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PERSPECTIVE

Calculating the configurational entropy of a landscape mosaic

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Abstract

Background Applications of entropy and the second law of thermodynamics in landscape ecology are rare and poorly developed. This is a fundamental limitation given the centrally important role the second law plays in all physical and biological processes. A critical first step to exploring the utility of thermodynamics in landscape ecology is to define the configurational entropy of a landscape mosaic. In this paper I attempt to link landscape ecology to the second law of thermodynamics and the entropy concept by showing how the configurational entropy of a landscape mosaic may be calculated.

Result I begin by drawing parallels between the configuration of a categorical landscape mosaic and the mixing of ideal gases. I propose that the idea of the thermodynamic microstate can be expressed as unique configurations of a landscape mosaic, and posit that the landscape metric Total Edge length is an effective measure of configuration for purposes of calculating configurational entropy.

Conclusions I propose that the entropy of a given landscape configuration can be calculated using the Boltzmann equation. Specifically, the configurational entropy can be defined as the logarithm of the number of ways a landscape of a given dimensionality, number of classes and proportionality can be arranged (microstates) that produce the observed amount of total edge (macrostate).

Keywords Entropy · Landscape · Configuration · Composition · Thermodynamics

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Introduction

Entropy and the second law of thermodynamics are central organizing principles of nature, but are poorly developed and integrated in the landscape ecology literature (but see Li 2000, 2002; Vranken et al. 2014). Descriptions of landscape patterns, processes of landscape change, and propagation of pattern-process relationships across scale and through time are all governed and constrained by the second law of thermodynamics (Cushman 2015). This direct linkage to thermodynamics and entropy was noted in several of the pioneering works in the field of landscape ecology (e.g. Forman and Godron 1986; O'neill et al. 1989). Yet in the subsequent decades our field has largely failed to embrace and utilize these relationships and concepts.

Vranken et al. (2014) presented an overview of the use of entropy in landscape ecology and identified three main uses of the entropy concept in past landscape ecology research, including spatial heterogeneity, unpredictability of pattern dynamics, and pattern dependence on scale. They conclude from their review that thermodynamic interpretations of spatial heterogeneity in the literature are not relevant, that thermodynamic interpretations related to scale dependence are highly questionable, and that, of all applications of entropy in landscape ecology, only unpredictability could be thermodynamically relevant if appropriate measurements were performed to test it.

In an editorial that was published concurrently to the Vranken et al. (2014) paper, I noted that the near absence of thermodynamic focus from the landscape ecology literature is particularly strange given the focus of landscape ecology on understanding pattern process relationships across scale and through time and the fundamental importance of entropy and the second law of thermodynamics to understanding in natural sciences (e.g. Atkins 1992; Craig 1992). Every interaction between entities leads to irreversible change which increases the entropy and decreases the free energy of the closed system in which they reside and descriptions of landscape patterns, processes of landscape change, propagation of pattern-process relationships across scale and through time are all governed, constrained, and in large part directed by thermodynamics. This direct linkage to thermodynamics and entropy was noted in several of the pioneering works in the field of landscape ecology (e.g. Forman and Godron 1986; O'neill et al. 1989), but in the subsequent decades research has largely abandoned this framework and focus, as demonstrated by the Vranken et al. (2014) review (but see Li 2000, 2002; Zaccarelli et al. 2013).

Given how central entropy and the second law of thermodynamics are to understanding nature, landscape ecologists should strenuously explore the connections between landscape ecological concepts, theories and methods and entropy (Cushman 2015). In particular, there is a critical need to define the configurational entropy of landscape mosaics to define a benchmark with which to compare changes in landscape patterns and to associate these patterns with processes across space and time.

In this paper I attempt small step in linking landscape ecology to the second law of thermodynamics and the entropy concept by showing how the configurational entropy of a landscape mosaic may be calculated. I begin by drawing parallels between the configuration of a categorical landscape mosaic and the mixing of ideal gases. I propose that the idea of the thermodynamic microstate can be expressed as unique configurations of a landscape mosaic, and propose that the landscape metric Total Edge length is an appropriate measurement with which to define thermodynamic microstates of a landscape mosaic. I posit that the entropy of a given landscape configuration can be calculated using Boltzmann's equation for entropy once the number of ways a landscape of that dimensionality, number of classes and proportionality can be arranged (microstates) that produce the observed amount of total edge (macrostate).

Landscape configurations and entropy

There are n! unique ways to arrange the cells of a landscape represented as a lattice of n cells. This does not mean, however, that there will be n! unique landscape configurations. For example, if the landscape is all of one class there is only 1 possible configuration, because every one of the n! arrangements of the cells produce the same landscape pattern of a single uniform patch of class 1. In a two class landscape with class 1 amounting to proportion p and class 2 proportion q of the landscape, there will be n!/((np)!(nq)!) unique configurations of the landscape pattern. Each of these unique configurations will occur (np)!(nq)! times. This can be generalized to a landscape of any number of classes by adding $(nx_i)!$ terms for each additional class where xi is the proportion of the landscape in the ith class.

The most basic measure of the entropy of a landscape could be called the "compositional entropy" which is equivalent to the entropy of mixing ideal gases (Craig 1992). Boltzmann entropy defines entropy of a system as the natural logarithm of the number of microstates in that macrostate multiplied by the Boltzmann constant, which scales the relationship to the ideal gas laws, giving units of Joules per Kelvin. A classic example is temperature gradients in a fluid causing heat to flow from hotter regions to colder ones, by the random movement of particles. In the modern literature the concept embodied in Boltzmann entropy is often used in a more general sense and refers to any kinetic equation that describes the change of a macroscopic quantity in a thermodynamic system, such as energy, charge or particle number. In landscape ecology this macroscopic could be defined as the arrangement of the cells of a landscape mosaic.

In calculating the entropy of a landscape no scaling constant is needed, since the system is not an ideal gas, and entropy may be calculated as simply the natural logarithm of the number of microstates in a given macrostate, producing a unitless measure of entropy (Eq. 1). Thus the entropy of a landscape is defined as:

$$S_{land} = \ln(W), \tag{1}$$

where W is the number of arrangements of a landscape (microstates) with the dimensionality, number of classes and proportionality of the focal landscape that produce the same value for the macrostate. Number of arrangements in this case refers to the number of ways the cells of a given dimensionality can be arranged. Dimensionality of a landscape refers to the number of cells in that landscape. Number of classes refers to the number of different types, or classes of cells, such as cover types in a land cover mosaic.

Given this definition of compositional entropy of a landscape we can readily calculate the entropy of landscapes of different dimensions, numbers of classes and different proportions of each class (Eq. 1). We can imagine the compositional entropy as a function of the number of microstates in a macrostate defined by the dimensionality of a landscape, the number of classes of a landscape and the proportion of e ach class. For example, one could define compositional entropy using the classic formula for combinations. Specifically, compositional entropy can be calculated:

$$\ln(n!/((np!)(nq!)\ldots nx_i!), \tag{2}$$

where n is the number of cells in the landscape, p is the proportion of cells belonging to the first class, q the proportion of cells belonging to the second class and x the proportion of cells belonging to the ith class (Craig 1992). Figure 1 shows calculated compositional entropy for maps of different dimensionality (3 × 3 to 12 × 12 cells in extent), different number of classes (2, 3, 4, 5) and different proportion of area in each class (equal amounts in each class Fig. 1a and 90 % in one class with equal amounts in the remaining classes Fig. 1b). Compositional entropy increases nonlinearly with increasing dimensionality, increases with increasing number of classes in the landscape, and increases with increasing evenness of proportionality among classes.

This way of thinking about the entropy of a landscape is directly analogous to the way that the entropy of mixtures of ideal gasses is calculated (Craig 1992). In the case of ideal gases one can know the proportions of each molecular species in the mixture, but it is infeasible to measure the unique states of arrangement themselves (positions and velocities of the individual gas molecules). As such, the entropy equation measures the disorder of the system given a particular number and proportion of different molecular species.

Calculating the configurational entropy of a landscape mosaic

In contrast to gaseous mixtures, it is readily possible to measure the characteristics of particular landscape configurations. That is the purpose of landscape pattern analysis and there are an array of landscape metrics available to quantify the configurational characteristics of landscape mosaics (e.g. Neel et al. 2004; Cushman et al. 2008). Quantitative measurement of landscape configuration enables us to define additional macrostates corresponding to unique landscape configurations. Indeed, distinguishing the patterns of different landscapes and understanding the implications of these patterns for particular ecological processes is the central focus of landscape ecology as a discipline. Thus, the "compositional" entropy defined above does not provide a definition that is sufficiently useful to landscape ecologists. What would be more useful is a way to measure the entropy of different landscape configurations at a particular level of dimensionality, number of classes, and proportionality of cover.

At any particular level of dimensionality, number of classes and proportionality there will be some arrangements that produce highly likely landscape configurations, while others will produce highly unlikely arrangements. For example, consider a two class landscape consisting of a 3 × 3 lattice with 5 cells of class 1 and 4 cells of class 2. There are 9! ways to arrange the cells of this landscape, and there are 126 unique configurations, each of which occurs 2880 times among the 9! arrangements. Based on the compositional entropy idea developed above (Eq. 2), all landscapes with this dimensionality, number of classes and proportionality would have an entropy of $\ln(9!/(5!4!)) = 4.836$.

Intuition tells us, however, that there are substantial differences in the disorder of different particular

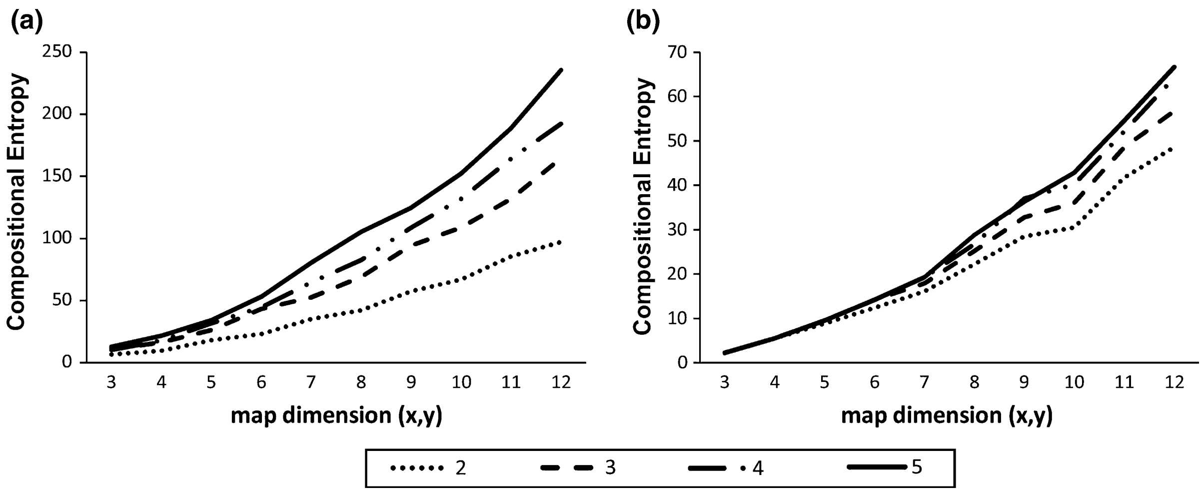

Fig. 1 Plots showing how the compositional entropy of a landscape changes as functions of map dimension (x-axis), number of classes (*2*, *3*, *4* or *5* shown as *lines* on each plot), and evenness of proportion of each class **a** even proportion in each class, **b** 90 % in one class and the remainder divided equally in the remaining classes

arrangements of this lattice. For example, placing cells in a maximally dispersed checkerboard (Fig. 2a) would seem to have less disorder and more organization than placing cells "at random". When placed at random it would be likely that cells would be more frequently adjacent to cells of the same type than in the checkerboard (where no like cells are adjacent; Fig. 2b). This higher frequency of adjacency in the random map would indicate more disorder than in the case of the checkerboard, and thus higher entropy. Conversely, when the cells are placed in a maximally clumped manner (e.g. Fig. 2c) the number of shared edges between cells of different classes will be fewer than expected by chance and would indicate a higher order and lower entropy than the random map.

There are few ways to arrange the cells which produce the maximal edge (Fig. 2a), few ways to produce the minimal edge (Fig. 2c) and relatively more ways to produce configurations with intermediate edge (Fig. 2b). Thus one can use the landscape metric Total Edge (McGarigal et al. 2012, which measures the total amount of edges between different cover classes in a landscape, to define a macrostate for a particular landscape in terms of that amount of edge (e.g. 12 units of edge, or 8 units of edge, or 4 units of edge). The entropy of a given landscape configuration

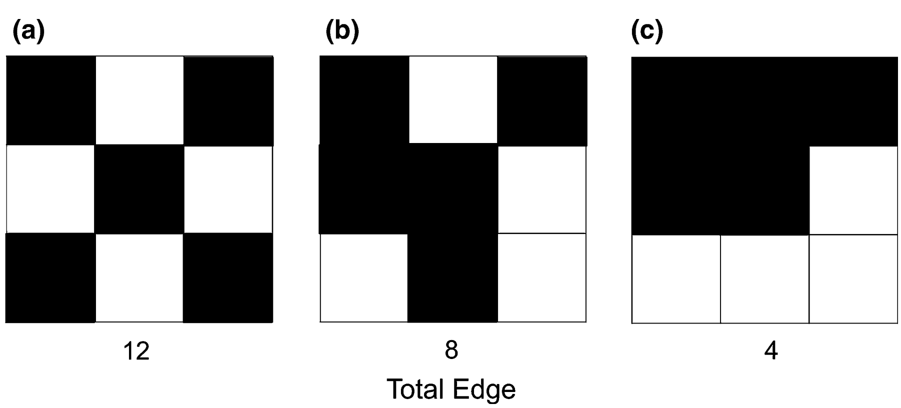

Fig. 2 Three possible configurations of a landscape mosaic with dimensions 3 × 3, two classes, 5 cells of class 1 and 4 cells of class 2. In its maximally dispersed state **a** there are 12 units of edge between cells of different class, while in its maximally aggregated sate, **c** there are only 4 units of edge. In one random permutation there were 8 units of edge (**b**)

would be calculated therefore as ln(W) where W is the number of microstates (unique landscape configurations) that produce a given value of the macrostate (length of total edge). This is conceptually very similar to the concept of "conditional entropy" as described in Vranken et al. (2014).

Using this logic, I calculated the entropy for all the different macrostates possible in a 3×3 landscape mosaic with two classes of 5 and 4 cells respectively. For this example landscape there are 126 unique configurations, and the total edge of these configurations ranges from 4 to 12 units (cell sides; Table 1). There is only one way to obtain the maximum edge of 12 (the perfect checkerboard pattern), and there are relatively few ways (12) to obtain the least edge of 4. In contrast, there are relatively many ways to arrange the mosaic to produce intermediate amounts of edge (e.g., 36 ways to produce Total Edge length of 6 or 7; Table 1). The natural logarithm of W, the number of unique arrangements (microstates) that produce a particular edge length (macrostate), is the configurational entropy of a given landscape configuration of a landscape with this dimensionality (3×3), two classes and this proportional area.

Discussion

In the sections above I propose that the configurational entropy of a landscape mosaic can be calculated using the equation for Boltzmann entropy, with entropy equal to the natural logarithm of the number of unique configurations of a landscape (microstates) that produce the same Total Edge length (macrostate) as the focal landscape. I showed using a simple example of a two class landscape of 3×3 cells in dimensionality that entropy is very low when a landscape is maximally dispersed (producing very few arrangements with maximal edge length) and is low when a landscape is maximally aggregated (producing few arrangements with minimal edge length). The entropy equation provides a means to quantitatively compare the order and disorder of any arrangement of a landscape in an explicitly thermodynamic framework.

Table 1 Total edge length (TE, macrostate), number of unique arrangements having that value of Total Edge (W, microstate), and the entropy of that macrostate (lnW) for a landscape mosaic of dimensionality 3×3, comprised of two class, with 5 and 4 cells in each class, respectively

TE	W	lnW
4	12	2.484907
5	12	2.484907
6	36	3.583519
7	36	3.583519
8	13	2.564949
9	12	2.484907
10	4	1.386294
12	1	0

This definition of the configurational entropy of landscape mosaics can serve as measuring stick which subsequently can be used to quantify entropy changes in dynamic landscape and the interactions of patterns and processes across scales of space and time, which are recognized as important issues in integrating thermodynamics with landscape ecology (Vranken et al. 2014). Specifically, ecological systems are driven by continual inflow of energy from the sun which enables photosynthesis to drive ecosystem energetics "uphill" against the current of entropy, with ecological food webs then providing a "cascade" back down the free energy ladder, reducing free energy and increasing thermodynamic disorder. Landscape ecologists should more formally associate landscape dynamics with changes in entropy and quantify the function of ecological dissipative structures (Cushman 2015) and a formal means to calculate the configurational entropy of landscape mosaics is essential to coherently investigate these processes.

A formal definition of configurational landscape entropy may be useful for addressing several additional topics. For example linking the scale dependence of landscape dynamics to thermodynamic constraints across different ecosystem types, and linkage of the second law of thermodynamics and the entropy principle with the concepts of resistance, resilience and recovery would be facilitated by a formal definition of landscape entropy. In addition, all increases in entropy result in increasing disorder and lower potential energy in the closed system. This decreases predictability, as there are more ways to be disordered than ordered and more ways to have dissipated energy than "concentrated" energy. Thus linking landscape structure, dynamics, predictability and entropy would be a fruitful enterprise. Attention should also be given to linkages between entropy, complexity theory and the organization of ecological systems as a multi-level or multi-scale systems of dissipative structures. The application of thermodynamic entropy concepts in landscape ecology has been limited by failure to measure energy transformations, changes in free energy, and changes in configurational entropy of landscape mosaics (Vranken et al. 2014), and as a result there has been a nebulous and inconsistent application and interpretation of these ideas in the field. It is my hope that a simple means to quantify the entropy of landscape mosaics could be a first step toward a broad exploration of the linkages between entropy, the second law of thermodynamics and the interaction of pattern process relationships across scales of space and time.

The definition of configurational entropy proposed here clarifies a debate in the landscape ecology literature regarding the relationship between entropy and landscape configuration. Vranken et al. (2014) note that the few authors who see a physical correspondence between heterogeneity and entropy presented two opposed interpretations. The first, based on information theory, is that higher heterogeneity would mean higher entropy (Bogaert et al. 2005). Conversely, some authors proposed that higher entropy corresponds to higher homogeneity, such as an extensive patch covering the full landscape (e.g. Forman and Godron 1986; Benson 1996; Harte 2011). The example presented here shows that neither conception is correct, and rather that entropy is low both in cases of maximally interspersed cells (maximum heterogeneity) and in cases of maximally aggregated cells (maximum homogeneity) and is highest at intermediate levels of aggregation and interspersion (the highest likelihood of random configurations) using the adaptation of the Boltzmann equation. An analogous idea in probability theory and statistics is the index of dispersion, which is a normalized measure of the dispersion of a probability distribution. It is used to quantify whether a set of observed occurrences are clustered or dispersed compared to a standard statistical model. In plant ecology the variance/mean ratio (a form of index of dispersion) has long been used to quantify non-random spatial patterning of plants (e.g. Dale 2000). In plant ecology dispersed patterns are considered "highly structured" and a result of competition leading to "regular spacing". Likewise aggregated patterns of plants are also "highly structured" as a result of conspecific association, leading to "spatial clustering". The same is true in landscapes. Patterns that are spatially random have high entropy, but patterns that are maximally clumped and patterns that are maximally dispersed have low entropy and high order.

Challenges in applying configurational entropy to landscapes

The example presented above was of a very simple landscape. Showing the linkages between landscape configuration and entropy on simple example landscapes such as this is useful to illustrate complex ideas in the most perspicuous ways. However, some challenges appear when one anticipates applying the method of calculating configurational entropy proposed here to landscapes of more realistic dimensionality. For example, the example landscape was only 3×3 cells, but produced 362,880 unique permutations of the landscape mosaic and 126 unique landscape configurations in the two class example. As the dimensionality and number of classes increases the number of configurations rapidly becomes intractably large. Given that calculating the entropy of a landscape is based on calculating the number of microstates in a given macrostate, this would seem to be a fundamental challenge. For example, as the dimensionality increases to 10×10 from 3×3 the number different permutations of the landscape lattice increases to 9.3326e+157 and the number of unique configurations for a two class landscape with equal proportionality increases to 9.2502e+128. A similar challenge was noted by Childress et al. (1996) in a paper about assessing rules to specify grid-based automata models, who noted that the number of "neighbhorhood states" increases dramatically as a function of both the number of dimensions of the grid and the number of states each cell could take. They suggested using voting rather than unique neighbor transition rules, reducing the number of possible states, and implementing ecologically-based heuristics to simplify the transition rule table. These recommendations for cellular automata models don't seem to apply to the simpler case of calculating configurational entropy of a landscape mosaic, which simply requires measurement of the number of microstates (configurations) in a macrostate (edge length) and application of the Boltzmann equation to quantify the entropy of that configuration.

The vast number of unique configurations makes it impossible, as presented here, to formally calculate each possible unique configuration and calculate the Total Edge length to quantify the number of microstates in a given macrostate. However, the effectively infinite number of unique configurations in landscapes of relatively realistic dimensionality also means that one can use neutral models such as QRULE (Gardner 1999) to generate spatially random maps to produce a large sample of the configurational distribution that can then be used to infer the number of microstates in a given macrostate.

In addition, as the dimensionality and number of classes in a landscape mosaic increase, the number of potential macrostates also increases extremely rapidly (Eq. 2). In the simple example presented there were only 8 possible macrostates defined by total edge length. As the dimensionality increases the number of possible values for the variable Total Edge length increases astronomically such that the chance that a particular randomized map has the same value as the observed focal landscape becomes vanishingly small. The same phenomenon occurs in physics when applying the second law of thermodynamics to such system properties which vary continuously. The solution is to define a meaningful increment over which to partition macrostates into a tractably small number of units (Craig 1992). The same thing can be done in landscape ecology, defining macrostates as bins that range between given values of the variable Total Edge length.

Next steps in applying configurational entropy in landscape ecology

This paper takes an initial step toward applying configurational entropy in landscape ecology by showing how configurational entropy may be measured and proposing how it may be applied in landscapes of larger dimensionality and more cover types. Several additional explorations might be fruitful as next steps. As I suggest above, it would be interesting to explore the use of neutral models to sample the permutation space for landscapes of larger dimensionality to develop distributions of microstate values. In addition, it would be valuable to explore how changing the definition of the macrostate, in terms of bin width, would affect the calculation of configurational entropy. Furthermore, meaningful application of entropy measurements in comparative analysis would depend on understanding how configurational entropy would change with dimensionality, number of classes, proportion of each class in the landscape, and the effects of resampling to coarser and finer grain.

Another area that deserves attention is how the entropy of landscapes that are not represented as raster lattices can be calculated. The patterns that are relevant to ecologists are often not best represented by categorical patch mosaics, and often gradient or point patterns are more appropriate (McGarigal and Cushman 2005; Cushman et al. 2010a). The entropy of a point pattern can be calculated in a similar way to the configurational entropy of a categorical mosaic landscape, but instead of drawing a large sample of random landscapes using a neutral model and then calculating the total edge length for each of these one could draw large samples of random point patterns with the same number of points and extent and calculate the point density of each within a tesselated grid of cells. Likewise, the entropy of a continuous landscape surface could be calculated by randomizing the surface by shuffling rows and columns a large number of times to produce a distribution of microstates with which to compare with the observed value of the focal landscape. Something of this kind was proposed by Cushman et al. (2010b).

One interesting extension of the approach I describe here would be to generalize calculating of landscape entropy to address nonequilibrium stationary states. For example, Li (2000) explored the mathematical formulation of the part–whole relation in ecological systems and showed analytically why the ecological system cannot be understood by reducing it to its parts. Specifically, using Prigogine's self-organization theory and Haken's synergetics, Li (2000) illustrated the necessity of such a holistic approach to study how the interaction of subsystems leads to the emergence of spatial, temporal and functional structures on macroscopic scales. This paper focuses on calculating the configurational entropy of a raster mosaic, with fixed dimensionality and number of map classes. In such cases, Li (2000) notes that stable states can be distinguished in a definite way, for example the amount of edge length in a raster of a given dimensionality and number of classes. However, it would be interesting to explore generalizing these ideas in systems of nonequilibrium stationary states where free energy need not have a minimum nor the entropy a maximum (Li 2000). One interesting application of this idea was presented by Li et al. (2000) who explored the theoretical thermodynamic basis for the self-thinning −3/2 power law in plant communities and used ecological field theory and showed how the stochastic nature of ecological interactions among individuals leads to self-thinning at the macroscopic level. Another interesting example of application of non-equilibrium thermodynamics in landscape ecological analysis was Li (2002) who described a theoretical framework of ecological phase transitions for modeling tree-grass dyanmics which combined percolation theory, fractal geometry, phase transition theory and Markov non-equilibrium thermodynamic stability theory. In a final example of applying non-equilibrium thermodynamic approaches in ecology, Zaccarelli et al. (2013) developed an entropy related index to measure the structural complexity of ecological time series and applied it to estimate spatio-temporal complexity of ecological systems.

Another key area of work will be to investigate whether the application of Boltzmann's entropy to landscape mosaics is more than just a convenient analogy providing an absolute measure of mosaic disorder, of if there is actually a true thermodynamic relationship between landscape configuration and entropy, per se. In physics W is the number of possible positions and momenta to the various molecules that can produce the macroscopic state of a system, such as the temperature or pressure of a gas. The Boltzmann constant, *k,* is used to translate the entropy into units of joules per Kelvin. There is a direct analogy in the calculation of entropy of a landscape mosaic in which we measure the number of ways (W) that a given macrostate (Edge Density) can be obtained. The usage proposed in this paper is unitless (not Joules per Kelvin) and so is not formally equivalent to true thermodynamic entropy. However, the usage of the entropy formula in the context of landscape pattern analysis is directly analogous to Boltzmann' entropy and, if not formally equivalent, provides an absolute measure of disorder of a landscape mosaic which is a theoretically important step forward from the merely relative measures of landscape structure previously employed in landscape pattern analysis.

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