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A Tree Taper Model Based on Similar Triangles and Use of Crown Ratio as a Measure of Form in Taper Equations for Longleaf Pine

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Abstract

Wc used data from 322 natural longleaf pine (*Pinus palustris* Mill.) trees to include crown ratio as a continuous variable in taper equations. The data were divided into 10 crown-ratio classes and fitted taper equations into each class to detect trends in the coefficients. For application to longleaf pine, we replaced coefficients that exhibited a trend with crown ratio with a function of crown ratio. The inclusion of crown ratio as a continuous variable improved by at least 16 percent the mean square residual for both models. The authors' model performed better on the modeling dataset based on fit statistics and on the validation dataset. It also contained fewer parameters and was easier to rearrange to solve for height to a given diameter.

Keywords: Crown ratio, longleaf pine, *Pinus palustris* Mill., tree taper equations, tree taper model.

Introduction

Taper has been defined as the change in stem diameter between two measurement points divided by the length of the stem between these two points (Morris and Forslund 1992). Taper equations attempt to describe it as a function of tree variables such as diameter at breast height (d.b.h.), total height, etc. Many forest scientists and foresters have demonstrated the importance of taper functions. According to Kozak (1988), taper equations can be used to provide: (1) predictions of inside bark diameters at any point on the stem; (2) estimates of total stem volume; (3) estimates of merchantable volume and merchantable height to any top diameter and from any stump height; and (4) estimates of individual log volumes.

Bruce and others (1968) believed that the inclusion of some measure of tree form, e.g., crown ratio (CR), as an additional independent variable might improve the fit of taper equations. Dell (1979), Feduccia and others (I 979), and Baldwin and Polmer (198 1) also discussed the potential of using CR to improve stem-taper curves. CR is defined as the percent of a tree's total height occupied by the live crown. CR can be calculated as the ratio of live-crown length to total tree height.

Several studies have explored the possibility of including CR in taper equations, although not all have had positive results. Valenti and Cao (1986) modified the three-segment taper equation introduced by Max and Burkhart (1976) to describe stem profile:

$$d^2 = D^2[\beta_1 z + \beta_2 z^2 + \beta_3(z-\alpha_1)^2 I_1 + \beta_4(z-\alpha_2)^2 I_2] + \varepsilon \qquad (1)$$

where d = diameter at height h, = d.b.h., z = 1 - (h/H), H = total height of the tree, α_i represents join points estimated from the data (a, = upper join point, α_2 = lower), the β_i's are the model parameters estimated from the data, I_i = 1 if $z > \alpha_i$ (i= 1,2), or 0 otherwise, and ε represents stochastic error. This equation will be referred to as model VC. The authors used data from a loblolly pine plantation and divided the data into 10 CR classes with approximately equal numbers of observations per class. Equation 1 was fitted to each CR class individually, and the resulting coefficients were plotted against the CR class medians to detect trends. Based on these plots, some of the coefficients were expressed as functions of CR. The resulting equation had the form:

$$d^2 = D^2[(\gamma_0 + \gamma_1 / CR) - z + \beta_2 z^2 + \beta_3(z-(\gamma_2+\gamma_3 \ln(CR))^2)I_1 + \beta_4(z-\alpha_2)^2 I_2] + \varepsilon \qquad (2)$$

where I, = 1 if $z > \gamma_2 + \gamma_3 \ln(CR)$, or 0 otherwise, and I_2 = 1 if z > a,, or 0 otherwise. The results of Valenti and Cao (1986) indicated that the addition of CR to the model was a significant improvement.

Our first objective was to determine if inclusion of CR as a continuous variable significantly could improve taper estimates for natural longleaf pine (***Pinuspalustris*** Mill.). As we examined various models, we noted problems with residuals, especially for the portion of the bole below breast height. To possibly remedy this problem, a new model was developed to better address that portion of the bole below breast height. This resultant model, along with Valenti and Cao's (1986) model, were then studied for inclusion of CR.

[1] Corresponding author

Data

Data for the modeling component of this study comprise two sets. The larger set, used in a taper study by Farrar (1987), consisted of 2,832 diameter measurements from 2 14 felled longleaf pine trees from naturally regenerated, even-aged stands in northwest Florida, southwest Georgia, central and south Alabama, and south Mississippi. Single-stemmed trees were selected over a range of diameters, total heights, and CR classes; all were free of excessive crook or sweep and visibly undamaged (Farrar 1987). Measurements included diameter outside bark (d.o.b.), bark thickness, and height to the diameter. Diameters were measured at 1-inch taper steps from a 0.2-foot stump.

The second datasct consisted of 1,376 diameter measurements on 108 felled longleaf pine trees from even-aged, naturally regenerated stands in Alabama and Florida that comprise a subset of plots in the USDA Forest Service Regional Longleaf Pine Growth Study (Kush and others 1987). Trees from all crown classes were selected following criteria of the Farrar study. Field measurements for each tree included crown class, d.b.h. to nearest 0.1 inch and total height and height to the base of the live crown to the nearest 0.1 foot. D.o.b. (using diameter tape), bark thickness, and height to the point where the diameter was measured were taken at the stump, between stump and breast height, at breast height, and above breast height at 5-foot (+/-) intervals to the tip of the tree. Length of the interval was adjusted as necessary to avoid limbs, knots, or other defects that abnormally affected the diameter.

Two independent datasets were obtained to conduct model validation. The first dataset contains measurements on 59 felled longleaf pine trees from naturally regenerated stands across southern Alabama. This dataset contains 684 diameter measurements.[1] The second model-validation dataset contains measurements of 33 felled longleaf pine trees from a site in southwest Alabama. It contains 49 1 diameter measurements.[2]

Model Development

We developed a new taper mode1 by starting with a linear mode1 containing three submodels represented by three triangles (fig. 1). The tree outline is represented by the shaded polygon ILMBNOK. Point B would occur at the tree's total height, ***H***. Point F would be at a height of ***H*** plus a constant, γ_1. Point J would be at a height of ***H*** minus a constant, γ_2. The upper join point, a,, would be represented by a line through points M and N. The lower join point, a,, would be represented by a line through points L and 0.

Upper stem diameters were related to d.b.h. by using the properties of similar triangles. Join points, ***a,*** and ***a,,*** are in terms of relative height (*h/H*). For a diameter, *d*, occurring above the upper join point, ***a,*** (using triangle ABC), the relationship to d.b.h. is given by:

$$d/(\beta_1 D) = (H-h)/(H-4.5) \tag{3}$$

where h is the height (foot) to diameter ***d*** (inch), and β_1 is a constant to account for the difference in the widths of triangles ABC and EFG at breast height.

For a diameter occurring between ***a,*** and ***a,*** (using triangle EFG), the relationship is given by:

$$d/D = (H+\gamma_1-h)/(H+\gamma_1-4.5) \tag{4}$$

For a diameter occurring below the lower join point, ***a, ,*** (using triangle IJK) the relationship is given by:

$$d/(\beta_2 D) = (H-\gamma_2-h)/(H-\gamma_2-4.5) \tag{5}$$

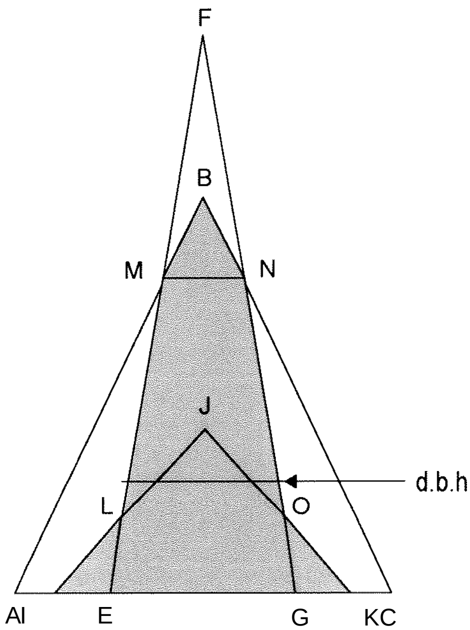

Figure 1—Linear model of a tree

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[1]Glover, Glenn, 1979. Unpublished dataset. On tile with Auburn University, School of Forestry & Wildlife Sciences, Auburn University, AL 36849.

[2]Meldahl, Ralph S., 1982. Unpublished dataset. On file with Auburn University, School of Forestry & Wildlife Sciences, Auburn University, AL 36849.

where β_2 is a constant to account for the differences in widths of triangles IJK and EFG at breast height.

Intercepts, to allow models to be continuous at the join points, and stochastic errors were added, coefficients renumbered, and ***H*** factored from the numerators to yield the model:

$$d/D = \begin{cases} \beta_0 + \beta_1(H(1-x)/(H-4.5)) + \varepsilon & 1 \geq x \geq \alpha_1 \\ \beta_2 + \beta_3(H(1+\gamma_1/H-x)/(H+\gamma_1-4.5)) + \varepsilon & \alpha_1 \geq x \geq \alpha_2 \\ \beta_4 + \beta_5(H(1-\gamma_2/H-x)/(H-\gamma_2-4.5)) + \varepsilon & \alpha_2 \geq x \geq 0 \end{cases} \quad (6)$$

wherex is the relative height (*h/H*) of the diameter, *d*.

As we expected, the linear mode1 was not satisfactory for describing tree taper, so exponents were added to the second term in each of the three segments. The exponents increased flexibility in the model as opposed to forcing a linear relationship on the model. This resulted in:

$$d/D = \begin{cases} \beta_0 + \beta_1(H(1-x)/(H-4.5))^{\eta_1} + \varepsilon & 1 \geq x \geq \alpha_1 \\ \beta_2 + \beta_3(H(1+\gamma_1/H-x)/(H+\gamma_1-4.5))^{\eta_2} + \varepsilon & \alpha_1 \geq x \geq \alpha_2 \\ \beta_4 + \beta_5(H(1-\gamma_2/H-x)/(H-\gamma_2-4.5))^{\eta_3} + \varepsilon & \alpha_2 \geq x \geq 0 \end{cases} \quad (7)$$

Imposing the condition that the diameter at the tip of the tree equals zero ($d = 0$ when $x = 1$) results in β_0 equaling zero. Further, imposing the conditions that the mode1 be continuous at the join points, and that the models have continuous first partial derivatives with respect to x at the join points, allowed $\beta_1, \beta_2, \beta_4$, and β_5 to be solved for in terms of β_3.

The resulting mode1 had the form:

$$d/D = \beta_3 \left(\frac{\eta_2 \, (H(1+\frac{\gamma_1}{H}-\alpha_1))^{\eta_2 - 1} \, (H\,(1-x))^{\eta_1}}{\eta_1 (H+\gamma_1-4.5)^{\eta_2} \, (H\,(1-\alpha_1))^{\eta_1 - 1}} \right) + \varepsilon \quad (8)$$

for $1 > x > \alpha_1$,

$$d/D = \frac{\eta_2 \, \beta_3 \, (H\,(1-\alpha_1))\,(H(1+\frac{\gamma_1}{H}-\alpha_1))^{(\eta_2-1)}}{\eta_1 (H+\gamma_1-4.5)^{\eta_2}} - \beta_3 \left(\frac{(H\,(1+\frac{\gamma_1}{H}-\alpha_1))}{(H+\gamma_1-4.5)} \right)^{\eta_2} + \beta_3 \left(\frac{(H\,(1+\frac{\gamma_1}{H}-x))}{(H+\gamma_1-4.5)} \right)^{\eta_2} + \varepsilon \quad (9)$$

for $\alpha_1 > x > \alpha_2$, and

$$d/D = \beta_3 \left(\frac{\eta_2 (H(1-\alpha_1))(H(1+\frac{\gamma_1}{H}-\alpha_1))^{(\eta_2-1)}}{\eta_1 (H+\gamma_1-4.5)^{\eta_2}} \right)$$

$$-\beta_3 \left(\frac{(H(1+\frac{\gamma_1}{H}-\alpha_1))}{(H+\gamma_1-4.5)} \right)^{\eta_2} + \beta_3 \left(\frac{(H(1+\frac{\gamma_1}{H}-\alpha_2))}{(H+\gamma_1-4.5)} \right)^{\eta_2}$$

$$-\beta_3 \left(\frac{\eta_2 (H(1-\frac{\gamma_2}{H}-\alpha_2)(H(1+\frac{\gamma_1}{H}-\alpha_2))^{(\eta_2-1)}}{\eta_3 (H+\gamma_1-4.5)^{\eta_2}} \right)$$

$$+\beta_3 \left(\frac{\eta_2 (H(1-\frac{\gamma_2}{H}-x))^{\eta_3} (H(1+\frac{\gamma_1}{H}-\alpha_2))^{(\eta_2-1)}}{\eta_3 (H+\gamma_1-4.5)^{\eta_2} (H(1-\frac{\gamma_2}{H}-\alpha_2))^{(\eta_3-1)}} \right) + \varepsilon \qquad (10)$$

for $\alpha_2 > x > 0$.

In fitting this model to the data, a value for γ_2 was obtained that was not significantly different from zero (95-percent confidence interval). γ_2 was replaced with zero, and the resulting model will be referred to as model A.

Methodology

The results from fitting mode1 VC (equation 1) and model A to the modeling data are given in table 1. The criteria used to select the best mode1 form were: (1) bias (l/n) $\Sigma(\gamma_i - \hat{\gamma}_i)$, (2) relative bias (l/n) $\Sigma(\gamma_i - \hat{\gamma}_i)/\gamma$, (3) absolute bias (1/n) $\Sigma|\gamma_i - \hat{\gamma}_i|$, (4) absolute relative bias (1/n) $\Sigma|\gamma_i - \hat{\gamma}_i|/\gamma_i$, and (5) fit index $1-\Sigma(\gamma_i - \hat{\gamma}_i)^2/\Sigma(\gamma_i - \bar{\gamma})^2$; index $1-\Sigma(\gamma_i - \hat{\gamma}_i)^2/\Sigma(\gamma_i - \hat{\gamma}_i)^2$; where γ_i and $\hat{\gamma}_i$ are the observed and predicted diameters for the i[th] observation and $\bar{\gamma}$ is the average of γ_i values, respectively, and n is the number of observations.

Table l-Fit statistics for models A, A/CR, VC, and VC/CR for inside bark and outside bark diameters

Model"	Parameters	Bias	Absolute bias	Relative bias	Absolute relative bias	Fit index
	Number					
			Inside bark			
A	7	0.01651	0.41348	0.00922	0.09343	0.98369
A/CR	7	-0.01272	0.36556	0.00324	0.08105	0.98680
VC	6	0.00797	0.41912	0.00304	0.09423	0.98336
VUCR	9	-0.00274	0.37229	0.00292	0.08053	0.98625
			Outside bark			
A	7	-0.00240	0.45670	0.00213	0.08653	0.98464
A/CR	7	0.00567	0.41938	0.01150	0.08339	0.98698
VC	6	0.02166	0.46723	0.00917	0.08604	0.98366
VC/CR	9	0.02012	0.42620	0.00940	0.07769	0.98615

[a] Model A = the authors' model (see equations 8, 9, and 10); model A/CR = the authors' model incorporating crown ratio (see equations 12, 13, and 14); model VC = Valenti and Cao's model (see equation 1); model VC/CR = Valenti and Cao's model incorporating crown ration (see equation 11).

The models then were examined for inclusion of CR as a continuous variable following the approach used by Valenti and Cao (1986). The modeling data were divided into 10 CR classes having approximately the same number of trees. The models were fitted separately to each CR class. The resulting coefficients then were plotted against CR class to see if any trends were observable. The plots for model VC indicated that the most obvious relationships with CR existed for β_2, β_3, and a,. The functions of CR found to give the best fit with the data were $\beta_2 = \gamma_1 + \gamma_2 CR$, $\beta_3 = \gamma_3 + \gamma_4 CR^2$, and $\alpha_1 = \rho_1 + \rho_2 CR$. The resulting model (VC/CR) had the form:

$$
\begin{aligned}
d^2 = D^2 \big(\beta_1 z + (\gamma_1 + \gamma_2 CR) z^2 \\
+ (\gamma_3 + \gamma_4 CR^2)\big(z - (\rho_1 + \rho_2 CR)\big)^2 I_1 \\
+ \beta_4 (z - a_2)^2 I_2
\end{aligned}
\tag{11}
$$

The plots for model A indicated that the most significant relationships with CR existed for parameters η_2 and a, . The functions of CR that were found to give the best fit with the data were $\eta_2 = \lambda_1 + \lambda_2 CR$ and $a, = \rho_1 + \rho_2 CR^2$. The resulting model (A/CR) had the form:

$$
d/D = \beta_3 \left(\frac{A B^{A-1} (H(1-x))}{F^A} \right) \tag{12}
$$

for $1 > x > (\rho_1 + \rho_2 CR^2)$, where $A = \lambda_1 + \lambda_2 CR$, $B = H (1 - \rho_1 - \rho_2 CR^2)$, and $F = H$-4.5,

$$
d/D = \beta_3 \left(\frac{(A-1) B^A + (H(1-x))^A}{F^A} \right) \tag{13}
$$

for $(\rho_1 + \rho_2 CR^2) > x > \alpha_2$, and

$$
\begin{aligned}
d/D = \beta_3 \left(\frac{(A-1)B^A + (H(1-\alpha_2)^A)}{F^A} \right) - \beta_3 \left(\frac{A\ E^A}{\eta_3\ F^A} \right) \\
+ \beta_3 \left(\frac{A(H(1-x))^{\eta_3} E^{(A-1)}}{\eta_3\ F^A E^{(\eta_3 - 1)}} \right)
\end{aligned}
\tag{14}
$$

for $\alpha_2 > x > 0$, where $E = H(1 - \alpha_2)$.

Validation of models A/CR and VC/CR was conducted using the independent dataset described above. The criteria used for comparing the models included model bias, or average residual $\left[\Sigma\ (\text{predicted} - \text{observed})\ /\ n\right]$, accuracy, or average absolute residual $\left(\Sigma\ [\text{predicted} - \text{observed}|\ /\ n\right)$, and root-mean-squared prediction error, or mean square residual (MSR) (square root of $\Sigma\ [\ (\text{predicted} - \text{observed})^2 / n\ ]$, where n is the number of observations. Bias, which measures expected error when several observations are to be combined by averaging or totaling, and accuracy, which measures the average error associated with the prediction of any one observation, provides nearly all the information necessary in validation (Burk 1986). Root-mean-squared prediction error, which is equal to the standard deviation of prediction error for unbiased models, is also recommended as validation criteria (Burk 1986). The results of models A, VC, A/CR, and VC/CR on the validation dataset are found in table 2. Parameter estimates for model A/CR for d.i.b. and d.o.b. are found in table 3.

Conclusions

The addition of CR as a continuous variable in both models A and VC was found to improve estimates of upper stem diameters. Model A/CR had a decrease in MSR of 20.5 percent when CR was included. Model A/CR also had a higher R_a^2 value, a lower average residual, and a lower average absolute residual, compared to model A [R_a^2 is the adjusted R^2 (Draper and Smith 1981)]. The idea is that the R_a^2 can be used to compare equations fitted not only to a specific set of data but also to two or more entirely different sets of data. Inclusion of CR in model VC resulted in an 18.25-percent decrease in MSR, an increase in R_a^2, a decrease in average residual, and a decrease in average absolute residual.

Although no significant differences in residual patterns were noted, model A was found to outperform model VC in terms of all four criteria on the modeling data for both d.i.b. and d.o.b. (table 1). Both models showed significant improvement with the addition of CR. Model A/CR was found to outperform model VC/CR in all four criteria for both d.i.b. and d.o.b. (table 1). Tests on the validation dataset resulted in both models again showing significant improvement with the addition of CR. Model A/CR was found to perform better than model VC/CR for both d.i.b. and d.o.b.

Further, while model A/CR has a complex final form, it contains fewer parameters than model VC/CR (7 vs. 9) and is derived from a simpler concept. In addition, this equation is easier to rearrange to solve for height to a given diameter because only linear functions of height (h) are used (see appendix). Rearranging model VC/CR to solve for h results in a quadratic equation.

Table 2—Validation statistics for models A, A/CR, VC, and VC/CR for inside bark and outside bark diameters

Model[a]	Parameters	Bias	Absolute bias	Relative bias	Absolute relative bias	Fit index
	Number					
			Inside bark			
A	7	-0.50935	0.63858	-0.08128	0.11084	0.95703
A/CR	7	-0.36443	0.50575	-0.05642	0.09217	0.97302
VC	6	-0.51034	0.64742	-0.07836	0.11061	0.95554
VUCR	9	-0.39525	0.53092	-0.06133	0.09352	0.97027
			Outside bark			
A	7	-0.24684	0.47683	-0.04101	0.08319	0.97152
A/CR	7	-0.10336	0.36799	-0.02788	0.07110	0.98233
VC	6	-0.19640	0.46690	-0.04144	0.08442	0.97238
VC/CR	9	-0.14503	0.39994	-0.03156	0.07382	0.97975

[a] Model A = the authors' model (see equations 8, 9, and 10); model A/CR = the authors' model incorporating crown ration (see equations 12, 13, and 14); model VC = Valenti and Cao's model (see equation 1); model VC/CR = Valenti and Cao's model incorporating crown ration (see equation 11).

Table 3-Parameter estimates, asymptotic standard errors, and confidence intervals for model A/CR for inside bark and outside bark diameters

Parameter	Estimate	Asymptotic standard error	95-percent confidence interval Lower	Upper
		Inside bark		
α_2	0.07194	0.00208	0.06787	0.07602
β_3	2.17988	0.27950	1.63190	2.72786
η_3	48.92647	1.83986	45.31928	52.53366
ρ_1	0.89646	0.00600	0.88470	0.90822
ρ_2	-0.38504	0.01157	-0.40773	-0.36235
λ_1	0.13536	0.01372	0.10845	0.16227
λ_2	0.12553	0.01328	0.09949	0.15157
		Outside bark		
α_2	0.07544	0.00181	0.07190	0.07898
β_3	1.42627	0.04070	1.34648	1.50607
η_3	46.90098	1.48181	43.99578	49.80618
ρ_1	0.93740	0.00443	0.92872	0.94608
ρ_2	-0.28840	0.01270	-0.31330	-0.26351
λ_1	0.25681	0.01169	0.23389	0.27973
λ_2	0.23635	0.01255	0.21175	0.26095

Model A/CR = the authors' model incorporating CR (see equations 12, 13, 14); β_3, η_3, α_2, λ_1, λ_2, ρ_1, and ρ_2 = regression coefficients.

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Appendix

Equation A/CR rearranged to solve for height, *h*, to a given diameter, *d*

$$h = H\left(\frac{(F^A)d}{(\beta_3)DA(B^{A-1})}\right)$$

for $0 < d < (\beta_3)(\beta^A)D(A/F^A)$,

$$h = H\left(\frac{(\beta_3)(F^A)d}{D} - (A-1)(B^A)\right)^{\frac{1}{A}}$$

for $(\beta_3)(B^A)D(A/F^A) < d < (\beta_3)D\left(\left(E^A + (A-1)(B^A)\right)/F^A\right)$, and

$$h = H\left(\left(\frac{(\eta_3)d(F^A)E^{(\eta_3 - A)}}{(\beta_3)DA} - \frac{(\eta_3)E^{(\eta_3 - A)}}{A}\right)[(A-1)B^A + H(1-\alpha_2)^A] + E^{\eta_3}\right)^{\frac{1}{\eta_3}}$$

for $(\beta_3)D\left(E^A + (A-1)B^A\right)/F^A < d < (\beta_3)DA\left(H/\left(F^A E^{(\eta^3 - A)}\right)\right) + (\beta_3)D[(A-1)B^A + E^A]/F^A - (\beta_3)DA\left(E^A/(\eta_3 F^A)\right)$.

where

$A = \lambda_1 + \lambda_2 CR$,

$B = H(1 - \rho_1 - \rho_2 CR)$,

$E = H(1 - \alpha_2)$,

$F = H\text{-}4.5$,

$D =$ d.b.h.,

$H =$ total height of tree,

$CR =$ crown ratio,

$d =$ diameter at a given height, *h*, and

β_3, η_3, α_2, λ_1, λ_2, ρ_1, and ρ_2 are regression coefficients.

Shaw, Dennis J.; Meldahl, Ralph S.; Kush, John S.; Somers, Greg L. 2003. A Tree taper model based on similar triangles and use of crown ratio as a measure of form in taper equations for longleaf pine. Gen. Tech. Rep. SRS-66. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station. 8 p.

We used data from 322 natural longleaf pine (*Pinus palustris* Mill.) trees to include crown ratio as a continuous variable in taper equations. The data were divided into 10 crown-ratio classes and fitted taper equations into each class to detect trends in the coefficients. For application to longleaf pine, we replaced coefficients that exhibited a trend with crown ratio with a function of crown ratio. The inclusion of crown ratio as a continuous variable improved by at least 16 percent the mean square residual for both models. The authors' model performed better on the modeling dataset based on fit statistics and on the validation dataset. It also contained fewer parameters and was easier to rearrange to solve for height to a given diameter.

Keywords: Crown ratio, longleaf pine, *Pinus palustris* Mill., tree taper equations, tree taper model.