a_3/n^{3/2}$$

for n.

It is clear from Tables 4 and 5 that we need *much* smaller sample sizes to yield good confidence interval coverages of the parameters when we have a full bivariate Gaussian–Weibull data set than when we have a .4,.8 truncated (on the Gaussian) bivariate Gaussian–Weibull data set (that is, a MSR-type data set).

5. Table 6 presents the biases and rmses in estimates of the fifth percentile of a .4,.8 PTW *population* from fits of corresponding full bivariate Gaussian–Weibull, and .1,.9, .2,.8, .4,.8, and .4,1 truncated (on the Gaussian) bivariate Gaussian–Weibull *data sets*. The manner in which we obtain estimates of the fifth percentile of a PTW from estimates of its parameters is described in the first subsection of Appendix B. (Recall from the discussion surrounding Equations (1), (3), and (4) that we can obtain estimates of the parameters of a univariate PTW population from fits of data sets from the corresponding full bivariate Gaussian–Weibull or truncated (on the Gaussian) bivariate Gaussian–Weibull populations.)

   It is interesting to note that there is relatively little difference among the full, .1,.9, .2,.8, .4,.8, and .4,.1 data sets in the qualities of their estimates of the fifth percentile of the .4,.8 population even though there are large differences in the qualities of their estimates of the population parameters (as displayed in Tables 2 - 5). This illustrates the fact that maximum likelihood estimation focuses on matching predicted probabilities of observations with observed densities of observations rather than on matching true and estimated parameters.

6. Table 7 presents the actual coverages of one-sided, lower nominal 75% confidence intervals on the 5th percentile of the PTW associated with a .4,.8 MSR *population* from fits to full or .1,.9, .2,.8, .4,.8, .4,1 *samples* from the corresponding full or truncated (on the Gaussian) Gaussian–Weibull populations. (The manner in which confidence intervals on the 5th percentile of the PTW are obtained is described in Appendix B.)

7. Table 8 presents results from a more careful analysis of a portion of the results presented in Table 7.

   For both full and .4,.8 data sets, and for ρ values .5, .6, .7, and .8, we again used least squares to fit the interpolating model (11) to the data. Two examples of the data and the associated fits to the data are given in Figures 3 and 4.

   Given these fits, we could calculate the n needed to ensure that coverages lay in, for example, [.73,.77] (the "Wide" case). In this case, if a curve approached the horizontal 0.77 line from above (as does the .4,.8 curve in Figure 3), then we would obtain the needed n by using a nonlinear equation solver to solve

$$0.77 = 0.75 + a_1/\sqrt{n} + a_2/n + a_3/n^{3/2}$$

   for n where a_1, a_2, and a_3 were obtained from the least squares fit of model (11). That is, we would find the n at which the upper curved line in Fig. 3 intersected the upper horizontal line. If a curve approached the horizontal 0.73 line from below (as does the full data curve in Fig. 4), then we would obtain the needed n by using a nonlinear equation solver to solve

$$0.73 = 0.75 + a_1/\sqrt{n} + a_2/n + a_3/n^{3/2}$$

   for n.

It is clear from Tables 7 and 8 that we need much smaller sample sizes to yield good confidence interval coverages on the fifth percentile of the PTW when we have a full bivariate Gaussian–Weibull data set than when we have a .4,.8 truncated (on the Gaussian) bivariate Gaussian–Weibull data set.

Tables 7 and 8 also illustrate the fact that fairly large sample sizes are required to ensure that the actual coverages of nominal 75% confidence intervals on the fifth percentile of the PTW closely approach the nominal value.

8. Tables 9,.1,.9; 9,.2,.8; 9,.4,.8; 9,.4,1. These four tables are related to tables 6-9 of Verrill et al. (2013). They assume that we want to estimate the probability, $p_{\mathrm{Br,true}}$ that a piece of lumber drawn from a .4,.8 MSR population (approximately a No. 2 population) will break when it is subjected to a strain equal to its true fifth percentile divided by 2.1 (approximately, the population "allowable property"). In the simulations, for each of 10,000 samples we calculated the ratio of this true probability of breakage to a probability of breakage calculated from the parameters estimated from the sample. In the four versions of Table 9, we report the fraction of the time that these ratios lay in the intervals [0,.01], (.01,.02], (.02,.1], (.1,.2], (.2,.5], (.5,1), (1,2), [2,5), [5,10), [10,50), [50,100), [100,∞). Ideally, these ratios would be close to 1. Values below 1 indicate overly conservative estimates. Values above 1 indicate nonconservative estimates.

   These tables illustrate the fact that, in general, a full bivariate Gaussian–Weibull data set yields better estimates of the probability of breakage at the allowable property than do truncated (on the Gaussian) bivariate Gaussian–Weibull data sets. However, the left-truncated .4,1 data sets yielded estimates of the probability of breakage that were very similar to the estimates obtained from the full bivariate Gaussian–Weibull data sets. This suggests that truncated data sets that are truncated only on one side might be an acceptable alternative to full data sets. We plan to perform an additional set of simulations to investigate this possibility.

3 Web Program to Estimate the Parameters of a Bivariate Gaussian–Weibull from MSR Data

We developed a computer program that obtains maximum likelihood estimates of the parameters of a bivariate Gaussian–Weibull population (and the associated PTW) from a sample of truncated (on the Gaussian) bivariate Gaussian–Weibull data. That is, it fits MSR-type data. The program also returns nominal 75%, 90%, 95%, and 99% sim and th (Eq. (8) and (9)) confidence intervals on the parameters. Finally, it performs simulations to obtain estimates of the actual coverages of these intervals. Algorithmic details of the program are provided in Appendices C and D. The Web program can be run at `http://www1.fpl.fs.fed.us/fit_tr_gauss_weib.html`. The code for a standalone FORTRAN program that performs these same functions can be found at `http://www1.fpl.fs.fed.us/fit_tr_gauss_weib_code.html`.

In Figures 5–9 we provide screen shots of the Web program.

A user can choose to fit one of four types of data: full bivariate Gaussian–Weibull data, left and right truncated (on the Gaussian) bivariate Gaussian–Weibull data, left truncated (on the Gaussian) bivariate Gaussian–Weibull data, or right truncated (on the Gaussian) bivariate Gaussian–Weibull data.

Six fields on the web page (five fields in the untruncated case) need to be filled:

1. The user must specify a data file name. The data file must be a txt file that contains two columns. (By default, Notepad and Emacs create txt files. Wordpad and Word can be directed to create txt files.) The first column must contain the values from the *Gaussian* distribution. The second column must contain the corresponding values from the *Weibull* distribution. The user must have previously sent the data file by FTP to our Web server. Directions for doing this are provided at the "provide the data file" link near the top of the Web page.

2. The user can specify a results file name. Directions for retrieving the results file appear above the results field on the Web page. In fact, however, the user need not specify a results file as the results are displayed in tabular form after the execute button is clicked. These results can be printed or saved to the user's machine.

3. The user must specify the sample size (n in this paper). Currently, the program cannot handle sample sizes greater than 6400 observations. If this presents a problem for you, please contact Steve Verrill at sverrill@fs.fed.us.

4. If the data set is doubly truncated, the user must provide the bounds c_{l} and c_{u} on the Gaussian (MOE in our applications). If the data set is left truncated, the user must provide the bound c_{l}. If the data set is right truncated, the user must provide the bound c_{u}.

5. The user must specify the number of trials in the simulation. Currently, this cannot exceed 10000. If this presents a problem for you, please contact Steve Verrill at sverrill@fs.fed.us.

6. The user must provide an integer starting value for the random number generator. This starting value cannot exceed $2^{31} - 1 = 2147483647$.

After filling the six (or five) fields, the user clicks the execute button and the program runs. Results are then displayed in tabular form. The program produces results for four confidence levels—75%, 90%, 95%, and 99%. For each confidence level, it produces two tables. The first table (see, for example, Fig. 7) contains the maximum likelihood estimates of the five parameters together with simulation-based and asymptotic theory-based confidence intervals on those parameters. The second table (see, for example, Figs. 7 and 8) contains simulation estimates of the actual coverages (rather than the nominal 75%, 90%, 95%, or 99%) of the two types of confidence intervals. It also contains confidence intervals (based on the arcsin square root transformation) on these actual coverages. If simulation estimates of actual coverages differ significantly from nominal coverages, then this fact should be reported and the simulation estimates of coverages should be used rather than the nominal coverages.

Important The response *will not* be immediate. Because simulations are being run, there will be a delay before the results appear. An approximate formula for the number of seconds needed to perform the simulations is $(n/2) \times N/10000$ where n is the sample size and N is the number of trials. For example, the time needed to run 10000 trials of samples of size 200 is approximately 100 seconds. Given equal sample sizes, trials for less-truncated data sets (e.g., a .1,.9 data set) take less time to complete than trials for more-truncated data sets (e.g., a .4,.8 data set).

If you encounter problems while running this program, please contact Steve Verrill at sverrill@fs.fed.us or 608-231-9375. As of September 2014, the program is a beta program. That is, we have tried to be very careful in its development. However, it might still contain bugs. If you believe that you have encountered a bug, please contact us.

4 Summary

In the context of wood strength modeling, Verrill et al. (2012a) introduced a bivariate Gaussian–Weibull distribution and the associated PTW distribution. In that paper, we also developed asymptotically efficient estimators of the parameters of the bivariate Gaussian–Weibull.

In this paper, we discuss a maximum likelihood method for fitting truncated (on the Gaussian) bivariate Gaussian–Weibull data. This method can be used to fit machine stress-rated MOE-MOR data sets. We describe a Web-based computer program that implements the maximum likelihood estimation technique. We also discuss computer simulations that investigate the small sample properties of this technique.

In the course of conducting these computer simulations we found that full bivariate Gaussian–Weibull data sets of size n yield much better estimates of parameters and probabilities of breakage at allowable properties than do truncated (on the Gaussian) bivariate Gaussian–Weibull data sets of the same size. That is, provided that the full bivariate population really is a bivariate Gaussian–Weibull, it is much more efficient to work with data from the full population than with data from the truncated (MSR) population *even if we are interested in the allowable property of the MOR from the MSR population.*

This implies that it would be very worthwhile to investigate whether *full* bivariate MOE-MOR populations (as opposed to visual grade populations or MSR populations) really are bivariate Gaussian–Weibull. If they are, we should be assigning allowable properties of MSR grades (and, perhaps, visual grades) based on estimates of the 5th percentiles of (for example) .4,.8 populations calculated from parameter estimates obtained from fits of *full* population data sets.

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6 Appendix A—Partial Derivatives of $\log(f_{\mathrm{tr}}(x,w))$

Given equality (3), we have

$$\log(f_{\mathrm{tr}}(x,w)) = \log(f(x,w)) - \log\left(\Phi((c_{\mathrm{u}}-\mu)/\sigma) - \Phi((c_{\mathrm{l}}-\mu)/\sigma)\right) \tag{12}$$

Thus the partials of the logs of the probability density functions in the full and truncated (on the Gaussian) cases do not differ if they involve the parameters ρ, γ, or β. These partials are provided in appendix C of Verrill et al. (2012a).

Partials involving μ and/or σ are equal to the corresponding values in appendix C of Verrill et al. (2012a) plus the appropriate partials of

$$T \equiv -\log\left(\Phi((c_{\mathrm{u}}-\mu)/\sigma) - \Phi((c_{\mathrm{l}}-\mu)/\sigma)\right) \tag{13}$$

The first and second partials of T with respect to μ and σ are provided below.

From (13) we have

$$\frac{\partial T}{\partial \mu} = \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)}{\sigma\left(\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)} \tag{14}$$

$$\frac{\partial T}{\partial \sigma} = \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)}{\sigma\left(\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)} \tag{15}$$

$$\begin{aligned}\frac{\partial^2 T}{\partial \mu^2} &= \frac{\left(\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)^2}{\sigma^2\left(\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)^2} \\ &\quad + \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)}{\sigma^2\left(\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)}\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial^2 T}{\partial \sigma^2} &= \frac{-2\left(\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)}{\sigma^2\left(\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)} \\ &\quad + \frac{\left(\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)^2}{\sigma^2\left(\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)^2} \\ &\quad + \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right)^3 - \phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)^3}{\sigma^2\left(\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)}\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{\partial^2 T}{\partial \mu \partial \sigma} &= \frac{\left(\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)\left(\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)}{\sigma^2\left(\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)^2} \\ &\quad + \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right)^2 - \phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)^2}{\sigma^2\left(\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)} \\ &\quad - \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)}{\sigma^2\left(\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right) - \Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)}\end{aligned} \tag{18}$$

The corresponding partials in the *left truncated* case (the case in which $c_{\mathrm{u}} = \infty$) are

$$\frac{\partial T}{\partial \mu} = \frac{-\phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)}{\sigma\left(1-\Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)} \tag{19}$$

$$\frac{\partial T}{\partial \sigma} = \frac{-\phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)}{\sigma\left(1-\Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)} \tag{20}$$

$$\frac{\partial^2 T}{\partial \mu^2} = \frac{\phi^2\left((c_{\mathrm{l}}-\mu)/\sigma\right)}{\sigma^2\left(1-\Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)^2} - \frac{\phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)}{\sigma^2\left(1-\Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)} \tag{21}$$

$$\begin{aligned}\frac{\partial^2 T}{\partial \sigma^2} &= \frac{2\phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)}{\sigma^2\left(1-\Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)} + \frac{\phi^2\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)^2}{\sigma^2\left(1-\Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)^2} \\ &\quad - \frac{\phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)^3}{\sigma^2\left(1-\Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)}\end{aligned} \tag{22}$$

$$\begin{aligned}\frac{\partial^2 T}{\partial \mu \partial \sigma} &= \frac{\phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)}{\sigma^2\left(1-\Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)} + \frac{\phi^2\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)}{\sigma^2\left(1-\Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)^2} \\ &\quad - \frac{\phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\left((c_{\mathrm{l}}-\mu)/\sigma\right)^2}{\sigma^2\left(1-\Phi\left((c_{\mathrm{l}}-\mu)/\sigma\right)\right)}\end{aligned} \tag{23}$$

The corresponding partials in the *right truncated* case (the case in which $c_{\mathrm{l}} = -\infty$) are

$$\frac{\partial T}{\partial \mu} = \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)}{\sigma\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)} \tag{24}$$

$$\frac{\partial T}{\partial \sigma} = \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right)}{\sigma\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)} \tag{25}$$

$$\begin{aligned}\frac{\partial^2 T}{\partial \mu^2} &= \frac{\phi^2\left((c_{\mathrm{u}}-\mu)/\sigma\right)}{\sigma^2\Phi^2\left((c_{\mathrm{u}}-\mu)/\sigma\right)} \\ &\quad + \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right)}{\sigma^2\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)}\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{\partial^2 T}{\partial \sigma^2} &= \frac{-2\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right)}{\sigma^2\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)} \\ &\quad + \frac{\phi^2\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right)^2}{\sigma^2\Phi^2\left((c_{\mathrm{u}}-\mu)/\sigma\right)} \\ &\quad + \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right)^3}{\sigma^2\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)}\end{aligned} \tag{27}$$

$$\begin{aligned}\frac{\partial^2 T}{\partial \mu \partial \sigma} &= \frac{\phi^2\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right)}{\sigma^2\Phi^2\left((c_{\mathrm{u}}-\mu)/\sigma\right)} \\ &\quad + \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)\left((c_{\mathrm{u}}-\mu)/\sigma\right)^2}{\sigma^2\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)} \\ &\quad - \frac{\phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)}{\sigma^2\Phi\left((c_{\mathrm{u}}-\mu)/\sigma\right)}\end{aligned} \tag{28}$$

7 Appendix B — Obtaining an Estimate of the Fifth Percentile of a Pseudo-Truncated Weibull and an Approximate One-sided, Lower 75% Confidence Bound on the Fifth Percentile

7.1 Estimation of the 5th percentile

Our simulation program estimates the fifth percentile, $\xi_{.05}$, by numerically solving the nonlinear equation

$$\int_0^{\xi_{.05}} f_{\mathrm{PTW}}(w;\hat{\mu},\hat{\sigma},\hat{\rho},\hat{\gamma},\hat{\beta},c_{\mathrm{l}},c_{\mathrm{u}})dw = 0.05$$

for $\xi_{.05}$ where c_{l} and c_{u} are the known truncation values, $\hat{\mu}$, $\hat{\sigma}$, $\hat{\rho}$, $\hat{\gamma}$, and $\hat{\beta}$ are the maximum likelihood parameter estimates, and $f_{\mathrm{PTW}}(w;\mu,\sigma,\rho,\gamma,\beta,c_{\mathrm{l}},c_{\mathrm{u}})$ is the probability density function of a PTW:

$$\begin{aligned}
&\gamma^{\beta}\beta w^{\beta-1}\exp\left(-(\gamma w)^{\beta}\right) \qquad (29)\\
&\times\left(\Phi\left((c_{\mathrm{u}}-\mu)/\left(\sigma\sqrt{1-\rho^2}\right)-\rho y/\sqrt{1-\rho^2}\right)-\Phi\left((c_{\mathrm{l}}-\mu)/\left(\sigma\sqrt{1-\rho^2}\right)-\rho y/\sqrt{1-\rho^2}\right)\right)\\
&\div\left(\Phi((c_{\mathrm{u}}-\mu)/\sigma)-\Phi((c_{\mathrm{l}}-\mu)/\sigma)\right)
\end{aligned}$$

where

$$y=\Phi^{-1}\left(1-\exp\left(-(\gamma w)^{\beta}\right)\right)$$

(See section 5 of Verrill et al. (2012a) for the derivation of this density.)

7.2 Estimation of the one-sided, lower 75% confidence bound on the 5th percentile

In accordance with ASTM D1990, to calculate the allowable MOR property of a population, we must obtain a one-sided, lower 75% confidence bound on the fifth percentile of that population. This can be done via nonparametric methods. However, it is also currently permissible to use a parametric Weibull assumption. We argued in Verrill et al. (2012a, 2013, 2014) that a parametric PTW assumption (or an assumption of a mixture of PTWs) would be more appropriate.

To obtain an approximate parametric one-sided, lower 75% confidence bound on the fifth percentile of a PTW, we use the "delta method." That is we start with the simple linear Taylor series approximation:

$$\hat{\xi}_{.05}\approx\xi_{.05}+\begin{pmatrix}\frac{\partial\xi_{.05}}{\partial\mu} & \frac{\partial\xi_{.05}}{\partial\sigma} & \frac{\partial\xi_{.05}}{\partial\rho} & \frac{\partial\xi_{.05}}{\partial\gamma} & \frac{\partial\xi_{.05}}{\partial\beta}\end{pmatrix}_{|\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta})$$

where

$$\hat{\boldsymbol{\theta}}=\begin{pmatrix}\hat{\mu}\\ \hat{\sigma}\\ \hat{\rho}\\ \hat{\gamma}\\ \hat{\beta}\end{pmatrix}$$

and

$$\boldsymbol{\theta}=\begin{pmatrix}\mu\\ \sigma\\ \rho\\ \gamma\\ \beta\end{pmatrix}$$

This immediately yields

$$
\begin{aligned}
\mathrm{Var}(\hat{\xi}_{.05}) \;\approx\; & \begin{pmatrix} \frac{\partial \xi_{.05}}{\partial \mu} & \frac{\partial \xi_{.05}}{\partial \sigma} & \frac{\partial \xi_{.05}}{\partial \rho} & \frac{\partial \xi_{.05}}{\partial \gamma} & \frac{\partial \xi_{.05}}{\partial \beta} \end{pmatrix}_{|\boldsymbol{\theta}} \\
& \times \left(\mathrm{Cov}(\hat{\boldsymbol{\theta}})\right) \begin{pmatrix} \frac{\partial \xi_{.05}}{\partial \mu} & \frac{\partial \xi_{.05}}{\partial \sigma} & \frac{\partial \xi_{.05}}{\partial \rho} & \frac{\partial \xi_{.05}}{\partial \gamma} & \frac{\partial \xi_{.05}}{\partial \beta} \end{pmatrix}^{\mathrm{T}}_{|\boldsymbol{\theta}}
\end{aligned}
\tag{30}
$$

In our program, in accordance with maximum likelihood theory, we approximate $\mathrm{Cov}(\hat{\boldsymbol{\theta}})$, the covariance matrix of the parameter vector estimate $\hat{\boldsymbol{\theta}}$, by the inverse of the approximate information matrix, $I(\hat{\boldsymbol{\theta}})$, where the j,kth element of $I(\hat{\boldsymbol{\theta}})$ is given by (7) in the truncated case, and $I(\boldsymbol{\theta})$ is given in appendix A of Verrill et al. (2012b) in the full case.

For $j = 1, \ldots, 5$, our simulation program approximates

$$
\frac{\partial \xi_{.05}}{\partial \theta_j}|_{\boldsymbol{\theta}}
$$

in Equation (30) by

$$
\frac{\partial \xi_{.05}}{\partial \theta_j}|_{\hat{\boldsymbol{\theta}}} \tag{31}
$$

Our simulation program approximates, for example,

$$
\frac{\partial \xi_{.05}}{\partial \mu}|_{\hat{\boldsymbol{\theta}}}
$$

by

$$
(P_2 - P_1)/h
$$

where

$$
\int_0^{P_1} f_{\mathrm{PTW}}(w; \hat{\boldsymbol{\theta}}, c_{\mathrm{l}}, c_{\mathrm{u}}) dw = 0.05
$$

(that is, P_1 is the fifth percentile when the true parameter vector is $\hat{\boldsymbol{\theta}}$), and

$$
\int_0^{P_2} f_{\mathrm{PTW}}(w; \hat{\boldsymbol{\theta}} + h\boldsymbol{u}_1, c_{\mathrm{l}}, c_{\mathrm{u}}) dw = 0.05
$$

where $\boldsymbol{u}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \end{pmatrix}^{\mathrm{T}}$ (that is P_2 is the fifth percentile when the true parameter vector is $\hat{\boldsymbol{\theta}} + h\boldsymbol{u}_1$).

Given the fact that (6) is an asymptotic (large sample) result, the fact that we are using an approximate information matrix, the fact that we are using a two-term Taylor series approximation to obtain (30), and the fact that we are using a finite difference (of values obtained from numerically solving equations involving numerically approximated integrals) to estimate (31), it is somewhat remarkable that we obtain reasonable fifth percentile coverages for (somewhat) reasonable sample sizes — see Tables 7 and 8.

If there is sufficient interest in one-sided, lower 75% confidence intervals on the fifth percentiles of PTWs, it might be worthwhile to attempt to reduce these sample sizes further by using a three-term Taylor series, and by being more careful with our numerical integrations, numerical equation solving, and numerical estimates of derivatives.

8 Appendix C—Algorithmic Details of the Web Program that Estimates the Parameters of a Bivariate Gaussian–Weibull from MSR Data

The algorithm for full data sets is described in appendix B of Verrill et al. (2012b).

The program to fit truncated (on the Gaussian) bivariate Gaussian–Weibull data is straightforward. It performs the following tasks:

1. It obtains initital estimates of μ and σ. These are simply the standard univariate estimates—$\bar{x}$ and $\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2/n}$. Note that these are not optimal for univariate truncated data. However, in practice, we found that we had occasional convergence problems in the bivariate optimization when we began with univariate, truncated data maximum likelihood estimates of μ and σ. Thus, we adopted this rough approach to obtaining starting values, and it appeared to eliminate the convergence problems.

2. Similarly, our initial estimates of γ and β are linear regression estimates that are appropriate for a true univariate Weibull distribution. These estimates are described in detail in appendix B of Verrill et al. (2012b) (and in appendix X.2 of ASTM D 5457 (ASTM 2010d)). Again, these estimates are not optimal for a PTW, but we found that they led to good convergence behavior.

3. Our initial estimate of ρ is simply the sample correlation coefficient between the corresponding Gaussian and Weibull values.

4. Given these initial estimates of μ, σ, ρ, γ, and β, we employ the public domain UNCMIN optimization routine. It makes use of the Newton method modified by a backtracking line search technique to find the $\hat{\mu}$, $\hat{\sigma}$, $\hat{\rho}$, $\hat{\gamma}$, $\hat{\beta}$ that yield a maximum of the of the log likelihood function (based on Eq. 1 or Eq. 3 depending upon whether we have a full or a truncated [on the Gaussian] bivariate Gaussian–Weibull data set).

5. We calculate the approximate information matrix via expression (7) and the results in Appendix A.

6. We invert the information matrix using the LINPACK routines dpofa and dpodi.

7. We perform a simulation that has two purposes. First, it permits us to calculate the σ_{sim} used to calculate the simulation-based confidence intervals (8). Second, it permits us to estimate the coverage of both the simulation-based and theory-based (9) confidence intervals.

   Let n denote the size of a sample provided by a user of the Web program. From the sample, the program first obtains maximum likelihood estimates $\hat{\mu}$, $\hat{\sigma}$, $\hat{\rho}$, $\hat{\gamma}$, and $\hat{\beta}$ as noted in point 4 above. As noted in points 5 and 6 above, it also obtains an approximation to the inverse of the information matrix. This yields $\hat{\sigma}_{\text{th}}$ for all five parameters. Then it generates N samples of size n from a truncated (on the Gaussian) bivariate Gaussian–Weibull distribution with parameters $\hat{\mu}$, $\hat{\sigma}$, $\hat{\rho}$, $\hat{\gamma}$, and $\hat{\beta}$. (Here, N is the number of trials specified by the user.) For the ith generated sample, it obtains estimates (as above) of the five parameters, $\hat{\hat{\mu}}_i$, $\hat{\hat{\sigma}}_i$, $\hat{\hat{\rho}}_i$, $\hat{\hat{\gamma}}_i$, and $\hat{\hat{\beta}}_i$. For each of the five parameters, the program calculates

$$\hat{\sigma}_{\text{sim}} = \sqrt{\sum_{i=1}^{N} \left(\hat{\hat{\theta}}_i - \bar{\hat{\hat{\theta}}} \right)^2 /(N-1)}$$

For all five parameters, the program reports the theory-based confidence intervals

$$\hat{\theta} \pm z_{1-\alpha/2} \times \hat{\sigma}_{\text{th}}/\sqrt{n}$$

and the simulation-based confidence intervals

$$\hat{\theta} \pm z_{1-\alpha/2} \times \hat{\sigma}_{\text{sim}}$$

It obtains estimates of the coverages of these intervals by going back through the N groups of $\hat{\hat{\mu}}_i$, $\hat{\hat{\sigma}}_i$, $\hat{\hat{\rho}}_i$, $\hat{\hat{\gamma}}_i$, and $\hat{\hat{\beta}}_i$ and calculating the fraction of the time in which (for the theory-based intervals)

$$\hat{\theta} \in \hat{\hat{\theta}}_i \pm z_{1-\alpha/2} \times \hat{\sigma}_{\text{th},i}/\sqrt{n} \tag{32}$$

or (for the simulation-based confidence intervals)

$$\hat{\theta} \in \hat{\hat{\theta}}_i \pm z_{1-\alpha/2} \times \hat{\sigma}_{\text{sim}}$$

Here, a new $\hat{\sigma}_{\text{th},i}$ is calculated for each of the N samples of size n (see points 5 and 6 above).

9 Appendix D — "Kludges" Used in the Optimization Programs

To ensure that the maximum likelihood programs converged, we placed constraints on the fitted bivariate Gaussian–Weibull parameters and on the fitted truncated (on the Gaussian) bivariate Gaussian–Weibull parameters in our computer programs. We do not claim that these constraints are necessary or optimal. However, in practice, they work. We describe them here. They are appropriate for the parameter ranges that we have fitted. However, the programs might have to be modified for other parameter ranges.

The fitbiv.f routine (see http://www1.fpl.fs.fed.us/sim_tr_gauss_weib.html) is used to fit a full bivariate Gaussian–Weibull distribution. It finds the parameters p_1, p_2, p_3, p_4, and p_5 that maximize the likelihood where

$$\begin{aligned}
\mu &= p_1 \\
\sigma &= p_2^2 \\
\rho &= \sin(p_3) \\
\gamma &= p_4^2 \\
\beta &= \sin(p_5) \times 24.795 + 25.795
\end{aligned} \tag{33}$$

Thus, σ and γ are constrained to be non-negative, ρ is constrained to lie between -1 and 1, and β is constrained to lie between 1 and 50.59. That is, the coefficient of variation of the Weibull is constrained to lie between 0.025 and 1 if we define it as a fraction (2.5% and 100% if we define it as a percent).

The fitbivtr.f, routine (see http://www1.fpl.fs.fed.us/sim_tr_gauss_weib.html) is used to fit a truncated (on the Gaussian) bivariate Gaussian–Weibull distribution. It finds the parameters p_1, p_2, p_3, p_4, and p_5 that maximize the likelihood where

$$\begin{aligned}
\mu &= \exp(\log(\text{mubase}) + \sin(p_1) \times \log(2)) \\
\sigma &= \exp(\log(\text{sigbase}) + \sin(p_2) \times \log(8)) \\
\rho &= \sin(p_3) \\
\gamma &= p_4^2 \\
\beta &= p_5^2
\end{aligned} \tag{34}$$

Thus, γ and β are constrained to be non-negative, ρ is constrained to lie between -1 and 1, μ is constrained to lie between 1/2 and 2 times "mubase," and σ is constrained to lie between 1/8 and 8 times "sigbase." In the current implementation, "mubase" is just the sample average of the truncated x values (MOE in the MOE,MOR case) and "sigbase" is just the sample standard deviation of the truncated x values.

These parameterizations lead to new partials of the $\log(f(x,w))$ values. Let θ_1, θ_2, θ_3, θ_4, θ_5 denote μ, σ, ρ, γ, β.

We have

$$\frac{\partial \log(f(x,w;\boldsymbol{\theta}))}{\partial p_i} = \frac{\partial \log(f)}{\partial \theta_i} \times \frac{\partial \theta_i}{\partial p_i} \tag{35}$$

$$\frac{\partial^2 \log(f(x,w;\boldsymbol{\theta}))}{\partial p_i^2} = \frac{\partial^2 \log(f)}{\partial \theta_i^2} \times \left(\frac{\partial \theta_i}{\partial p_i}\right)^2 + \frac{\partial \log(f)}{\partial \theta_i} \times \frac{\partial^2 \theta_i}{\partial p_i^2} \tag{36}$$

and

$$\frac{\partial^2 \log(f(x,w;\boldsymbol{\theta}))}{\partial p_i \partial p_j} = \frac{\partial \log(f)}{\partial \theta_i \theta_j} \times \frac{\partial \theta_i}{\partial p_i} \times \frac{\partial \theta_j}{\partial p_j} \tag{37}$$

The $\frac{\partial \log(f)}{\partial \theta_i}$ and $\frac{\partial^2 \log(f)}{\partial \theta_i \partial \theta_j}$ values can be found in (or calculated from) appendix C of Verrill et al. (2012a) and Appendix A.

For fitbiv.f the $\frac{\partial \theta_i}{\partial p_i}$ values are

$$\begin{aligned}
\frac{\partial \mu}{\partial p_1} &= 1 \\
\frac{\partial \sigma}{\partial p_2} &= 2 \times p_2 \\
\frac{\partial \rho}{\partial p_3} &= \cos(p_3) \\
\frac{\partial \gamma}{\partial p_4} &= 2 \times p_4 \\
\frac{\partial \beta}{\partial p_5} &= \cos(p_5) \times 24.795
\end{aligned} \tag{38}$$

For fitbiv.f the $\frac{\partial^2 \theta_i}{\partial p_i^2}$ values are

$$\begin{aligned}
\frac{\partial^2 \mu}{\partial p_1^2} &= 0 \\
\frac{\partial^2 \sigma}{\partial p_2^2} &= 2 \\
\frac{\partial^2 \rho}{\partial p_3^2} &= -\sin(p_3) \\
\frac{\partial^2 \gamma}{\partial p_4^2} &= 2 \\
\frac{\partial^2 \beta}{\partial p_5^2} &= -\sin(p_5) \times 24.795
\end{aligned} \tag{39}$$

For fitbivtr.f, the $\frac{\partial \theta_i}{\partial p_i}$ values are

$$\begin{aligned}
\frac{\partial \mu}{\partial p_1} &= \exp(\log(\text{mubase}) + \sin(p_1) \times \log(2)) \times \cos(p_1) \times \log(2) \\
\frac{\partial \sigma}{\partial p_2} &= \exp(\log(\text{sigbase}) + \sin(p_2) \times \log(8)) \times \cos(p_2) \times \log(8) \\
\frac{\partial \rho}{\partial p_3} &= \cos(p_3) \\
\frac{\partial \gamma}{\partial p_4} &= 2 \times p_4 \\
\frac{\partial \beta}{\partial p_5} &= 2 \times p_5
\end{aligned} \tag{40}$$

For fitbivtr.f, the $\frac{\partial^2 \theta_i}{\partial p_i^2}$ values are

$$\begin{aligned}
\frac{\partial^2 \mu}{\partial p_1^2} &= \exp(\log(\text{mubase}) + \sin(p_1) \times \log(2)) \times (\cos(p_1) \times \log(2))^2 \\
&\quad - \exp(\log(\text{mubase}) + \sin(p_1) \times \log(2)) \times \sin(p_1) \times \log(2) \\
\frac{\partial^2 \sigma}{\partial p_2^2} &= \exp(\log(\text{sigbase}) + \sin(p_2) \times \log(8)) \times (\cos(p_2) \times \log(8))^2 \\
&\quad - \exp(\log(\text{sigbase}) + \sin(p_2) \times \log(8)) \times \sin(p_2) \times \log(8) \\
\frac{\partial^2 \rho}{\partial p_3^2} &= -\sin(p_3) \qquad (41) \\
\frac{\partial^2 \gamma}{\partial p_4^2} &= 2 \\
\frac{\partial^2 \beta}{\partial p_5^2} &= 2
\end{aligned} \tag{42}$$

ρ	n	Estimated percent bias of $\hat{\mu}$					Estimated percent rmse of $\hat{\mu}$				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.0	0.2	0.8	-3.0	-1.1	2.0	7.0	14.7	16.7	8.7
	200	0.0	0.0	0.3	-2.7	-0.5	1.4	2.6	7.3	12.6	5.6
	400	0.0	0.0	0.1	-2.1	-0.2	1.0	1.6	3.2	9.0	3.7
	800	0.0	0.0	0.0	-1.3	-0.1	0.7	1.1	1.8	5.7	2.6
	1600	0.0	0.0	0.0	-0.6	0.0	0.5	0.8	1.1	3.4	1.8
	3200	0.0	0.0	0.0	-0.3	0.0	0.4	0.5	0.8	2.2	1.3
	6400	0.0	0.0	0.0	-0.1	0.0	0.2	0.4	0.6	1.5	0.9
.6	100	0.0	0.2	0.8	-2.7	-1.1	2.0	6.2	12.9	16.5	8.5
	200	0.0	0.0	0.3	-2.5	-0.5	1.4	2.6	6.3	11.7	5.4
	400	0.0	0.0	0.0	-1.5	-0.2	1.0	1.6	3.0	7.6	3.7
	800	0.0	0.0	0.0	-0.8	-0.1	0.7	1.1	1.7	4.5	2.6
	1600	0.0	0.0	0.0	-0.4	-0.1	0.5	0.8	1.1	2.9	1.8
	3200	0.0	0.0	0.0	-0.2	0.0	0.4	0.5	0.8	1.9	1.3
	6400	0.0	0.0	0.0	-0.1	0.0	0.2	0.4	0.5	1.3	0.9
.7	100	0.0	0.2	0.5	-2.7	-1.1	2.0	5.4	11.8	15.8	8.4
	200	0.0	0.0	0.2	-1.9	-0.5	1.4	2.4	5.5	10.3	5.3
	400	0.0	0.0	0.0	-1.2	-0.2	1.0	1.6	2.7	6.6	3.6
	800	0.0	0.0	0.0	-0.5	-0.1	0.7	1.1	1.7	3.9	2.5
	1600	0.0	0.0	0.0	-0.2	0.0	0.5	0.8	1.1	2.5	1.7
	3200	0.0	0.0	0.0	-0.1	0.0	0.4	0.5	0.8	1.7	1.2
	6400	0.0	0.0	0.0	-0.1	0.0	0.2	0.4	0.5	1.2	0.9
.8	100	0.0	0.2	0.4	-2.5	-0.8	2.0	4.8	11.1	15.3	7.8
	200	0.0	0.0	0.1	-1.8	-0.5	1.4	2.3	5.1	9.9	5.2
	400	0.0	0.0	0.1	-1.0	-0.2	1.0	1.6	2.6	6.1	3.5
	800	0.0	0.0	0.0	-0.4	-0.1	0.7	1.1	1.6	3.5	2.4
	1600	0.0	0.0	0.0	-0.2	0.0	0.5	0.8	1.1	2.3	1.7
	3200	0.0	0.0	0.0	-0.1	0.0	0.4	0.5	0.8	1.6	1.2
	6400	0.0	0.0	0.0	-0.1	0.0	0.2	0.4	0.5	1.1	0.8

Table 2,μ: Estimated percent biases and rmse's of $\hat{\mu}$ (from 10,000 trials)

ρ	n	Estimated percent bias of $\hat{\sigma}$					Estimated percent rmse of $\hat{\sigma}$				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	-0.8	9.9	25.8	14.2	0.3	7.1	51.2	82.9	68.5	18.1
	200	-0.4	3.2	15.7	13.0	0.0	5.0	22.9	57.5	59.5	12.4
	400	-0.2	1.1	7.3	9.8	0.0	3.5	12.3	33.7	46.4	8.7
	800	-0.1	0.5	3.3	6.6	0.0	2.5	8.0	19.9	33.5	6.1
	1600	0.0	0.4	1.4	2.9	-0.1	1.8	5.6	12.4	22.2	4.3
	3200	0.0	0.1	0.7	1.7	0.0	1.2	3.9	8.4	15.6	3.1
	6400	0.0	0.1	0.5	0.8	0.1	0.9	2.7	5.9	10.6	2.2
.6	100	-0.8	9.0	22.0	14.4	0.2	7.1	46.2	74.7	66.4	18.1
	200	-0.4	2.7	11.2	12.3	0.0	4.9	19.8	48.2	55.1	12.3
	400	-0.2	1.2	5.5	7.2	0.1	3.5	12.0	29.0	39.8	8.6
	800	-0.1	0.6	2.4	3.9	0.0	2.5	8.0	17.2	27.3	6.1
	1600	-0.1	0.2	1.2	2.1	0.0	1.8	5.5	11.4	18.8	4.2
	3200	0.0	0.1	0.6	0.8	0.0	1.2	3.9	7.8	12.8	3.0
	6400	0.0	0.1	0.3	0.5	0.0	0.9	2.7	5.5	8.9	2.1
.7	100	-0.7	6.6	18.2	14.5	0.3	7.0	40.3	67.8	63.9	17.7
	200	-0.4	2.6	9.2	9.1	0.1	5.0	18.9	42.2	48.8	12.1
	400	-0.2	0.9	4.2	6.3	0.0	3.5	11.6	24.6	35.3	8.5
	800	-0.1	0.4	1.9	2.8	0.0	2.5	7.7	15.6	23.6	6.0
	1600	-0.1	0.2	0.9	1.2	0.0	1.8	5.4	10.3	15.6	4.1
	3200	0.0	0.0	0.7	0.7	0.0	1.2	3.7	7.2	10.9	2.9
	6400	0.0	0.0	0.3	0.4	0.0	0.9	2.6	5.0	7.7	2.1
.8	100	-0.9	5.5	16.8	14.4	-0.2	7.0	35.8	64.7	62.2	16.8
	200	-0.4	1.7	7.6	9.8	0.0	4.9	16.9	37.2	46.6	11.7
	400	-0.2	0.9	3.4	5.3	0.0	3.4	10.9	22.0	31.5	8.1
	800	-0.1	0.4	1.4	2.3	-0.1	2.4	7.3	14.0	20.4	5.7
	1600	0.0	0.2	0.8	1.3	0.0	1.7	5.2	9.5	13.9	4.1
	3200	0.0	0.1	0.4	0.5	0.0	1.2	3.6	6.6	9.7	2.9
	6400	0.0	0.1	0.1	0.3	0.0	0.9	2.5	4.6	6.7	2.0

Table 2,σ: Estimated percent biases and rmse's of $\hat{\sigma}$ (from 10,000 trials)

		Estimated percent bias of $\hat{\rho}$					Estimated percent rmse of $\hat{\rho}$				
ρ	n	full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	-0.4	1.6	2.5	-7.7	-1.7	15.0	27.7	39.6	51.4	23.6
	200	-0.1	0.8	2.7	-3.1	-1.0	10.7	18.6	30.1	38.4	16.7
	400	-0.2	0.3	1.6	-0.6	-0.4	7.5	12.8	21.6	28.2	11.8
	800	-0.1	0.0	0.6	0.3	-0.2	5.2	9.1	15.1	20.8	8.3
	1600	0.0	0.1	0.2	0.1	-0.2	3.7	6.4	10.5	15.3	5.8
	3200	0.0	-0.1	0.1	0.0	-0.1	2.6	4.5	7.3	11.0	4.2
	6400	0.0	0.1	0.1	0.0	0.0	1.9	3.2	5.3	7.7	2.9
.6	100	-0.3	1.2	0.8	-6.0	-1.4	10.7	20.0	28.7	37.6	16.8
	200	-0.2	0.2	0.6	-2.7	-0.9	7.6	13.7	21.2	27.7	12.1
	400	-0.1	0.1	0.6	-1.3	-0.4	5.3	9.5	15.7	20.6	8.3
	800	0.0	0.1	0.3	-0.6	-0.2	3.8	6.6	10.9	14.9	5.9
	1600	0.0	0.0	0.1	-0.3	-0.1	2.6	4.7	7.7	10.9	4.1
	3200	0.0	0.0	0.2	-0.2	-0.1	1.9	3.3	5.5	7.7	2.9
	6400	0.0	0.0	0.1	-0.1	-0.1	1.3	2.3	3.9	5.4	2.1
.7	100	-0.3	0.0	-0.4	-5.0	-1.1	7.3	13.8	20.3	27.2	11.7
	200	-0.1	0.2	-0.1	-2.9	-0.5	5.2	9.6	15.1	20.0	8.2
	400	-0.1	-0.1	0.1	-1.1	-0.3	3.6	6.8	10.8	14.5	5.7
	800	-0.1	0.0	0.0	-0.7	-0.2	2.6	4.7	7.6	10.5	4.1
	1600	0.0	0.0	0.0	-0.4	-0.1	1.8	3.3	5.4	7.3	2.8
	3200	0.0	0.0	0.1	-0.1	0.0	1.3	2.3	3.8	5.2	2.0
	6400	0.0	0.0	0.0	-0.1	0.0	0.9	1.7	2.7	3.7	1.4
.8	100	-0.2	-0.2	-0.9	-4.1	-0.8	4.5	8.7	13.3	18.8	7.3
	200	-0.1	-0.2	-0.3	-1.8	-0.4	3.1	6.1	9.5	13.4	5.0
	400	-0.1	-0.1	-0.2	-0.9	-0.2	2.2	4.2	6.8	9.5	3.5
	800	0.0	0.0	-0.2	-0.4	-0.1	1.6	3.0	4.8	6.6	2.5
	1600	0.0	0.0	0.0	-0.2	-0.1	1.1	2.1	3.4	4.6	1.7
	3200	0.0	0.0	0.0	-0.1	0.0	0.8	1.5	2.4	3.3	1.2
	6400	0.0	0.0	0.0	0.0	0.0	0.6	1.0	1.7	2.3	0.9

Table 2,ρ: Estimated percent biases and rmse's of $\hat{\rho}$ (from 10,000 trials)

ρ	n	Estimated percent bias of $\hat{\gamma}$					Estimated percent rmse of $\hat{\gamma}$				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.1	-0.1	-0.3	1.1	0.6	1.8	3.2	5.5	7.7	4.6
	200	0.0	0.0	-0.2	0.9	0.2	1.3	1.6	3.1	5.2	2.8
	400	0.0	0.0	-0.1	0.6	0.1	0.9	1.1	1.6	3.3	1.9
	800	0.0	0.0	0.0	0.4	0.1	0.6	0.8	1.0	2.1	1.3
	1600	0.0	0.0	0.0	0.2	0.0	0.5	0.5	0.7	1.3	0.9
	3200	0.0	0.0	0.0	0.1	0.0	0.3	0.4	0.5	0.9	0.6
	6400	0.0	0.0	0.0	0.0	0.0	0.2	0.3	0.3	0.6	0.5
.6	100	0.1	-0.1	-0.4	1.5	0.7	1.8	3.3	6.0	8.9	5.0
	200	0.0	0.0	-0.2	1.0	0.3	1.3	1.7	3.3	5.6	3.1
	400	0.0	0.0	-0.1	0.6	0.2	0.9	1.1	1.8	3.4	2.1
	800	0.0	0.0	0.0	0.3	0.1	0.6	0.8	1.1	2.1	1.4
	1600	0.0	0.0	0.0	0.2	0.0	0.5	0.6	0.7	1.3	1.0
	3200	0.0	0.0	0.0	0.1	0.0	0.3	0.4	0.5	0.9	0.7
	6400	0.0	0.0	0.0	0.0	0.0	0.2	0.3	0.4	0.6	0.5
.7	100	0.1	-0.1	-0.3	1.7	0.9	1.8	3.5	6.6	9.8	5.6
	200	0.0	0.0	-0.2	1.0	0.4	1.3	1.8	3.4	5.9	3.4
	400	0.0	0.0	0.0	0.5	0.2	0.9	1.2	1.9	3.5	2.3
	800	0.0	0.0	-0.1	0.2	0.1	0.6	0.8	1.2	2.1	1.5
	1600	0.0	0.0	0.0	0.1	0.0	0.5	0.6	0.8	1.3	1.1
	3200	0.0	0.0	0.0	0.1	0.0	0.3	0.4	0.6	0.9	0.7
	6400	0.0	0.0	0.0	0.0	0.0	0.2	0.3	0.4	0.6	0.5
.8	100	0.1	-0.2	-0.4	1.9	0.8	1.8	3.5	7.3	11.1	5.6
	200	0.0	0.0	-0.2	1.1	0.4	1.3	1.9	3.7	6.8	3.6
	400	0.0	0.0	-0.1	0.6	0.2	0.9	1.3	2.0	4.0	2.3
	800	0.0	0.0	-0.1	0.2	0.1	0.6	0.9	1.3	2.2	1.6
	1600	0.0	0.0	0.0	0.1	0.0	0.5	0.6	0.9	1.4	1.1
	3200	0.0	0.0	0.0	0.1	0.0	0.3	0.4	0.6	1.0	0.8
	6400	0.0	0.0	0.0	0.0	0.0	0.2	0.3	0.4	0.7	0.6

Table 2,γ: Estimated percent biases and rmse's of $\hat{\gamma}$ (from 10,000 trials)

ρ	n	Estimated percent bias of $\hat{\beta}$					Estimated percent rmse of $\hat{\beta}$				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	1.4	-0.7	-2.7	-0.8	1.2	8.2	12.9	17.5	21.6	13.8
	200	0.6	-0.1	-2.2	-1.5	0.7	5.6	8.1	12.8	17.3	9.6
	400	0.4	0.0	-1.2	-1.4	0.4	3.9	5.4	8.7	12.9	6.8
	800	0.2	0.0	-0.5	-1.2	0.1	2.8	3.7	5.8	9.6	4.8
	1600	0.1	0.0	-0.2	-0.5	0.1	2.0	2.6	3.9	6.9	3.4
	3200	0.0	0.0	-0.1	-0.3	0.0	1.4	1.9	2.7	4.9	2.4
	6400	0.0	0.0	-0.1	-0.1	0.0	1.0	1.3	1.9	3.4	1.7
.6	100	1.4	-0.8	-2.4	-0.1	1.5	8.0	14.1	19.6	24.8	15.1
	200	0.7	-0.1	-1.5	-0.8	0.9	5.6	9.0	14.0	19.4	10.6
	400	0.3	0.0	-1.0	-0.7	0.4	3.9	6.0	9.9	14.6	7.5
	800	0.2	0.0	-0.5	-0.5	0.2	2.8	4.2	6.6	10.6	5.2
	1600	0.1	0.0	-0.2	-0.3	0.1	2.0	3.0	4.6	7.7	3.7
	3200	0.1	0.0	-0.2	-0.1	0.1	1.4	2.1	3.3	5.4	2.6
	6400	0.0	0.0	-0.1	0.0	0.0	1.0	1.5	2.3	3.8	1.9
.7	100	1.4	-0.3	-1.8	1.2	1.9	8.1	15.3	22.3	28.9	16.8
	200	0.7	-0.2	-1.2	0.2	1.0	5.6	10.1	16.0	22.1	11.7
	400	0.4	0.1	-0.8	-0.3	0.5	3.9	6.9	11.2	16.5	8.2
	800	0.2	0.0	-0.3	-0.1	0.2	2.8	4.8	7.7	12.0	5.8
	1600	0.1	0.0	-0.1	0.0	0.2	1.9	3.4	5.3	8.4	4.1
	3200	0.0	0.0	-0.2	-0.1	0.1	1.4	2.4	3.8	6.0	2.9
	6400	0.0	0.0	-0.1	0.0	0.0	0.9	1.7	2.6	4.3	2.1
.8	100	1.4	0.1	-1.1	3.0	2.6	8.1	16.7	25.5	34.2	18.0
	200	0.8	0.2	-0.8	0.6	1.1	5.5	11.2	18.0	26.3	12.5
	400	0.3	0.1	-0.3	0.1	0.6	3.8	7.7	12.6	19.2	8.7
	800	0.2	0.0	-0.1	0.1	0.3	2.7	5.4	8.8	13.6	6.3
	1600	0.1	0.0	-0.1	-0.1	0.2	1.9	3.8	6.2	9.6	4.5
	3200	0.0	0.0	-0.1	0.0	0.0	1.4	2.7	4.4	6.8	3.1
	6400	0.0	0.0	0.0	0.0	0.0	1.0	1.9	3.0	4.7	2.2

Table 2,β: Estimated percent biases and rmse's of $\hat{\beta}$ (from 10,000 trials)

ρ	n	(full rmse)/(tr rmse) .1,.9	.2,.8	.4,.8	.4,1	(tr n needed)/(full n needed) .1,.9	.2,.8	.4,.8	.4,1
.5	100	.29	.14	.12	.23	12.0	53.8	69.2	18.2
	200	.54	.19	.11	.25	3.4	26.5	79.0	15.7
	400	.62	.31	.11	.27	2.6	10.4	79.4	13.9
	800	.64	.40	.12	.27	2.4	6.4	66.0	13.4
	1600	.65	.44	.15	.28	2.4	5.1	45.8	12.9
	3200	.67	.45	.16	.27	2.2	5.0	39.4	13.3
	6400	.65	.46	.17	.28	2.4	4.8	34.6	13.2
.6	100	.32	.16	.12	.23	10.0	41.0	68.8	18.4
	200	.55	.22	.12	.26	3.4	20.0	66.9	14.6
	400	.63	.34	.13	.27	2.6	8.6	56.8	13.9
	800	.64	.41	.16	.28	2.4	5.9	40.7	13.1
	1600	.66	.44	.17	.28	2.3	5.2	33.9	12.7
	3200	.67	.46	.19	.28	2.2	4.8	28.6	12.5
	6400	.66	.46	.19	.29	2.3	4.8	26.5	12.2
.7	100	.38	.17	.13	.24	7.1	35.0	62.3	17.7
	200	.58	.25	.14	.27	3.0	15.5	53.7	14.2
	400	.63	.37	.15	.28	2.5	7.4	43.1	13.2
	800	.64	.42	.18	.28	2.4	5.7	31.1	12.9
	1600	.66	.44	.20	.28	2.3	5.1	24.3	12.4
	3200	.66	.46	.21	.29	2.3	4.8	23.0	11.7
	6400	.66	.46	.21	.29	2.3	4.7	21.9	11.8
.8	100	.42	.18	.13	.26	5.8	30.4	58.7	15.4
	200	.60	.28	.14	.27	2.7	12.9	49.4	13.2
	400	.64	.38	.16	.28	2.4	6.8	37.2	12.5
	800	.65	.43	.20	.30	2.4	5.4	25.1	11.4
	1600	.65	.45	.21	.29	2.4	5.0	21.7	12.0
	3200	.65	.46	.23	.29	2.4	4.8	19.4	11.6
	6400	.65	.46	.23	.30	2.3	4.8	18.9	11.2

Table 3,μ: (rmse in the full case) divided by (rmse in the truncated case), and (sample size needed to obtain a given rmse in the truncated case) divided by (sample size needed to obtain the same rmse in the full case) (from 10,000 trials)

		(full rmse)/(tr rmse)				(tr n needed)/(full n needed)			
ρ	n	.1,.9	.2,.8	.4,.8	.4,1	.1,.9	.2,.8	.4,.8	.4,1
.5	100	.14	.09	.10	.39	52.3	135.6	93.3	6.5
	200	.22	.09	.08	.40	21.0	131.3	139.2	6.1
	400	.28	.11	.08	.41	12.3	90.1	173.7	6.0
	800	.31	.13	.07	.40	10.4	62.1	181.7	6.1
	1600	.31	.14	.08	.41	10.2	49.3	159.3	6.1
	3200	.32	.15	.08	.41	10.0	44.9	155.6	5.9
	6400	.32	.15	.08	.40	9.7	45.1	143.1	6.2
.6	100	.15	.09	.11	.39	42.1	111.2	88.4	6.6
	200	.25	.10	.09	.41	16.1	92.6	122.9	6.0
	400	.29	.12	.09	.42	11.9	67.7	125.8	5.8
	800	.31	.14	.09	.41	10.4	47.7	121.7	5.8
	1600	.32	.15	.09	.41	9.6	42.4	113.0	5.9
	3200	.31	.16	.10	.40	10.1	39.8	104.6	6.1
	6400	.33	.16	.10	.42	9.4	39.3	105.2	5.8
.7	100	.17	.10	.11	.40	33.1	94.0	83.5	6.4
	200	.26	.12	.10	.41	14.5	72.0	97.8	5.9
	400	.30	.14	.10	.42	10.8	50.6	102.6	5.8
	800	.32	.16	.10	.41	9.5	39.9	92.0	5.9
	1600	.33	.17	.11	.42	9.4	35.0	80.5	5.6
	3200	.33	.17	.11	.42	9.1	33.8	79.8	5.7
	6400	.33	.17	.11	.42	9.0	33.4	76.0	5.7
.8	100	.19	.11	.11	.41	26.5	86.1	79.5	5.9
	200	.29	.13	.11	.41	12.0	56.6	88.6	5.9
	400	.32	.16	.11	.43	10.0	38.9	82.9	5.5
	800	.33	.18	.12	.43	8.9	32.0	67.5	5.5
	1600	.33	.18	.12	.43	9.0	30.3	65.5	5.5
	3200	.34	.19	.13	.43	8.8	28.4	62.5	5.4
	6400	.34	.19	.13	.43	8.5	27.7	59.9	5.4

Table 3,σ: (rmse in the full case) divided by (rmse in the truncated case), and (sample size needed to obtain a given rmse in the truncated case) divided by (sample size needed to obtain the same rmse in the full case) (from 10,000 trials)

ρ	n	(full rmse)/(tr rmse)				(tr n needed)/(full n needed)			
		.1,.9	.2,.8	.4,.8	.4,1	.1,.9	.2,.8	.4,.8	.4,1
.5	100	.54	.38	.30	.65	3.4	6.8	11.4	2.4
	200	.57	.36	.28	.64	3.1	7.9	12.9	2.4
	400	.58	.35	.27	.64	2.9	8.4	14.1	2.5
	800	.58	.35	.25	.64	3.0	8.2	15.5	2.5
	1600	.58	.36	.25	.64	3.0	7.9	16.5	2.4
	3200	.59	.36	.24	.64	2.9	7.8	17.4	2.5
	6400	.59	.35	.24	.63	2.9	8.0	17.1	2.6
.6	100	.53	.37	.29	.64	3.5	7.1	12.2	2.4
	200	.55	.36	.27	.63	3.3	7.7	13.4	2.6
	400	.56	.34	.26	.64	3.2	8.8	14.9	2.4
	800	.57	.34	.25	.64	3.1	8.4	15.7	2.5
	1600	.56	.35	.24	.64	3.2	8.4	17.1	2.4
	3200	.57	.34	.24	.63	3.1	8.5	17.0	2.5
	6400	.57	.34	.25	.64	3.1	8.6	16.5	2.5
.7	100	.53	.36	.27	.64	3.5	7.6	13.8	2.5
	200	.54	.34	.26	.63	3.4	8.8	14.8	2.5
	400	.54	.34	.25	.64	3.5	8.8	16.1	2.5
	800	.55	.34	.24	.63	3.3	8.8	16.8	2.5
	1600	.54	.33	.24	.64	3.5	8.9	16.7	2.5
	3200	.54	.34	.24	.63	3.4	8.7	16.8	2.5
	6400	.54	.33	.24	.63	3.5	8.9	17.0	2.6
.8	100	.52	.34	.24	.63	3.7	8.6	17.1	2.5
	200	.52	.33	.24	.63	3.8	9.1	18.0	2.5
	400	.53	.33	.24	.64	3.6	9.3	18.0	2.4
	800	.53	.32	.24	.62	3.6	9.5	17.5	2.6
	1600	.52	.32	.24	.64	3.7	9.6	17.3	2.5
	3200	.53	.32	.24	.64	3.6	9.5	17.8	2.5
	6400	.53	.33	.24	.64	3.6	9.2	17.6	2.5

Table 3,ρ: (rmse in the full case) divided by (rmse in the truncated case), and (sample size needed to obtain a given rmse in the truncated case) divided by (sample size needed to obtain the same rmse in the full case) (from 10,000 trials)

ρ	n	(full rmse)/(tr rmse)				(tr n needed)/(full n needed)			
		.1,.9	.2,.8	.4,.8	.4,1	.1,.9	.2,.8	.4,.8	.4,1
.5	100	.58	.33	.24	.39	2.9	9.3	17.8	6.4
	200	.78	.42	.24	.45	1.6	5.6	17.0	4.9
	400	.84	.57	.28	.48	1.4	3.1	12.9	4.3
	800	.86	.65	.31	.49	1.4	2.4	10.5	4.1
	1600	.86	.69	.35	.50	1.3	2.1	8.3	4.0
	3200	.88	.70	.37	.50	1.3	2.1	7.3	4.0
	6400	.86	.69	.39	.50	1.4	2.1	6.7	4.0
.6	100	.55	.31	.20	.37	3.3	10.5	24.2	7.3
	200	.74	.40	.23	.43	1.8	6.4	18.8	5.5
	400	.80	.52	.26	.43	1.6	3.8	14.5	5.4
	800	.79	.60	.31	.45	1.6	2.8	10.3	4.9
	1600	.82	.61	.34	.47	1.5	2.7	8.6	4.6
	3200	.82	.64	.36	.46	1.5	2.5	7.7	4.7
	6400	.82	.63	.38	.47	1.5	2.5	7.1	4.6
.7	100	.52	.28	.19	.32	3.7	13.0	29.0	9.7
	200	.71	.37	.21	.38	2.0	7.2	21.8	7.1
	400	.75	.49	.26	.40	1.8	4.2	15.0	6.2
	800	.76	.53	.30	.41	1.8	3.6	11.0	5.9
	1600	.78	.56	.34	.42	1.7	3.2	8.5	5.6
	3200	.77	.56	.35	.43	1.7	3.2	8.2	5.3
	6400	.76	.58	.36	.43	1.7	3.0	7.8	5.4
.8	100	.52	.25	.16	.33	3.7	15.7	37.1	9.5
	200	.67	.35	.19	.37	2.2	8.3	28.7	7.4
	400	.71	.45	.23	.38	2.0	5.0	19.1	6.8
	800	.71	.48	.29	.40	2.0	4.3	11.9	6.4
	1600	.72	.51	.32	.39	1.9	3.9	9.8	6.5
	3200	.71	.51	.34	.40	2.0	3.8	8.8	6.2
	6400	.72	.52	.35	.41	1.9	3.6	8.4	6.1

Table 3,γ: (rmse in the full case) divided by (rmse in the truncated case), and (sample size needed to obtain a given rmse in the truncated case) divided by (sample size needed to obtain the same rmse in the full case) (from 10,000 trials)

ρ	n	(full rmse)/(tr rmse) .1,.9	.2,.8	.4,.8	.4,1	(tr n needed)/(full n needed) .1,.9	.2,.8	.4,.8	.4,1
.5	100	.64	.46	.38	.59	2.5	4.7	6.9	2.9
	200	.70	.44	.33	.58	2.1	5.1	9.5	3.0
	400	.72	.45	.30	.58	1.9	4.9	11.2	3.0
	800	.74	.47	.29	.58	1.8	4.4	11.8	3.0
	1600	.74	.51	.28	.57	1.8	3.9	12.7	3.0
	3200	.74	.50	.28	.58	1.8	3.9	12.8	3.0
	6400	.73	.50	.29	.56	1.9	4.0	12.0	3.1
.6	100	.57	.41	.33	.54	3.0	5.9	9.4	3.4
	200	.62	.40	.29	.53	2.6	6.3	12.0	3.6
	400	.66	.39	.27	.53	2.3	6.5	13.6	3.6
	800	.65	.42	.26	.52	2.3	5.8	15.3	3.6
	1600	.66	.42	.25	.53	2.3	5.7	15.6	3.6
	3200	.65	.42	.25	.52	2.4	5.7	15.6	3.7
	6400	.66	.42	.25	.52	2.3	5.6	16.0	3.7
.7	100	.53	.36	.28	.49	3.6	7.5	13.1	4.2
	200	.55	.35	.25	.48	3.3	8.2	15.6	4.4
	400	.57	.35	.24	.47	3.1	8.2	18.0	4.5
	800	.58	.36	.23	.47	3.0	7.7	19.2	4.4
	1600	.57	.36	.23	.48	3.1	7.7	19.0	4.4
	3200	.57	.37	.23	.48	3.0	7.5	19.0	4.4
	6400	.56	.37	.23	.47	3.2	7.5	19.5	4.5
.8	100	.48	.32	.24	.44	4.3	10.0	17.8	5.1
	200	.49	.31	.21	.44	4.1	10.6	22.5	5.1
	400	.50	.31	.20	.44	4.0	10.3	24.2	5.1
	800	.50	.31	.20	.43	3.9	10.3	25.0	5.3
	1600	.50	.31	.20	.43	4.1	10.4	25.0	5.4
	3200	.51	.31	.20	.43	3.9	10.3	25.9	5.4
	6400	.51	.32	.20	.44	3.9	9.9	24.8	5.2

Table 3,β: (rmse in the full case) divided by (rmse in the truncated case), and (sample size needed to obtain a given rmse in the truncated case) divided by (sample size needed to obtain the same rmse in the full case) (from 10,000 trials)

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.746	0.810	0.904	0.719	0.778	0.751	0.946	0.898	0.836	0.862
	200	0.749	0.780	0.868	0.721	0.770	0.750	0.819	0.907	0.860	0.810
	400	0.746	0.755	0.827	0.753	0.761	0.750	0.766	0.857	0.872	0.783
	800	0.743	0.749	0.781	0.769	0.747	0.744	0.756	0.799	0.870	0.769
	1600	0.753	0.748	0.766	0.760	0.750	0.747	0.749	0.767	0.850	0.755
	3200	0.747	0.755	0.757	0.746	0.746	0.749	0.752	0.758	0.808	0.750
	6400	0.751	0.745	0.750	0.750	0.746	0.750	0.753	0.754	0.777	0.750
.6	100	0.755	0.797	0.901	0.730	0.774	0.750	0.931	0.902	0.835	0.858
	200	0.754	0.773	0.856	0.747	0.764	0.749	0.805	0.902	0.859	0.811
	400	0.754	0.758	0.808	0.753	0.760	0.749	0.767	0.838	0.872	0.777
	800	0.753	0.754	0.778	0.760	0.754	0.749	0.755	0.786	0.848	0.766
	1600	0.744	0.751	0.762	0.757	0.756	0.747	0.754	0.765	0.818	0.757
	3200	0.744	0.755	0.758	0.751	0.752	0.751	0.753	0.757	0.785	0.754
	6400	0.751	0.759	0.759	0.750	0.746	0.751	0.755	0.753	0.762	0.750
.7	100	0.737	0.800	0.897	0.756	0.775	0.750	0.912	0.905	0.839	0.856
	200	0.740	0.772	0.849	0.761	0.767	0.746	0.787	0.885	0.864	0.810
	400	0.748	0.764	0.793	0.769	0.751	0.747	0.764	0.814	0.870	0.786
	800	0.752	0.750	0.770	0.760	0.750	0.749	0.757	0.781	0.840	0.765
	1600	0.746	0.750	0.749	0.762	0.747	0.752	0.750	0.764	0.793	0.756
	3200	0.752	0.753	0.753	0.754	0.749	0.752	0.754	0.756	0.778	0.750
	6400	0.749	0.751	0.747	0.754	0.745	0.747	0.750	0.748	0.765	0.750
.8	100	0.745	0.793	0.887	0.767	0.773	0.750	0.889	0.905	0.845	0.848
	200	0.744	0.772	0.834	0.782	0.766	0.748	0.783	0.876	0.867	0.806
	400	0.745	0.759	0.792	0.778	0.763	0.750	0.760	0.808	0.865	0.777
	800	0.749	0.757	0.771	0.764	0.754	0.746	0.756	0.774	0.821	0.760
	1600	0.756	0.753	0.762	0.757	0.739	0.751	0.755	0.761	0.785	0.757
	3200	0.756	0.746	0.746	0.750	0.751	0.751	0.750	0.749	0.768	0.757
	6400	0.753	0.749	0.752	0.755	0.745	0.751	0.754	0.753	0.755	0.750

Table 4,75,μ: Actual coverages (in 10,000 trials) of nominal 75% confidence intervals on μ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.743	0.783	0.748	0.690	0.764	0.744	0.932	0.851	0.802	0.793
	200	0.747	0.793	0.752	0.709	0.755	0.745	0.897	0.872	0.819	0.770
	400	0.749	0.768	0.755	0.722	0.758	0.745	0.801	0.875	0.839	0.760
	800	0.748	0.760	0.759	0.724	0.749	0.749	0.770	0.838	0.829	0.756
	1600	0.750	0.753	0.760	0.733	0.748	0.749	0.761	0.791	0.797	0.751
	3200	0.752	0.754	0.756	0.729	0.744	0.747	0.755	0.767	0.777	0.749
	6400	0.758	0.759	0.749	0.742	0.745	0.755	0.754	0.758	0.765	0.753
.6	100	0.741	0.793	0.758	0.709	0.751	0.749	0.927	0.860	0.801	0.781
	200	0.750	0.782	0.749	0.726	0.750	0.749	0.858	0.880	0.825	0.764
	400	0.750	0.764	0.746	0.724	0.756	0.748	0.795	0.866	0.832	0.756
	800	0.755	0.760	0.755	0.720	0.745	0.754	0.771	0.810	0.800	0.750
	1600	0.748	0.760	0.752	0.735	0.755	0.752	0.762	0.780	0.781	0.752
	3200	0.759	0.750	0.751	0.741	0.752	0.755	0.754	0.768	0.760	0.754
	6400	0.745	0.755	0.747	0.756	0.750	0.749	0.757	0.755	0.760	0.746
.7	100	0.743	0.779	0.763	0.728	0.754	0.743	0.920	0.867	0.808	0.789
	200	0.740	0.773	0.763	0.740	0.757	0.742	0.850	0.874	0.837	0.769
	400	0.744	0.752	0.751	0.739	0.745	0.749	0.791	0.839	0.820	0.762
	800	0.745	0.755	0.754	0.735	0.739	0.751	0.767	0.797	0.791	0.755
	1600	0.744	0.746	0.754	0.744	0.753	0.748	0.756	0.776	0.764	0.750
	3200	0.752	0.752	0.756	0.749	0.749	0.750	0.750	0.763	0.760	0.748
	6400	0.752	0.752	0.755	0.749	0.745	0.754	0.750	0.757	0.753	0.746
.8	100	0.740	0.783	0.773	0.738	0.755	0.746	0.918	0.874	0.812	0.773
	200	0.747	0.760	0.780	0.767	0.754	0.750	0.830	0.872	0.835	0.767
	400	0.749	0.756	0.759	0.752	0.754	0.748	0.789	0.826	0.819	0.759
	800	0.751	0.760	0.751	0.749	0.750	0.748	0.764	0.788	0.782	0.756
	1600	0.746	0.749	0.752	0.755	0.748	0.747	0.757	0.771	0.768	0.755
	3200	0.757	0.753	0.747	0.747	0.749	0.753	0.751	0.758	0.755	0.752
	6400	0.751	0.747	0.753	0.754	0.752	0.752	0.747	0.752	0.753	0.749

Table 4,75,σ: Actual coverages (in 10,000 trials) of nominal 75% confidence intervals on σ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.744	0.736	0.683	0.639	0.739	0.755	0.770	0.736	0.753	0.755
	200	0.740	0.752	0.699	0.643	0.748	0.751	0.760	0.741	0.731	0.759
	400	0.747	0.756	0.724	0.684	0.745	0.751	0.756	0.754	0.734	0.753
	800	0.756	0.751	0.741	0.708	0.744	0.750	0.752	0.760	0.742	0.749
	1600	0.753	0.753	0.750	0.718	0.751	0.753	0.752	0.753	0.742	0.749
	3200	0.750	0.752	0.751	0.735	0.746	0.749	0.750	0.753	0.750	0.753
	6400	0.753	0.757	0.747	0.748	0.747	0.750	0.751	0.748	0.751	0.751
.6	100	0.744	0.726	0.677	0.640	0.743	0.756	0.759	0.730	0.751	0.758
	200	0.743	0.744	0.709	0.671	0.739	0.754	0.757	0.745	0.734	0.752
	400	0.747	0.749	0.715	0.696	0.743	0.753	0.753	0.749	0.740	0.742
	800	0.748	0.754	0.742	0.713	0.749	0.752	0.752	0.752	0.740	0.751
	1600	0.757	0.750	0.742	0.726	0.755	0.755	0.751	0.746	0.747	0.751
	3200	0.748	0.749	0.742	0.741	0.745	0.751	0.750	0.749	0.752	0.746
	6400	0.751	0.752	0.741	0.749	0.753	0.754	0.750	0.748	0.748	0.753
.7	100	0.741	0.731	0.685	0.648	0.740	0.757	0.759	0.735	0.766	0.764
	200	0.744	0.744	0.703	0.688	0.743	0.752	0.755	0.742	0.748	0.755
	400	0.749	0.740	0.718	0.714	0.750	0.754	0.751	0.743	0.751	0.754
	800	0.739	0.752	0.740	0.726	0.748	0.750	0.752	0.752	0.751	0.756
	1600	0.748	0.748	0.747	0.740	0.749	0.749	0.751	0.749	0.750	0.750
	3200	0.753	0.752	0.750	0.745	0.751	0.754	0.752	0.752	0.749	0.750
	6400	0.751	0.746	0.753	0.747	0.747	0.751	0.751	0.751	0.747	0.751
.8	100	0.742	0.731	0.689	0.663	0.743	0.760	0.759	0.743	0.802	0.771
	200	0.747	0.740	0.718	0.694	0.748	0.748	0.758	0.747	0.766	0.761
	400	0.741	0.750	0.729	0.725	0.754	0.745	0.755	0.747	0.764	0.757
	800	0.748	0.748	0.736	0.734	0.742	0.753	0.749	0.746	0.754	0.751
	1600	0.752	0.747	0.741	0.748	0.751	0.751	0.750	0.751	0.755	0.750
	3200	0.745	0.750	0.743	0.741	0.752	0.745	0.749	0.749	0.752	0.750
	6400	0.752	0.745	0.751	0.752	0.749	0.751	0.747	0.748	0.750	0.747

Table 4,75,ρ: Actual coverages (in 10,000 trials) of nominal 75% confidence intervals on ρ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.740	0.781	0.852	0.828	0.793	0.750	0.883	0.881	0.869	0.861
	200	0.754	0.764	0.824	0.799	0.769	0.756	0.784	0.874	0.872	0.813
	400	0.745	0.753	0.800	0.795	0.760	0.746	0.755	0.814	0.866	0.780
	800	0.743	0.746	0.768	0.789	0.760	0.746	0.750	0.775	0.849	0.768
	1600	0.752	0.754	0.758	0.767	0.751	0.748	0.748	0.758	0.820	0.751
	3200	0.745	0.748	0.752	0.754	0.752	0.752	0.747	0.755	0.792	0.755
	6400	0.750	0.743	0.740	0.759	0.751	0.750	0.748	0.748	0.772	0.752
.6	100	0.748	0.785	0.854	0.824	0.793	0.754	0.871	0.884	0.862	0.864
	200	0.748	0.761	0.821	0.805	0.776	0.748	0.780	0.871	0.869	0.810
	400	0.745	0.753	0.786	0.781	0.761	0.747	0.754	0.813	0.859	0.782
	800	0.760	0.749	0.776	0.773	0.754	0.754	0.749	0.775	0.830	0.767
	1600	0.748	0.751	0.758	0.765	0.758	0.750	0.748	0.756	0.803	0.758
	3200	0.748	0.754	0.752	0.757	0.750	0.750	0.754	0.752	0.776	0.756
	6400	0.755	0.747	0.753	0.755	0.755	0.753	0.744	0.754	0.760	0.756
.7	100	0.737	0.779	0.854	0.827	0.794	0.752	0.873	0.886	0.854	0.872
	200	0.739	0.765	0.818	0.813	0.775	0.746	0.771	0.860	0.869	0.827
	400	0.750	0.760	0.782	0.801	0.764	0.750	0.756	0.799	0.861	0.791
	800	0.749	0.753	0.768	0.773	0.755	0.748	0.756	0.773	0.821	0.770
	1600	0.745	0.755	0.753	0.765	0.748	0.750	0.754	0.756	0.785	0.759
	3200	0.751	0.746	0.752	0.760	0.749	0.751	0.748	0.755	0.774	0.752
	6400	0.758	0.751	0.744	0.753	0.746	0.748	0.749	0.749	0.759	0.749
.8	100	0.740	0.787	0.847	0.838	0.787	0.752	0.858	0.889	0.861	0.864
	200	0.743	0.769	0.812	0.825	0.778	0.750	0.778	0.862	0.878	0.818
	400	0.741	0.756	0.788	0.807	0.764	0.744	0.759	0.794	0.870	0.783
	800	0.751	0.753	0.761	0.778	0.757	0.750	0.752	0.774	0.814	0.766
	1600	0.753	0.752	0.755	0.759	0.748	0.750	0.752	0.760	0.784	0.761
	3200	0.753	0.742	0.751	0.753	0.753	0.750	0.747	0.755	0.765	0.754
	6400	0.751	0.749	0.754	0.755	0.744	0.752	0.751	0.750	0.757	0.747

Table 4,75,γ: Actual coverages (in 10,000 trials) of nominal 75% confidence intervals on γ

		Theory-based confidence intervals					Simulation-based confidence intervals				
ρ	n	full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.739	0.766	0.754	0.676	0.750	0.751	0.805	0.793	0.757	0.752
	200	0.742	0.768	0.750	0.666	0.750	0.751	0.779	0.800	0.761	0.748
	400	0.757	0.756	0.750	0.689	0.752	0.760	0.759	0.784	0.760	0.750
	800	0.748	0.754	0.754	0.718	0.754	0.750	0.754	0.779	0.766	0.755
	1600	0.746	0.746	0.757	0.722	0.749	0.747	0.747	0.759	0.757	0.750
	3200	0.743	0.756	0.751	0.731	0.745	0.746	0.755	0.753	0.755	0.745
	6400	0.753	0.754	0.748	0.746	0.746	0.749	0.752	0.755	0.753	0.748
.6	100	0.757	0.764	0.735	0.659	0.754	0.759	0.796	0.780	0.740	0.755
	200	0.747	0.761	0.740	0.677	0.744	0.749	0.770	0.790	0.742	0.749
	400	0.744	0.762	0.736	0.695	0.748	0.750	0.758	0.782	0.748	0.747
	800	0.750	0.760	0.751	0.714	0.750	0.753	0.757	0.765	0.746	0.749
	1600	0.741	0.759	0.749	0.733	0.758	0.746	0.756	0.756	0.753	0.751
	3200	0.750	0.747	0.749	0.740	0.749	0.750	0.748	0.757	0.751	0.751
	6400	0.746	0.757	0.741	0.751	0.752	0.747	0.754	0.746	0.749	0.751
.7	100	0.750	0.752	0.715	0.664	0.745	0.758	0.787	0.770	0.718	0.753
	200	0.748	0.757	0.729	0.692	0.744	0.752	0.763	0.775	0.731	0.751
	400	0.748	0.754	0.733	0.714	0.752	0.753	0.755	0.764	0.743	0.757
	800	0.748	0.757	0.752	0.727	0.744	0.752	0.756	0.765	0.748	0.750
	1600	0.750	0.747	0.749	0.738	0.743	0.748	0.748	0.751	0.748	0.745
	3200	0.750	0.757	0.756	0.743	0.753	0.745	0.753	0.754	0.747	0.752
	6400	0.758	0.753	0.754	0.749	0.744	0.748	0.753	0.754	0.749	0.749
.8	100	0.745	0.746	0.706	0.683	0.742	0.755	0.775	0.754	0.710	0.745
	200	0.753	0.746	0.732	0.699	0.750	0.755	0.761	0.763	0.728	0.749
	400	0.753	0.754	0.737	0.726	0.755	0.752	0.758	0.757	0.749	0.751
	800	0.754	0.755	0.743	0.729	0.746	0.751	0.756	0.752	0.747	0.749
	1600	0.754	0.739	0.746	0.745	0.745	0.751	0.749	0.749	0.750	0.750
	3200	0.750	0.752	0.747	0.744	0.750	0.751	0.754	0.751	0.750	0.748
	6400	0.749	0.750	0.753	0.752	0.749	0.750	0.748	0.752	0.751	0.747

Table 4,75,β: Actual coverages (in 10,000 trials) of nominal 75% confidence intervals on β

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.891	0.951	0.986	0.796	0.865	0.902	0.972	0.931	0.882	0.929
	200	0.897	0.930	0.979	0.801	0.886	0.901	0.930	0.945	0.902	0.924
	400	0.896	0.908	0.966	0.832	0.900	0.899	0.903	0.934	0.921	0.917
	800	0.898	0.905	0.942	0.861	0.901	0.901	0.901	0.910	0.920	0.907
	1600	0.905	0.903	0.921	0.876	0.895	0.902	0.898	0.907	0.924	0.902
	3200	0.900	0.905	0.911	0.884	0.897	0.902	0.898	0.902	0.919	0.901
	6400	0.903	0.896	0.901	0.889	0.897	0.902	0.900	0.900	0.915	0.902
.6	100	0.897	0.947	0.986	0.809	0.862	0.900	0.967	0.935	0.884	0.927
	200	0.900	0.927	0.977	0.827	0.889	0.897	0.923	0.947	0.909	0.920
	400	0.903	0.914	0.960	0.836	0.896	0.902	0.905	0.924	0.922	0.916
	800	0.905	0.904	0.941	0.866	0.894	0.901	0.900	0.913	0.922	0.908
	1600	0.895	0.904	0.917	0.884	0.902	0.899	0.898	0.902	0.923	0.906
	3200	0.897	0.904	0.912	0.890	0.900	0.905	0.902	0.901	0.914	0.905
	6400	0.900	0.899	0.908	0.901	0.898	0.900	0.899	0.904	0.908	0.899
.7	100	0.889	0.947	0.984	0.828	0.867	0.900	0.962	0.938	0.887	0.925
	200	0.896	0.930	0.975	0.839	0.888	0.899	0.918	0.941	0.916	0.925
	400	0.902	0.913	0.953	0.864	0.899	0.900	0.901	0.915	0.926	0.915
	800	0.901	0.902	0.931	0.881	0.896	0.900	0.901	0.907	0.923	0.909
	1600	0.896	0.904	0.910	0.894	0.899	0.902	0.902	0.900	0.919	0.903
	3200	0.900	0.903	0.907	0.896	0.901	0.900	0.900	0.901	0.913	0.901
	6400	0.900	0.901	0.902	0.896	0.899	0.899	0.899	0.902	0.906	0.905
.8	100	0.894	0.944	0.982	0.843	0.868	0.900	0.959	0.941	0.892	0.931
	200	0.898	0.926	0.974	0.859	0.894	0.901	0.913	0.944	0.919	0.921
	400	0.894	0.910	0.950	0.879	0.900	0.898	0.902	0.913	0.928	0.911
	800	0.901	0.912	0.923	0.888	0.898	0.900	0.906	0.903	0.923	0.907
	1600	0.900	0.903	0.911	0.896	0.896	0.897	0.898	0.900	0.914	0.908
	3200	0.901	0.896	0.905	0.899	0.898	0.900	0.899	0.902	0.911	0.902
	6400	0.901	0.897	0.905	0.904	0.897	0.900	0.899	0.904	0.908	0.900

Table 4,90,μ: Actual coverages (in 10,000 trials) of nominal 90% confidence intervals on μ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.894	0.873	0.824	0.758	0.883	0.902	0.958	0.893	0.863	0.922
	200	0.896	0.894	0.853	0.800	0.894	0.900	0.951	0.916	0.878	0.909
	400	0.902	0.903	0.875	0.836	0.901	0.899	0.924	0.925	0.898	0.907
	800	0.900	0.907	0.889	0.860	0.900	0.903	0.908	0.925	0.911	0.904
	1600	0.899	0.904	0.897	0.874	0.896	0.899	0.906	0.918	0.917	0.900
	3200	0.902	0.903	0.901	0.876	0.902	0.900	0.902	0.908	0.914	0.901
	6400	0.904	0.909	0.899	0.888	0.897	0.901	0.905	0.906	0.907	0.901
.6	100	0.886	0.879	0.832	0.776	0.875	0.899	0.957	0.901	0.865	0.921
	200	0.899	0.890	0.854	0.818	0.890	0.899	0.936	0.923	0.882	0.910
	400	0.901	0.900	0.879	0.846	0.897	0.899	0.920	0.926	0.910	0.905
	800	0.898	0.903	0.889	0.863	0.897	0.897	0.913	0.922	0.919	0.904
	1600	0.896	0.900	0.898	0.881	0.901	0.899	0.905	0.912	0.913	0.901
	3200	0.899	0.896	0.900	0.883	0.898	0.896	0.902	0.906	0.905	0.900
	6400	0.897	0.901	0.897	0.895	0.903	0.900	0.900	0.904	0.903	0.902
.7	100	0.890	0.872	0.837	0.792	0.882	0.901	0.953	0.910	0.868	0.912
	200	0.895	0.893	0.864	0.832	0.893	0.900	0.936	0.926	0.902	0.908
	400	0.894	0.898	0.884	0.869	0.898	0.898	0.920	0.925	0.916	0.908
	800	0.895	0.899	0.893	0.880	0.894	0.900	0.911	0.917	0.917	0.902
	1600	0.899	0.896	0.901	0.893	0.899	0.903	0.905	0.909	0.909	0.900
	3200	0.903	0.900	0.902	0.902	0.901	0.901	0.902	0.904	0.905	0.903
	6400	0.899	0.901	0.902	0.902	0.902	0.901	0.903	0.904	0.904	0.903
.8	100	0.889	0.871	0.842	0.809	0.880	0.895	0.953	0.915	0.872	0.918
	200	0.897	0.890	0.875	0.851	0.893	0.900	0.929	0.926	0.903	0.911
	400	0.903	0.902	0.893	0.884	0.897	0.899	0.917	0.923	0.915	0.902
	800	0.900	0.903	0.894	0.891	0.896	0.899	0.907	0.915	0.914	0.902
	1600	0.900	0.897	0.901	0.898	0.897	0.902	0.904	0.908	0.908	0.901
	3200	0.906	0.903	0.899	0.896	0.901	0.902	0.904	0.904	0.905	0.898
	6400	0.900	0.903	0.902	0.902	0.899	0.900	0.901	0.899	0.902	0.899

Table 4,90,σ: Actual coverages (in 10,000 trials) of nominal 90% confidence intervals on σ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.893	0.878	0.820	0.785	0.891	0.904	0.900	0.890	0.929	0.904
	200	0.892	0.901	0.839	0.793	0.896	0.901	0.909	0.892	0.912	0.903
	400	0.897	0.899	0.864	0.825	0.897	0.900	0.900	0.900	0.900	0.903
	800	0.902	0.900	0.887	0.852	0.899	0.900	0.902	0.902	0.897	0.903
	1600	0.903	0.898	0.893	0.864	0.899	0.902	0.899	0.898	0.899	0.899
	3200	0.900	0.904	0.901	0.876	0.894	0.898	0.901	0.901	0.899	0.896
	6400	0.900	0.904	0.898	0.893	0.898	0.898	0.900	0.901	0.898	0.901
.6	100	0.890	0.875	0.818	0.782	0.890	0.900	0.901	0.894	0.920	0.902
	200	0.889	0.889	0.850	0.813	0.889	0.900	0.902	0.896	0.908	0.904
	400	0.894	0.897	0.862	0.835	0.901	0.901	0.900	0.900	0.904	0.905
	800	0.896	0.901	0.886	0.857	0.902	0.902	0.903	0.900	0.902	0.901
	1600	0.900	0.902	0.895	0.874	0.900	0.899	0.901	0.899	0.900	0.900
	3200	0.897	0.897	0.895	0.886	0.902	0.901	0.899	0.900	0.902	0.901
	6400	0.896	0.903	0.892	0.896	0.904	0.897	0.901	0.898	0.902	0.903
.7	100	0.893	0.878	0.827	0.789	0.891	0.907	0.898	0.904	0.908	0.903
	200	0.894	0.889	0.848	0.825	0.895	0.902	0.900	0.900	0.910	0.904
	400	0.900	0.893	0.870	0.854	0.902	0.903	0.901	0.902	0.905	0.904
	800	0.893	0.896	0.888	0.869	0.900	0.902	0.899	0.903	0.898	0.903
	1600	0.901	0.895	0.896	0.888	0.898	0.899	0.898	0.899	0.901	0.899
	3200	0.898	0.899	0.899	0.895	0.904	0.900	0.898	0.900	0.898	0.901
	6400	0.900	0.896	0.898	0.894	0.895	0.899	0.898	0.896	0.899	0.898
.8	100	0.891	0.877	0.826	0.803	0.894	0.904	0.901	0.915	0.902	0.908
	200	0.898	0.886	0.864	0.834	0.899	0.903	0.900	0.903	0.915	0.905
	400	0.899	0.896	0.874	0.863	0.903	0.904	0.902	0.899	0.904	0.900
	800	0.901	0.899	0.888	0.882	0.893	0.904	0.901	0.898	0.904	0.903
	1600	0.901	0.894	0.896	0.894	0.898	0.900	0.898	0.900	0.901	0.900
	3200	0.899	0.902	0.894	0.892	0.902	0.900	0.901	0.900	0.903	0.903
	6400	0.901	0.902	0.901	0.901	0.902	0.900	0.901	0.901	0.902	0.900

Table 4,90,ρ: Actual coverages (in 10,000 trials) of nominal 90% confidence intervals on ρ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.890	0.931	0.954	0.913	0.890	0.897	0.958	0.926	0.913	0.930
	200	0.901	0.916	0.949	0.896	0.900	0.900	0.914	0.932	0.919	0.924
	400	0.895	0.907	0.938	0.888	0.901	0.900	0.903	0.924	0.922	0.912
	800	0.897	0.901	0.922	0.899	0.901	0.899	0.901	0.908	0.922	0.910
	1600	0.901	0.904	0.911	0.898	0.895	0.898	0.899	0.901	0.919	0.901
	3200	0.896	0.903	0.903	0.899	0.899	0.901	0.902	0.903	0.914	0.900
	6400	0.896	0.896	0.895	0.900	0.900	0.897	0.898	0.899	0.908	0.901
.6	100	0.899	0.931	0.954	0.910	0.885	0.903	0.952	0.925	0.909	0.933
	200	0.897	0.910	0.949	0.898	0.897	0.901	0.908	0.938	0.920	0.921
	400	0.898	0.908	0.935	0.888	0.903	0.901	0.900	0.919	0.921	0.915
	800	0.905	0.901	0.926	0.898	0.902	0.900	0.901	0.908	0.920	0.907
	1600	0.898	0.902	0.908	0.899	0.907	0.900	0.898	0.904	0.916	0.905
	3200	0.895	0.903	0.908	0.901	0.903	0.899	0.901	0.904	0.911	0.904
	6400	0.901	0.900	0.902	0.900	0.897	0.899	0.898	0.902	0.904	0.899
.7	100	0.891	0.927	0.960	0.917	0.880	0.902	0.954	0.929	0.907	0.930
	200	0.896	0.918	0.951	0.907	0.898	0.898	0.906	0.933	0.923	0.929
	400	0.902	0.908	0.931	0.905	0.904	0.900	0.899	0.914	0.928	0.914
	800	0.902	0.902	0.919	0.905	0.897	0.899	0.903	0.904	0.923	0.910
	1600	0.899	0.903	0.909	0.909	0.896	0.902	0.899	0.899	0.913	0.906
	3200	0.899	0.896	0.902	0.899	0.900	0.898	0.900	0.902	0.909	0.904
	6400	0.907	0.904	0.899	0.900	0.899	0.901	0.905	0.901	0.904	0.905
.8	100	0.892	0.933	0.953	0.925	0.871	0.900	0.948	0.934	0.909	0.928
	200	0.896	0.918	0.942	0.920	0.900	0.899	0.910	0.938	0.929	0.924
	400	0.898	0.912	0.932	0.915	0.906	0.901	0.902	0.914	0.932	0.915
	800	0.901	0.907	0.917	0.913	0.899	0.899	0.903	0.905	0.922	0.910
	1600	0.902	0.905	0.905	0.907	0.896	0.900	0.903	0.904	0.910	0.907
	3200	0.898	0.897	0.901	0.909	0.899	0.896	0.901	0.900	0.905	0.901
	6400	0.903	0.898	0.908	0.909	0.896	0.904	0.900	0.906	0.906	0.903

Table 4,90,γ: Actual coverages (in 10,000 trials) of nominal 90% confidence intervals on γ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.900	0.912	0.899	0.826	0.901	0.901	0.919	0.899	0.901	0.900
	200	0.897	0.915	0.896	0.815	0.899	0.899	0.912	0.906	0.904	0.899
	400	0.901	0.906	0.895	0.835	0.903	0.899	0.899	0.906	0.907	0.902
	800	0.897	0.903	0.902	0.854	0.901	0.898	0.899	0.903	0.907	0.900
	1600	0.902	0.901	0.905	0.869	0.898	0.901	0.901	0.903	0.902	0.899
	3200	0.900	0.901	0.903	0.877	0.900	0.901	0.899	0.905	0.902	0.900
	6400	0.904	0.899	0.898	0.895	0.896	0.901	0.899	0.901	0.900	0.898
.6	100	0.903	0.910	0.879	0.801	0.899	0.898	0.915	0.897	0.904	0.899
	200	0.901	0.911	0.883	0.820	0.898	0.903	0.909	0.909	0.907	0.896
	400	0.899	0.913	0.884	0.840	0.899	0.903	0.903	0.912	0.905	0.900
	800	0.898	0.901	0.895	0.862	0.901	0.897	0.899	0.903	0.905	0.899
	1600	0.896	0.899	0.896	0.874	0.900	0.898	0.898	0.900	0.900	0.896
	3200	0.901	0.899	0.897	0.884	0.900	0.901	0.903	0.901	0.901	0.900
	6400	0.899	0.903	0.895	0.896	0.894	0.899	0.899	0.899	0.900	0.896
.7	100	0.903	0.904	0.862	0.804	0.886	0.904	0.914	0.901	0.905	0.897
	200	0.899	0.904	0.868	0.831	0.897	0.898	0.904	0.905	0.899	0.902
	400	0.898	0.903	0.884	0.854	0.898	0.900	0.901	0.905	0.900	0.900
	800	0.898	0.901	0.893	0.868	0.898	0.899	0.899	0.905	0.901	0.902
	1600	0.899	0.897	0.898	0.887	0.903	0.899	0.899	0.903	0.900	0.903
	3200	0.903	0.902	0.900	0.896	0.901	0.901	0.901	0.901	0.904	0.900
	6400	0.908	0.898	0.901	0.898	0.900	0.901	0.898	0.900	0.899	0.903
.8	100	0.897	0.896	0.853	0.815	0.895	0.896	0.909	0.898	0.903	0.895
	200	0.900	0.900	0.875	0.842	0.900	0.898	0.901	0.905	0.894	0.900
	400	0.902	0.898	0.888	0.866	0.902	0.897	0.900	0.906	0.896	0.897
	800	0.902	0.900	0.890	0.883	0.898	0.898	0.899	0.902	0.901	0.901
	1600	0.902	0.900	0.894	0.892	0.895	0.900	0.905	0.899	0.897	0.898
	3200	0.899	0.903	0.898	0.896	0.903	0.900	0.903	0.899	0.903	0.903
	6400	0.897	0.901	0.900	0.902	0.903	0.898	0.900	0.898	0.901	0.899

Table 4,90,β: Actual coverages (in 10,000 trials) of nominal 90% confidence intervals on β

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
	100	0.945	0.983	0.996	0.836	0.904	0.949	0.978	0.944	0.903	0.947
	200	0.946	0.974	0.995	0.839	0.918	0.949	0.964	0.957	0.923	0.951
	400	0.947	0.959	0.992	0.864	0.938	0.950	0.950	0.956	0.938	0.954
.5	800	0.950	0.954	0.983	0.894	0.945	0.951	0.951	0.946	0.941	0.952
	1600	0.952	0.955	0.971	0.910	0.943	0.951	0.951	0.947	0.945	0.951
	3200	0.949	0.952	0.959	0.923	0.948	0.950	0.948	0.950	0.946	0.953
	6400	0.951	0.948	0.949	0.936	0.950	0.950	0.950	0.948	0.951	0.950
	100	0.949	0.980	0.998	0.847	0.898	0.950	0.975	0.947	0.906	0.947
	200	0.950	0.971	0.993	0.861	0.924	0.950	0.954	0.961	0.929	0.950
	400	0.954	0.963	0.991	0.871	0.934	0.952	0.947	0.950	0.940	0.953
.6	800	0.951	0.956	0.982	0.903	0.940	0.950	0.950	0.948	0.945	0.952
	1600	0.951	0.953	0.966	0.920	0.950	0.952	0.949	0.947	0.948	0.950
	3200	0.948	0.955	0.960	0.931	0.951	0.951	0.950	0.947	0.950	0.949
	6400	0.948	0.951	0.955	0.944	0.948	0.948	0.948	0.951	0.952	0.950
	100	0.943	0.981	0.997	0.861	0.904	0.950	0.975	0.952	0.908	0.946
	200	0.949	0.975	0.995	0.876	0.924	0.951	0.954	0.959	0.936	0.953
	400	0.951	0.962	0.988	0.897	0.939	0.952	0.949	0.948	0.946	0.949
.7	800	0.952	0.956	0.974	0.917	0.943	0.950	0.950	0.944	0.948	0.951
	1600	0.948	0.955	0.962	0.934	0.948	0.950	0.951	0.946	0.952	0.953
	3200	0.947	0.952	0.956	0.939	0.948	0.949	0.949	0.951	0.952	0.952
	6400	0.949	0.951	0.956	0.942	0.951	0.949	0.951	0.951	0.952	0.956
	100	0.945	0.981	0.995	0.878	0.903	0.949	0.975	0.954	0.914	0.949
	200	0.947	0.971	0.994	0.896	0.930	0.950	0.952	0.961	0.939	0.951
	400	0.946	0.962	0.987	0.910	0.942	0.949	0.951	0.947	0.948	0.949
.8	800	0.953	0.957	0.972	0.923	0.945	0.952	0.949	0.947	0.950	0.951
	1600	0.952	0.952	0.960	0.939	0.947	0.948	0.948	0.947	0.948	0.952
	3200	0.950	0.950	0.957	0.945	0.947	0.948	0.952	0.951	0.952	0.948
	6400	0.951	0.949	0.953	0.953	0.950	0.950	0.951	0.949	0.952	0.952

Table 4,95,μ: Actual coverages (in 10,000 trials) of nominal 95% confidence intervals on μ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.941	0.904	0.860	0.788	0.918	0.948	0.966	0.913	0.891	0.952
	200	0.948	0.926	0.887	0.832	0.933	0.951	0.968	0.934	0.904	0.952
	400	0.951	0.935	0.909	0.871	0.944	0.951	0.952	0.944	0.924	0.953
	800	0.950	0.948	0.927	0.901	0.947	0.951	0.949	0.947	0.936	0.951
	1600	0.947	0.951	0.937	0.919	0.947	0.948	0.952	0.951	0.947	0.950
	3200	0.953	0.951	0.944	0.926	0.949	0.952	0.952	0.951	0.953	0.951
	6400	0.950	0.956	0.945	0.939	0.948	0.948	0.951	0.950	0.951	0.952
.6	100	0.936	0.911	0.866	0.807	0.911	0.949	0.966	0.918	0.898	0.954
	200	0.947	0.924	0.889	0.851	0.934	0.952	0.956	0.940	0.913	0.952
	400	0.948	0.937	0.914	0.882	0.940	0.949	0.952	0.946	0.934	0.952
	800	0.949	0.945	0.929	0.909	0.946	0.949	0.951	0.947	0.945	0.953
	1600	0.948	0.949	0.942	0.927	0.947	0.950	0.948	0.950	0.951	0.949
	3200	0.952	0.945	0.947	0.933	0.946	0.950	0.950	0.950	0.951	0.948
	6400	0.950	0.950	0.945	0.943	0.950	0.951	0.950	0.951	0.949	0.950
.7	100	0.939	0.907	0.868	0.822	0.919	0.949	0.964	0.929	0.898	0.950
	200	0.947	0.930	0.896	0.863	0.933	0.952	0.956	0.942	0.926	0.952
	400	0.946	0.937	0.923	0.900	0.946	0.950	0.954	0.948	0.943	0.951
	800	0.946	0.946	0.932	0.921	0.943	0.951	0.952	0.951	0.949	0.952
	1600	0.950	0.948	0.943	0.937	0.949	0.953	0.951	0.949	0.953	0.951
	3200	0.951	0.950	0.949	0.946	0.952	0.951	0.950	0.949	0.951	0.950
	6400	0.948	0.949	0.951	0.948	0.951	0.949	0.951	0.950	0.952	0.952
.8	100	0.939	0.907	0.871	0.837	0.916	0.948	0.965	0.933	0.899	0.956
	200	0.947	0.926	0.907	0.881	0.936	0.951	0.954	0.949	0.927	0.953
	400	0.951	0.942	0.924	0.914	0.944	0.952	0.954	0.948	0.944	0.948
	800	0.951	0.948	0.939	0.935	0.944	0.950	0.951	0.949	0.950	0.949
	1600	0.950	0.945	0.948	0.943	0.949	0.951	0.949	0.950	0.949	0.949
	3200	0.952	0.949	0.948	0.944	0.950	0.949	0.950	0.950	0.949	0.951
	6400	0.951	0.952	0.949	0.951	0.950	0.951	0.951	0.951	0.951	0.950

Table 4,95,σ: Actual coverages (in 10,000 trials) of nominal 95% confidence intervals on σ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.943	0.927	0.876	0.852	0.941	0.950	0.942	0.952	0.962	0.950
	200	0.945	0.948	0.897	0.856	0.942	0.948	0.953	0.946	0.966	0.951
	400	0.947	0.949	0.918	0.884	0.945	0.952	0.949	0.945	0.956	0.951
	800	0.951	0.948	0.935	0.900	0.946	0.951	0.949	0.947	0.954	0.949
	1600	0.952	0.950	0.944	0.918	0.949	0.951	0.950	0.949	0.953	0.949
	3200	0.950	0.953	0.949	0.928	0.949	0.951	0.951	0.951	0.950	0.951
	6400	0.950	0.952	0.948	0.944	0.949	0.947	0.948	0.951	0.952	0.951
.6	100	0.940	0.926	0.874	0.844	0.938	0.953	0.945	0.963	0.957	0.951
	200	0.938	0.940	0.903	0.870	0.941	0.947	0.950	0.947	0.960	0.950
	400	0.946	0.948	0.916	0.894	0.952	0.948	0.949	0.950	0.956	0.953
	800	0.948	0.950	0.937	0.917	0.949	0.952	0.948	0.948	0.956	0.949
	1600	0.953	0.951	0.947	0.925	0.951	0.949	0.951	0.949	0.951	0.951
	3200	0.947	0.948	0.943	0.939	0.951	0.950	0.949	0.951	0.951	0.950
	6400	0.946	0.950	0.947	0.945	0.951	0.948	0.950	0.950	0.951	0.951
.7	100	0.940	0.931	0.881	0.853	0.940	0.952	0.948	0.966	0.946	0.951
	200	0.945	0.938	0.901	0.884	0.944	0.951	0.950	0.951	0.949	0.951
	400	0.950	0.942	0.924	0.902	0.950	0.950	0.949	0.951	0.953	0.950
	800	0.945	0.947	0.938	0.919	0.946	0.949	0.948	0.951	0.949	0.951
	1600	0.952	0.947	0.945	0.936	0.950	0.952	0.948	0.950	0.951	0.951
	3200	0.947	0.951	0.946	0.944	0.950	0.950	0.950	0.950	0.953	0.951
	6400	0.951	0.947	0.949	0.947	0.948	0.951	0.950	0.950	0.949	0.950
.8	100	0.941	0.929	0.881	0.859	0.941	0.950	0.948	0.959	0.936	0.948
	200	0.948	0.940	0.916	0.888	0.947	0.954	0.948	0.953	0.948	0.950
	400	0.951	0.947	0.925	0.914	0.950	0.953	0.952	0.950	0.950	0.949
	800	0.948	0.949	0.938	0.933	0.945	0.951	0.952	0.949	0.954	0.949
	1600	0.950	0.947	0.942	0.939	0.950	0.951	0.950	0.949	0.951	0.952
	3200	0.951	0.950	0.949	0.943	0.951	0.952	0.949	0.950	0.949	0.950
	6400	0.950	0.953	0.951	0.948	0.954	0.950	0.954	0.950	0.950	0.951

Table 4,95,ρ: Actual coverages (in 10,000 trials) of nominal 95% confidence intervals on ρ

		Theory-based confidence intervals					Simulation-based confidence intervals				
ρ	n	full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.942	0.967	0.979	0.946	0.923	0.949	0.971	0.942	0.932	0.950
	200	0.949	0.961	0.980	0.934	0.932	0.947	0.958	0.950	0.939	0.954
	400	0.951	0.954	0.975	0.926	0.942	0.951	0.949	0.952	0.941	0.953
	800	0.951	0.951	0.966	0.934	0.949	0.951	0.950	0.950	0.943	0.951
	1600	0.954	0.956	0.957	0.936	0.945	0.952	0.950	0.948	0.947	0.950
	3200	0.946	0.952	0.951	0.938	0.950	0.949	0.950	0.949	0.949	0.948
	6400	0.947	0.946	0.946	0.942	0.950	0.948	0.949	0.949	0.951	0.948
.6	100	0.947	0.969	0.982	0.944	0.919	0.952	0.969	0.943	0.931	0.951
	200	0.948	0.961	0.977	0.933	0.932	0.950	0.951	0.957	0.940	0.949
	400	0.950	0.956	0.976	0.926	0.942	0.949	0.949	0.949	0.943	0.952
	800	0.953	0.952	0.968	0.934	0.947	0.951	0.949	0.948	0.948	0.951
	1600	0.948	0.952	0.958	0.940	0.951	0.950	0.951	0.948	0.951	0.951
	3200	0.950	0.952	0.956	0.941	0.951	0.953	0.951	0.951	0.952	0.951
	6400	0.950	0.953	0.951	0.950	0.948	0.949	0.951	0.950	0.951	0.950
.7	100	0.944	0.969	0.984	0.949	0.912	0.948	0.973	0.947	0.929	0.948
	200	0.951	0.961	0.981	0.944	0.928	0.953	0.948	0.955	0.942	0.951
	400	0.950	0.956	0.969	0.942	0.942	0.951	0.948	0.947	0.948	0.949
	800	0.952	0.956	0.963	0.942	0.945	0.949	0.952	0.948	0.950	0.952
	1600	0.949	0.952	0.956	0.947	0.948	0.950	0.950	0.948	0.948	0.952
	3200	0.949	0.950	0.951	0.949	0.954	0.949	0.950	0.950	0.950	0.951
	6400	0.954	0.952	0.950	0.950	0.948	0.950	0.951	0.949	0.949	0.953
.8	100	0.946	0.974	0.978	0.952	0.907	0.950	0.968	0.949	0.929	0.949
	200	0.947	0.964	0.977	0.951	0.928	0.951	0.951	0.960	0.945	0.950
	400	0.949	0.959	0.967	0.948	0.943	0.951	0.949	0.949	0.953	0.949
	800	0.951	0.955	0.961	0.948	0.943	0.951	0.951	0.946	0.952	0.953
	1600	0.949	0.953	0.954	0.951	0.947	0.949	0.950	0.949	0.950	0.952
	3200	0.950	0.949	0.954	0.956	0.947	0.948	0.952	0.951	0.951	0.948
	6400	0.948	0.950	0.954	0.956	0.950	0.948	0.951	0.952	0.952	0.952

Table 4,95,γ: Actual coverages (in 10,000 trials) of nominal 95% confidence intervals on γ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.950	0.961	0.944	0.885	0.950	0.949	0.952	0.933	0.946	0.947
	200	0.948	0.960	0.943	0.871	0.947	0.947	0.952	0.939	0.946	0.949
	400	0.951	0.954	0.944	0.890	0.951	0.948	0.949	0.944	0.948	0.950
	800	0.952	0.954	0.949	0.908	0.951	0.952	0.947	0.944	0.946	0.952
	1600	0.952	0.951	0.950	0.921	0.947	0.950	0.949	0.950	0.950	0.947
	3200	0.950	0.949	0.951	0.930	0.952	0.950	0.948	0.951	0.952	0.953
	6400	0.952	0.950	0.949	0.943	0.948	0.951	0.950	0.948	0.950	0.950
.6	100	0.953	0.957	0.930	0.863	0.945	0.944	0.950	0.934	0.956	0.947
	200	0.952	0.955	0.933	0.872	0.947	0.950	0.950	0.943	0.956	0.950
	400	0.950	0.958	0.936	0.895	0.948	0.950	0.948	0.947	0.952	0.952
	800	0.948	0.954	0.944	0.916	0.952	0.948	0.951	0.947	0.955	0.948
	1600	0.950	0.950	0.946	0.924	0.953	0.950	0.948	0.949	0.952	0.950
	3200	0.948	0.949	0.948	0.933	0.951	0.948	0.950	0.948	0.952	0.951
	6400	0.950	0.952	0.951	0.943	0.949	0.951	0.950	0.952	0.949	0.948
.7	100	0.952	0.951	0.917	0.859	0.942	0.947	0.948	0.940	0.970	0.946
	200	0.951	0.951	0.922	0.882	0.947	0.946	0.948	0.947	0.958	0.949
	400	0.947	0.951	0.932	0.908	0.946	0.948	0.948	0.949	0.951	0.949
	800	0.948	0.949	0.943	0.922	0.945	0.948	0.949	0.949	0.951	0.949
	1600	0.951	0.949	0.947	0.937	0.953	0.950	0.949	0.951	0.952	0.953
	3200	0.954	0.950	0.950	0.945	0.950	0.951	0.947	0.949	0.952	0.950
	6400	0.957	0.949	0.950	0.945	0.948	0.952	0.950	0.948	0.949	0.949
.8	100	0.949	0.948	0.909	0.869	0.944	0.944	0.949	0.943	0.975	0.946
	200	0.949	0.948	0.926	0.896	0.948	0.946	0.948	0.949	0.959	0.950
	400	0.949	0.946	0.936	0.917	0.950	0.947	0.947	0.949	0.949	0.949
	800	0.952	0.952	0.943	0.935	0.947	0.949	0.950	0.952	0.950	0.950
	1600	0.953	0.948	0.949	0.942	0.947	0.951	0.952	0.952	0.950	0.949
	3200	0.948	0.949	0.945	0.945	0.949	0.950	0.950	0.950	0.952	0.950
	6400	0.950	0.950	0.951	0.949	0.953	0.950	0.951	0.952	0.951	0.952

Table 4,95,β: Actual coverages (in 10,000 trials) of nominal 95% confidence intervals on β

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.987	0.999	1.000	0.897	0.948	0.990	0.983	0.959	0.935	0.969
	200	0.989	0.997	1.000	0.897	0.962	0.990	0.987	0.972	0.949	0.977
	400	0.989	0.996	1.000	0.915	0.974	0.990	0.985	0.977	0.959	0.982
	800	0.989	0.993	0.999	0.939	0.983	0.989	0.988	0.979	0.966	0.985
	1600	0.992	0.993	0.998	0.953	0.985	0.991	0.991	0.984	0.971	0.987
	3200	0.989	0.992	0.994	0.965	0.988	0.990	0.991	0.986	0.975	0.990
	6400	0.990	0.988	0.991	0.977	0.989	0.990	0.989	0.989	0.981	0.989
.6	100	0.990	0.998	1.000	0.903	0.945	0.991	0.985	0.963	0.936	0.970
	200	0.990	0.998	1.000	0.920	0.964	0.989	0.984	0.975	0.955	0.976
	400	0.991	0.996	1.000	0.924	0.975	0.991	0.987	0.976	0.964	0.982
	800	0.990	0.992	0.999	0.947	0.981	0.990	0.989	0.981	0.969	0.986
	1600	0.991	0.993	0.998	0.961	0.986	0.991	0.990	0.987	0.976	0.988
	3200	0.988	0.992	0.995	0.968	0.987	0.990	0.990	0.987	0.981	0.988
	6400	0.990	0.990	0.992	0.980	0.989	0.990	0.989	0.988	0.985	0.991
.7	100	0.988	0.998	1.000	0.910	0.947	0.992	0.985	0.966	0.940	0.970
	200	0.990	0.999	1.000	0.925	0.965	0.991	0.984	0.977	0.959	0.978
	400	0.991	0.996	1.000	0.946	0.976	0.991	0.985	0.978	0.970	0.981
	800	0.991	0.991	0.999	0.955	0.982	0.990	0.988	0.982	0.974	0.983
	1600	0.989	0.991	0.996	0.971	0.987	0.990	0.988	0.987	0.978	0.989
	3200	0.990	0.989	0.993	0.979	0.989	0.990	0.989	0.988	0.983	0.989
	6400	0.990	0.991	0.992	0.984	0.988	0.990	0.990	0.989	0.985	0.989
.8	100	0.988	0.998	1.000	0.924	0.950	0.990	0.988	0.969	0.941	0.974
	200	0.989	0.997	1.000	0.940	0.969	0.991	0.985	0.979	0.962	0.977
	400	0.990	0.996	1.000	0.950	0.976	0.992	0.987	0.980	0.969	0.982
	800	0.991	0.993	0.999	0.964	0.982	0.990	0.989	0.984	0.976	0.987
	1600	0.993	0.991	0.996	0.976	0.985	0.993	0.989	0.986	0.979	0.988
	3200	0.990	0.991	0.994	0.983	0.985	0.989	0.990	0.990	0.984	0.988
	6400	0.990	0.988	0.991	0.988	0.989	0.989	0.989	0.988	0.988	0.991

Table 4,99,μ: Actual coverages (in 10,000 trials) of nominal 99% confidence intervals on μ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.982	0.948	0.908	0.838	0.961	0.990	0.975	0.939	0.998	0.977
	200	0.986	0.967	0.933	0.878	0.972	0.991	0.982	0.958	0.961	0.985
	400	0.989	0.972	0.950	0.916	0.982	0.991	0.978	0.966	0.959	0.987
	800	0.988	0.979	0.965	0.942	0.985	0.989	0.983	0.972	0.968	0.988
	1600	0.990	0.985	0.974	0.960	0.987	0.990	0.988	0.980	0.977	0.990
	3200	0.990	0.989	0.981	0.972	0.988	0.990	0.989	0.983	0.981	0.991
	6400	0.990	0.990	0.985	0.981	0.989	0.989	0.990	0.986	0.984	0.989
.6	100	0.982	0.951	0.911	0.854	0.956	0.990	0.976	0.947	0.993	0.980
	200	0.987	0.964	0.933	0.895	0.972	0.990	0.978	0.965	0.952	0.985
	400	0.989	0.975	0.956	0.926	0.980	0.989	0.978	0.971	0.968	0.987
	800	0.989	0.981	0.967	0.951	0.986	0.989	0.984	0.978	0.976	0.988
	1600	0.989	0.985	0.979	0.968	0.987	0.990	0.986	0.980	0.980	0.989
	3200	0.991	0.986	0.984	0.976	0.988	0.990	0.988	0.985	0.986	0.990
	6400	0.990	0.987	0.985	0.982	0.990	0.991	0.988	0.989	0.988	0.990
.7	100	0.983	0.951	0.916	0.863	0.958	0.989	0.979	0.953	0.986	0.979
	200	0.988	0.967	0.937	0.909	0.973	0.991	0.978	0.967	0.958	0.986
	400	0.988	0.974	0.960	0.942	0.982	0.991	0.981	0.975	0.970	0.986
	800	0.989	0.983	0.974	0.961	0.986	0.989	0.984	0.977	0.979	0.987
	1600	0.989	0.986	0.981	0.976	0.988	0.989	0.988	0.983	0.985	0.989
	3200	0.988	0.989	0.987	0.983	0.990	0.988	0.990	0.986	0.986	0.990
	6400	0.990	0.990	0.988	0.986	0.989	0.991	0.988	0.987	0.988	0.989
.8	100	0.985	0.951	0.917	0.881	0.961	0.990	0.981	0.955	0.977	0.981
	200	0.986	0.967	0.949	0.922	0.975	0.989	0.978	0.971	0.959	0.985
	400	0.989	0.977	0.960	0.954	0.983	0.990	0.982	0.974	0.972	0.987
	800	0.991	0.984	0.974	0.971	0.983	0.990	0.985	0.979	0.980	0.990
	1600	0.990	0.984	0.983	0.982	0.988	0.989	0.987	0.985	0.982	0.990
	3200	0.991	0.988	0.987	0.985	0.988	0.990	0.990	0.987	0.987	0.990
	6400	0.990	0.991	0.988	0.989	0.989	0.990	0.991	0.989	0.989	0.990

Table 4,99,σ: Actual coverages (in 10,000 trials) of nominal 99% confidence intervals on σ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.984	0.972	0.936	0.927	0.983	0.988	0.981	0.997	0.988	0.987
	200	0.987	0.988	0.956	0.935	0.986	0.990	0.985	0.991	0.994	0.988
	400	0.989	0.990	0.971	0.949	0.986	0.988	0.988	0.988	0.995	0.989
	800	0.990	0.988	0.981	0.963	0.988	0.990	0.990	0.986	0.994	0.989
	1600	0.990	0.989	0.987	0.972	0.991	0.990	0.990	0.988	0.992	0.992
	3200	0.990	0.990	0.989	0.979	0.991	0.990	0.990	0.989	0.991	0.991
	6400	0.990	0.992	0.990	0.985	0.989	0.988	0.990	0.990	0.990	0.989
.6	100	0.982	0.974	0.935	0.924	0.983	0.988	0.984	0.994	0.988	0.986
	200	0.984	0.986	0.958	0.944	0.984	0.990	0.989	0.992	0.992	0.989
	400	0.988	0.988	0.972	0.954	0.989	0.989	0.988	0.990	0.992	0.992
	800	0.989	0.989	0.982	0.970	0.988	0.988	0.989	0.990	0.993	0.989
	1600	0.991	0.988	0.986	0.976	0.990	0.990	0.990	0.989	0.992	0.990
	3200	0.989	0.989	0.988	0.982	0.990	0.990	0.989	0.990	0.990	0.990
	6400	0.989	0.990	0.990	0.988	0.989	0.989	0.989	0.991	0.990	0.989
.7	100	0.983	0.975	0.938	0.928	0.981	0.987	0.987	0.992	0.983	0.986
	200	0.987	0.986	0.958	0.948	0.986	0.989	0.988	0.994	0.986	0.988
	400	0.988	0.989	0.974	0.961	0.987	0.989	0.989	0.992	0.987	0.987
	800	0.988	0.990	0.982	0.970	0.987	0.990	0.989	0.989	0.989	0.988
	1600	0.989	0.990	0.986	0.982	0.990	0.989	0.991	0.990	0.988	0.990
	3200	0.989	0.990	0.989	0.987	0.989	0.990	0.990	0.990	0.991	0.988
	6400	0.991	0.989	0.990	0.989	0.989	0.990	0.990	0.990	0.991	0.990
.8	100	0.980	0.975	0.940	0.933	0.983	0.987	0.987	0.988	0.975	0.982
	200	0.987	0.984	0.964	0.946	0.985	0.990	0.988	0.992	0.981	0.986
	400	0.990	0.987	0.977	0.965	0.988	0.991	0.988	0.991	0.984	0.988
	800	0.988	0.989	0.985	0.979	0.988	0.988	0.990	0.991	0.988	0.989
	1600	0.989	0.989	0.986	0.984	0.990	0.989	0.990	0.990	0.990	0.990
	3200	0.990	0.990	0.989	0.985	0.990	0.991	0.990	0.990	0.989	0.990
	6400	0.990	0.992	0.991	0.988	0.991	0.989	0.991	0.992	0.990	0.992

Table 4,99,ρ: Actual coverages (in 10,000 trials) of nominal 99% confidence intervals on ρ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.989	0.995	0.997	0.981	0.965	0.990	0.984	0.962	0.959	0.972
	200	0.988	0.993	0.998	0.977	0.970	0.989	0.987	0.971	0.963	0.978
	400	0.991	0.992	0.997	0.972	0.978	0.990	0.988	0.978	0.966	0.982
	800	0.990	0.990	0.996	0.974	0.985	0.990	0.989	0.984	0.969	0.983
	1600	0.990	0.992	0.993	0.972	0.986	0.990	0.991	0.989	0.975	0.989
	3200	0.988	0.990	0.990	0.979	0.989	0.989	0.989	0.987	0.979	0.989
	6400	0.989	0.989	0.991	0.982	0.989	0.990	0.989	0.991	0.983	0.991
.6	100	0.988	0.995	0.997	0.982	0.962	0.988	0.984	0.964	0.959	0.971
	200	0.989	0.994	0.996	0.975	0.971	0.990	0.986	0.975	0.963	0.976
	400	0.989	0.994	0.997	0.974	0.977	0.991	0.989	0.978	0.970	0.981
	800	0.991	0.991	0.997	0.974	0.984	0.991	0.990	0.984	0.974	0.985
	1600	0.990	0.992	0.994	0.978	0.988	0.990	0.991	0.988	0.978	0.986
	3200	0.991	0.989	0.992	0.980	0.989	0.991	0.988	0.987	0.985	0.988
	6400	0.990	0.991	0.991	0.985	0.989	0.990	0.990	0.990	0.987	0.989
.7	100	0.987	0.996	0.998	0.982	0.956	0.990	0.986	0.965	0.958	0.969
	200	0.992	0.995	0.996	0.979	0.968	0.991	0.985	0.977	0.964	0.976
	400	0.989	0.993	0.995	0.979	0.975	0.990	0.989	0.982	0.971	0.979
	800	0.992	0.992	0.995	0.979	0.982	0.991	0.990	0.983	0.976	0.984
	1600	0.989	0.991	0.993	0.983	0.987	0.990	0.989	0.987	0.982	0.988
	3200	0.991	0.992	0.990	0.986	0.989	0.990	0.990	0.988	0.984	0.991
	6400	0.993	0.990	0.991	0.986	0.988	0.991	0.990	0.989	0.987	0.989
.8	100	0.986	0.996	0.997	0.985	0.953	0.989	0.984	0.967	0.955	0.971
	200	0.989	0.994	0.996	0.985	0.970	0.990	0.985	0.978	0.965	0.976
	400	0.990	0.993	0.995	0.983	0.976	0.991	0.987	0.980	0.972	0.982
	800	0.991	0.992	0.992	0.984	0.981	0.991	0.989	0.983	0.979	0.984
	1600	0.991	0.990	0.992	0.985	0.985	0.991	0.988	0.986	0.980	0.987
	3200	0.992	0.991	0.991	0.988	0.985	0.992	0.992	0.990	0.986	0.989
	6400	0.988	0.990	0.991	0.989	0.989	0.989	0.990	0.990	0.989	0.992

Table 4,99,γ: Actual coverages (in 10,000 trials) of nominal 99% confidence intervals on γ

ρ	n	Theory-based confidence intervals					Simulation-based confidence intervals				
		full	.1,.9	.2,.8	.4,.8	.4,1	full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.989	0.992	0.987	0.960	0.989	0.985	0.977	0.971	0.989	0.988
	200	0.991	0.993	0.986	0.944	0.988	0.988	0.984	0.974	0.986	0.990
	400	0.990	0.993	0.984	0.951	0.989	0.986	0.989	0.978	0.985	0.989
	800	0.990	0.992	0.986	0.962	0.989	0.990	0.989	0.981	0.982	0.989
	1600	0.989	0.991	0.988	0.970	0.990	0.990	0.990	0.987	0.987	0.990
	3200	0.989	0.990	0.991	0.977	0.991	0.990	0.989	0.987	0.987	0.991
	6400	0.990	0.991	0.990	0.983	0.989	0.990	0.990	0.990	0.988	0.989
.6	100	0.991	0.990	0.979	0.933	0.987	0.986	0.977	0.977	0.996	0.987
	200	0.989	0.990	0.979	0.938	0.987	0.987	0.984	0.977	0.992	0.989
	400	0.990	0.992	0.983	0.950	0.990	0.989	0.988	0.980	0.990	0.991
	800	0.990	0.991	0.985	0.968	0.990	0.989	0.988	0.985	0.989	0.990
	1600	0.992	0.991	0.989	0.976	0.992	0.991	0.990	0.989	0.989	0.991
	3200	0.990	0.989	0.989	0.982	0.990	0.990	0.989	0.987	0.990	0.990
	6400	0.990	0.990	0.990	0.987	0.991	0.990	0.989	0.991	0.991	0.991
.7	100	0.990	0.988	0.971	0.926	0.985	0.984	0.980	0.982	0.999	0.989
	200	0.990	0.990	0.974	0.941	0.986	0.987	0.987	0.983	0.996	0.988
	400	0.989	0.987	0.979	0.960	0.987	0.988	0.989	0.986	0.991	0.989
	800	0.991	0.989	0.982	0.971	0.989	0.989	0.988	0.987	0.990	0.990
	1600	0.991	0.989	0.987	0.983	0.989	0.991	0.989	0.988	0.991	0.990
	3200	0.992	0.990	0.990	0.986	0.991	0.991	0.990	0.988	0.991	0.991
	6400	0.991	0.989	0.990	0.988	0.989	0.989	0.989	0.989	0.990	0.990
.8	100	0.991	0.986	0.966	0.928	0.986	0.986	0.984	0.989	0.998	0.989
	200	0.992	0.988	0.977	0.957	0.990	0.986	0.986	0.986	0.998	0.990
	400	0.991	0.990	0.981	0.967	0.990	0.989	0.988	0.989	0.989	0.991
	800	0.990	0.990	0.985	0.980	0.987	0.988	0.989	0.989	0.991	0.988
	1600	0.989	0.989	0.989	0.986	0.988	0.989	0.989	0.990	0.990	0.989
	3200	0.990	0.988	0.990	0.986	0.989	0.989	0.989	0.991	0.990	0.989
	6400	0.991	0.991	0.990	0.989	0.991	0.990	0.992	0.989	0.990	0.991

Table 4,99,β: Actual coverages (in 10,000 trials) of nominal 99% confidence intervals on β

Parameter	Actual coverage	ρ	Theory based Full	Theory based .4,.8	Simulation based Full	Simulation based .4,.8
μ	$.88 - .92$	.5	< 100	2184	< 100	2737
		.6	< 100	1347	< 100	1972
		.7	< 100	697	< 100	1517
		.8	< 100	446	< 100	1050
	$.89 - .91$	.5	< 100	5815	< 100	> 12000
		.6	< 100	2388	< 100	4884
		.7	106	1360	< 100	4028
		.8	< 100	806	< 100	3626
σ	$.88 - .92$	.5	< 100	2794	< 100	214
		.6	< 100	1816	< 100	185
		.7	< 100	656	< 100	121
		.8	< 100	414	< 100	114
	$.89 - .91$	.5	< 100	9381	< 100	4693
		.6	111	5565	< 100	2034
		.7	< 100	1109	< 100	1373
		.8	106	706	< 100	1261
ρ	$.88 - .92$	.5	< 100	3386	< 100	131
		.6	< 100	2250	< 100	100
		.7	< 100	1111	< 100	< 100
		.8	< 100	800	< 100	< 100
	$.89 - .91$	.5	< 100	10530	< 100	202
		.6	< 100	6658	< 100	165
		.7	< 100	2499	< 100	203
		.8	< 100	1389	< 100	247
γ	$.88 - .92$	.5	< 100	< 100	< 100	1240
		.6	< 100	< 100	< 100	912
		.7	< 100	< 100	< 100	976
		.8	< 100	170	< 100	824
	$.89 - .91$	.5	< 100	429	< 100	5565
		.6	< 100	436	< 100	3017
		.7	< 100	150	< 100	2462
		.8	< 100	1670	< 100	1947
β	$.88 - .92$	.5	< 100	2886	< 100	< 100
		.6	< 100	1988	< 100	< 100
		.7	< 100	1088	< 100	< 100
		.8	< 100	678	< 100	< 100
	$.89 - .91$	.5	< 100	8529	< 100	< 100
		.6	< 100	4949	< 100	< 100
		.7	< 100	2414	< 100	< 100
		.8	< 100	1274	< 100	< 100

Table 5: *Approximate* sample sizes needed to obtain actual confidence interval coverages that lie between .88 and .92 ("wide") or between .89 and .91 ("narrow") for nominal two-sided, 90% confidence intervals on the parameter

ρ	n	Bias (%)					rmse (%)				
		Full	.1,.9	.2,.8	.4,.8	.4,1	Full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.49	0.56	0.55	0.75	0.63	4.10	4.20	4.41	4.20	4.05
	200	0.24	0.31	0.28	0.45	0.37	2.87	2.98	3.18	3.03	2.84
	400	0.12	0.16	0.16	0.30	0.21	2.03	2.09	2.22	2.17	1.98
	800	0.04	0.06	0.10	0.15	0.07	1.43	1.46	1.59	1.54	1.41
	1600	0.03	0.04	0.03	0.08	0.04	1.01	1.04	1.10	1.09	0.99
	3200	0.02	0.01	0.01	0.05	0.02	0.72	0.73	0.77	0.78	0.71
	6400	0.01	0.01	0.00	0.02	0.02	0.51	0.52	0.56	0.55	0.50
.6	100	0.49	0.51	0.45	0.58	0.57	3.69	3.76	3.97	3.80	3.64
	200	0.23	0.23	0.25	0.38	0.30	2.59	2.61	2.77	2.74	2.51
	400	0.07	0.13	0.15	0.22	0.13	1.82	1.86	1.99	1.97	1.77
	800	0.06	0.07	0.06	0.10	0.07	1.27	1.31	1.40	1.41	1.27
	1600	0.03	0.02	0.02	0.07	0.04	0.89	0.93	0.98	1.00	0.87
	3200	0.01	0.01	0.02	0.03	0.02	0.64	0.66	0.69	0.71	0.63
	6400	0.01	0.01	0.02	0.02	0.01	0.45	0.46	0.49	0.50	0.44
.7	100	0.34	0.34	0.34	0.47	0.44	3.08	3.14	3.29	3.32	3.11
	200	0.17	0.20	0.18	0.16	0.24	2.18	2.21	2.35	2.40	2.19
	400	0.08	0.11	0.08	0.17	0.10	1.54	1.59	1.66	1.72	1.56
	800	0.03	0.05	0.04	0.06	0.04	1.10	1.11	1.17	1.23	1.09
	1600	0.01	0.02	0.02	0.03	0.03	0.76	0.78	0.82	0.85	0.77
	3200	0.01	0.00	0.02	0.02	0.01	0.54	0.55	0.58	0.61	0.54
	6400	0.01	0.01	0.00	0.01	0.01	0.38	0.39	0.41	0.43	0.39
.8	100	0.27	0.25	0.20	0.30	0.32	2.42	2.47	2.52	2.73	2.50
	200	0.17	0.11	0.12	0.19	0.17	1.70	1.73	1.78	1.96	1.77
	400	0.05	0.06	0.06	0.09	0.08	1.20	1.21	1.25	1.41	1.25
	800	0.06	0.03	0.02	0.04	0.04	0.84	0.86	0.90	0.99	0.87
	1600	0.02	0.01	0.02	0.04	0.02	0.60	0.61	0.63	0.70	0.62
	3200	0.01	0.01	0.01	0.01	0.01	0.42	0.43	0.44	0.49	0.45
	6400	0.00	0.00	0.01	0.00	0.01	0.30	0.30	0.31	0.35	0.31

Table 6: Fifth percentile estimates

ρ	n	Theory based					Simulation based				
		Full	.1,.9	.2,.8	.4,.8	.4,1	Full	.1,.9	.2,.8	.4,.8	.4,1
.5	100	0.701	0.702	0.695	0.682	0.693	0.711	0.775	0.890	0.942	0.743
	200	0.714	0.705	0.712	0.697	0.702	0.717	0.727	0.833	0.911	0.727
	400	0.728	0.723	0.719	0.707	0.717	0.730	0.732	0.772	0.876	0.725
	800	0.744	0.742	0.723	0.723	0.736	0.744	0.745	0.749	0.826	0.741
	1600	0.742	0.736	0.741	0.731	0.738	0.742	0.738	0.755	0.789	0.740
	3200	0.738	0.745	0.747	0.729	0.738	0.740	0.747	0.753	0.763	0.741
	6400	0.746	0.740	0.747	0.740	0.735	0.746	0.743	0.753	0.755	0.737
.6	100	0.694	0.688	0.690	0.694	0.687	0.704	0.780	0.895	0.946	0.738
	200	0.709	0.717	0.710	0.703	0.715	0.717	0.738	0.828	0.911	0.734
	400	0.733	0.720	0.713	0.716	0.726	0.737	0.733	0.772	0.867	0.733
	800	0.734	0.733	0.722	0.726	0.732	0.735	0.739	0.754	0.813	0.737
	1600	0.734	0.739	0.741	0.729	0.739	0.733	0.743	0.757	0.787	0.736
	3200	0.742	0.737	0.743	0.736	0.740	0.743	0.740	0.752	0.765	0.743
	6400	0.743	0.744	0.738	0.741	0.744	0.744	0.744	0.745	0.753	0.743
.7	100	0.700	0.703	0.699	0.693	0.693	0.712	0.785	0.912	0.952	0.742
	200	0.716	0.711	0.708	0.724	0.712	0.723	0.737	0.835	0.918	0.736
	400	0.728	0.719	0.726	0.715	0.728	0.734	0.737	0.782	0.868	0.740
	800	0.735	0.733	0.733	0.733	0.736	0.741	0.742	0.763	0.817	0.739
	1600	0.745	0.741	0.730	0.739	0.735	0.745	0.747	0.749	0.783	0.735
	3200	0.743	0.752	0.740	0.735	0.747	0.742	0.755	0.750	0.759	0.747
	6400	0.746	0.743	0.747	0.743	0.743	0.745	0.744	0.753	0.756	0.745
.8	100	0.697	0.696	0.702	0.703	0.698	0.712	0.793	0.934	0.967	0.749
	200	0.706	0.721	0.715	0.711	0.714	0.715	0.751	0.846	0.919	0.738
	400	0.735	0.726	0.726	0.723	0.719	0.741	0.743	0.783	0.868	0.731
	800	0.728	0.734	0.740	0.732	0.733	0.729	0.744	0.769	0.804	0.736
	1600	0.734	0.739	0.735	0.730	0.737	0.735	0.748	0.755	0.772	0.739
	3200	0.747	0.736	0.741	0.740	0.735	0.749	0.743	0.752	0.760	0.738
	6400	0.748	0.749	0.743	0.747	0.740	0.749	0.753	0.749	0.758	0.743

Table 7: Actual coverages of nominal one-sided, lower 75% confidence intervals on the 5th percentile of the .4,.8 PTW population

Actual coverage	ρ	Theory based		Simulation based	
		Full	.4,.8	Full	.4,.8
.73 – .77	.5	441	1910	403	2653
	.6	566	1391	405	2665
	.7	454	753	297	2287
	.8	586	815	421	1702
.74 – .76	.5	1203	8121	1087	4106
	.6	2183	5895	2134	4064
	.7	1248	5375	914	3728
	.8	1655	3188	1294	3694

Table 8: *Approximate* sample sizes needed to obtain actual confidence interval coverages that lie between .73 and .77 ("wide") or between .74 and .76 ("narrow") for nominal one-sided, lower 75% confidence intervals on the 5th percentile of the .4,.8 PTW population

ρ	n		Median p_{Br} ratio	p_{Br} ratio frequencies [0, .01]	(.01, .02]	(.02, .1]	(.1, .2]	(.2, .5]	(.5, 1)	(1, 2)	[2, 5)	[5, 10)	[10, 50)	[50, 100)	[100, ∞)
		tr	1.200	0.0000	0.0000	0.0017	0.0147	0.1447	0.2646	0.2744	0.2019	0.0542	0.0336	0.0035	0.0067
	100	full	1.141	0.0000	0.0000	0.0007	0.0132	0.1408	0.2805	0.3069	0.2094	0.0372	0.0112	0.0001	0.0000
		tr	1.100	0.0000	0.0000	0.0001	0.0015	0.1022	0.3382	0.3638	0.1696	0.0191	0.0048	0.0006	0.0001
	200	full	1.058	0.0000	0.0000	0.0000	0.0009	0.0938	0.3664	0.3885	0.1429	0.0072	0.0003	0.0000	0.0000
		tr	1.048	0.0000	0.0000	0.0000	0.0001	0.0414	0.4163	0.4515	0.0892	0.0014	0.0001	0.0000	0.0000
	400	full	1.032	0.0000	0.0000	0.0000	0.0001	0.0360	0.4329	0.4681	0.0628	0.0001	0.0000	0.0000	0.0000
		tr	1.016	0.0000	0.0000	0.0000	0.0000	0.0113	0.4691	0.4944	0.0252	0.0000	0.0000	0.0000	0.0000
.5	800	full	1.011	0.0000	0.0000	0.0000	0.0000	0.0060	0.4778	0.5030	0.0132	0.0000	0.0000	0.0000	0.0000
		tr	1.013	0.0000	0.0000	0.0000	0.0000	0.0009	0.4777	0.5195	0.0019	0.0000	0.0000	0.0000	0.0000
	1600	full	1.009	0.0000	0.0000	0.0000	0.0000	0.0003	0.4827	0.5163	0.0007	0.0000	0.0000	0.0000	0.0000
		tr	1.002	0.0000	0.0000	0.0000	0.0000	0.0000	0.4950	0.5049	0.0001	0.0000	0.0000	0.0000	0.0000
	3200	full	1.005	0.0000	0.0000	0.0000	0.0000	0.0000	0.4856	0.5144	0.0000	0.0000	0.0000	0.0000	0.0000
		tr	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4911	0.5089	0.0000	0.0000	0.0000	0.0000	0.0000
	6400	full	1.002	0.0000	0.0000	0.0000	0.0000	0.0000	0.4915	0.5085	0.0000	0.0000	0.0000	0.0000	0.0000
		tr	1.257	0.0000	0.0000	0.0044	0.0265	0.1565	0.2288	0.2411	0.2053	0.0700	0.0523	0.0057	0.0094
	100	full	1.142	0.0000	0.0000	0.0030	0.0208	0.1523	0.2677	0.2603	0.2139	0.0588	0.0228	0.0004	0.0000
		tr	1.100	0.0000	0.0000	0.0001	0.0057	0.1263	0.3186	0.3194	0.1866	0.0327	0.0100	0.0002	0.0004
	200	full	1.079	0.0000	0.0000	0.0000	0.0033	0.1161	0.3375	0.3568	0.1700	0.0145	0.0018	0.0000	0.0000
		tr	1.051	0.0000	0.0000	0.0000	0.0001	0.0677	0.3958	0.4078	0.1217	0.0062	0.0007	0.0000	0.0000
	400	full	1.026	0.0000	0.0000	0.0000	0.0001	0.0592	0.4200	0.4349	0.0851	0.0007	0.0000	0.0000	0.0000
		tr	1.024	0.0000	0.0000	0.0000	0.0000	0.0210	0.4537	0.4793	0.0459	0.0001	0.0000	0.0000	0.0000
.6	800	full	1.021	0.0000	0.0000	0.0000	0.0000	0.0141	0.4616	0.5011	0.0232	0.0000	0.0000	0.0000	0.0000
		tr	1.012	0.0000	0.0000	0.0000	0.0000	0.0023	0.4823	0.5091	0.0063	0.0000	0.0000	0.0000	0.0000
	1600	full	1.006	0.0000	0.0000	0.0000	0.0000	0.0003	0.4892	0.5087	0.0018	0.0000	0.0000	0.0000	0.0000
		tr	1.006	0.0000	0.0000	0.0000	0.0000	0.0000	0.4854	0.5143	0.0003	0.0000	0.0000	0.0000	0.0000
	3200	full	1.004	0.0000	0.0000	0.0000	0.0000	0.0000	0.4907	0.5093	0.0000	0.0000	0.0000	0.0000	0.0000
		tr	1.004	0.0000	0.0000	0.0000	0.0000	0.0000	0.4867	0.5133	0.0000	0.0000	0.0000	0.0000	0.0000
	6400	full	1.002	0.0000	0.0000	0.0000	0.0000	0.0000	0.4935	0.5065	0.0000	0.0000	0.0000	0.0000	0.0000

Table 9, .1, .9: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for bivariate Gaussian–Weibulls with generating correlations .5 and .6 (tr: truncated (on the Gaussian) bivariate Gaussian–Weibull, $p_{\mathrm{l}} = .1$, $p_{\mathrm{u}} = .9$; full: full bivariate Gaussian–Weibull)

ρ	n		Median p_{Br} ratio	p_{Br} ratio frequencies											
				$[0, .01]$	$(.01, .02]$	$(.02, .1]$	$(.1, .2]$	$(.2, .5]$	$(.5, 1)$	$(1, 2)$	$[2, 5)$	$[5, 10)$	$[10, 50)$	$[50, 100)$	$[100, \infty)$
.7	100	tr	1.232	0.0000	0.0000	0.0113	0.0458	0.1772	0.2024	0.1962	0.1902	0.0820	0.0689	0.0104	0.0156
		full	1.164	0.0000	0.0002	0.0067	0.0391	0.1722	0.2289	0.2293	0.2080	0.0748	0.0386	0.0018	0.0004
	200	tr	1.132	0.0000	0.0000	0.0007	0.0175	0.1584	0.2714	0.2678	0.2096	0.0494	0.0230	0.0014	0.0008
		full	1.093	0.0000	0.0000	0.0007	0.0115	0.1439	0.3046	0.3125	0.1895	0.0327	0.0046	0.0000	0.0000
	400	tr	1.061	0.0000	0.0000	0.0000	0.0026	0.1141	0.3444	0.3650	0.1544	0.0173	0.0021	0.0001	0.0000
		full	1.044	0.0000	0.0000	0.0000	0.0006	0.0899	0.3795	0.4012	0.1244	0.0042	0.0002	0.0000	0.0000
	800	tr	1.035	0.0000	0.0000	0.0000	0.0000	0.0512	0.4175	0.4434	0.0871	0.0007	0.0001	0.0000	0.0000
		full	1.012	0.0000	0.0000	0.0000	0.0000	0.0351	0.4519	0.4628	0.0501	0.0001	0.0000	0.0000	0.0000
	1600	tr	1.014	0.0000	0.0000	0.0000	0.0000	0.0113	0.4691	0.4996	0.0199	0.0001	0.0000	0.0000	0.0000
		full	1.004	0.0000	0.0000	0.0000	0.0000	0.0037	0.4913	0.4954	0.0096	0.0000	0.0000	0.0000	0.0000
	3200	tr	1.005	0.0000	0.0000	0.0000	0.0000	0.0004	0.4918	0.5055	0.0023	0.0000	0.0000	0.0000	0.0000
		full	1.007	0.0000	0.0000	0.0000	0.0000	0.0001	0.4847	0.5147	0.0005	0.0000	0.0000	0.0000	0.0000
	6400	tr	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4932	0.5068	0.0000	0.0000	0.0000	0.0000	0.0000
		full	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4917	0.5083	0.0000	0.0000	0.0000	0.0000	0.0000
.8	100	tr	1.333	0.0002	0.0012	0.0488	0.0744	0.1625	0.1471	0.1550	0.1587	0.0891	0.1069	0.0204	0.0357
		full	1.228	0.0000	0.0007	0.0307	0.0599	0.1695	0.1834	0.1814	0.1936	0.0854	0.0820	0.0070	0.0064
	200	tr	1.120	0.0000	0.0000	0.0126	0.0468	0.1885	0.2131	0.2157	0.1803	0.0789	0.0546	0.0057	0.0038
		full	1.118	0.0000	0.0000	0.0050	0.0325	0.1675	0.2477	0.2547	0.2076	0.0610	0.0239	0.0001	0.0000
	400	tr	1.068	0.0000	0.0000	0.0010	0.0134	0.1640	0.2905	0.2876	0.1940	0.0364	0.0127	0.0004	0.0000
		full	1.049	0.0000	0.0000	0.0000	0.0070	0.1443	0.3248	0.3339	0.1708	0.0171	0.0021	0.0000	0.0000
	800	tr	1.043	0.0000	0.0000	0.0000	0.0019	0.1059	0.3654	0.3772	0.1406	0.0082	0.0008	0.0000	0.0000
		full	1.037	0.0000	0.0000	0.0000	0.0004	0.0720	0.3988	0.4271	0.0997	0.0017	0.0003	0.0000	0.0000
	1600	tr	1.019	0.0000	0.0000	0.0000	0.0000	0.0423	0.4427	0.4513	0.0632	0.0005	0.0000	0.0000	0.0000
		full	1.015	0.0000	0.0000	0.0000	0.0000	0.0228	0.4602	0.4843	0.0327	0.0000	0.0000	0.0000	0.0000
	3200	tr	1.012	0.0000	0.0000	0.0000	0.0000	0.0075	0.4779	0.4987	0.0159	0.0000	0.0000	0.0000	0.0000
		full	1.007	0.0000	0.0000	0.0000	0.0000	0.0027	0.4871	0.5054	0.0048	0.0000	0.0000	0.0000	0.0000
	6400	tr	1.003	0.0000	0.0000	0.0000	0.0000	0.0004	0.4937	0.5051	0.0008	0.0000	0.0000	0.0000	0.0000
		full	0.997	0.0000	0.0000	0.0000	0.0000	0.0001	0.5053	0.4945	0.0001	0.0000	0.0000	0.0000	0.0000

Table 9, .1, .9 continued: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for bivariate Gaussian–Weibulls with generating correlations .7 and .8 (tr: truncated (on the Gaussian) bivariate Gaussian–Weibull, $p_{\mathrm{l}} = .1$, $p_{\mathrm{u}} = .9$; full: full bivariate Gaussian–Weibull)

ρ	n		Median p_{Br} ratio	p_{Br} ratio frequencies											
				$[0,.01]$	$(.01,.02]$	$(.02,.1]$	$(.1,.2]$	$(.2,.5]$	$(.5,1)$	$(1,2)$	$[2,5)$	$[5,10)$	$[10,50)$	$[50,100)$	$[100,\infty)$
.5	100	tr	1.259	0.0000	0.0000	0.0017	0.0192	0.1596	0.2364	0.2354	0.1851	0.0650	0.0686	0.0122	0.0168
		full	1.150	0.0000	0.0000	0.0003	0.0112	0.1393	0.2833	0.3106	0.2064	0.0393	0.0096	0.0000	0.0000
	200	tr	1.130	0.0000	0.0000	0.0000	0.0043	0.1226	0.3087	0.3030	0.1815	0.0439	0.0310	0.0033	0.0017
		full	1.079	0.0000	0.0000	0.0000	0.0014	0.0877	0.3596	0.4008	0.1422	0.0079	0.0004	0.0000	0.0000
	400	tr	1.063	0.0000	0.0000	0.0000	0.0000	0.0662	0.3871	0.3878	0.1358	0.0183	0.0047	0.0001	0.0000
		full	1.031	0.0000	0.0000	0.0000	0.0000	0.0371	0.4333	0.4683	0.0611	0.0002	0.0000	0.0000	0.0000
	800	tr	1.034	0.0000	0.0000	0.0000	0.0000	0.0194	0.4483	0.4657	0.0646	0.0019	0.0001	0.0000	0.0000
		full	1.008	0.0000	0.0000	0.0000	0.0000	0.0059	0.4826	0.4996	0.0119	0.0000	0.0000	0.0000	0.0000
	1600	tr	1.013	0.0000	0.0000	0.0000	0.0000	0.0023	0.4797	0.5057	0.0123	0.0000	0.0000	0.0000	0.0000
		full	1.009	0.0000	0.0000	0.0000	0.0000	0.0003	0.4837	0.5156	0.0004	0.0000	0.0000	0.0000	0.0000
	3200	tr	1.004	0.0000	0.0000	0.0000	0.0000	0.0000	0.4923	0.5069	0.0008	0.0000	0.0000	0.0000	0.0000
		full	1.004	0.0000	0.0000	0.0000	0.0000	0.0000	0.4871	0.5129	0.0000	0.0000	0.0000	0.0000	0.0000
	6400	tr	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4907	0.5093	0.0000	0.0000	0.0000	0.0000	0.0000
		full	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4856	0.5144	0.0000	0.0000	0.0000	0.0000	0.0000
.6	100	tr	1.290	0.0000	0.0000	0.0083	0.0393	0.1702	0.2089	0.1897	0.1828	0.0715	0.0820	0.0152	0.0321
		full	1.132	0.0000	0.0000	0.0025	0.0219	0.1582	0.2620	0.2716	0.2068	0.0558	0.0209	0.0003	0.0000
	200	tr	1.117	0.0000	0.0000	0.0002	0.0117	0.1531	0.2840	0.2653	0.1805	0.0554	0.0403	0.0057	0.0038
		full	1.063	0.0000	0.0000	0.0000	0.0031	0.1175	0.3411	0.3572	0.1644	0.0153	0.0014	0.0000	0.0000
	400	tr	1.076	0.0000	0.0000	0.0000	0.0007	0.1044	0.3540	0.3440	0.1599	0.0260	0.0099	0.0005	0.0006
		full	1.046	0.0000	0.0000	0.0000	0.0000	0.0507	0.4084	0.4498	0.0898	0.0013	0.0000	0.0000	0.0000
	800	tr	1.037	0.0000	0.0000	0.0000	0.0000	0.0438	0.4259	0.4310	0.0953	0.0038	0.0001	0.0001	0.0000
		full	1.017	0.0000	0.0000	0.0000	0.0000	0.0139	0.4663	0.4959	0.0239	0.0000	0.0000	0.0000	0.0000
	1600	tr	1.017	0.0000	0.0000	0.0000	0.0000	0.0087	0.4725	0.4902	0.0285	0.0001	0.0000	0.0000	0.0000
		full	1.007	0.0000	0.0000	0.0000	0.0000	0.0008	0.4870	0.5100	0.0022	0.0000	0.0000	0.0000	0.0000
	3200	tr	1.010	0.0000	0.0000	0.0000	0.0000	0.0004	0.4812	0.5154	0.0030	0.0000	0.0000	0.0000	0.0000
		full	0.999	0.0000	0.0000	0.0000	0.0000	0.0000	0.5022	0.4978	0.0000	0.0000	0.0000	0.0000	0.0000
	6400	tr	1.009	0.0000	0.0000	0.0000	0.0000	0.0000	0.4779	0.5220	0.0001	0.0000	0.0000	0.0000	0.0000
		full	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4920	0.5080	0.0000	0.0000	0.0000	0.0000	0.0000

Table 9, .2, .8: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for bivariate Gaussian–Weibulls with generating correlations .5 and .6 (tr: truncated (on the Gaussian) bivariate Gaussian–Weibull, $p_{\mathrm{l}} = .2$, $p_{\mathrm{u}} = .8$; full: full bivariate Gaussian–Weibull)

ρ	n		Median	p_{Br} ratio frequencies											
			p_{Br} ratio	$[0,.01]$	$(.01,.02]$	$(.02,.1]$	$(.1,.2]$	$(.2,.5]$	$(.5,1)$	$(1,2)$	$[2,5)$	$[5,10)$	$[10,50)$	$[50,100)$	$[100,\infty)$
.7	100	tr	1.301	0.0000	0.0002	0.0232	0.0626	0.1759	0.1706	0.1634	0.1571	0.0762	0.0936	0.0239	0.0533
		full	1.159	0.0000	0.0000	0.0090	0.0402	0.1702	0.2280	0.2302	0.2078	0.0713	0.0413	0.0018	0.0002
	200	tr	1.153	0.0000	0.0000	0.0033	0.0343	0.1814	0.2323	0.2150	0.1863	0.0660	0.0596	0.0090	0.0128
		full	1.081	0.0000	0.0000	0.0005	0.0101	0.1406	0.3117	0.3091	0.1930	0.0291	0.0059	0.0000	0.0000
	400	tr	1.081	0.0000	0.0000	0.0000	0.0078	0.1458	0.3080	0.2941	0.1809	0.0438	0.0177	0.0012	0.0007
		full	1.042	0.0000	0.0000	0.0000	0.0010	0.0827	0.3881	0.3973	0.1266	0.0041	0.0002	0.0000	0.0000
	800	tr	1.030	0.0000	0.0000	0.0000	0.0006	0.0830	0.3976	0.3685	0.1370	0.0115	0.0018	0.0000	0.0000
		full	1.011	0.0000	0.0000	0.0000	0.0000	0.0296	0.4608	0.4623	0.0473	0.0000	0.0000	0.0000	0.0000
	1600	tr	1.021	0.0000	0.0000	0.0000	0.0000	0.0322	0.4503	0.4546	0.0624	0.0004	0.0001	0.0000	0.0000
		full	1.006	0.0000	0.0000	0.0000	0.0000	0.0045	0.4883	0.4977	0.0095	0.0000	0.0000	0.0000	0.0000
	3200	tr	1.019	0.0000	0.0000	0.0000	0.0000	0.0037	0.4697	0.5109	0.0157	0.0000	0.0000	0.0000	0.0000
		full	1.004	0.0000	0.0000	0.0000	0.0000	0.0001	0.4917	0.5079	0.0003	0.0000	0.0000	0.0000	0.0000
	6400	tr	1.006	0.0000	0.0000	0.0000	0.0000	0.0000	0.4893	0.5099	0.0008	0.0000	0.0000	0.0000	0.0000
		full	1.005	0.0000	0.0000	0.0000	0.0000	0.0000	0.4874	0.5126	0.0000	0.0000	0.0000	0.0000	0.0000
.8	100	tr	1.361	0.0011	0.0036	0.0762	0.0811	0.1559	0.1271	0.1200	0.1282	0.0709	0.1061	0.0284	0.1014
		full	1.258	0.0000	0.0009	0.0301	0.0591	0.1680	0.1830	0.1765	0.1965	0.0867	0.0851	0.0081	0.0060
	200	tr	1.178	0.0000	0.0003	0.0270	0.0685	0.1874	0.1745	0.1633	0.1619	0.0742	0.0952	0.0180	0.0297
		full	1.109	0.0000	0.0000	0.0051	0.0330	0.1709	0.2468	0.2542	0.2066	0.0577	0.0253	0.0004	0.0000
	400	tr	1.071	0.0000	0.0000	0.0035	0.0392	0.1875	0.2433	0.2254	0.1815	0.0622	0.0493	0.0039	0.0042
		full	1.048	0.0000	0.0000	0.0001	0.0073	0.1370	0.3294	0.3311	0.1734	0.0196	0.0021	0.0000	0.0000
	800	tr	1.028	0.0000	0.0000	0.0000	0.0071	0.1592	0.3179	0.3035	0.1708	0.0317	0.0093	0.0004	0.0001
		full	1.029	0.0000	0.0000	0.0000	0.0005	0.0705	0.4064	0.4223	0.0976	0.0027	0.0000	0.0000	0.0000
	1600	tr	1.028	0.0000	0.0000	0.0000	0.0004	0.0869	0.3914	0.3973	0.1166	0.0071	0.0003	0.0000	0.0000
		full	1.017	0.0000	0.0000	0.0000	0.0000	0.0231	0.4549	0.4896	0.0323	0.0001	0.0000	0.0000	0.0000
	3200	tr	1.016	0.0000	0.0000	0.0000	0.0000	0.0293	0.4562	0.4594	0.0550	0.0001	0.0000	0.0000	0.0000
		full	1.005	0.0000	0.0000	0.0000	0.0000	0.0024	0.4901	0.5035	0.0040	0.0000	0.0000	0.0000	0.0000
	6400	tr	1.010	0.0000	0.0000	0.0000	0.0000	0.0035	0.4822	0.5039	0.0104	0.0000	0.0000	0.0000	0.0000
		full	1.006	0.0000	0.0000	0.0000	0.0000	0.0001	0.4879	0.5120	0.0000	0.0000	0.0000	0.0000	0.0000

Table 9, .2, .8 continued: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for bivariate Gaussian–Weibulls with generating correlations .7 and .8 (tr: truncated (on the Gaussian) bivariate Gaussian–Weibull, $p_{\mathrm{l}} = .2$, $p_{\mathrm{u}} = .8$; full: full bivariate Gaussian–Weibull)

ρ	n		Median p_{Br} ratio	p_{Br} ratio frequencies											
				$[0, .01]$	$(.01, .02]$	$(.02, .1]$	$(.1, .2]$	$(.2, .5]$	$(.5, 1)$	$(1, 2)$	$[2, 5)$	$[5, 10)$	$[10, 50)$	$[50, 100)$	$[100, \infty)$
.5	100	tr	1.214	0.0000	0.0000	0.0009	0.0151	0.1594	0.2439	0.2503	0.1695	0.0642	0.0721	0.0137	0.0109
		full	1.124	0.0000	0.0000	0.0008	0.0124	0.1426	0.2906	0.2914	0.2103	0.0406	0.0113	0.0000	0.0000
	200	tr	1.095	0.0000	0.0000	0.0000	0.0024	0.1211	0.3294	0.2834	0.1665	0.0506	0.0416	0.0034	0.0016
		full	1.075	0.0000	0.0000	0.0001	0.0011	0.0916	0.3556	0.4057	0.1377	0.0078	0.0004	0.0000	0.0000
	400	tr	1.074	0.0000	0.0000	0.0000	0.0002	0.0607	0.3954	0.3537	0.1532	0.0286	0.0082	0.0000	0.0000
		full	1.037	0.0000	0.0000	0.0000	0.0000	0.0366	0.4291	0.4734	0.0605	0.0004	0.0000	0.0000	0.0000
	800	tr	1.034	0.0000	0.0000	0.0000	0.0000	0.0214	0.4485	0.4231	0.1006	0.0060	0.0004	0.0000	0.0000
		full	1.019	0.0000	0.0000	0.0000	0.0000	0.0063	0.4675	0.5119	0.0143	0.0000	0.0000	0.0000	0.0000
	1600	tr	1.021	0.0000	0.0000	0.0000	0.0000	0.0035	0.4691	0.4881	0.0391	0.0002	0.0000	0.0000	0.0000
		full	1.010	0.0000	0.0000	0.0000	0.0000	0.0003	0.4791	0.5199	0.0007	0.0000	0.0000	0.0000	0.0000
	3200	tr	1.009	0.0000	0.0000	0.0000	0.0000	0.0001	0.4864	0.5056	0.0079	0.0000	0.0000	0.0000	0.0000
		full	1.007	0.0000	0.0000	0.0000	0.0000	0.0000	0.4801	0.5199	0.0000	0.0000	0.0000	0.0000	0.0000
	6400	tr	1.002	0.0000	0.0000	0.0000	0.0000	0.0000	0.4949	0.5048	0.0003	0.0000	0.0000	0.0000	0.0000
		full	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4871	0.5129	0.0000	0.0000	0.0000	0.0000	0.0000
.6	100	tr	1.166	0.0000	0.0000	0.0050	0.0382	0.1978	0.2114	0.1843	0.1482	0.0727	0.0931	0.0218	0.0275
		full	1.141	0.0000	0.0000	0.0014	0.0231	0.1589	0.2614	0.2719	0.2100	0.0506	0.0222	0.0004	0.0001
	200	tr	1.103	0.0000	0.0000	0.0002	0.0117	0.1706	0.2806	0.2292	0.1688	0.0687	0.0604	0.0052	0.0046
		full	1.071	0.0000	0.0000	0.0001	0.0043	0.1121	0.3385	0.3617	0.1657	0.0158	0.0018	0.0000	0.0000
	400	tr	1.056	0.0000	0.0000	0.0000	0.0008	0.1259	0.3463	0.2932	0.1717	0.0409	0.0203	0.0007	0.0002
		full	1.038	0.0000	0.0000	0.0000	0.0001	0.0528	0.4148	0.4378	0.0936	0.0009	0.0000	0.0000	0.0000
	800	tr	1.024	0.0000	0.0000	0.0000	0.0000	0.0654	0.4155	0.3696	0.1340	0.0138	0.0017	0.0000	0.0000
		full	1.024	0.0000	0.0000	0.0000	0.0000	0.0131	0.4578	0.5005	0.0286	0.0000	0.0000	0.0000	0.0000
	1600	tr	1.017	0.0000	0.0000	0.0000	0.0000	0.0192	0.4639	0.4491	0.0656	0.0022	0.0000	0.0000	0.0000
		full	1.010	0.0000	0.0000	0.0000	0.0000	0.0006	0.4810	0.5167	0.0017	0.0000	0.0000	0.0000	0.0000
	3200	tr	1.007	0.0000	0.0000	0.0000	0.0000	0.0023	0.4872	0.4944	0.0160	0.0001	0.0000	0.0000	0.0000
		full	1.005	0.0000	0.0000	0.0000	0.0000	0.0000	0.4885	0.5115	0.0000	0.0000	0.0000	0.0000	0.0000
	6400	tr	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4943	0.5048	0.0009	0.0000	0.0000	0.0000	0.0000
		full	1.002	0.0000	0.0000	0.0000	0.0000	0.0000	0.4940	0.5060	0.0000	0.0000	0.0000	0.0000	0.0000

Table 9, .4, .8: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for bivariate Gaussian–Weibulls with generating correlations .5 and .6 (tr: truncated (on the Gaussian) bivariate Gaussian–Weibull, $p_{\mathrm{l}} = .4$, $p_{\mathrm{u}} = .8$; full: full bivariate Gaussian–Weibull)

ρ	n		Median p_{Br} ratio	p_{Br} ratio frequencies											
				$[0, .01]$	$(.01, .02]$	$(.02, .1]$	$(.1, .2]$	$(.2, .5]$	$(.5, 1)$	$(1, 2)$	$[2, 5)$	$[5, 10)$	$[10, 50)$	$[50, 100)$	$[100, \infty)$
.7	100	tr	1.166	0.0001	0.0002	0.0290	0.0772	0.2036	0.1578	0.1314	0.1277	0.0664	0.1101	0.0299	0.0666
		full	1.215	0.0000	0.0000	0.0064	0.0357	0.1681	0.2196	0.2384	0.2113	0.0743	0.0431	0.0028	0.0003
	200	tr	1.028	0.0000	0.0000	0.0063	0.0544	0.2146	0.2167	0.1709	0.1544	0.0700	0.0779	0.0168	0.0180
		full	1.085	0.0000	0.0000	0.0002	0.0122	0.1445	0.3052	0.3177	0.1866	0.0284	0.0052	0.0000	0.0000
	400	tr	1.070	0.0000	0.0000	0.0003	0.0145	0.1922	0.2680	0.2373	0.1788	0.0589	0.0432	0.0038	0.0030
		full	1.033	0.0000	0.0000	0.0001	0.0012	0.0854	0.3923	0.3980	0.1178	0.0049	0.0003	0.0000	0.0000
	800	tr	1.029	0.0000	0.0000	0.0000	0.0024	0.1358	0.3471	0.3152	0.1584	0.0303	0.0105	0.0002	0.0001
		full	1.019	0.0000	0.0000	0.0000	0.0000	0.0318	0.4498	0.4681	0.0503	0.0000	0.0000	0.0000	0.0000
	1600	tr	1.002	0.0000	0.0000	0.0000	0.0000	0.0623	0.4358	0.3882	0.1082	0.0047	0.0008	0.0000	0.0000
		full	1.011	0.0000	0.0000	0.0000	0.0000	0.0049	0.4787	0.5073	0.0091	0.0000	0.0000	0.0000	0.0000
	3200	tr	1.012	0.0000	0.0000	0.0000	0.0000	0.0141	0.4722	0.4715	0.0419	0.0002	0.0001	0.0000	0.0000
		full	1.002	0.0000	0.0000	0.0000	0.0000	0.0002	0.4954	0.5043	0.0001	0.0000	0.0000	0.0000	0.0000
	6400	tr	1.001	0.0000	0.0000	0.0000	0.0000	0.0012	0.4974	0.4942	0.0072	0.0000	0.0000	0.0000	0.0000
		full	1.001	0.0000	0.0000	0.0000	0.0000	0.0000	0.4976	0.5024	0.0000	0.0000	0.0000	0.0000	0.0000
.8	100	tr	1.123	0.0024	0.0103	0.1234	0.0990	0.1455	0.1055	0.0866	0.0868	0.0581	0.0999	0.0356	0.1469
		full	1.222	0.0000	0.0003	0.0328	0.0616	0.1695	0.1822	0.1820	0.1926	0.0894	0.0753	0.0072	0.0071
	200	tr	1.113	0.0000	0.0008	0.0646	0.0986	0.1809	0.1337	0.1244	0.1215	0.0685	0.1056	0.0294	0.0720
		full	1.104	0.0000	0.0000	0.0057	0.0329	0.1775	0.2470	0.2554	0.2023	0.0550	0.0230	0.0011	0.0001
	400	tr	1.059	0.0000	0.0000	0.0226	0.0707	0.2051	0.1866	0.1638	0.1606	0.0758	0.0792	0.0148	0.0208
		full	1.061	0.0000	0.0000	0.0001	0.0062	0.1321	0.3247	0.3397	0.1753	0.0198	0.0021	0.0000	0.0000
	800	tr	1.036	0.0000	0.0000	0.0025	0.0286	0.2029	0.2528	0.2296	0.1842	0.0574	0.0379	0.0030	0.0011
		full	1.028	0.0000	0.0000	0.0000	0.0003	0.0751	0.4012	0.4164	0.1053	0.0017	0.0000	0.0000	0.0000
	1600	tr	1.036	0.0000	0.0000	0.0000	0.0055	0.1460	0.3308	0.3170	0.1657	0.0283	0.0067	0.0000	0.0000
		full	1.024	0.0000	0.0000	0.0000	0.0000	0.0223	0.4520	0.4924	0.0333	0.0000	0.0000	0.0000	0.0000
	3200	tr	1.009	0.0000	0.0000	0.0000	0.0002	0.0784	0.4146	0.3872	0.1141	0.0054	0.0001	0.0000	0.0000
		full	1.006	0.0000	0.0000	0.0000	0.0000	0.0027	0.4900	0.5030	0.0043	0.0000	0.0000	0.0000	0.0000
	6400	tr	1.001	0.0000	0.0000	0.0000	0.0000	0.0229	0.4757	0.4617	0.0395	0.0002	0.0000	0.0000	0.0000
		full	1.005	0.0000	0.0000	0.0000	0.0000	0.0001	0.4898	0.5100	0.0001	0.0000	0.0000	0.0000	0.0000

Table 9, .4, .8 continued: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for bivariate Gaussian–Weibulls with generating correlations .7 and .8 (tr: truncated (on the Gaussian) bivariate Gaussian–Weibull, $p_{\mathrm{l}} = .4$, $p_{\mathrm{u}} = .8$; full: full bivariate Gaussian–Weibull)

ρ	n		Median p_{Br} ratio	p_{Br} ratio frequencies $[0, .01]$	$(.01, .02]$	$(.02, .1]$	$(.1, .2]$	$(.2, .5]$	$(.5, 1)$	$(1, 2)$	$[2, 5)$	$[5, 10)$	$[10, 50)$	$[50, 100)$	$[100, \infty)$
		tr	1.148	0.0000	0.0000	0.0004	0.0082	0.1404	0.2857	0.2975	0.2056	0.0466	0.0148	0.0007	0.0001
	100	full	1.150	0.0000	0.0000	0.0008	0.0142	0.1426	0.2761	0.3025	0.2122	0.0411	0.0104	0.0001	0.0000
		tr	1.064	0.0000	0.0000	0.0000	0.0004	0.0848	0.3724	0.3832	0.1470	0.0112	0.0010	0.0000	0.0000
	200	full	1.066	0.0000	0.0000	0.0000	0.0008	0.0910	0.3652	0.3896	0.1452	0.0077	0.0005	0.0000	0.0000
		tr	1.030	0.0000	0.0000	0.0000	0.0000	0.0332	0.4376	0.4619	0.0666	0.0007	0.0000	0.0000	0.0000
	400	full	1.031	0.0000	0.0000	0.0000	0.0000	0.0368	0.4339	0.4671	0.0619	0.0003	0.0000	0.0000	0.0000
		tr	1.019	0.0000	0.0000	0.0000	0.0000	0.0046	0.4698	0.5124	0.0132	0.0000	0.0000	0.0000	0.0000
.5	800	full	1.019	0.0000	0.0000	0.0000	0.0000	0.0064	0.4677	0.5143	0.0116	0.0000	0.0000	0.0000	0.0000
		tr	1.005	0.0000	0.0000	0.0000	0.0000	0.0002	0.4910	0.5082	0.0006	0.0000	0.0000	0.0000	0.0000
	1600	full	1.005	0.0000	0.0000	0.0000	0.0000	0.0004	0.4906	0.5087	0.0003	0.0000	0.0000	0.0000	0.0000
		tr	1.004	0.0000	0.0000	0.0000	0.0000	0.0000	0.4896	0.5104	0.0000	0.0000	0.0000	0.0000	0.0000
	3200	full	1.004	0.0000	0.0000	0.0000	0.0000	0.0000	0.4893	0.5107	0.0000	0.0000	0.0000	0.0000	0.0000
		tr	1.002	0.0000	0.0000	0.0000	0.0000	0.0000	0.4922	0.5078	0.0000	0.0000	0.0000	0.0000	0.0000
	6400	full	1.002	0.0000	0.0000	0.0000	0.0000	0.0000	0.4921	0.5079	0.0000	0.0000	0.0000	0.0000	0.0000
		tr	1.141	0.0000	0.0000	0.0007	0.0134	0.1657	0.2601	0.2734	0.2028	0.0567	0.0259	0.0012	0.0001
	100	full	1.143	0.0000	0.0000	0.0017	0.0176	0.1682	0.2513	0.2776	0.2084	0.0538	0.0210	0.0003	0.0001
		tr	1.086	0.0000	0.0000	0.0000	0.0031	0.1089	0.3423	0.3585	0.1691	0.0156	0.0025	0.0000	0.0000
	200	full	1.086	0.0000	0.0000	0.0000	0.0045	0.1134	0.3358	0.3631	0.1693	0.0125	0.0014	0.0000	0.0000
		tr	1.036	0.0000	0.0000	0.0000	0.0000	0.0515	0.4188	0.4316	0.0964	0.0016	0.0001	0.0000	0.0000
	400	full	1.037	0.0000	0.0000	0.0000	0.0000	0.0566	0.4127	0.4376	0.0923	0.0008	0.0000	0.0000	0.0000
		tr	1.025	0.0000	0.0000	0.0000	0.0000	0.0117	0.4568	0.5039	0.0276	0.0000	0.0000	0.0000	0.0000
.6	800	full	1.025	0.0000	0.0000	0.0000	0.0000	0.0135	0.4544	0.5067	0.0254	0.0000	0.0000	0.0000	0.0000
		tr	1.011	0.0000	0.0000	0.0000	0.0000	0.0011	0.4806	0.5160	0.0023	0.0000	0.0000	0.0000	0.0000
	1600	full	1.011	0.0000	0.0000	0.0000	0.0000	0.0011	0.4804	0.5168	0.0017	0.0000	0.0000	0.0000	0.0000
		tr	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4923	0.5076	0.0001	0.0000	0.0000	0.0000	0.0000
	3200	full	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4920	0.5079	0.0001	0.0000	0.0000	0.0000	0.0000
		tr	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4915	0.5085	0.0000	0.0000	0.0000	0.0000	0.0000
	6400	full	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4915	0.5085	0.0000	0.0000	0.0000	0.0000	0.0000

Table 9, .4, 1: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for bivariate Gaussian–Weibulls with generating correlations .5 and .6 (tr: truncated (on the Gaussian) bivariate Gaussian–Weibull, $p_{\mathrm{l}} = .4$, $p_{\mathrm{u}} = 1$; full: full bivariate Gaussian–Weibull)

			Median	p_{Br} ratio frequencies											
ρ	n		p_{Br} ratio	$[0,.01]$	$(.01,.02]$	$(.02,.1]$	$(.1,.2]$	$(.2,.5]$	$(.5,1)$	$(1,2)$	$[2,5)$	$[5,10)$	$[10,50)$	$[50,100)$	$[100,\infty)$
		tr	1.171	0.0000	0.0000	0.0037	0.0340	0.1754	0.2353	0.2324	0.1969	0.0733	0.0449	0.0032	0.0009
	100	full	1.175	0.0000	0.0000	0.0073	0.0377	0.1732	0.2292	0.2360	0.2029	0.0716	0.0402	0.0012	0.0007
		tr	1.082	0.0000	0.0000	0.0002	0.0083	0.1443	0.3078	0.3125	0.1833	0.0346	0.0090	0.0000	0.0000
	200	full	1.082	0.0000	0.0000	0.0005	0.0121	0.1444	0.3035	0.3165	0.1843	0.0323	0.0064	0.0000	0.0000
		tr	1.042	0.0000	0.0000	0.0000	0.0004	0.0826	0.3876	0.3969	0.1268	0.0055	0.0002	0.0000	0.0000
	400	full	1.043	0.0000	0.0000	0.0000	0.0006	0.0870	0.3824	0.4013	0.1243	0.0042	0.0002	0.0000	0.0000
		tr	1.019	0.0000	0.0000	0.0000	0.0000	0.0264	0.4567	0.4658	0.0509	0.0002	0.0000	0.0000	0.0000
.7	800	full	1.019	0.0000	0.0000	0.0000	0.0000	0.0297	0.4532	0.4694	0.0475	0.0002	0.0000	0.0000	0.0000
		tr	1.007	0.0000	0.0000	0.0000	0.0000	0.0042	0.4861	0.4991	0.0106	0.0000	0.0000	0.0000	0.0000
	1600	full	1.007	0.0000	0.0000	0.0000	0.0000	0.0046	0.4852	0.5004	0.0098	0.0000	0.0000	0.0000	0.0000
		tr	1.003	0.0000	0.0000	0.0000	0.0000	0.0000	0.4928	0.5069	0.0003	0.0000	0.0000	0.0000	0.0000
	3200	full	1.004	0.0000	0.0000	0.0000	0.0000	0.0000	0.4928	0.5069	0.0003	0.0000	0.0000	0.0000	0.0000
		tr	1.005	0.0000	0.0000	0.0000	0.0000	0.0000	0.4831	0.5169	0.0000	0.0000	0.0000	0.0000	0.0000
	6400	full	1.005	0.0000	0.0000	0.0000	0.0000	0.0000	0.4828	0.5172	0.0000	0.0000	0.0000	0.0000	0.0000
		tr	1.261	0.0000	0.0001	0.0256	0.0572	0.1694	0.1857	0.1857	0.1807	0.0887	0.0875	0.0105	0.0089
	100	full	1.269	0.0001	0.0004	0.0310	0.0591	0.1636	0.1832	0.1866	0.1867	0.0896	0.0843	0.0092	0.0062
		tr	1.117	0.0000	0.0000	0.0034	0.0279	0.1858	0.2421	0.2436	0.2031	0.0632	0.0299	0.0009	0.0001
	200	full	1.118	0.0000	0.0000	0.0053	0.0312	0.1842	0.2378	0.2469	0.2052	0.0628	0.0259	0.0006	0.0001
		tr	1.057	0.0000	0.0000	0.0002	0.0063	0.1320	0.3317	0.3353	0.1695	0.0214	0.0036	0.0000	0.0000
	400	full	1.059	0.0000	0.0000	0.0004	0.0078	0.1335	0.3282	0.3378	0.1700	0.0197	0.0026	0.0000	0.0000
		tr	1.017	0.0000	0.0000	0.0000	0.0005	0.0694	0.4157	0.4146	0.0979	0.0019	0.0000	0.0000	0.0000
.8	800	full	1.018	0.0000	0.0000	0.0000	0.0006	0.0721	0.4130	0.4175	0.0955	0.0013	0.0000	0.0000	0.0000
		tr	1.007	0.0000	0.0000	0.0000	0.0000	0.0189	0.4740	0.4704	0.0367	0.0000	0.0000	0.0000	0.0000
	1600	full	1.007	0.0000	0.0000	0.0000	0.0000	0.0203	0.4724	0.4720	0.0353	0.0000	0.0000	0.0000	0.0000
		tr	1.006	0.0000	0.0000	0.0000	0.0000	0.0013	0.4890	0.5038	0.0059	0.0000	0.0000	0.0000	0.0000
	3200	full	1.006	0.0000	0.0000	0.0000	0.0000	0.0015	0.4885	0.5045	0.0055	0.0000	0.0000	0.0000	0.0000
		tr	1.004	0.0000	0.0000	0.0000	0.0000	0.0000	0.4918	0.5080	0.0002	0.0000	0.0000	0.0000	0.0000
	6400	full	1.004	0.0000	0.0000	0.0000	0.0000	0.0000	0.4917	0.5081	0.0002	0.0000	0.0000	0.0000	0.0000

Table 9, .4, 1 continued: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for bivariate Gaussian–Weibulls with generating correlations .7 and .8 (tr: truncated (on the Gaussian) bivariate Gaussian–Weibull, $p_{\mathrm{l}} = .4$, $p_{\mathrm{u}} = 1$; full: full bivariate Gaussian–Weibull)

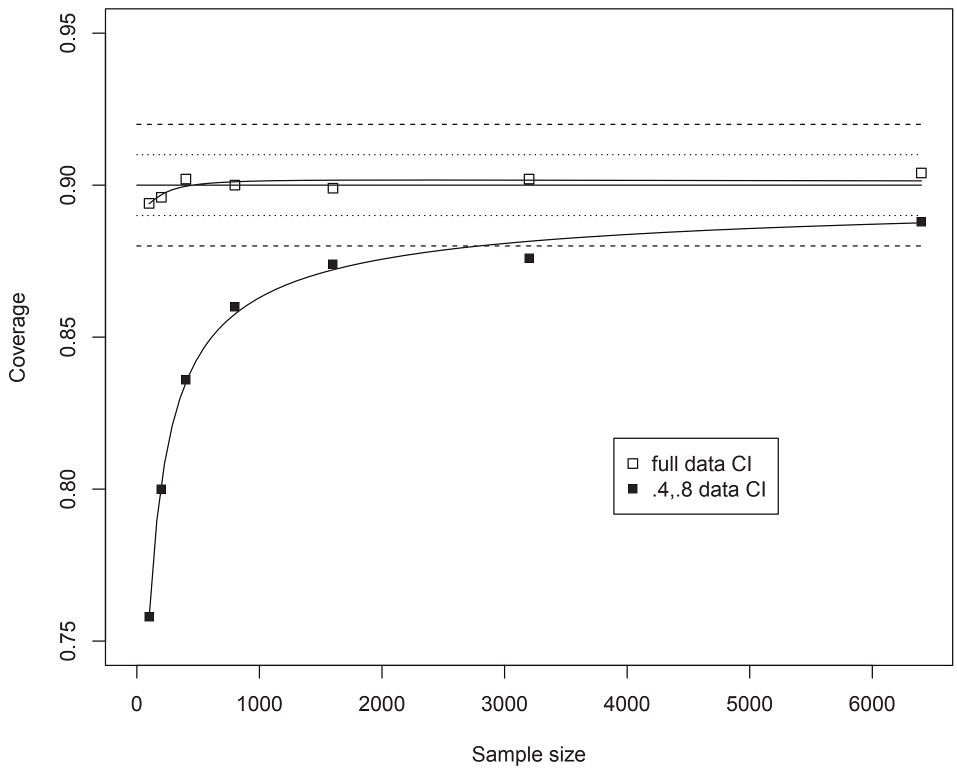

Figure 1: Actual coverages of nominal 90% theory-based confidence intervals on σ versus sample size. $\rho = 0.50$. The empty squares indicate the coverages of confidence intervals based on full bivariate Gaussian–Weibull data sets. The solid squares indicate the coverages of confidence intervals based on .4,.8 truncated (on the Gaussian) bivariate Gaussian–Weibull data sets. The solid horizontal line is at the nominal 0.90 coverage level. The dashed horizontal lines are at the 0.92 and 0.88 coverage levels. The dotted horizontal lines are at the 0.91 and 0.89 coverage levels. The curved solid lines indicate the fitted interpolations to the full and .4,.8 data set coverages.

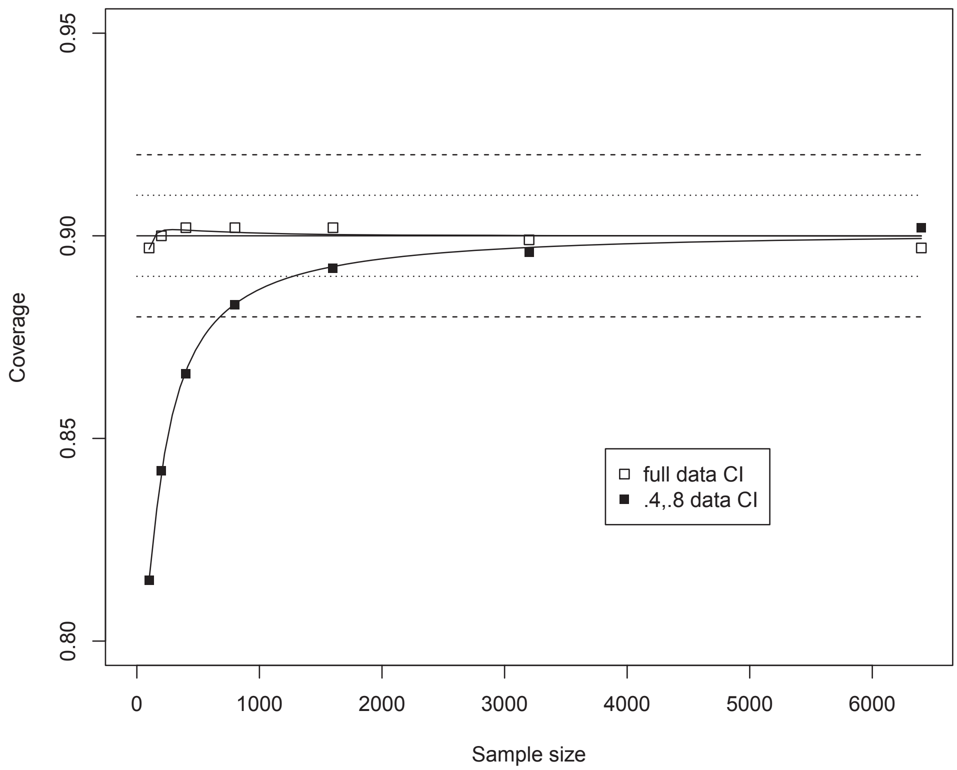

Figure 2: Actual coverages of nominal 90% theory-based confidence intervals on β versus sample size. $\rho = 0.80$. The empty squares indicate the coverages of confidence intervals based on full bivariate Gaussian–Weibull data sets. The solid squares indicate the coverages of confidence intervals based on .4,.8 truncated (on the Gaussian) bivariate Gaussian–Weibull data sets. The solid horizontal line is at the nominal 0.90 coverage level. The dashed horizontal lines are at the 0.92 and 0.88 coverage levels. The dotted horizontal lines are at the 0.91 and 0.89 coverage levels. The curved solid lines indicate the fitted interpolations to the full and .4,.8 data set coverages.

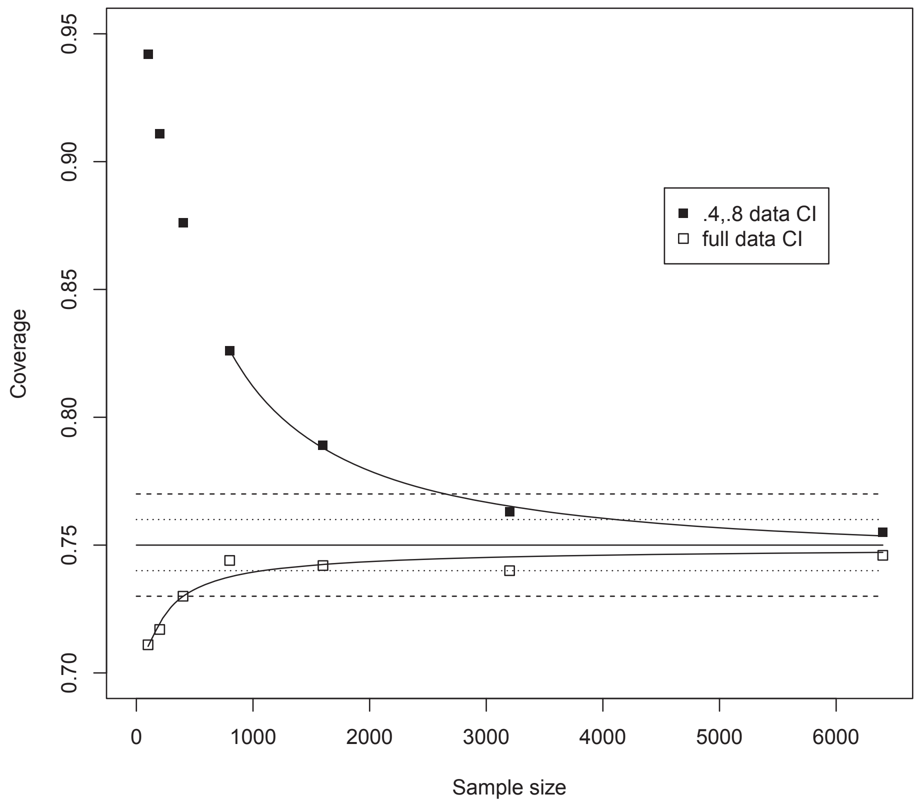

Figure 3: Actual coverages of nominal one-sided, lower 75% simulation-based confidence intervals on the .4,.8 PTW fifth percentile versus sample size. $\rho = 0.50$. The solid squares indicate the coverages of confidence intervals based on .4,.8 truncated (on the Gaussian) bivariate Gaussian–Weibull data sets. The empty squares indicate the coverages of confidence intervals based on full bivariate Gaussian–Weibull data sets. The solid horizontal line is at the nominal 0.75 coverage level. The dashed horizontal lines are at the 0.77 and 0.73 coverage levels. The dotted horizontal lines are at the 0.76 and 0.74 coverage levels. The curved solid lines indicate the fitted interpolations to the .4,.8 and full data set coverages.

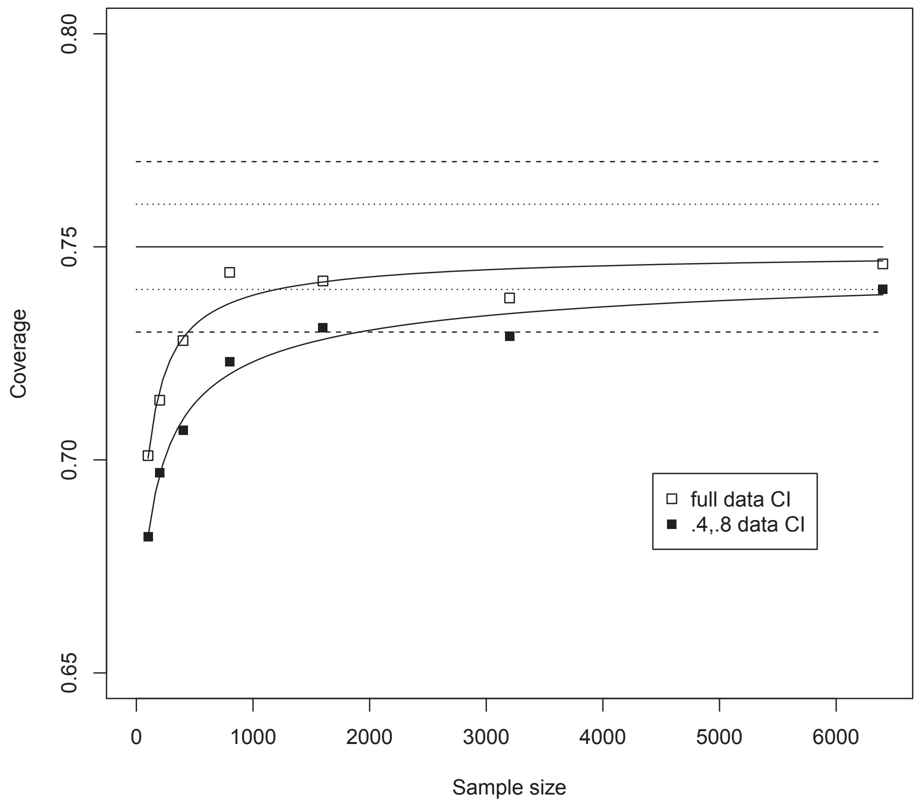

Figure 4: Actual coverages of nominal one-sided, lower 75% theory-based confidence intervals on the .4,.8 PTW fifth percentile versus sample size. $\rho = 0.50$. The empty squares indicate the coverages of confidence intervals based on full bivariate Gaussian–Weibull data sets. The solid squares indicate the coverages of confidence intervals based on .4,.8 truncated (on the Gaussian) bivariate Gaussian–Weibull data sets. The solid horizontal line is at the nominal 0.75 coverage level. The dashed horizontal lines are at the 0.77 and 0.73 coverage levels. The dotted horizontal lines are at the 0.76 and 0.74 coverage levels. The curved solid lines indicate the fitted interpolations to the full and .4,.8 data set coverages.

FPL Statistics Group

Estimation of a left and right truncated (on the Gaussian) bivariate Gaussian-Weibull distribution

Disclaimer of Warranties

Welcome to the *left and right truncated* (on the Gaussian) bivariate Gaussian-Weibull estimation program.

We also have programs that will handle the following kinds of data:

- Full (untruncated) bivariate Gaussian-Weibull data
- Left truncated (on the Gaussian) bivariate Gaussian-Weibull data
- Right truncated (on the Gaussian) bivariate Gaussian-Weibull data

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Documentation for the program can be found in section 3 of USDA Forest Products Laboratory Research Paper xxx --- "Maximum Likelihood Estimation of the Parameters of a Bivariate Gaussian--Weibull Distribution from Machine Stress-Rated Data" (pdf).

As currently written this program can handle at most 6400 bivariate observations. If this is insufficient for your purposes, please contact Steve Verrill at 608-231-9375 or at sverrill@fs.fed.us.

Before proceeding with the analysis you must provide the data file. If you have already done so, you may proceed with the form below.

What is the name of the data file?

tr_bivdat_4_8_rhop7_cvp2_n400.dat

What is the name of the results file?

Figure 5: Page 1 of sample input to the fit_tr_gauss_weib Web program.

The name should be unique to you to prevent the file from being accidentally overwritten by another user.

Since you will see an html document that presents the results as soon as you execute the program, you may not want an ascii copy. However, we do provide this option.

The ascii file will be written in the pub/data anonymous ftp directory. If a file by the same name already exists in the directory, the new results will be appended to the existing file. Check the anonymous ftp link for directions about retrieving the file.

delme

What is the sample size?
(6400 or fewer)

400

What are the lower and upper truncation values?

94.933 116.832

How many trials in the simulation?
(10000 or fewer)

10000

Important The response *will not* be immediate. Because simulations are being run, there will be a delay before the results appear. An approximate formula for the number of seconds needed to perform the simulations is (n/2) x N/10000 where n is the sample size and N is the number of trials. For example, the time needed to run 10000 trials of samples of size 200 is approximately 100 seconds. Given equal sample sizes, trials for less-truncated data sets (e.g., a .1,.9 data set) take less time to complete than trials for a more-truncated data set (e.g., a .4,.8 data set).

What is the istart value for the random number generator?
(e.g., 6324542)

132242

Execute the program

For further information, please contact Steve Verrill at sverrill@fs.fed.us or 608-231-9375.

Figure 6: Page 2 of sample input to the fit_tr_gauss_weib Web program.

Disclaimer of Warranties

The results from this run will be appended to the file `/export/home/ftp/pub/data/delme`
To see how to download this file, read this description of anonymous ftp.

Alternatively, you should be able to do a "Save As" or "Print" in your browser.

The maximum likelihood estimates of mu, sigma, rho, gamma, and beta are:

0.9762E+02, 0.1631E+02, 0.6826E+00, 0.9635E-02, 0.5762E+01

Parameter	MLE Estimate	75% Confidence Intervals			
		Simulation-based		Theory-based	
mu	0.9762E+02	0.9057E+02	0.1047E+03	0.9228E+02	0.1030E+03
sigma	0.1631E+02	0.1083E+02	0.2179E+02	0.1123E+02	0.2138E+02
rho	0.6826E+00	0.5784E+00	0.7868E+00	0.5662E+00	0.7990E+00
gamma	0.9635E-02	0.9177E-02	0.1009E-01	0.9298E-02	0.9972E-02
beta	0.5762E+01	0.4723E+01	0.6801E+01	0.4607E+01	0.6916E+01

Actual coverages of nominal 75% confidence intervals

Parameter	Type of Confidence Interval	Simulation Estimate of Actual Coverage	95% Confidence Interval on Actual Coverage
mu	simulation-based	0.866	[0.860,0.873]
	theory-based	0.765	[0.757,0.773]
sigma	simulation-based	0.821	[0.814,0.829]
	theory-based	0.738	[0.729,0.747]

Figure 7: Page 1 of sample output from the fit_tr_gauss_weib Web program.

rho	simulation-based	0.747	[0.738,0.755]
	theory-based	0.719	[0.710,0.727]
gamma	simulation-based	0.875	[0.869,0.882]
	theory-based	0.783	[0.775,0.791]
beta	simulation-based	0.753	[0.744,0.761]
	theory-based	0.722	[0.713,0.730]

Parameter	MLE Estimate	90% Confidence Intervals			
		Simulation-based		Theory-based	
mu	0.9762E+02	0.8754E+02	0.1077E+03	0.8999E+02	0.1053E+03
sigma	0.1631E+02	0.8467E+01	0.2414E+02	0.9052E+01	0.2356E+02
rho	0.6826E+00	0.5335E+00	0.8317E+00	0.5161E+00	0.8491E+00
gamma	0.9635E-02	0.8980E-02	0.1029E-01	0.9153E-02	0.1012E-01
beta	0.5762E+01	0.4275E+01	0.7248E+01	0.4275E+01	0.7248E+01

Actual coverages of nominal 90% confidence intervals

Parameter	Type of Confidence Interval	Simulation Estimate of Actual Coverage	95% Confidence Interval on Actual Coverage
mu	simulation-based	0.923	[0.917,0.928]
	theory-based	0.858	[0.851,0.864]
sigma	simulation-based	0.917	[0.911,0.922]
	theory-based	0.874	[0.867,0.880]
rho	simulation-based	0.899	[0.893,0.905]
	theory-based	0.862	[0.855,0.868]
gamma	simulation-based	0.929	[0.924,0.934]
	theory-based	0.873	[0.866,0.879]

Figure 8: Page 2 of sample output from the fit_tr_gauss_weib Web program.

beta	simulation-based	0.902	[0.897,0.908]
	theory-based	0.865	[0.858,0.871]

Parameter	MLE Estimate	95% Confidence Intervals			
		Simulation-based		Theory-based	
mu	0.9762E+02	0.8561E+02	0.1096E+03	0.8852E+02	0.1067E+03
sigma	0.1631E+02	0.6966E+01	0.2564E+02	0.7663E+01	0.2495E+02
rho	0.6826E+00	0.5049E+00	0.8602E+00	0.4842E+00	0.8810E+00
gamma	0.9635E-02	0.8855E-02	0.1042E-01	0.9061E-02	0.1021E-01
beta	0.5762E+01	0.3991E+01	0.7532E+01	0.3793E+01	0.7730E+01

Actual coverages of nominal 95% confidence intervals

Parameter	Type of Confidence Interval	Simulation Estimate of Actual Coverage	95% Confidence Interval on Actual Coverage
mu	simulation-based	0.943	[0.939,0.948]
	theory-based	0.890	[0.884,0.896]
sigma	simulation-based	0.944	[0.939,0.948]
	theory-based	0.911	[0.905,0.917]
rho	simulation-based	0.952	[0.947,0.956]
	theory-based	0.913	[0.908,0.919]
gamma	simulation-based	0.948	[0.943,0.952]
	theory-based	0.907	[0.901,0.913]
beta	simulation-based	0.952	[0.947,0.956]
	theory-based	0.916	[0.910,0.921]

Figure 9: Page 3 of sample output from the fit_tr_gauss_weib Web program.

Parameter	MLE Estimate	99% Confidence Intervals			
		Simulation-based		Theory-based	
mu	0.9762E+02	0.8183E+02	0.1134E+03	0.8566E+02	0.1096E+03
sigma	0.1631E+02	0.4031E+01	0.2858E+02	0.4946E+01	0.2766E+02
rho	0.6826E+00	0.4491E+00	0.9160E+00	0.4218E+00	0.9433E+00
gamma	0.9635E-02	0.8609E-02	0.1066E-01	0.8880E-02	0.1039E-01
beta	0.5762E+01	0.3434E+01	0.8089E+01	0.3175E+01	0.8348E+01

Actual coverages of nominal 99% confidence intervals

Parameter	Type of Confidence Interval	Simulation Estimate of Actual Coverage	95% Confidence Interval on Actual Coverage
mu	simulation-based	0.968	[0.965,0.972]
	theory-based	0.931	[0.926,0.936]
sigma	simulation-based	0.972	[0.968,0.975]
	theory-based	0.951	[0.946,0.955]
rho	simulation-based	0.991	[0.989,0.993]
	theory-based	0.967	[0.963,0.970]
gamma	simulation-based	0.971	[0.967,0.974]
	theory-based	0.950	[0.946,0.954]
beta	simulation-based	0.989	[0.987,0.991]
	theory-based	0.965	[0.962,0.969]

Figure 10: Page 4 of sample output from the fit_tr_gauss_weib Web program.