

Theory of Precipitation Effects on Dead Cylindrical Fuels

MICHAEL A. FOSBERG

Abstract. Numerical and analytical solutions of the Fickian diffusion equation were used to determine the effects of precipitation on dead cylindrical forest fuels. The analytical solution provided a physical framework. The numerical solutions were then used to refine the analytical solution through a similarity argument. The theoretical solutions predicted realistic rates of water exchange and the internal distribution of the water. The theory also (1) predicted that precipitation duration rather than amount or rate determined the amount of water uptake and (2) provided a function useful in practical applications. *Forest Sci.* 18:98–108.

Additional key words. Fuel moisture, fire hazard, forest fuels, weather effects.

WHEN PRECIPITATION increases moisture content of dead forest fuels, fire behavior and fire-danger rating are profoundly affected. While a great deal of work has been done on the duff and litter layers, there has been little definitive research on cylindrical fuels such as logs, stems, and twigs. In this paper, I have attempted to help fill this void in fuel moisture understanding through a series of three theoretical steps.

The first step is a linearized analytical solution of the Fickian diffusion equation for a highly idealized case. This solution is not applicable in the field, but it provides the general functional form for data evaluation.

The second step consists of obtaining a set of particular solutions of the nonlinear differential form of Fick's equation. This set may be treated as a data set to evaluate the coefficients of the analytical solution.

In the third step, which yields an expression for general use, the analytical solution is combined with the data of the particular solutions by use of similarity theory. This involves constructing a set of nondimensional groups of the relevant variables, and relating them through function coefficients. This final expression can then be used to predict changes in moisture

content of dead cylindrical fuels due to precipitation.

Background

Previous work on cylindrical fuels has been concerned with the drying process following rain (Williams 1962, Storey 1965, and Brackebusch¹). These papers were primarily concerned with the qualitative aspects of precipitation effects. Morris² considered the quantitative aspects, but was limited to the drying process for the most part. Although he considered the effects of precipitation immediately preceding fuel moisture determination, the independent variable was precipitation amount over predetermined intervals of time. Simard (1968) examined the wet-

The author is Principal Meteorologist, Rocky Mountain Forest and Range Exp. Sta., USDA Forest Serv., with central headquarters at Fort Collins, Colorado, maintained in cooperation with Colorado State University. Manuscript received Dec. 22, 1970.

¹ Brackebusch, Arthur P. 1962. Gain and loss of moisture in large forest fuels. Unpublished manuscript on file at Intermountain Forest and Range Exp. Sta., N. Forest Fire Lab, USDA Forest Serv., Missoula, Mont.

² Personal correspondence in 1969 with W. G. Morris on progress of studies in fuel moisture on July 3, 14, and 17.

ting process for a number of fuels but did not provide a cohesive structure for data interpretation. He did, however, imply that precipitation duration rather than amount may be the important precipitation variable.

Nissan and Hansen (1961) described the diffusion processes in porous media as falling into two classes: one, the mass loss taking place from a free liquid surface, involves a constant mass exchange process. The second, involving a variable mass change rate, may take place either above or below the fiber saturation point in wood, since free water as well as hindered and bound water must diffuse through the internal cavities of the wood. Although the diffusivity will be constant at and above the fiber saturation point, it depends on the permeability of the wood. This implies that the mass change rate is gradient dependent, since there is a discontinuity between the diffusivity of water vapor in air and wood at the fuel surface. Thus the constant exchange rate does not apply to wood.

The maximum moisture content of wood may be determined from Stamm (1938). This moisture content, the point at which the fibers are saturated and all voids are filled with free water, is near 200 percent for softwoods. It is also the point separating the constant exchange process from the variable mass change rate.

The movement of moisture through wood is governed by the Fickian diffusion equation (Crank 1956, Stamm 1964), and several solutions exist for such movement below fiber saturation. The mathematical methods used to obtain these solutions may be applied to water movement above fiber saturation by utilizing a similar but different set of constraining conditions. The solutions thus obtained are valid both above and below the fiber saturation point.

The Analytical Solution

The Fickian diffusion equation for moisture content (Fosberg 1970) is

$$\frac{\partial m}{\partial t} = \nu \frac{\partial^2 m}{\partial r^2} + \frac{\nu}{r} \frac{\partial m}{\partial r}, \quad (1)$$

where ν is the diffusivity, held constant in this analysis, m is the moisture content, and r and t are the radius and time coordinates respectively. The variation of moisture content is dependent only on radius and time in this portion of the analysis. Relative moisture content (ϕ) can be defined as

$$\phi = (m - m_j) / (m_i - m_j), \quad (2)$$

where m_i is the initial moisture content, and m_j is the moisture content at the surface of the fuel. Both are held constant for a solution, but not necessarily equal between solutions. Eq. (2) may be solved for m and the resulting function substituted into eq. (1). The resulting equation is

$$\frac{\partial \phi}{\partial t} = \nu \frac{\partial^2 \phi}{\partial r^2} + \frac{\nu}{r} \frac{\partial \phi}{\partial r}, \quad (3)$$

since those derivatives involving m_j with t and m_i with r are zero. Thus from the conditions given, $\phi(r, t)$ has two conditions imposed on it.

$$\phi(r, 0) = 1 \quad \text{and} \quad \phi(R, t) = 0. \quad (4)$$

The first part of eq. (4) states that before the diffusion begins, ϕ is constant, while the second part states that at the surface of the cylinder $m = m_j$ for all time.

Now by separation of variables,

$$\phi(r, t) = S(t) B(r). \quad (5)$$

Eq. (5) may be substituted into eq. (3) such that

$$\frac{1}{\nu S} \frac{\partial S}{\partial t} = \frac{1}{B} \frac{\partial^2 B}{\partial r^2} + \frac{1}{rB} \frac{\partial B}{\partial r} = -\chi^2, \quad (6)$$

where χ^2 is a third arbitrary function.

Eq. (6) may be written as

$$\frac{1}{S} \frac{dS}{dt} = -\nu \chi^2, \quad (7a)$$

$$\frac{d^2 B}{dr^2} + \frac{1}{r} \frac{dB}{dr} + B \chi^2 = 0. \quad (7b)$$

Integrating eq. (7a)

$$\int_1^S \frac{1}{S} dS = \int_0^{t_*} -\nu \chi^2 dt, \quad (8)$$

$$S = \exp(-\nu \chi^2 t_*).$$

Eq. (7b) is a Bessel equation of the first kind and zero order (Watson 1966). Thus

$$B = J_0(\chi r). \quad (9)$$

Combining eq. (8) and (9)

$$\phi(r, t) = SB = \exp(-\nu \chi^2 t_*) J_0(\chi r). \quad (10)$$

Imposing the constraints introduced by eq. (4),

$$\phi(R, t) = 0 = \sum_n C_n J_0(\chi_n R) \exp(-\nu \chi_n^2 t_*).$$

Thus $\chi_n = K_n/R$, where the K_n values are the roots of J_0 . Similarly

$$\begin{aligned} \phi(r, 0) = 1 &= \sum_n C_n J_0[K_n(r/R)] \\ &= \sum_n \frac{2J_0[K_n(r/R)]}{K_n J_1(K_n)} \end{aligned}$$

or

$$\phi_1(r, t) = 2 \sum_n \frac{J_0[K_n(r/R)]}{K_n J_1(K_n)} \exp[-\nu(K_n^2/R^2)t_*]. \quad (11)$$

In eq. (11) ϕ_1 now represents a particular solution.

A more general solution may now be obtained by permitting the moisture content obtained by eq. (11) to be a new set of initial conditions. If at the same time a new boundary condition is defined and an arbitrary time interval is used to solve the problem, a second particular solution can be obtained. A series type of solution is obtained by repeating this procedure for a large number of time intervals to obtain the general solution. The new relative moisture content is

$$\phi_2(r, t) = \frac{m_2(r, t) - m_{t_2}}{m_1(r, t_*) - m_{t_2}}. \quad (12)$$

Since $m_1(r, t_*) = m_2(r, 0)$, the initial and boundary conditions expressed in eq. (4) still apply. Eq. (3) is still the governing equation, so the methodology of eq. (5 through 7) is valid. Integration of the time dependent portion is now over the arbitrary limits S_* to S .

$$\int_{S_*}^S \frac{1}{S_2} dS_2 = \int_{t_*}^t -\nu \chi^2 dt = \int_0^{t_d} -\nu \chi^2 dt, \quad (13)$$

where $t_d = t - t_*$, and

$$\frac{d^2 B}{dr^2} + \frac{1}{r} \frac{dB}{dr} + \chi^2 B = 0,$$

$$B = J_0(\chi r),$$

so that

$$\phi_2(r, t) = S_* J_0(\chi r) \exp(-\chi^2 \nu t). \quad (14)$$

Now, by the initial and boundary conditions of eq. (4) for time

$$\phi_2(R, t) = 0 = \sum_n C_n S_* J_0(K_n) \exp(-\nu K_n^2 t_d)$$

and for radius

$$\begin{aligned} \phi_2(r, 0) = 1 &= \sum_n S_* \frac{2J_0[K_n(r/R)]}{K_n J_1(K_n)} \\ &= S_* \sum_n C_n J_0[K_n(r/R)], \end{aligned}$$

the resultant equation is

$$\phi_2(r, t) = \sum_n \frac{2}{S_*} \frac{J_0[K_n(r/R)]}{K_n J_1(K_n)} \exp[-\nu K_n^2 t_d/R^2],$$

or by using the space dependent boundary condition

$$\phi_2(r, t) = \sum_n 2C_n \frac{\{J_0[K_n(r/R)]\}^2}{K_n J_1(K_n)} \exp(-\nu K_n^2 t_d/R^2). \quad (15)$$

Integrating eq. (15) over the volume of the cylinder

$$\begin{aligned} \bar{\phi}(t_d) &= \frac{c}{R^2} \int_0^R r \phi_2(r, t_d) dr \\ &= \frac{c}{R^2} \int_0^R \sum_n \frac{\{J_0[K_n(r/R)]\}^2}{K_n J_1(K_n)} \\ &\quad \exp(-\nu K_n^2 t_d/R^2) r dr \\ &= \sum_n c [J_1(K_n)/K_n] \exp(-\nu t_d K_n^2/R^2) \end{aligned}$$

The Similarity Conditions

If t_d is taken to be the duration of precipitation, the increase in moisture content is then

$$\frac{\overline{\delta m}}{m_j - m_i} = \frac{\bar{m} - m_i}{m_j - m_i} = 1 - \sum_n c \frac{J_1(K_n)}{K_n} \exp [(-\nu K_n^2 t_d) / R^2] \quad (16)$$

when the bars imply integration over r . The traditional solution of moisture movement is obtained by integrating eq. (11) over the volume of the cylinder, yielding

$$\delta m / \Delta m = 1 - \sum_n (1 / K_n^2) \exp (-\nu K_n^2 t / R^2) = 1 - e^{-\lambda t},$$

which differs from eq. (16) slightly. This difference is due to the nonhomogeneous initial conditions used to obtain eq. (16). Knowing the governing equation for δm , however, provides a basis for obtaining a unique similarity condition (Long 1963).

This condition is in general

$$\overline{\delta m} / (m_j - m_i) = 1 - \xi \exp (-\lambda t_d), \quad (17)$$

where the mean value theorem has been used to obtain ξ after the substitution of the solution for λ given by standard drying conditions.

Thus for homogeneous initial conditions and fixed boundary conditions $\xi = 1$, but in general it will differ from unity because of the non-uniform initial conditions.

The Numerical Solutions

Inclusion of the nonlinear terms in the Fickian diffusion eq. (1) provides in cylindrical coordinates

$$\frac{\partial m}{\partial t} = \nu_r \frac{\partial^2 m}{\partial r^2} + \frac{\nu_r}{r} \frac{\partial m}{\partial r} + \frac{\partial \nu_r}{\partial r} \frac{\partial m}{\partial r} + \frac{\nu}{r^2} \frac{\partial^2 m}{\partial \theta^2} + \frac{1}{r^2} \frac{\partial \nu_\theta}{\partial \theta} \frac{\partial m}{\partial \theta}. \quad (18)$$

In eq. (18), r is the radial coordinate, θ is the tangential coordinate, and ν_r , ν_θ are the diffusivities along r and θ . The diffusivities, ν_r and ν_θ , used previously (Fosberg 1970), are taken from Stamm (1946) and Stamm and Nelson (1961). The radial and tangential diffusivities differ because the numbers of fibers per unit distance differ.

The diffusion equation was solved for isothermal conditions. The finite difference analog of eq. (18) contained centered space differences at all but the central point and the Adams-Bashforth (Lilly 1965) time step. The central point was expressed by the average of the uncentered differences along each ray. A 9×10 -grid mesh with a center point was used to reduce truncation error. Computational stability (Luikov 1966) was maintained by requiring that the finite time step be

$$\delta t < \frac{R^2 (\delta \theta)^2}{2 \nu_{\max}}, \frac{(\delta r)^2}{2 \nu_{\max}},$$

where R is the radius, $\delta \theta$ is the finite angle between rays, δr is the finite radius interval, and $\nu_{\max}$ is the maximum value of ν_r or ν_θ . To prevent spurious long-period time changes, δt had to satisfy the stability criteria.

The boundary conditions at the surface of the fuel were determined from a water balance analysis. The balance is based on the behavior over a unit area of fuel surface. Because the effects of surface characteristics of cellulosic materials on water (such as contact angle, etc.) are not well defined, a boundary layer was assumed to exist in the surface layer of the wood. The water balance could then be formulated as a rate process involving adsorption in the boundary layer, precipitation, runoff, growth of a storage layer, and evaporation. The balance equation is then

$$\rho_w A_0 h' = P \rho_w A_0 - Ru - Ad - Ev,$$

where ρ_w is the density of water, A_0 is a unit area, h' is the rate of growth of the free-standing water storage on the surface, Ru is the runoff of water in excess of that maintained by surface tension, Ad is the adsorption rate of the water by wood, and Ev is the water loss from the system through evaporation. The storage layer was obtained by assuming that its maximum depth was small compared to the radius of the cylinder, so that surface tension and gravity were the only relevant forces. This water layer was allowed to exist only on the upper half of the cylinder. These computations gave a maximum

depth of 7.56×10^{-2} cm. When the layer reached this depth, any excess would run off and h' then is zero. During the rainfall, evaporation was assumed to be zero. The adsorption into the wood was obtained by assuming an internal boundary layer (Lui-kov 1966).

The adsorption was assumed to be a steady-state gradient process proportional to

$$\kappa \frac{\partial \rho_m}{\partial r} \Big|_s \approx \kappa \frac{\Delta \rho_m}{d_f},$$

where d_f is the thickness of the interface boundary layer and $\Delta \rho_m$ is the moisture density gradient between the liquid layer and the wood. Thus the adsorption becomes

$$Ad = (-\kappa/d_f)(A_0 d_f m \rho_f - A_0 d_w \rho_w),$$

where d_w is the depth of the free water layer.

The boundary layer depth represents the depth to which a pulse of precipitation will produce a rapid increase of fuel moisture.

The depth of the boundary layer is obtained by assuming that the cylinder is composed of two moisture regions. The internal region $0 \leq r \leq R - d_f$ has a constant moisture content of m_0 . The outer layer $R - d_f \leq r \leq R$ has a linear moisture profile of

$$m = m_0 + (m_m - m_0) [(r - R + d_f)/d_f],$$

where m_m is the moisture content at the surface. The integral moisture content under these conditions is

$$\begin{aligned} \bar{m} = m_0 & \left[1 - \frac{2d_f}{R} + \left(\frac{d_f}{R} \right)^2 \right] \\ & + (m_m - m_0) \left[\frac{d_f}{R} - \frac{5}{3} \left(\frac{d_f}{R} \right)^2 \right]. \end{aligned}$$

The integral moisture content can be turned around to give a quadratic equation for d_f .

$$\begin{aligned} d_f^2 + d_f & \left[\frac{3R(m_m - 3m_0)}{(8m_0 - 5m_m)} \right] \\ & + \left[\frac{3R^2(m_0 - \bar{m})}{(8m_0 - 5m_m)} \right] = 0. \end{aligned}$$

The magnitude of d_f may be obtained from the data from Tables 7 and 8 of the appendix in Simard (1968). Only the data for the largest samples without bark were selected. These restrictions were imposed because of the assumptions in the derivation of the diffusion equation and those used to obtain the depth of the surface tension layer.

Thickness of the boundary layer d_f may be computed if we assume that the initial moisture in Simard's tables represent m_0 , the boundary moisture content m_m is 200 percent, and that the mean moisture content after a time interval is $\bar{m}$. The length of the precipitation pulse was taken to be 10 minutes. Linear interpolation was used to find $\bar{m}$ at the end of 10 minutes. These procedures and assumptions give values of 2 to 3 mm for d_f .

The rate of water uptake by the wood was assumed to be proportional to the diffusivity at and above the fiber saturation point. Thus, the water balance equation may now be written as

$$\begin{aligned} \rho_w h' A_0 = P \rho_w A_0 \\ - (\nu A_0 / d_f^2) (d_f m \rho_f - d_w \rho_w) - Ru. \end{aligned}$$

The development of the internal boundary layer involves such a large number of assumptions that it is valid only to the order of magnitude. The development applies to large diameter cylinders, but the results are valid for small diameter cylinders because the water uptake rate is limited by the diffusivity in wood and the final results are based on Simard's (1968) experiments.

The maximum effective precipitation rate may now be determined by allowing $h' = 0$ and runoff to equal zero so that

$$P = \nu / d_f [m(\rho_f / \rho_w) - (d_w / d_f)]. \quad (19)$$

Allowing d_f to be 3 mm for the sake of argument, the maximum effective precipitation is of the order of a millimeter per day. This analysis supports the idea that duration, not amount, of precipitation is the controlling factor in wetting of fuels.

These results are consistent with the analytical solution obtained in eq. (17). The surface moisture content was deter-

mined from Simard's (1968) wetting experiments by solving eq. (17) for m_j . These data were the same as used to define the boundary layer. The value of ζ was obtained from the analytical solution. These data gave a linear time-dependent value of m_j of

$$m_j = 76\% + (2.7\%/hr)t$$

as a first approximation. This experimentally defined boundary condition removed the difficulty of determining d_f for small diameter fuels and permitted a check on the maximum effective precipitation.

Combined Numerical and Analytical Solutions

The set of numerical solutions was obtained by considering wood cylinders whose response times were 1, 10, and 100 hr under standard drying conditions of 26.7°C and 20 percent relative humidity and precipitation durations of 1, 2, 6, and 24 hr. These numerical solutions were then used to refine the analytical solution by introducing the nonlinear processes and variable boundary conditions. The maximum adsorption rate was chosen by using the experimentally defined boundary condition. The fuels were initially assumed to have a homogeneous moisture content of 30 percent. The fuels were dried $\frac{1}{2}$, 1, and 2 response time periods before precipitation was started. At the end of the precipitation, the fuels were again allowed to dry at standard drying conditions. The boundary conditions for the numerical solutions during the precipitation period were chosen to be fiber-saturated on the lower half and the linear time-dependent boundary moisture on the topmost point. A linear decrease to fiber saturation was assumed between the top point and the limb points.

Results

Two cases of the 36 numerical simulations were selected to illustrate the physical processes predicted by the theory. The chief features of the theoretical prediction are (1) the more rapid moisture exchange

rate associated with precipitation than predicted for an initially homogeneous case, (2) an independent theoretical solution with experimentally determined boundary conditions demonstrating the low value of the maximum effective precipitation rate, and (3) the asymmetry of moisture distribution within a wood cylinder during and following precipitation.

Wood will gain or lose moisture at a rate proportional to $e^{-\lambda^2 t}$ if the internal distribution of moisture is initially homogeneous. If the internal distribution is not homogeneous at the time a new environmental condition is imposed, however, the gain or loss of moisture will be governed by the local internal gradients. In principle, the resultant gain or loss may be either greater or less than that obtained from an initially homogeneous distribution. However, during precipitation the local gradient near the surface is sharply increased and moisture increase will be more rapid.

As the moisture content increased in the center layers, the rate of increase slowed down because the local gradients were weakened. The increase rate asymptotically approached zero as the internal gradients approached zero and the moisture content approached the boundary moisture content (Fig. 1).

Conversely, following precipitation the moisture maximum was concentrated near the surface. Thus a strong local gradient existed near the surface and the moisture loss was rapid. These processes are illustrated in Figure 1a and b. The drying rate following the precipitation was greater than that obtained from the initially homogeneous conditions at the beginning of each simulation. These theoretically predicted rates of moisture exchange agree with the observations in Simard (1968).

The effective precipitation rate may be computed from the two examples. Since the boundary conditions used in the numerical solutions were determined experimentally, the calculations provide a partial verification on the low value of the maximum effective precipitation rate.

The effective precipitation rate is com-

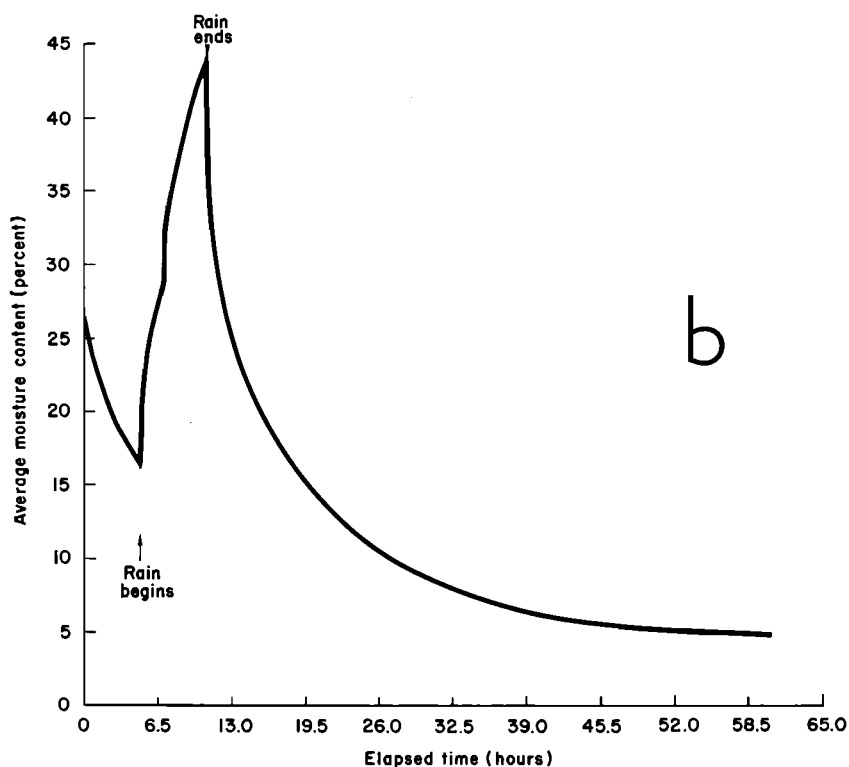
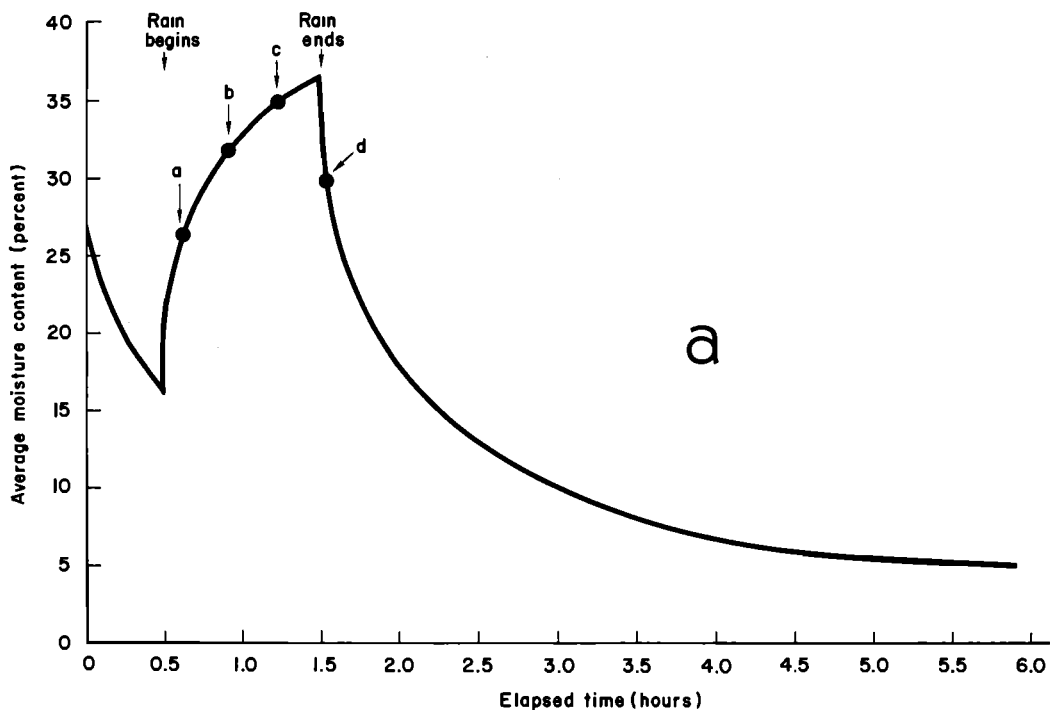


FIGURE 1. Moisture content of wood cylinders during a drying, precipitation, and post-precipitation sequence. (a) A 0.14-cm-radius dowel with precipitation duration of 1 hour. Points a through d refer to the internal distributions depicted in Figure 2. (b) A 0.41-cm-radius dowel with a precipitation duration of 6 hours.

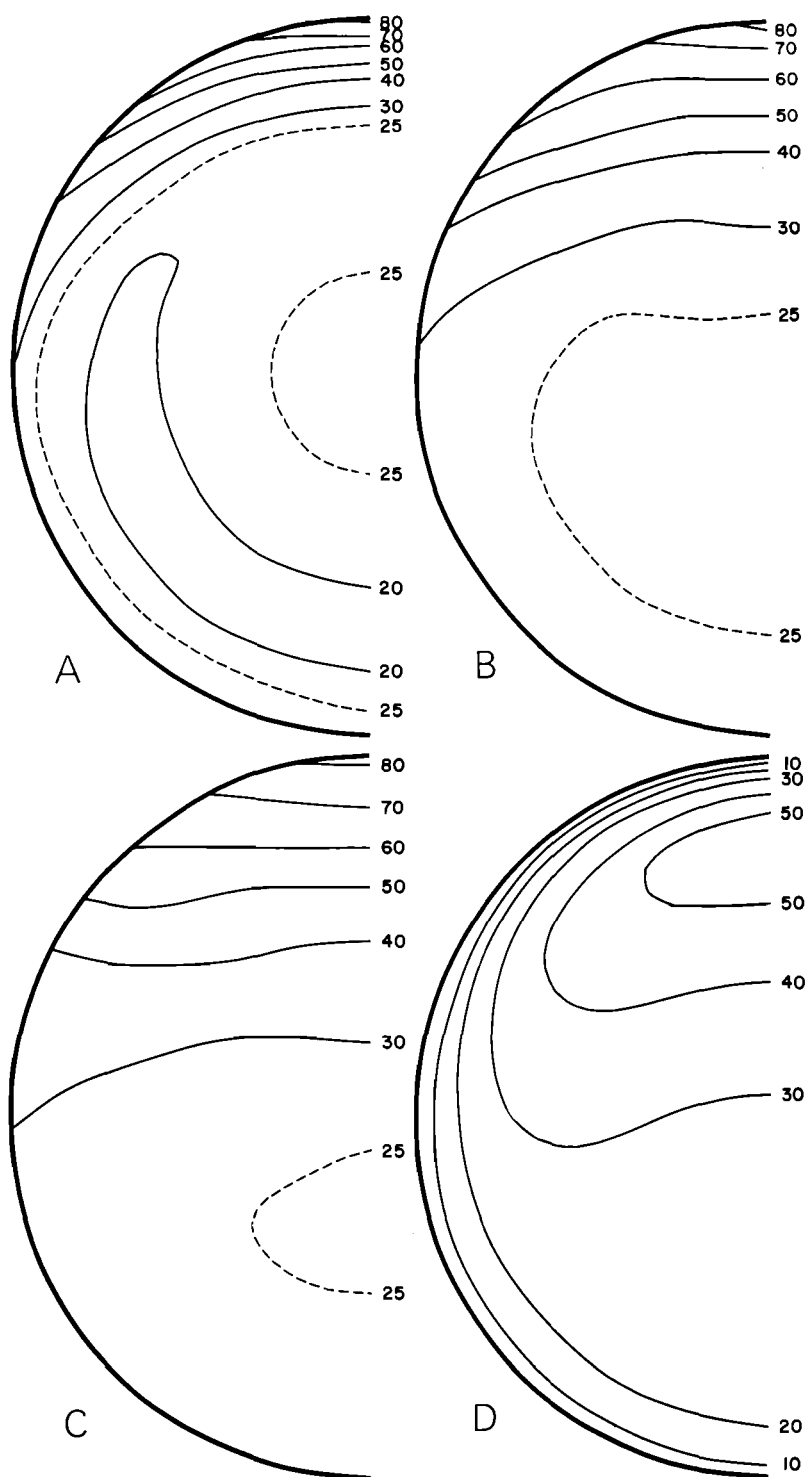


FIGURE 2. Distribution of moisture content within the cylinder for the 0.14-cm-radius dowel at times a through d of Figure 1a. The half planes are symmetrical from right to left.

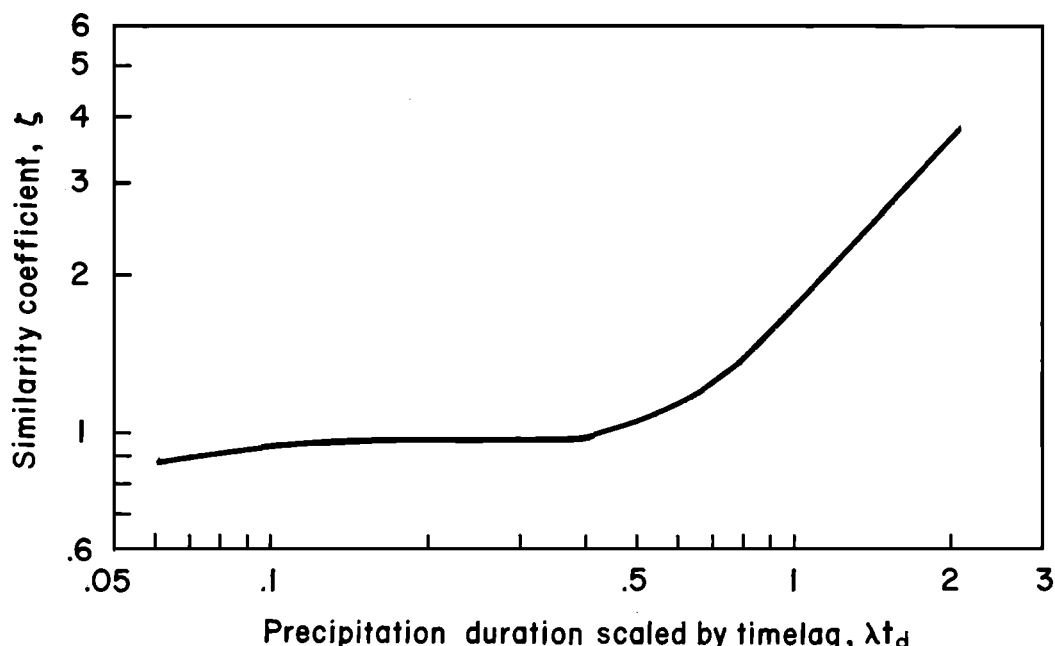


FIGURE 3. Relationship of the similarity coefficient ζ to the nondimensional time scale defined by the product of the real time and the inverse of the response time.

puted from the moisture increase rate and the surface area of the upper half of the cylinder. This effective precipitation rate is then

$$P = [(\Delta m / \Delta t) \rho R^2 \pi L] / R \pi L,$$

where $(\Delta m / \Delta t)$ is moisture increase rate, ρ is the dry density of the wood, R is the radius, and L is the length.

For the case illustrated in Figure 1a, the moisture increase rate was 20 gm water per 100 gm of wood in 1 hr. The radius of the dowel was 0.14 cm. This gives an effective precipitation rate of 2.88 mm per day. The second case (Fig. 1b) had 20 percent moisture content increase in 6 hr and a radius of 0.41 cm. This gives an effective precipitation rate of 1.92 mm per day. Both of these values are sufficiently close to the value of 1 mm per day predicted by the water balance equation.

The internal distribution of moisture in the cylinders is illustrated in Figure 2 by half planes cut along the vertical axis. The four parts correspond to specific times in-

dicated by the letters in Figure 1a. The moisture contents at points *b* and *d* in Figure 1a are nearly the same, but the internal distributions are markedly different. During the wetting process (Parts A, B, and C of Fig. 2), the free water input was on the top half of the cylinder only. Thus the quarter plane distribution for the top half was markedly different from the lower half. This asymmetric distribution is due to the fact that the top half had a liquid water input while the lower half had only a vapor input. In reality, the asymmetry between the top half and bottom half would be decreased because water does run down the sides and underneath. This departure from reality was invoked because of mathematical difficulties of incorporating rundown and drip in the solution. The D part of Figure 2 illustrates the distribution of moisture after the cylinder has begun to dry. There is some experimental verification for this distribution. Bramhall's (1970) experiments show a similar pattern, although the moisture contents in his analysis are much higher.

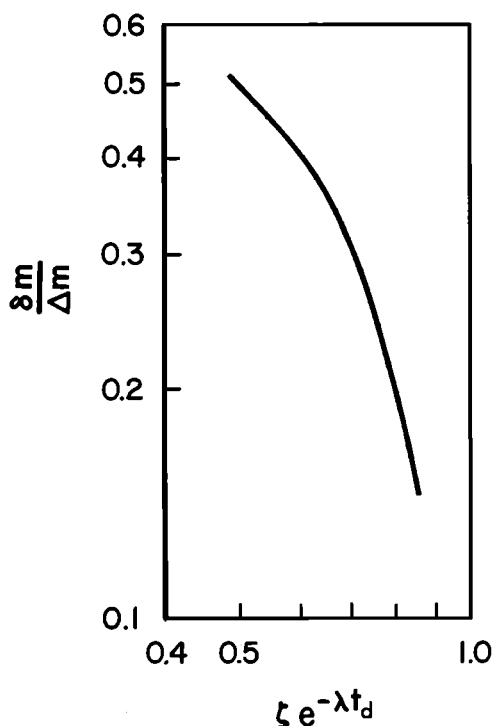


FIGURE 4. Relationship between change in moisture content, response time, and precipitation duration.

The theoretical solution also provided a relationship useful in practical problems. Eq. 17 may be used to estimate and predict the moisture content of wood. The 36 numerical simulations were used to define the coefficient ζ in the function. The initial, final, and boundary moisture contents were known, as were the response time and duration of precipitation. Thus, ζ was defined over a large interval of a nondimensional time scale as defined by the product λt (Fig. 3). Eq. 17 may be solved for any set of initial and boundary conditions over the interval in which ζ is defined.

The gain and loss of moisture predicted by eq. 17 differs from the traditional solutions of moisture gain and loss in wood in that the ratio of the actual moisture content change to the potential moisture change is not a simple exponential function (Fig. 4). A solution with an initially

homogeneous moisture content would be represented as a straight line in Fig. 4.

Conclusions

The theoretical solution of the influence of precipitation on fuel moisture predicted two results not normally considered in fuel moisture processes associated with liquid water uptake. A linear solution of the water balance at the wood surface predicted that the rate of water uptake by wood is limited by the rate at which wood can absorb water. It further predicted that the limiting precipitation rate is of the order of 1 mm per day. Precipitation rates in excess of this rate will in fact influence the moisture content, but only through secondary effects such as the fuels lying in a pool of water or the resultant higher humidities produced by microclimate interactions. The important implication of this prediction is that the duration of a liquid film on the fuel surface, not the total amount of precipitation, is the dominant factor in water uptake.

The theory also predicts realistic adsorption and desorption rates associated with precipitation. The shorter response time associated with both precipitation input and post-precipitation drying are due to the strong local gradients of moisture within the cylinders. These variations from traditional theoretical solutions compare favorably with experimental evidence.

The final feature of the predictions is the internal moisture distribution. While only limited experimental evidence is available to verify the predictions, none is contradictory, and all tends to support the theory.

Perhaps the most important feature of the mathematical development is the resultant theoretical equation itself. This result, eq. 17, is

$$\delta m / \Delta m = 1 - \zeta e^{-\lambda t},$$

where δm is the actual moisture content change, Δm is the potential moisture content change, λ^{-1} is the response time of the fuel, t is the time period involved, and ζ is a varying parameter as defined in Fig. 3. This relationship contains the results

of the theoretical solution in a form that (1) can be experimentally verified, (2) can be improved on by experimental evidence, and (3) is readily applied to practical problems with great efficiency.

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