SIMPLE TAPER: TAPER EQUATIONS FOR THE FIELD FORESTER

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Abstract.—"Simple taper" is set of linear equations that are based on stem taper rates; the intent is to provide taper equation functionality to field foresters. The equation parameters are two taper rates based on differences in diameter outside bark at two points on a tree. The simple taper equations are statistically equivalent to more complex equations. The linear equations in simple taper were developed with data from 11,610 stem analysis trees of 34 species from across the southern United States. A Visual Basic program (Microsoft®) is also provided in the appendix.

INTRODUCTION

Taper equations are used to predict the diameter of a tree stem at any height of interest. These can be useful for predicting unmeasured stem diameters, for making volume estimates with variable merchantability limits (stump height, minimum top diameter, log lengths, etc.), and for creating log size tables from sampled tree measurements. Forest biometricians know the value of these equations in forest management; however, as currently formulated, taper equations appear to be very complex. Although standard taper equations are flexible and produce appropriate estimates, they can be difficult for many foresters to calibrate or use. Most foresters would not use taper equations unless they are embedded within a computer program. Additionally, very few would consider collecting the data to parameterize the equations for their local conditions.

The typical taper equation formulations used today are a splined set of two or three polynomials that are usually quadratic (2nd-order polynomial) or cubic (3rd-order polynomial) (Max and Burkhart 1976). These polynomials are usually constrained to be equal at the join points (i.e., at specific heights on the tree stem). This is accomplished by constraining the first derivatives of the polynomial equations. If the second derivatives are constrained to be equal at the join points, the lines will be smooth through the join points. This process often generates equations that are quite complex and difficult to use. Some formulations can be solved as closed-form equations, but not all functions can be solved in this form. The complexity of taper equations has likely limited their use.

This paper proposes an approach to taper estimation that maintains many of the advantages and concepts of taper equations, yet uses a simpler quantitative approach that is easier to parameterize. This approach invokes assumptions from forest stand dynamics (Oliver and Larson 1996) and a number of ideas from Jensen's matchacurve papers (Jensen 1973, Jensen and Homeyer 1971). When tested against equations that were developed using the standard methods of splined sets of polynomials to predict volume from large regional stem analysis data sets, the simple taper equations produced statistically equivalent results.

BACKGROUND

There is a large body of literature on taper equations, and they have been parameterized for most commercially important tree species in many regions worldwide. Taper equations have many advantages over volume equations in that they allow the user to choose the merchantability limits for the tree stem.

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The shape of a tree stem is such that a single parameter-based equation is difficult to fit to the shape. The exterior profile of a tree usually bends sharply near its base, is linear along the central portion of the bole, and is variable along the upper stem (Shaw et al. 2003). The shape can vary greatly by species. To accommodate this shape variation, biometricians have used splining equations, which use separate equations in different parts of a curve. The equations are constrained to pass through a common point by requiring the first derivatives to be equal at those points. Most authors also constrain the second derivatives to be equal, which makes the modeled taper line smooth at the join point. Generally two join points are used that are commonly forced to be a point of information (e.g., diameter at breast height [d.b.h.] and diameter in the upper stem) or adjusted iteratively to a point that produces the best fit to the observed data.

When developing a growth model, I wanted a taper equation that could be used easily with very simple data requirements for the end user but in a format that could use available data. The central portion of the stem is very nearly a linear equation, and the slope of the line could be described as an average taper rate; this information could come from either a rate of change between two points on the tree or an assumed taper rate selected by the end user. These are basic measurements that are often already taken and can provide the data needed to calibrate the equation for local conditions. To produce a function that approximates the shape of the tree stem, I followed the work of Jensen (1973) and Jensen and Homeyer (1971) for combining multiple-component models to develop a curve that can reproduce the same results of the standard modern taper equations. Jensen's works suggested that data could be standardized, graphed, and compared to standardized curves, and he provided papers on the exponential and sigmoidal equations and a paper about combining equations to produce compound curves. These methods were cumbersome to use in the 1970s because they required tracing graphs on paper using a standard curve. They are quite easy to use today with graphical computer programs. These papers provided the inspiration for the following approach.

METHODS

Equations

As stated previously, the simple taper relies on some assumptions from stand dynamics. The shape of the central part of the stem (between breast height and crown base) can be described as a tapered cylinder or the frustum of a cone (Fig. 1). The volume of the frustum can be calculated with equation 1:

$$V = \frac{L}{3}\left(A_l + \sqrt{A_l A_s} + A_s\right) \tag{1}$$

Where

V = the cubic volume in the units of L and $\sqrt{A}$,

L = the length of the stem section,

A_l = the area of the large end of the log, and

A_s = the area of the small end of the log.

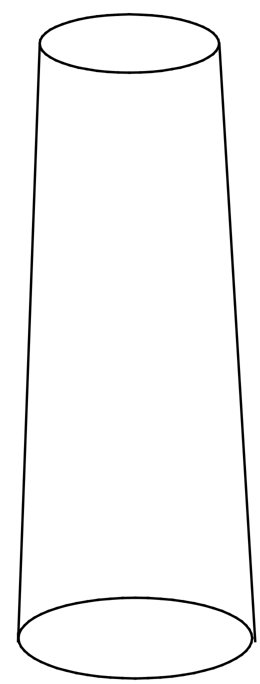

Figure 1.—A frustum of a cone using equation 1.

Using this idea, a tree profile is described as three parts: (1) below breast height, (2) breast height to crown base, and (3) stem within the crown.

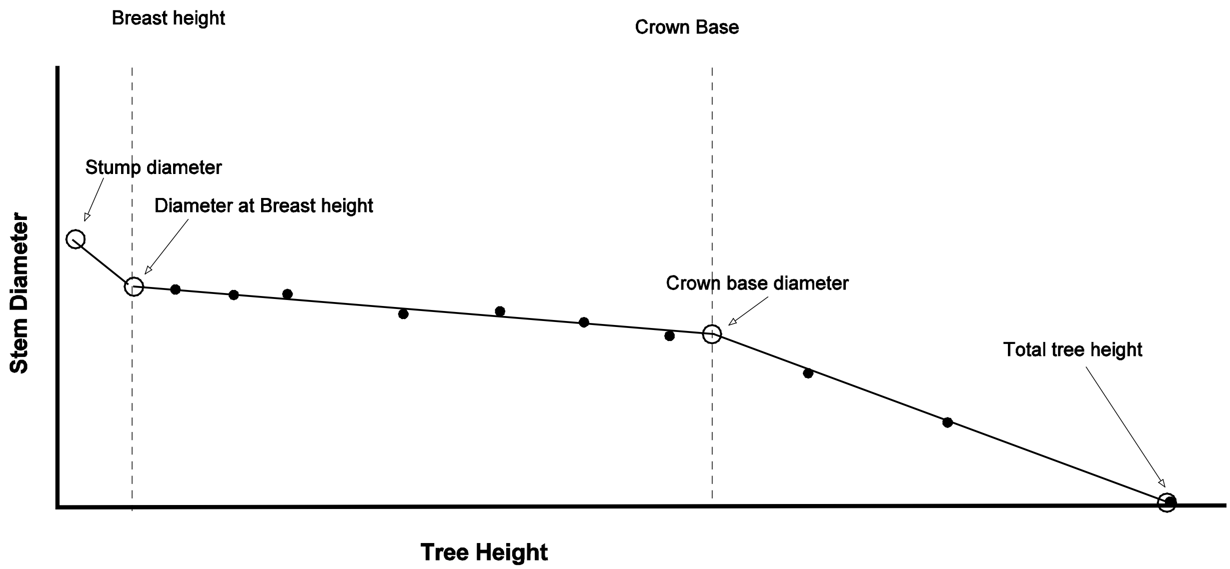

Figure 2.—Illustration of a simple taper profile. Diameter at breast height and diameter at crown base are used as the step between equations.

The typical method of illustrating taper equations is to describe the stem profile relative to the center of the stem (Fig. 2). In this diagram, y dimension is the radius of the stem and the x dimension is the height of the tree. When interpreting taper equations one can think of revolving the taper line around the stem center to estimate stem volume. This is simply the integral of the taper line with respect to height.

In most trees, the portion of the stem from breast height to the base of the crown produces a linear or nearly linear stem profile. This is supported by the long history and wide application of simple taper rates to describe trees and logs. Taper rates describe the change in the stem diameter per one unit of height (height and diameter units are the same). This can be derived from the slope parameter in a simple linear regression. The taper rate on trees is relatively easy to estimate from either standing or recently harvested trees. It requires two diameters measured on the stem at a known distance apart. These data can be collected with a dendrometer, a d-tape and a ladder, a caliper or log-scale stick on a cut log, or stem analysis on felled trees.

In all trees examined in this study, the taper rate below breast height differed from the taper rate above breast height, so a taper rate below breast height is also estimated. The taper rate for the crown portion of the tree does not need to be estimated from data because the diameter at the base of the crown is presumably predicted with the above breast height taper rate estimated with the diameter at crown base, and the diameter at the top of the tree is assumed to be 0. These are assumed to form a cone.

To use the simple taper equation, the user needs only two parameters: a taper rate below breast height and a taper rate above breast height.

The functions assume that at breast height the tree should equal diameter at breast height. For the section below breast height the equation is

$$d_h = dbh + p_b * (h - bh) \qquad (2)$$

Where

d_h = the diameter at height h on the tree,

dbh = the diameter at breast height,

p_b = the stump taper rate from Table 1 for the tree species,

h = the height at which the user desires a diameter prediction, and

bh = the breast height.

Note: all dimensions are in the same units.

The parameters are negative and are defined as the decrease in diameter for one unit increase in tree height with all dimensions in the same units. In equation 2, (h–bh) will be negative and will add to the breast height diameter.

Table 1.—Average taper rates by species (unitless)

Name	Code	n	Stump taper rate	Stem taper rate
Sand pine	107	237	–0.0257	–0.0115
Shortleaf pine	110	430	–0.0390	–0.0132
Slash pine	111	2867	–0.0460	–0.0128
Spruce pine	115	18	–0.0354	–0.0112
Longleaf pine	121	1104	–0.0373	–0.0103
Table Mountain pine	123	74	–0.0326	–0.0097
Pitch pine	126	30	–0.0375	–0.0132
Pond pine	128	488	–0.0414	–0.0124
White pine	129	170	–0.0209	–0.0107
Loblolly pine	131	1706	–0.0496	–0.0142
Virginia pine	132	203	–0.0268	–0.0117
Bald cypress	221	103	–0.1208	–0.0173
Pond cypress	222	351	–0.1645	–0.0199
Hemlock	260	25	–0.0170	–0.0149
Red maple	316	385	–0.0422	–0.0142
Hickory	400	380	–0.0518	–0.0158
Beech	531	18	–0.0529	–0.0172
Sweetgum	611	728	–0.0427	–0.0146
Yellow-poplar	621	212	–0.0453	–0.0121
Sycamore	731	97	–0.0568	–0.0140
Cherry	762	79	–0.0210	–0.0098
White oak	802	181	–0.0713	–0.0169
Scarlet oak	806	57	–0.0682	–0.0153
Southern red oak	812	274	–0.0624	–0.0211
Cherrybark oak	813	96	–0.0635	–0.0152
Laurel oak	820	450	–0.0722	–0.0172
Overcup oak	822	5	–0.1688	–0.0212
Water oak	827	446	–0.0531	–0.0168
Willow oak	831	57	–0.0625	–0.0202
Chestnut oak	832	81	–0.0307	–0.0125
Northern red oak	833	22	–0.0603	–0.0122
Post oak	835	154	–0.0565	–0.0196
Black oak	837	36	–0.0762	–0.0165
Black locust	901	46	–0.0216	–0.0100

For the section above breast height and below crown base the equation is:

$$d_h = dbh + p_s * (h - bh) \quad (3)$$

Where all values are the same except for p_s, which is the stem taper rate from Table 1 for the tree species.

For the stem above crown base, a cone is assumed with the following equation:

$$d_h = (ht - h) * top\_ht\_rate \quad (4)$$

Where

d_h = the diameter at height h on the tree,

ht = total tree height,

h = the height at which the user desires a diameter prediction, and

top_ht_rate = the implicit parameter calculated in the following steps:

1. Determine the diameter at crown base with equation 3.
2. Determine crown length as *total height – height to crown base.*
3. Calculate the *top_ht_rate* as *crown base diameter/crown length.*

The additions to the linear equation between breast height and the stump can presumably be described as the frustum of a cone as well, but with a different taper rate. To describe the volume between stump height and breast height, a frustum of a neiloid with top diameter d.b.h. and bottom diameter stump diameter works very well (Walters and Hann 1986). The differences between the frustum of a cone and the frustum of a neiliod are small, and the formula for a neiliod is more complex. A continuation of the main stem cone to stump height will capture most of the usable volume within this space.

Upper stem shape varies more than the other parts of the tree, and the author left three options for the user: (1) model the upper stem as a cone; (2) model the upper stem as a paraboloid, which adds volume to the upper stem compared to using a cone; or (3) model the upper stem as a neiloid, which removes volume from the upper stem compared to using a cone. The reasons for selecting a cone, paraboloid, or neiloid are covered in the discussion.

Data and Approaches

This study used the U.S. Forest Service, Southern Research Station, Forest Inventory and Analysis stem analysis data set. Cost (1978) described data collection. This study used only a subset of the data to demonstrate the methods (Table 2). Tree measurements on 11,610 trees and 34 species included d.b.h., total tree height, and crown ratio. The trees were sectioned along the stem at 4- or 5-foot intervals. At the ends of each section, diameter outside and inside of the bark and the section length were recorded.

From the stem profile data, outside bark values (i.e., the diameters at various heights on the stem) were calculated to demonstrate the methods. The taper rate, which is the change in diameter per unit of height, was calculated for each section of each tree. Mean taper rates were calculated for the two lower sections. For the top section the appropriate taper required to

Table 2.—Summary of the data used in this study including diameters (cm), heights (m), and height to crown base (m)

Name	Code	n	Minimum diameter	Maximum diameter	Minimum height	Maximum height	Minimum crown base	Maximum crown base
Sand pine	107	156	10.4	46.7	8.53	24.38	1.89	18.38
Shortleaf pine	110	418	13.0	55.1	7.32	29.26	1.95	18.04
Slash pine	111	2458	2.8	60.5	3.96	28.96	1.89	19.51
Spruce pine	115	17	19.6	56.4	17.68	25.30	6.58	16.00
Longleaf pine	121	1048	5.1	67.1	3.96	27.13	1.98	18.29
Table Mountain pine	123	73	12.4	34.5	10.67	19.51	6.55	16.18
Pitch pine	126	25	10.4	38.9	8.23	19.51	4.02	12.44
Pond pine	128	467	11.2	51.8	7.32	24.99	2.13	17.50
White pine	129	144	3.0	39.1	3.35	26.52	1.95	14.30
Loblolly pine	131	1648	13.0	88.1	5.79	29.87	1.89	19.51
Virginia pine	132	109	4.6	54.9	6.10	23.77	2.38	15.09
Bald cypress	221	97	2.8	51.1	4.27	29.57	3.20	19.63
Pond cypress	222	254	10.2	56.6	6.10	28.35	2.68	22.68
Hemlock	260	11	4.8	26.9	5.18	18.29	1.95	4.57
Red maple	316	204	4.3	66.0	7.62	27.43	2.19	17.07
Hickory	400	244	4.3	70.6	5.79	30.78	2.13	18.38
Beech	531	6	8.9	41.7	9.14	21.95	3.29	6.49
Sweetgum	611	508	4.1	60.7	4.27	32.00	1.92	21.46
Yellow-poplar	621	166	2.8	74.4	5.18	28.04	2.07	18.35
Sycamore	731	97	12.7	67.6	8.53	29.26	4.39	19.84
Cherry	762	38	3.6	42.2	5.79	24.69	2.32	14.63
White oak	802	179	13.0	70.1	7.62	30.78	2.29	18.47
Scarlet oak	806	55	13.0	70.4	10.06	26.52	3.57	15.91
Southern red oak	812	216	4.8	81.8	5.49	28.04	1.95	14.08
Cherrybark oak	813	96	13.0	66.0	12.80	28.35	2.19	15.58
Laurel oak	820	242	3.0	78.0	5.49	28.96	2.01	17.92
Overcup oak	822	5	24.4	84.3	15.85	29.87	4.75	14.30
Water oak	827	261	3.3	76.5	5.18	27.74	1.95	17.92
Willow oak	831	48	8.9	96.3	7.01	32.00	2.10	13.87
Chestnut oak	832	48	4.1	53.6	4.88	28.04	2.38	14.02
Northern red oak	833	22	13.0	39.1	12.80	22.86	6.22	11.89
Post oak	835	108	4.1	62.0	4.88	24.38	2.13	12.19
Black oak	837	36	13.0	61.7	8.53	24.99	4.48	13.87
Black locust	901	42	5.6	34.3	7.32	22.86	4.02	15.15

connect the crown base diameter to the top of the tree was selected (Fig. 2). From the data set, volumetric calculations using a frustum of a cone for each tree section were generated for each tree and considered to be the true volume.

Four approaches for estimating the taper rate were compared. The first approach was to calculate two taper rates for each tree by averaging the appropriate sections to create the two required taper rates. This approach requires stem analysis data for the tree being predicted; it is usually useful only for developing taper equations, but it could also be very useful for collecting optical dendrometry data. This method is labeled "tree taper" in Table 3. Figure 3 illustrates the difference between the volume calculated by the stem sections and the volume by the tree taper approach.

The second approach is for people who would like to use locally collected taper data without a formal stem analysis study. This method uses data from only four points on a tree: stump diameter, d.b.h, crown base diameter, and total tree height. This method could be implemented very easily during timber harvest to develop a local taper data set for a forest. This method is labeled "four point taper" in Table 3. Figure 4 illustrates the difference between the volume calculated by the stem sections and the volume derived using the four point taper approach. This approach works almost as well as the tree taper approach without the need to conduct a stem analysis study.

The third approach is to average the per-tree taper rates by species. This is the most practical use of these data because two taper rates are used: one below breast height and one above breast height for each species. This method is labeled as "average taper" in Table 3, and Table 1 provides a list of the values for the data set used in this study. Figure 5 illustrates the difference between the volume calculated by the stem sections and the volume by the average taper approach. This approach is the most practical for most foresters because only two taper rates per species are needed. The predictions from this approach are still very similar to those produced by the other methods.

The fourth approach is to use the stem profile equations published by Clark et al. (1991). This data set was developed from the same data set and used six parameters to predict the same trees. This method is labeled "Clark et al." in Table 3. Figure 6 illustrates the difference between the volume calculated by the stem sections and the volume by the approach described by Clark et al. (1991).

In all methods, the taper equation was used to predict the two end diameters for each of the three sections, and the cubic volume outside bark was calculated using the frustum of a cone. For these four approaches, the predicted cubic volume was compared to the volume from observed stem sections. The statistic for comparing this difference is root mean squared error (RMSE; see Table 3).

Table 3.—Root mean square error (RMSE) (m^3) by four ways of fitting the data including tree taper, the four point taper, average taper, and the six-parameter equations by Clark et al. (1991)

		RMSE			
Name	Code	Tree taper	Four point taper	Average taper	Clark et al.
Sand pine	107	0.0205	0.0224	0.0334	0.0206
Shortleaf pine	110	0.0310	0.0333	0.0542	0.0850
Slash pine	111	0.0385	0.0401	0.0670	0.0358
Spruce pine	115	0.0516	0.0524	0.0388	0.1346
Longleaf pine	121	0.0435	0.0460	0.0563	0.0526
Table Mountain pine	123	0.0408	0.0421	0.0446	0.0579
Pitch pine	126	0.0060	0.0135	0.0387	0.0359
Pond pine	128	0.0394	0.0406	0.0641	0.0224
White pine	129	0.0243	0.0248	0.0267	0.0160
Loblolly pine	131	0.0278	0.0336	0.0640	0.0221
Virginia pine	132	0.0147	0.0194	0.0221	0.0065
Bald cypress	221	0.0100	0.0146	0.0658	0.1784
Pond cypress	222	0.0403	0.0414	0.0947	0.1562
Hemlock	260	0.0171	0.0171	0.0205	0.0126
Red maple	316	0.0108	0.0213	0.0071	0.0944
Hickory	400	0.0063	0.0233	0.0441	0.1951
Beech	531	0.0003	0.0003	0.0089	0.2415
Sweetgum	611	0.0249	0.0359	0.0451	0.0752
Yellow-poplar	621	0.0247	0.0316	0.0121	0.1392
Sycamore	731	0.0038	0.0243	0.0039	0.0383
Cherry	762	0.0013	0.0089	0.0109	0.0393
White oak	802	0.0048	0.0453	0.0679	0.2523
Scarlet oak	806	0.0033	0.0130	0.0250	0.2908
Southern red oak	812	0.0007	0.0176	0.0482	0.1366
Cherrybark oak	813	0.0484	0.0652	0.0684	0.2063
Laurel oak	820	0.0136	0.0285	0.0136	0.1943
Overcup oak	822	0.3224	0.3224	0.5133	0.4014
Water oak	827	0.0113	0.0304	0.0365	0.1353
Willow oak	831	0.0048	0.0572	0.0369	0.2134
Chestnut oak	832	0.0183	0.0047	0.0129	0.1023
Northern red oak	833	0.0238	0.0006	0.0224	0.1181
Post oak	835	0.0356	0.0413	0.0556	0.1242
Black oak	837	0.0109	0.0207	0.0209	0.1908
Black locust	901	0.0085	0.0152	0.0010	0.0594

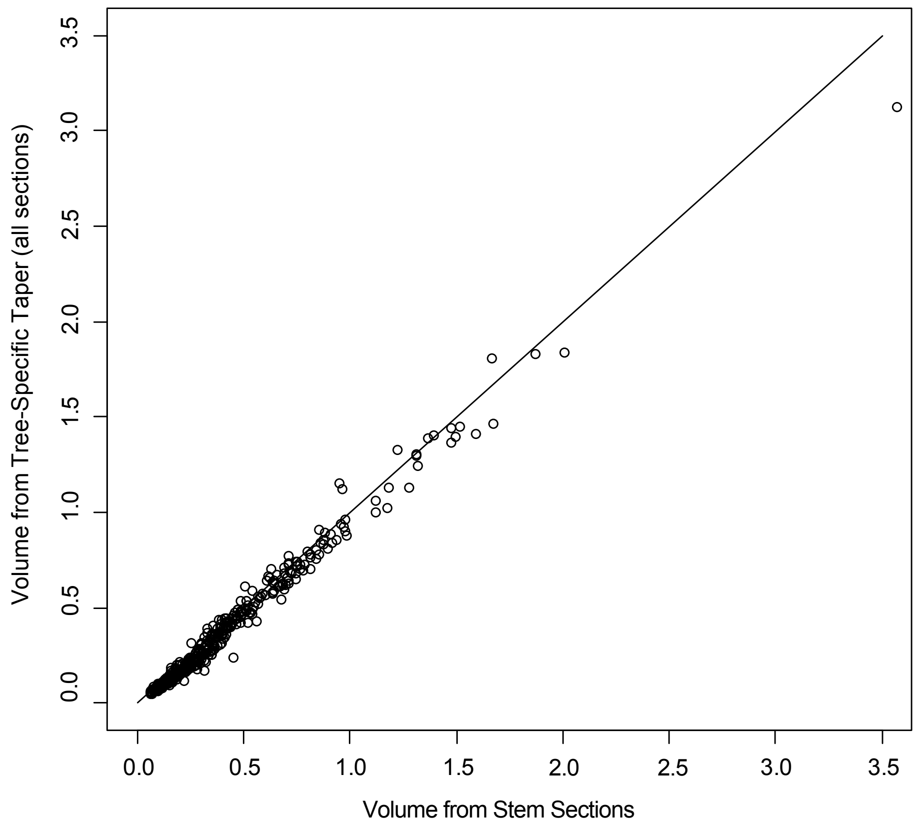

Figure 3.—A comparison of the volume (m^3) calculated from observed stem sections and the tree taper approach in Table 3. The data were from 418 shortleaf pine trees.

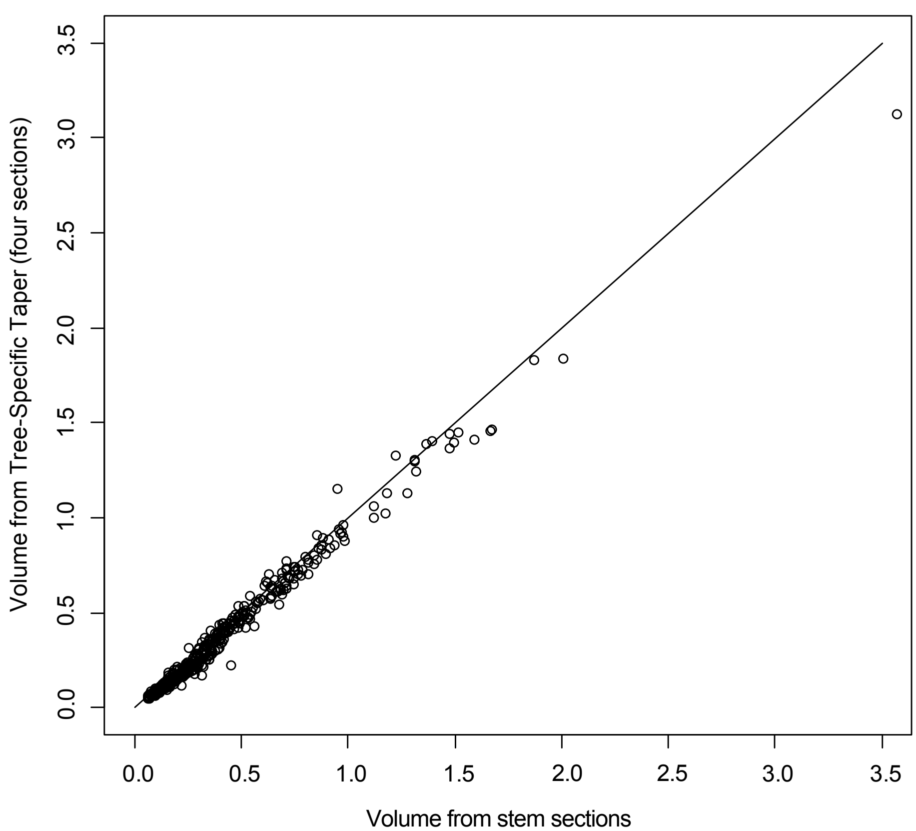

Figure 4.—A comparison of the volume (m^3) calculated from observed stem sections and the four point taper approach in Table 3. The data were from 418 shortleaf pine trees.

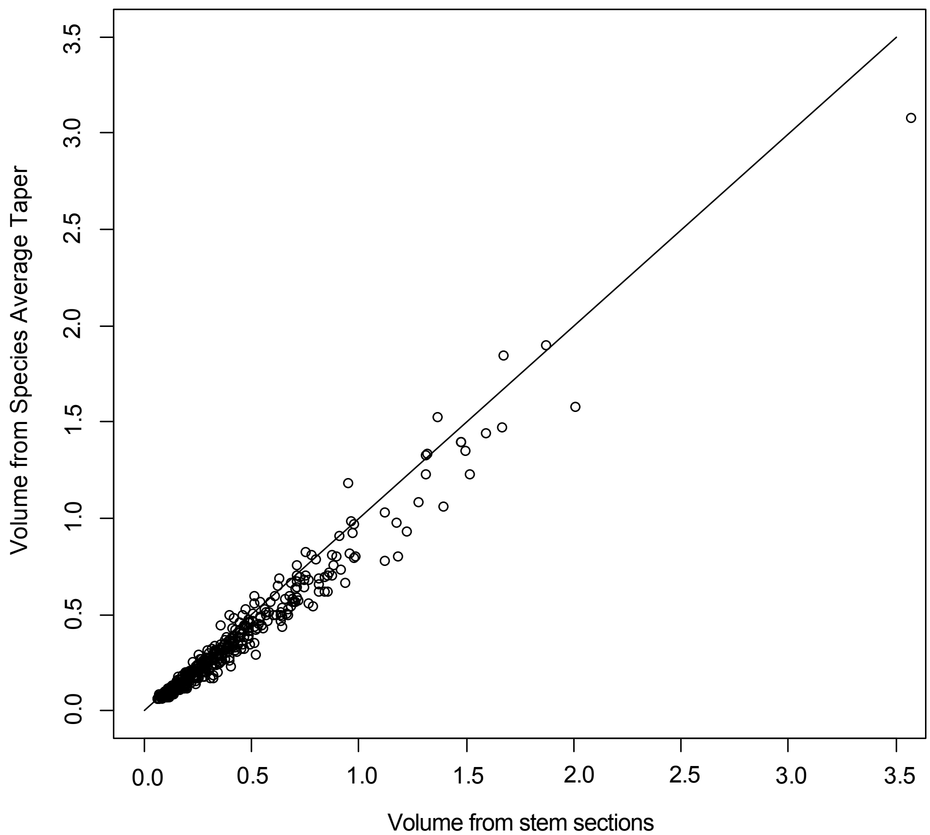

Figure 5.—A comparison of the volume (m^3) calculated from observed stem sections and the average taper approach in Table 3. The data were from 418 shortleaf pine trees.

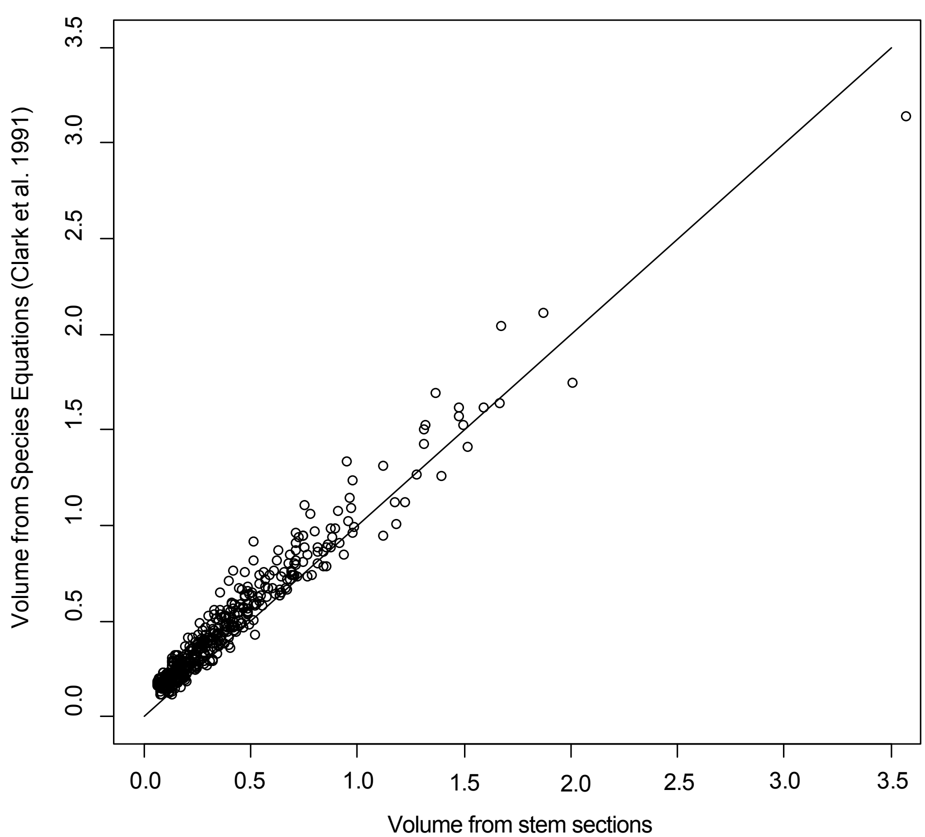

Figure 6.—A comparison of the volume (m^3) calculated from observed stem sections and the Clark et al. approach in Table 3. The data were from 418 shortleaf pine trees.

RESULTS

On average, for the 34 species in this study the tree taper method produced a 0.028 m^3 difference, the four point taper method produced a 0.051 m^3 difference, the average taper produced a 0.037 m^3 difference, and the Clark et al. (1991) equations produced a 0.120 m^3 difference. The Clark method works much better on most of the pines, and the methods proposed in this paper work better on the hardwoods. In general, these methods all work very well and could be considered statistically equivalent. The poor estimates seem to be from small sample sizes (e.g., overcup oak with only five trees).

Overall, the tree taper method generally provided the best fit of the simple taper approaches, but the data needs are greater than most people are willing to collect. In terms of bias, both the tree taper and the four point taper methods produce nearly identical patterns. The average taper approach tends to underpredict volume, and the Clark et al. (1991) equations tend to overpredict volume.

DISCUSSION

The beauty of the simple taper approach is its simplicity. It requires parameters that are simple to determine and logical for practicing foresters to collect. The rates are unitless and based on outside bark, so the same rate can be applied to all unit systems.

In Figs. 3 through 6, the tree taper approach (Fig. 3) works the best but requires stem diameter data on the tree of interest. Also, this approach slightly underpredicts the true volume. The four point taper (Fig. 4) works nearly as well but requires fewer measurements along the stem. This approach also slightly underpredicts the true volume. The average taper (Fig. 5) is the most practical approach and is presented in Table 3. Estimates made with this approach are similar to those made with the previous two methods, but are slightly underpredicted. All of the first three approaches are basically equivalent to the Clark et al. (1991) method, which slightly overpredicts the true volume (Fig. 6).

One of the important aspects of this approach is the estimation of crown base. This study used the crown base provided in the data set. Based on the stem profiles, there is usually a bend in the stem profile on any given tree that has experienced crown recession (Fig. 7). The crown base in this way of thinking is the base of the functional crown and usually does not include the residual branches on the stem below the crown. Many studies have shown that within the crown, the diameter increment is proportional to the leaf area above the point of interest (Jensen 1983). This means that the point of maximum diameter increment is at the crown base and the increment on the stem below the crown base is smaller and proportional to the size of the crown and the distance below the crown base. These observations are consistent with pipe model theory and observation (Valentine 1988).

This study showed that most of the differences were in the portions above the crown base. This portion of the stem is usually of low value or left in the woods as residue. The pine profiles within the crown tended to be slightly parabolic and hardwood profiles tended to be slightly neiloid. Some of these differences come from the fact that pines tend to exhibit excurrent branching and hardwoods tend to exhibit decurrent branching. These differences did not seem to be the same across the range of tree sizes of the various species. The current data set is insufficient to explore relationships between volume prediction and assumptions about shape for these species. The volume differences, however, are smaller than a few hundredths of a cubic meter if a paraboloid or neiloid shape is assumed.

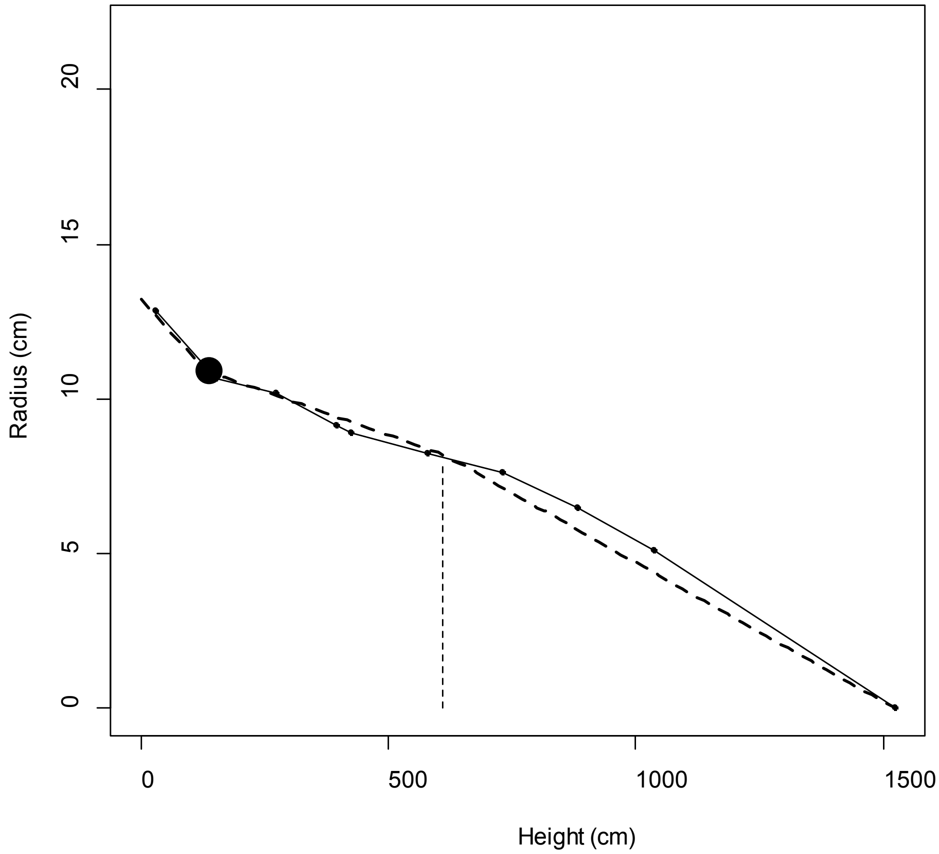

Figure 7.—Illustration of simple taper applied to a shortleaf pine tree. The solid thin line and points are the observed data; the thicker dashed line is the simple taper prediction. The large dot at breast height is the observed d.b.h. and the thin dashed vertical line indicated the crown base. All dimensions are in centimeters.

CONCLUSION

The simple taper equations are simple to use, require relatively few tree measurements, and provide very accurate volume predictions. The approach presented here allows managers to collect data on trees in their forest and can be used to develop local taper equations.

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APPENDIX

A Microsoft Visual Basic function that can be imported into Excel to calculate simple taper

```
Function simpleTaper(h As Double, dbh As Double, ht As Double,
htcb As Double, stumpR As Double, stemR As Double, bh As Double)
As Double
 ' Function to calculate a simple taper equation
 ' Copyright by David R. Larsen, August 14, 2012
 '
 diameterCrownBase = dbh + stemR * (htcb - bh)
 crownLength = ht - htcb
 topRate = diameterCrownBase / crownLength

 simpleTaper = 0#

 If (h < bh) Then
 simpleTaper = dbh + stumpR * (h - bh)
 ElseIf ((h >= bh) And (h < htcb)) Then
 simpleTaper = dbh + stemR * (h - bh)
 Else
 simpleTaper = (ht - h) * topRate
 End If

End Function
```

The content of this paper reflects the views of the author, who is responsible for the facts and accuracy of the information presented herein.