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# SIDE SLOPE STABILITY OF ARTICULATED-FRAME LOGGING TRACTORS

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## INTRODUCTION

MANY log or pulpwood transporting machines have hinged or articulated frames for steering. The articulated frame offers advantages for these machines, but the design introduces some problems in stability. We formulated and analyzed a mathematical model simulating stability of a 4-wheel-drive, articulated frame logging tractor (wheeled skidder) at static or low constant velocity. The model was designed to be versatile so that the model data might be used for stability determinations of various machines.

Machines with articulated frames—such as the wheeled skidder, the pulpwood feller-delimber-bucket-transporter, and the pulpwood feller-transporter—have other design features in common, including rubber tires and 4-wheel-drive (Fig. 1). The hinged frame permits good maneuverability in small spaces. Rubber tires have an advantage in speed and running gear maintenance costs in comparison with tracked vehicles [1].

Among disadvantages, the instability on steep slopes is noted: "... their (wheeled skidders) higher center-of-gravity, which is an advantage on level ground in greater clearance over stumps, rocks, and other obstacles, becomes a hazard" [1].

In a test of different tractor steering and traction methods, utilizing a common test tractor, it was discovered that frame steering resulted in lowered slope stability when turning uphill [2].

A study of wheeled skidders modified to carry chemical tanks revealed that determination of sideslope stability was complicated by the 3-point suspension system [3]. The front axle is pinned to the frame at the axle center (Fig. 2), creating a walking beam action. The pin is the front suspension point. The rear tire ground contact points are the other two points.

## THE ANALYTICAL METHOD

### *Stability criteria*

Three non-linear, supporting contact points constitute stability. Once a vehicle has only two contact points, it becomes unstable. Lateral instability for a wheeled skidder with pinned front axle is defined as when a rear tire contact force is zero, that is, when one of the rear wheels is about to lift off the ground.

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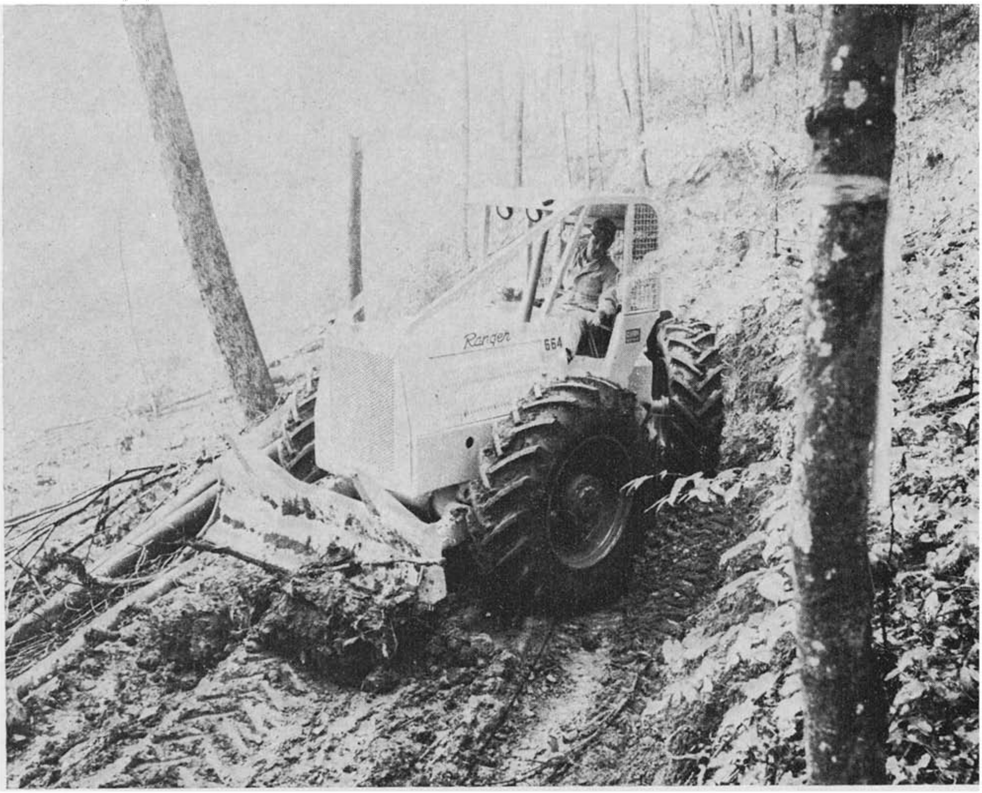


FIG. 1. A 4-wheel-drive, articulated-frame logging tractor (wheeled skidder) working on logging road. Note articulated frame used for steering.

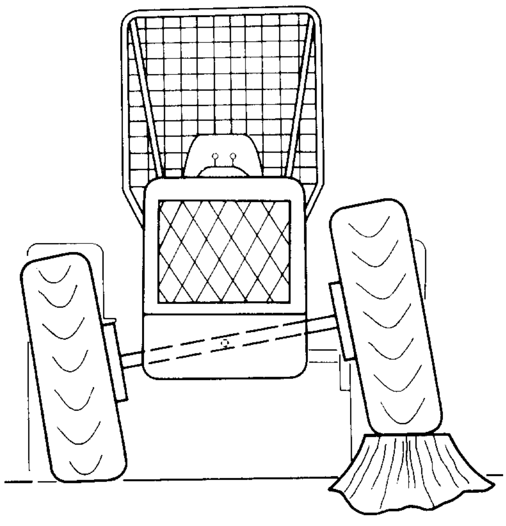

FIG. 2. Pinned front axle action of this wheeled skidder is shown as tractor runs one wheel over stump.

*Center-of-gravity location*

The center-of-gravities (c.g.) locations used are approximations. The level of fuel and hydraulic fluids will effect the c.g. locations, but the effect was considered negligible as was operator weight and location. Four tractors in common use were checked, and the weight of fluids and operator were found to be about 4–5 per cent of the total tractor weight. The items are scattered throughout the tractor frame and most cannot vary much in capacity without seriously damaging the tractor, hence only a fraction of the 4–5 per cent can ever be effective at one time. Baffling of fluid tanks also minimizes the dynamic action of fluids on the tractor.

*Slope*

The major factor in tractor instability is the ground slope. The ground surface slope and the angle between the tractor frame and a horizontal plane are not the same. More of the weight is carried by the downhill-side tires, increasing tire deflection and resulting in frame tilting to a greater angle than the ground slope. Adding to this increased slope angle is tire sinkage into the soil on the downhill side. On soft soils, this could be significant.

*Definition of the tractor bodies*

This study was restricted to an articulated frame, 4-wheel-drive, logging tractor that utilized a walking-beam type front axle and a rigidly connected rear axle (Fig. 2).

The front axle could not resist any overturning moment until the tractor frame tipped to contact with stops on the axle. Hence the tractor c.g. was that of the tractor without the front axle, wheels, and tires [4]. The front and rear tractor frames were considered a rigid body separate from the front axle assembly.

*Tipping phases*

When the tractor tips sideways, the front and rear frames act together. During tipping, the frame rotates around the lower rear wheel ground contact point and the front axle hinge pin. The main force preventing tipping is the tractor weight acting through the c.g. A small restoring moment is applied to the main frame by the front axle if the axle is oriented to the slope and rear frame so that tipping does not occur at right angles to the longitudinal axis of the front frame, but this moment is unpredictable and small and therefore is disregarded. If the side slope is sufficiently steep, the tractor weight vector will act outside of the line between the lower rear wheel contact point and the front axle pin and cause the tractor frame to tip until it contacts the front axle housing. This was called Phase I tipping. During this phase, the front axle can only supply a small restoring moment to the frame.

For one production tractor that is considered typical, the front axle weighs 1600 lb, and the total tractor weight is 12,860 lb. The axle constitutes about 12 per cent of the total tractor weight. The axle supplies no restoring moment to the main frame when the axle is parallel to the major slope axis. When it is at some other angle to the major slope axis, it would only supply a fraction of its weight times the moment arm for that condition. This moment would be small in relation to the tractor weight times a similar moment arm.

When the frame contacts the axle stop, three contact points again occur: the two front wheels and the downhill rear wheel. For the tipping to proceed, the vehicle must rotate around a line between the downhill front and rear wheel ground contact points. This was defined as Phase II tipping. The weight of the front axle and the front axle acting as a moment arm helps to counteract further tipping.

Only Phase I tipping was analyzed. We feel that a tractor having proceeded through the first phase of tipping will have enough momentum to overcome the resistance to tipping developed in the Phase II situation to completely overturn.

*Forces acting on driving wheels*

Soil–wheel interaction affects stability. High sinkage contributes to instability on a side slope. Under some circumstances, the vehicle will slide sidewise rather than tip.

Physical tire characteristics bear on any stability relation. Flexible sidewalls will roll under on a side slope to a degree dependent on the slope angle, further reducing tractor stability. Large diameter, narrow cross-section tires tend to gouge into the soil, and the sidewalls are less flexible; hence do not roll under. However, they also raise the tractor c.g. and are not used on steep slopes for that reason. Tires with a greater number of plies and tires with metal cord belts have stiffer sidewalls.

A wheel at rest or rolling without resistance has only the supporting force $R_y$ (equivalent to the weight $W$ carried by the wheel) acting (Fig. 3). See Notation for definition of symbols.

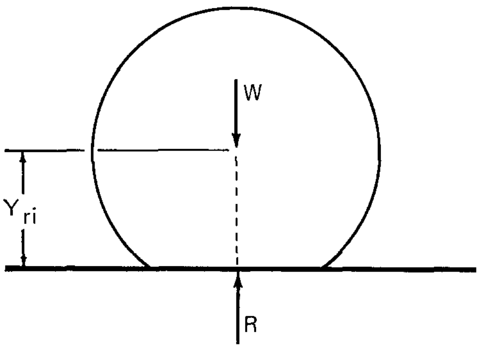


FIG. 3. Basic forces on a wheel at rest or rolling without resistance.

When the wheel has a driving torque $T$, a pull $G$, and a rolling resistance applied and when a concept of net traction is used, the support point of the wheel may be described by the distances $Z_r$ in front and $Y_r$ below the center [7]. $Z_r$ is the "distance of rolling resistance" and $Y_r$ the static rolling radius (Fig. 4). The distance $Z_r$ can be found from

$$Z_r = \mu_r \, Y_r \tag{1}$$

where $\mu_r$ = coefficient of rolling resistance, and $Y_r$ = static rolling radius. Values for $\mu$ were assumed. The equation for the necessary torque $T$ is

$$T = R_z \, Y_r + R_y \, Z_r \tag{2}$$

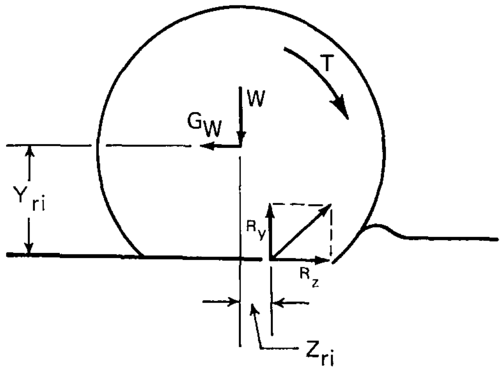


FIG. 4. Forces on a wheel with a driving torque, rolling resistance and a drawbar load.

The ratio between the net pull $G$ and the vertical force $W$ has been defined as the "coefficient of net traction" $\mu$ [7] and is also

$$\mu = \frac{R_z}{R_y} \tag{3}$$

When the tractor is used on a side slope or travels around a curve, or when the skidded load pulls the winch line to one side, a side thrust on the tires exists. This is denoted $R_x$ and is the reaction force to $F$ (Fig. 5). The equal and opposite forces $F$ and $R_x$ cause lateral tire deflection. This lateral wheel shift in relation to the ground contact point creates an additional moment formed by the weight $W$, the vertical ground reaction $R_y$, and the lateral shift $X_r$. This latter couple reduces the stability of the skidder. If other moments act in the same direction and are large, the tractor might overturn. To evaluate this influence, the sidewall deflection rates must be known. This information was not available. The force $R_y$ was therefore considered as acting in the same plane as $W$, disregarding the distance of lateral displacement $X_r$.

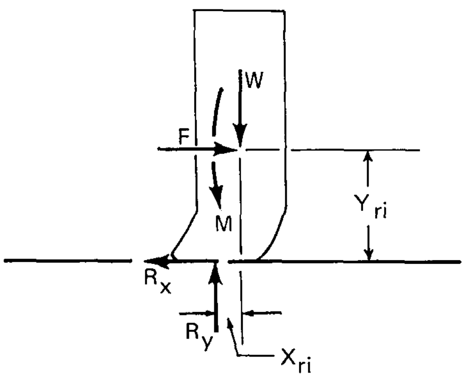


FIG. 5. Lateral force on a wheel causing tire roll-under.

*Variability of c.g. coordinates with steering angle*

The location of front frame c.g. is a function of the angle β between the front and rear frames (Fig. 6). The $X$ and $Z$ axes were chosen fixed to the rear frame, and the location of the front frame c.g. was denoted as $X_F$ and $Z_F$.

$$X_F = Z_4 \sin \beta \tag{4}$$

$$Z_F = Z_1 + Z_4 \cos \beta \tag{5}$$

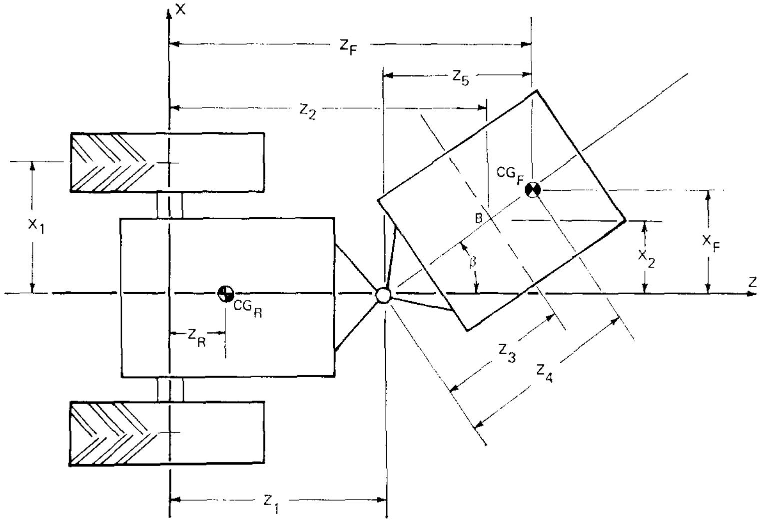


FIG. 6. Location of front and rear frame c.g. in $Z$–$X$ plane.

*Resultant vector and stability triangle conditions for stability*

To maintain stability, the weight vector of a vehicle must pass within the limits of a bounded plane constructed by connecting the supporting points. For this tractor, the stability plane is a triangle defined by the axle pin location [5] and the two rear tire ground contact points (Figs. 7 and 8). The problem is now to determine the slope—given a steering angle and an orientation-to-slop angle—that causes the weight vector to intersect the line forming a side of the stability triangle. The total weight of the tipping body is:

$W = W_R + W_F$ where $W$ = Total tractor weight minus front axle-tire assembly, lb.

$W_R$ = Weight of rear frame, lb.

$W_F$ = Weight of front frame (minus front axle-tire assembly), lb.

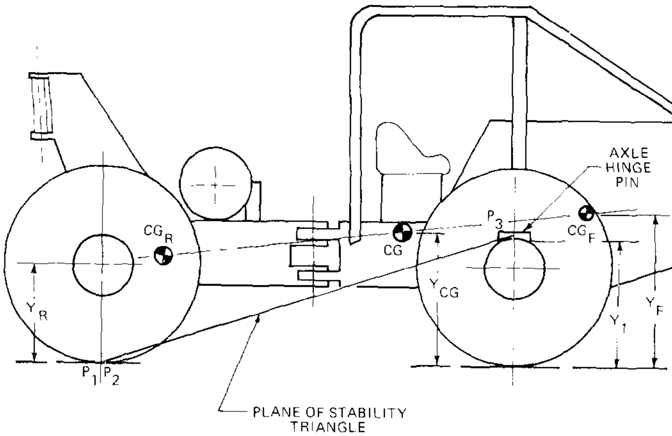


FIG. 7. Side view of articulated frame tractor showing plane of stability triangle and c.g. locations

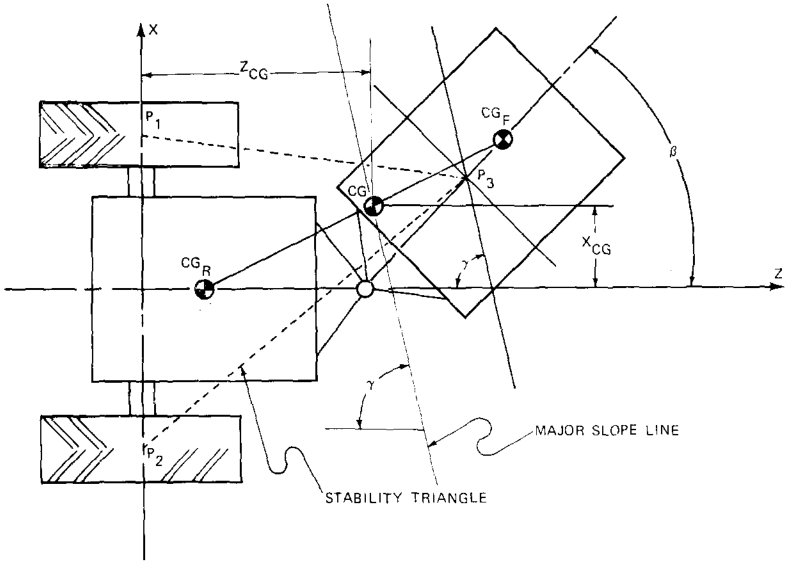


FIG. 8. Stability triangle, steering angle β, orientation-to-slope angle γ and c.g. locations.

By summing moments about each axis, the following terms were developed:

$$X_{c.g.} = \frac{W_F X_F}{W} \tag{6}$$

$$Z_{c.g.} = \frac{W_R Z_R + W_F Z_F}{W} \tag{7}$$

and

$$Y_{c.g.} = \frac{W_R Y_R + W_F Y_F}{W} \tag{8}$$

The terms $X_F$ and $Z_F$ are determined from equations (4) and (5). The distance to the rear c.g. location, $Z_R$, is a measured value, as are $Y_R$ and $Y_F$.

The weight vector $W$ was defined in terms of direction cosines. With reference to Fig. 9, it is seen that the direction cosines are:

$$\left.\begin{array}{lll} X: & - & \sin\alpha \sin\gamma \\ Y: & - & \cos\alpha \\ Z: & + & \sin\alpha\cos\gamma \end{array}\right] \tag{9}$$

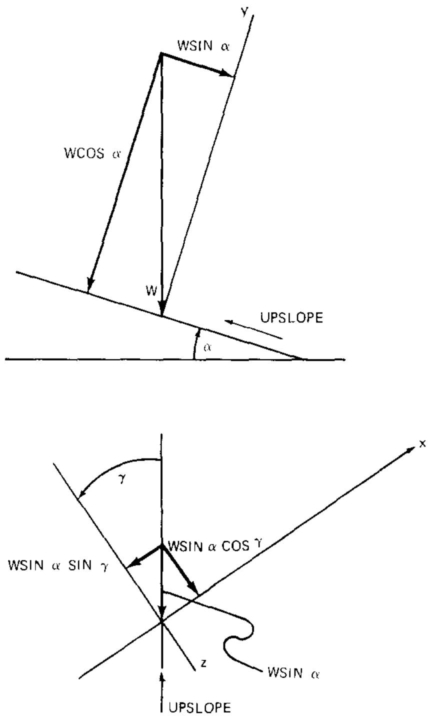


FIG. 9. Direction cosines illustrated with reference to slope and coordinated axes.

With a tractor having a steering angle β, but without driving torque or tire roll-under, and situated on a slope, the stability triangle has the form shown in Fig. 10.

The parametric equations for the lower of the three lines (line $P_2 P_3$) that make up the sides of the stability triangle shown in Fig. 10 are:

$$\left.\begin{array}{l} X = -x_1 + (x_2 + x_1)\, t_2 \\ Y = y_1\, t_2 \\ Z = z_2\, t_2 \end{array}\right] \tag{10}$$

The term $t_2$ is a variable parameter of proportionality [8]. When the weight vector intersects line $P_2 P_3$, the tractor becomes unstable, and tipping impends.

The weight vector $W$ is as a line with originating point ($X_{c.g.}$, $Y_{c.g.}$, $Z_{c.g.}$), and has the parametric equations:

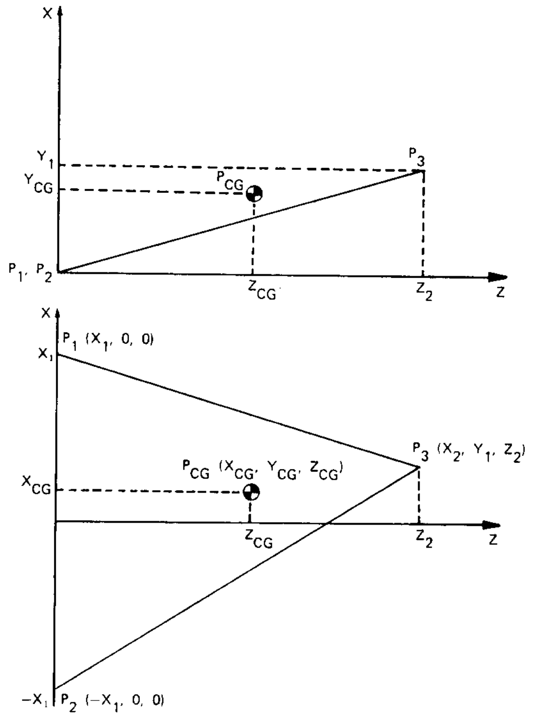


FIG. 10. Stability plane and center of gravity location.

$$\begin{aligned} X &= x_{c.g.} - (\sin \alpha \sin \gamma)\, t_4 \\ Y &= y_{c.g.} - (\cos \alpha)\, t_4 \\ Z &= z_{c.g.} + (\sin \alpha \sin \gamma)\, t_4 \end{aligned} \tag{11}$$

The problem is to determine the values of $\alpha$ (for selected $\gamma$ and $\beta$ angles) when the weight line $W$ intersects the lower side stability triangle line $P_2 P_3$.

To satisfy the condition of intersecting lines, the coordinates must be equal where the lines intersect. The coordinates of lines $P_2 P_3$ and $W$ were equated as:

$$\begin{aligned} -x_1 + (x_2 + x_1)\, t_2 &= x_{c.g.} - (\sin \alpha \sin \gamma)\, t_4 \\ y_1\, t_2 &= y_{c.g.} - (\cos \alpha)\, t_4 \\ z_2\, t_2 &= z_{c.g.} + (\sin \alpha \cos \gamma)\, t_4 \end{aligned}$$

Thus, there are three equations in three unknowns; $\alpha$, $t_2$, and $t_4$. The solution may be written

$$t_2 = \frac{(x_{c.g.} + x_1) \cos \gamma + z_{c.g.} \sin \gamma}{(x_2 + x_1) \cos \gamma + z_2 \sin \gamma} \tag{12}$$

$$\tan \alpha = \frac{-z_{c.g.} + z_2\, t_2}{(\cos \gamma)\,(y_{c.g.} - y_1\, t_2)} \tag{13}$$

The tan α was found by substituting the value for $t_2$ from equation (12) into equation (13).

The computer was programmed for solution of this problem using a step-by-step method, outlined in [9].

*Torque and roll-under effect on wheel support points*

Forward torque application to a wheel moves the support point forward an amount $Z_r$ [7]. This moves the stability triangle forward, while the weight vector remains at its regular position (Fig. 11a). The c.g. is moved in a relative (but not a literal) sense to a more stable location in the stability triangle. Since the distance from the c.g. to the closest side of the triangle is an indication of the vehicle stability, the change in the wheel support points stabilizes the machine.

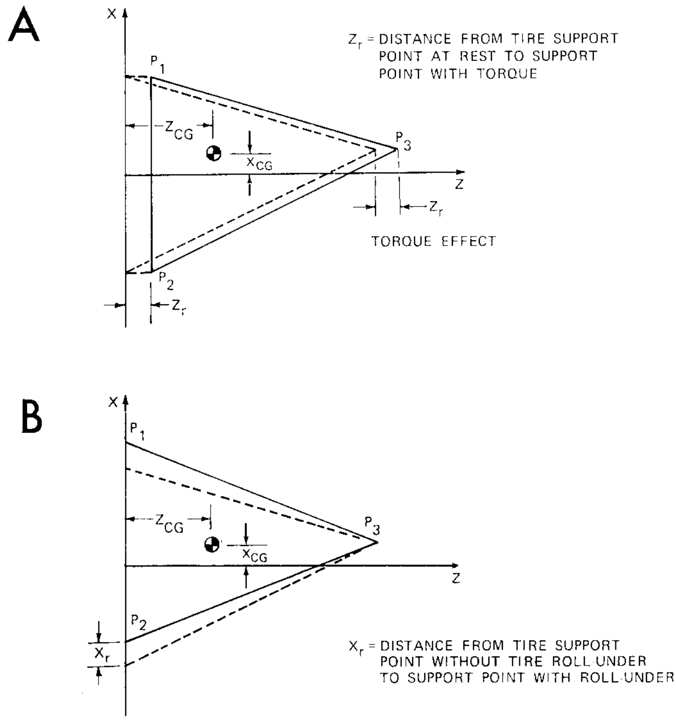


FIG. 11. Effect on stability triangle of torque and tire roll-under.

Tire roll-under on a slope has an opposite effect on the stability as torque application.

With roll-under, the stability triangle base is side displaced an amount equal to roll-under ($X_r$ in Fig. 11b). The weight vector $W$ is closer to the side of the stability triangle than if no roll-under existed, and the roll-under will result in lower slope tipping angles.

The mathematical model can include torque and tire roll-under effects using a stability triangle where changes in the point coordinated $P_1$, $P_2$, and $P_3$ have been introduced corresponding to torque and tire roll-under.

## RESULTS

*Tipping angle curves from the mathematical model*

Equation (13) was solved on the computer for given steering angles (β), and orientation-to-slope angles (γ) using input dimensions from a scale model of a typical logging tractor.

The resultant tipping angles were plotted vs. the orientation-to-slope angles, with steering angle as a third parameter (Fig. 12). A point on any of the curves indicates that tipping impends for that condition. For each curve, there is a minimum stability point (shown connected by a dotted line). For the machine studied, the critical orientation-to-slope for minimum stability follows approximately the equation:

$$\gamma_{crit} + \beta/2 = 80^{\circ}$$

This equation appears logical on inspection of the diagram showing the total machine c.g., orientation-to-slope angle, and the stability triangle (Fig. 8). From this diagram, it would appear that stability should decrease as a function of the distance from the c.g. location to the line $P_2$–$P_3$ for all steering angles. From the plain view of Fig. 8, the minimum distance between the c.g. and $P_2$–$P_3$ is a line perpendicular to $P_2$–$P_3$. When this line falls within the slope plane, minimum stability occurs. The sum of the critical orientation angle and one-half the steering angle results in the minimum stability geometry. For machines with different wheelbase to track ratios (and also $Z_1 : Z_3$ ratios), this relation would change, i.e., $\gamma_{crit,} + \beta/2$ may not equal 80°.

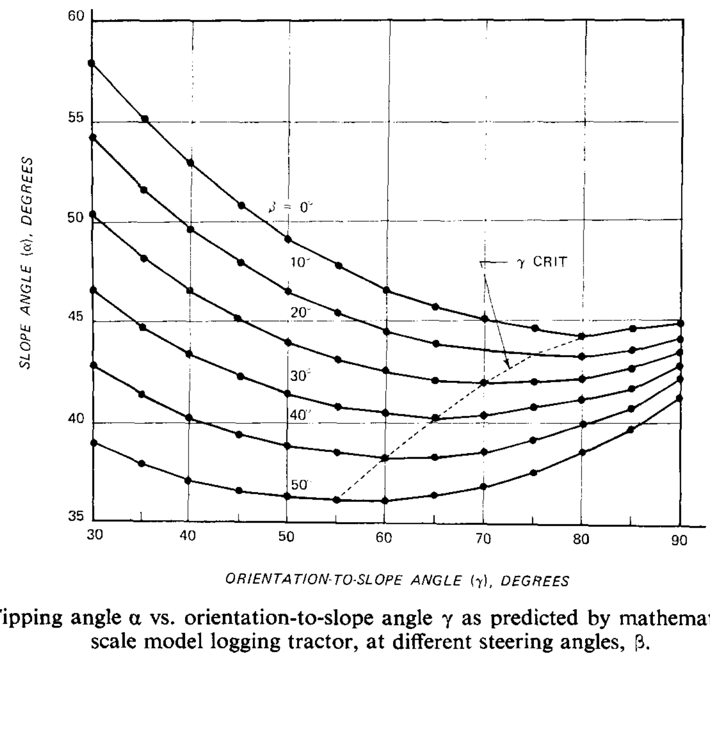


FIG. 12. Tipping angle α vs. orientation-to-slope angle γ as predicted by mathematical model for scale model logging tractor, at different steering angles, β.

*Scale model tipping tests and results*

The mathematical model results were compared with results from actual tipping tests with a scale model. The model (1 in. = 8·65 in.) was scaled from a typical logging tractor. The weight distribution ratio, front-to-rear, was 0·818 : 1. This does not include the front wheel/axle assembly. The ratio of $Z_1 : Z_3$ (Fig. 6) was 1·225 : 1 and wheelbase to track ratio ($[Z_1 + Z_3] : 2X_1$) was 1·185 : 1.

This model was placed on a tilt table which permitted the slope to be varied. The table was laid out so that angle $\gamma$ could be determined. The model was set at a steering angle ($\beta$) and placed on the table at a pre-determined orientation-to-slope angle ($\gamma$). The table was tilted until the upper rear wheel lifted (Fig. 13). This was slope angle, $\alpha$. Tests were replicated three times for each $\gamma$ and $\beta$ condition (those for $\beta = 10°$ were replicated twice).

Actual results were compared to the mathematical model predictions. The mean deviation was 1·6° with a maximum deviation of 3·3°.

An important criteria for comparing the mathematical results to the scale model tipping is the correlation for minimum stability conditions. For a given steering angle $\beta$, the minimum stability point (lowest slope value when tipping was predicted or occurred) should occur at this same orientation-to-slope angle $\gamma$, if the mathematical prediction values agree with the actual tipping results.

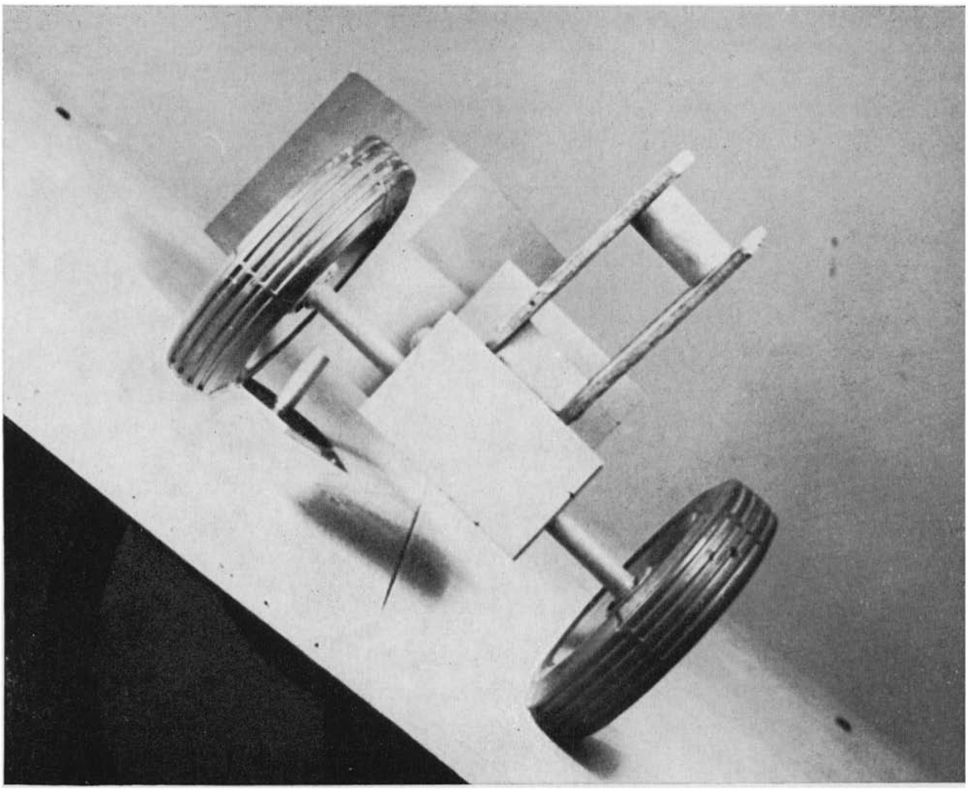

FIG. 13. Scale model shown as wheel lifts from ground when slope ($\alpha$) reaches tipping angle.

For all $\beta$ angles except 30°, minimum stability occurred at the same orientation angle $\gamma$ for both prediction and the actual scale model test. At a steering angle of 30°, the mathematical model predicted minimum stability at 60° orientation angle while the model tests showed 75°. The difference in slope angle was small ($\alpha = 40{\cdot}6°$ @ $\gamma = 60°$ for mathematical model and $\alpha = 42{\cdot}0°$ @ $\gamma = 75°$ for scale model). This difference could be due to thc assumptions that were not considered for the mathematical model, or due to experimental error in tipping the scale model.

## CONCLUSIONS

The geometry of an articulated-frame vehicle has a significant effect on slope stability. Stability varies with the vehicle's steering angle and orientation on a side slope. The vehicle is most unstable when the rear frame is orientated 60° to 80° to the slope. Tipping occurred at a slope angle of 36° for a steering angle of 50° and an orientation-to-slope angle of 60°. In comparison, the minimum stability slope for zero steering angle to slope was 44·5°, a difference of 8·5°. The critical orientation angle (minimum for each steering angle) follows the equation:

$$\gamma_{crit} = 80° - \beta/2.$$

The tipping angles ($\alpha$) might not be construed as realistic for machines working in the woods. Engine vibration, accelerations, tire spring rates, and other dynamic factors were not within the scope of this study and could have an effect on stability. Thus, the derived $\alpha$ values might not be achieved with a logging tractor. The dynamic factors might cause tip at a lower slope angle. However, the minimum stability orientation, $\gamma_{crit}$, would apply to a real machine, even if tipping angles would be lower.

## APPLICATION OF RESULTS

The method can be used to determine side slope stability of a mechanized timber harvester having an articulated frame. Once the c.g., wheelbase, track, and range of steering angle are known, the data can be added to the computer program, and with selected orientation-to-slope angles, the tipping angles can be calculated. Stability curves can be plotted using the resultant values, and one vehicle can be compared to another for stability. Since dynamic conditions are not included in the method, it would be best to calculate the stability curves for a "standard" vehicle of acceptable stability determined from operating experience. Then designs can be compared to this standard after stability curves have been developed for both.

Pulpwood forwarders can be analyzed for side slope stability when unloaded and loaded. For the loaded condition, the rear frame c.g. would have to be calculated using the load c.g. and the unloaded rear frame c.g. This new c.g. location would be used in the computer program as the rear frame c.g., and the program would calculate the c.g. location for given steering angles and stability curves for the loaded forwarder.

## NOTATION

| | |
|---|---|
| $R_y$ | Wheel supporting force parallel to $y$-axis |
| $W$ | Weight acting through wheel or total weight of tractor |
| $Y_{ri}$ | Static rolling radius of $i^{th}$ tire (wheel) |
| $T$ | Wheel driving torque |

| | |
|---|---|
| $G$ | Drawbar pull of a wheel |
| $Z_r$ | Theoretical "distance of rolling resistance" |
| $\mu_r$ | Coefficient of rolling resistance |
| $R_z$ | Rolling resistance force |
| $\mu$ | Coefficient of net traction |
| $R_x$ | Lateral force on a tire (wheel) |
| $F$ | Force acting through wheel laterally either from outside loading on tractor or from side slope component of tractor weight |
| $X_r$ | Lateral displacement of wheel from center of tire tread |
| $\beta$ | Steering angle of tractor measured between $Z$-axis and longitudinal axis of front frame |
| $X_F$ | Distance from $Z$-axis to front frame c.g. location measured parallel to $X$-axis in $X$–$Z$ plane |
| $Z_F$ | Distance from $X$-axis to front frame c.g. location measured parallel to $Z$-axis in $X$–$Z$ plane |
| $Z_4$ | Distance from steering hinge pin $C/L$ parallel to $Y$-axis to front frame c.g. location measured along longitudinal axis of front frame in $X$–$Z$ plane |
| $Z_1$ | Distance from $Z$-axis to $C/L$ of steering hinge pin parallel to $Y$-axis measured along $Z$-axis in $X$–$Z$ plane |
| $M$ | Moment necessary to counter overturning couples on wheel |
| $X_1$ | Distance from $Z$-axis to center of tire tread (1/2 track width) measured along $X$-axis in $X$–$Z$ plane |
| $Z_R$ | Distance from $X$-axis to rear frame c.g. measured along $Z$-axis in $X$–$Z$ plane |
| $Z_2, Z_3, X_2, Y_1$ | Location coordinates of front axle pivot pin |
| $W_F$ | Weight of front frame excluding front axle, tires, and wheels |
| $W_R$ | Weight of rear frame |
| $X_{c.g.}, Z_{c.g.}, Y_{c.g.}$ | Location coordinates of full tractor frame (front and rear) center-of-gravity |
| $\gamma$ | Angle between lines, one of which is formed between intersection of plane in which slope is measured and $X$–$Z$ plane, the other of which is formed by the intersection of a plane parallel to the horizontal and the $X$–$Z$ plane |
| $\alpha$ | Angle of slope measured in plane formed by weight vector which passes through the center of the earth and horizontal line intersecting weight vector. Slope is measured between horizontal plane and $X$–$Z$ plane |

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