POSSIBLE EFFECTS OF FREE CONVECTION ON FIRE BEHAVIOR--LAMINAR AND TURBULENT LINE AND POINT SOURCES OF HEAT

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by

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SUMMARY

The transfer theory is applied to the problem of atmospheric diffusion of momentum and heat induced by line and point sources of heat on the surface of the earth. In order that the validity of the approximations of the boundary layer theory be realized, the thickness of the layer in which the temperatures and velocities differ appreciably from the values at infinity must be small compared to the height of the region of influence. Variations of four dependent variables--inflow rate, inflow velocities, maximum velocities, and maximum temperature--as a function of elevation for representative heat source strengths are compared for the four cases investigated. In addition, the shape of the region of influence and the factors affecting the shape are investigated.

Results indicate that laminar sources tend to become uniform in cross-section with large vertical velocities and temperatures, thereby indicating a low lateral tendency to diffuse bouyancy and momentum. The turbulent sources diverge with a rapid diffusion of bouyancy and momentum depending upon the mixing length proportionality constant.

Application of the turbulent point source results to a finite area source has been presented by postulating that the dependent variables of indraft velocity, maximum temperature and maximum vertical velocity of the finite area source can be produced by a point source of equivalent heat strength functioning at a specified distance below the finite source. Data concerning the behavior of the aforementioned variables induced by a finite area heat source do not exist. Therefore, experimental investigations are necessary to provide guidance for further analysis.

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INTRODUCTION

A zone of convection is clearly evident when a fire burns in the free atmosphere. This convection zone is of major interest in understanding the behavior of forest fires. The transport of mass, heat and momentum occurs within the convection column. Knowledge of these transport processes will provide some explanation of fire behavior. Quantitative measures of fire behavior must stem from basic explanations of behavior which are consistent with the theories of heat and mass transfer, fluid flow, and the chemistry of combustion of solid fuels. The combustion and convection zones are inter-related. The oxygen required for combustion must be supplied by the convective velocities. In turn the convection zone is induced by the heat liberated by the combustion process. Despite this interaction, the convection zone may be studied by utilizing heat transfer and fluid flow theories. If a quasi-steady state process is postulated, the mechanism governing convective flow is amenable to analysis. The present investigation explains the free convective mechanism.

Density differences which are associated with temperature differences due to thermal expansion cause motion of a fluid which is called natural flow. The natural flow is termed free convection if it is not subjected to constraining boundaries. Such a phenomena exists when a fire occurs in the atmosphere under the influence of negligible wind velocities. In free convection the velocity and temperature distributions are interdependent, necessitating the simultaneous solution of the momentum and energy equations. The cases investigated here are for the condition of zero velocity and uniform potential temperature in the ambient region. The effect of variable potential temperature on the basic system of equations is presented for the laminar line source by an approximate method.

The earliest study concerning the effect of a heated source was undertaken by Schmidt (4,5)[1] in 1940. He presented theoretical investigations for the turbulent line and point sources. In addition, he obtained experimental velocity and temperature profiles for a finite electrically heated source 55 mm in diameter by use of a hot wire and thermocouple traversing probe. Correlation of the theoretical and experimental results was obtained.

Rouse et al (3) conducted experiments to determine velocity and temperature profiles for line and point sources. The two dimensional

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1/ Underlined numbers in parentheses refer to Literature Cited, page 45.

investigation was confined between parallel walls 4 feet high, 8 feet long and 4 feet apart with the source across the midsection of a low platform extending the length of the walls. The axisymmetrical case was conducted in a room 25 feet in diameter with an over-all height of 11 feet, wherein the source was located at the center of a platform 8 feet in diameter. Measurements for temperature and velocity were obtained by means of a copper-constantan thermocouple and a vane anemometer. Recessed gas burners provided rather low heat source output.

Yih (7,8) presented theoretical investigations for laminar free convection due to line and point sources of heat for small variations of pressure and temperature.

The present investigation presents a detailed review of the theoretical analysis of laminar and turbulent heat sources as effecting a free convective flow. The final objective is to establish a working model for the convection-column behavior induced by a fire on the surface of the earth. The basic analysis presented in Appendix A has been formulated with micrometeorology in mind and with heat source strengths representative of moderate forest fires. Appendix B presents the investigation of the horizontal pressure gradient since this gradient is taken as zero according to the boundary layer theory.

The mathematical investigations are essentially of a simple and idealized nature which by no means solves the complex problems of the behavior of fire-convection columns.

RESULTS

Since the solutions obtained here do not account for variable properties, the magnitudes involved in the results were taken at an average ambient condition for air at atmospheric pressure. Thus by choosing the properties for the equations and selecting reasonable heat rates, a comparative study of the variables was undertaken. The property values taken were

$$\nu_\infty = 1.80 \times 10^{-4} \text{ sq.ft./sec.}$$

$$\rho_\infty = 2.34 \times 10^{-3} \text{ lb.sec.}^2/\text{ft.}^4$$

$$c_p = 0.24 \text{ BTU/lb. }^{\circ}\text{R}$$

$$\theta_\infty = 530 \ {}^{\circ}\text{R}$$

The dependent variables obtained from the theory are presented in table 1. The mixing length proportionality constant was taken as 0.0417, which was experimentally determined by Schmidt (5). This value was taken in order to effect the comparison.

Table 1 - Dependent variables for laminar and turbulent line and point sources

Line Source	Point Source
Laminar $\sigma = 0.66$	Laminar $\sigma = 1.0$
$\psi_{\delta/L} = 1.12\,\nu_\infty \phi^{1/5} z^{3/5}$	$\psi_\delta = 6.00\,\nu_\infty z$
$v_\delta = -0.67\,\nu_\infty \phi^{1/5} z^{-2/5}$	$v_\delta = -0.50\,\nu_\infty \phi^{1/4} z^{-1/2}$
$u_m = 6.25\,\nu_\infty \phi^{2/5} z^{1/5}$	$u_m = 0.80\,\nu_\infty \phi^{1/2}$
$\theta_m - \theta_\infty = 0.67 \dfrac{\nu_\infty^2 \phi_\infty}{g} \phi^{4/5} z^{-3/5}$	$\theta_m - \theta_\infty = 0.11 \dfrac{\nu_\infty^2 \theta_\infty}{g} \phi z^{-1}$
$\delta = 3.00\,\phi^{-1/5} z^{2/5}$	$\delta = 12.00\,\phi^{-1/4} z^{1/2}$
Turbulent	Turbulent
$\psi_{\delta/L} = 0.97\,\nu_\infty \phi^{1/3} z$	$\psi_\delta = 0.51\,\nu_\infty \phi^{1/3} z^{5/3}$
$v_\delta = -0.97\,\nu_\infty \phi^{1/3}$	$v_\delta = -2.40\,\nu_\infty \phi^{1/3} z^{-1/3}$
$u_m = 7.40\,\nu_\infty \phi^{1/3}$	$u_m = 38.50\,\nu_\infty \phi^{1/3} z^{-1/3}$
$\theta_m - \theta_\infty = 44.44 \dfrac{\nu_\infty^2 \theta_\infty}{g} \phi^{2/3} z^{-1}$	$\theta_m - \theta_\infty = 35.70 \dfrac{\nu_\infty^2 \theta_\infty}{g} \phi^{2/3} z^{-5/3}$
$\delta = 0.30\,z$	$\delta = 0.36\,z$

Comparison of the variables is presented in figures 1 to 5 for two values of heat-source strength. These rates of 1.74 and 1740 BTU/sec.-ft. for the line sources, and 1.74 and 1740 BTU/sec. for the point sources were taken as representative values of energy release as experienced by actual fires. Only one heat output has been indicated in some of the curves since the omitted heat output yields either too large or small a magnitude of the quantity concerned.

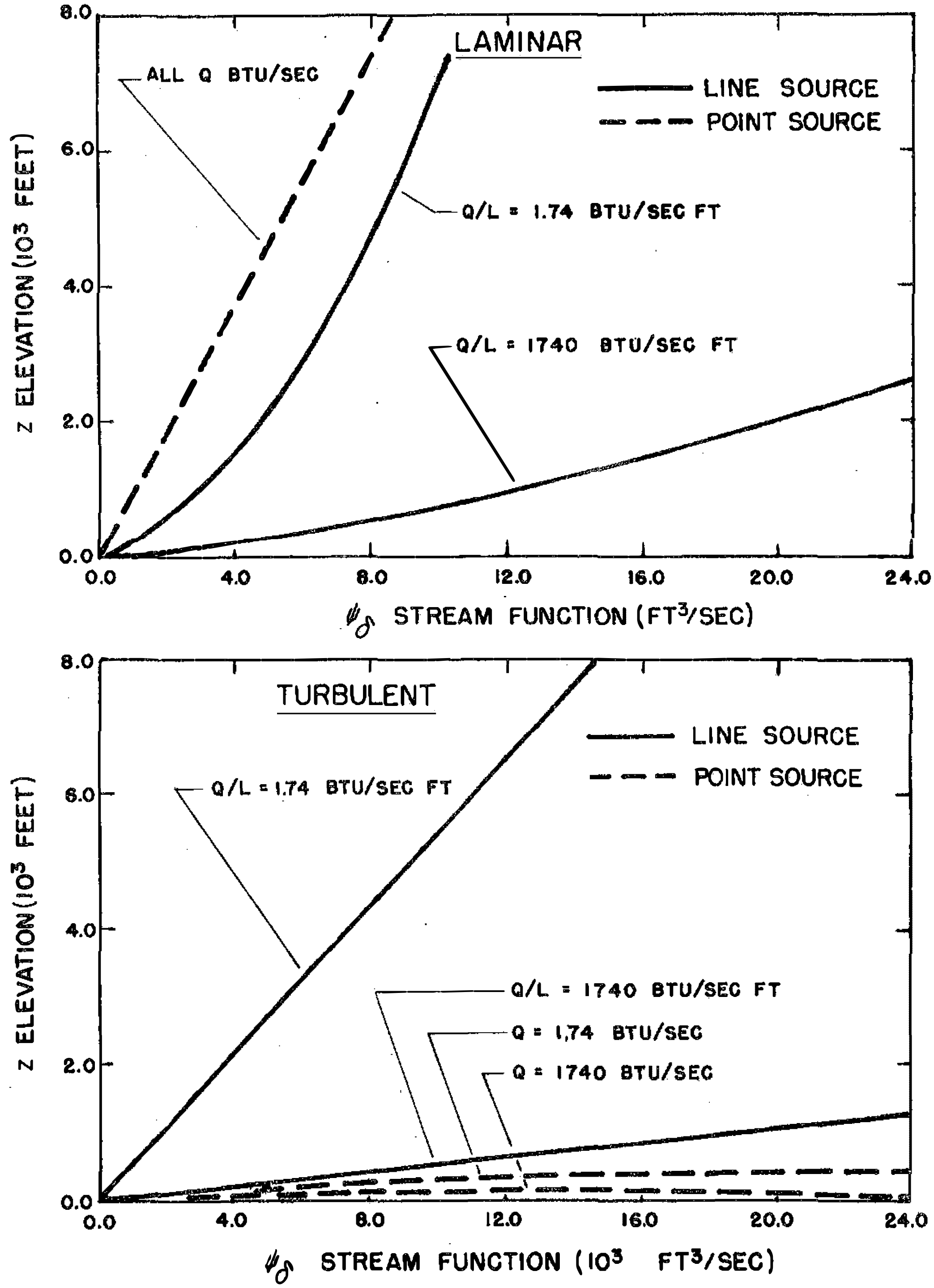


Figure 1.--Theoretical vertical variation of stream function for laminar and turbulent line and point sources.

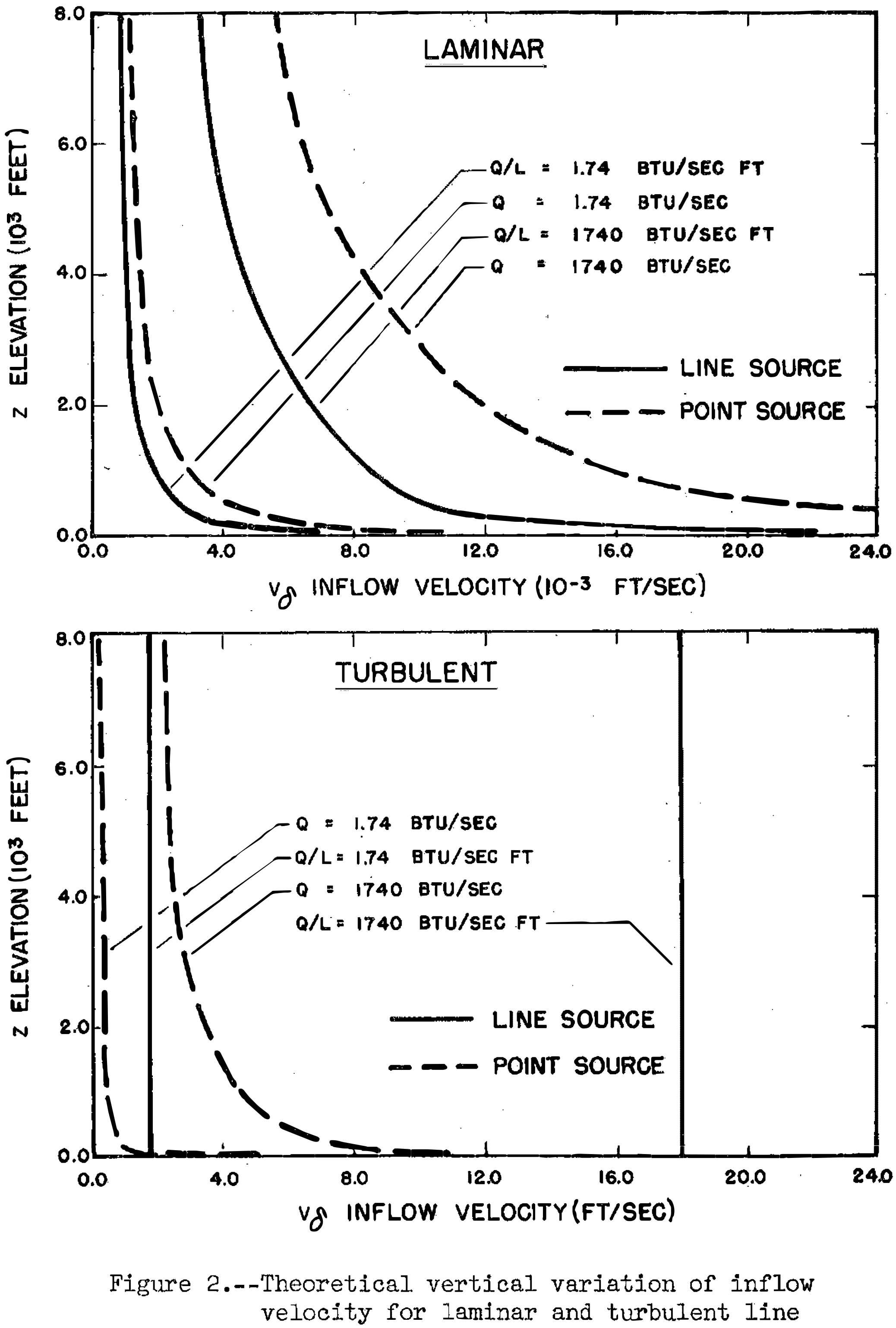


Figure 2.--Theoretical vertical variation of inflow velocity for laminar and turbulent line and point sources.

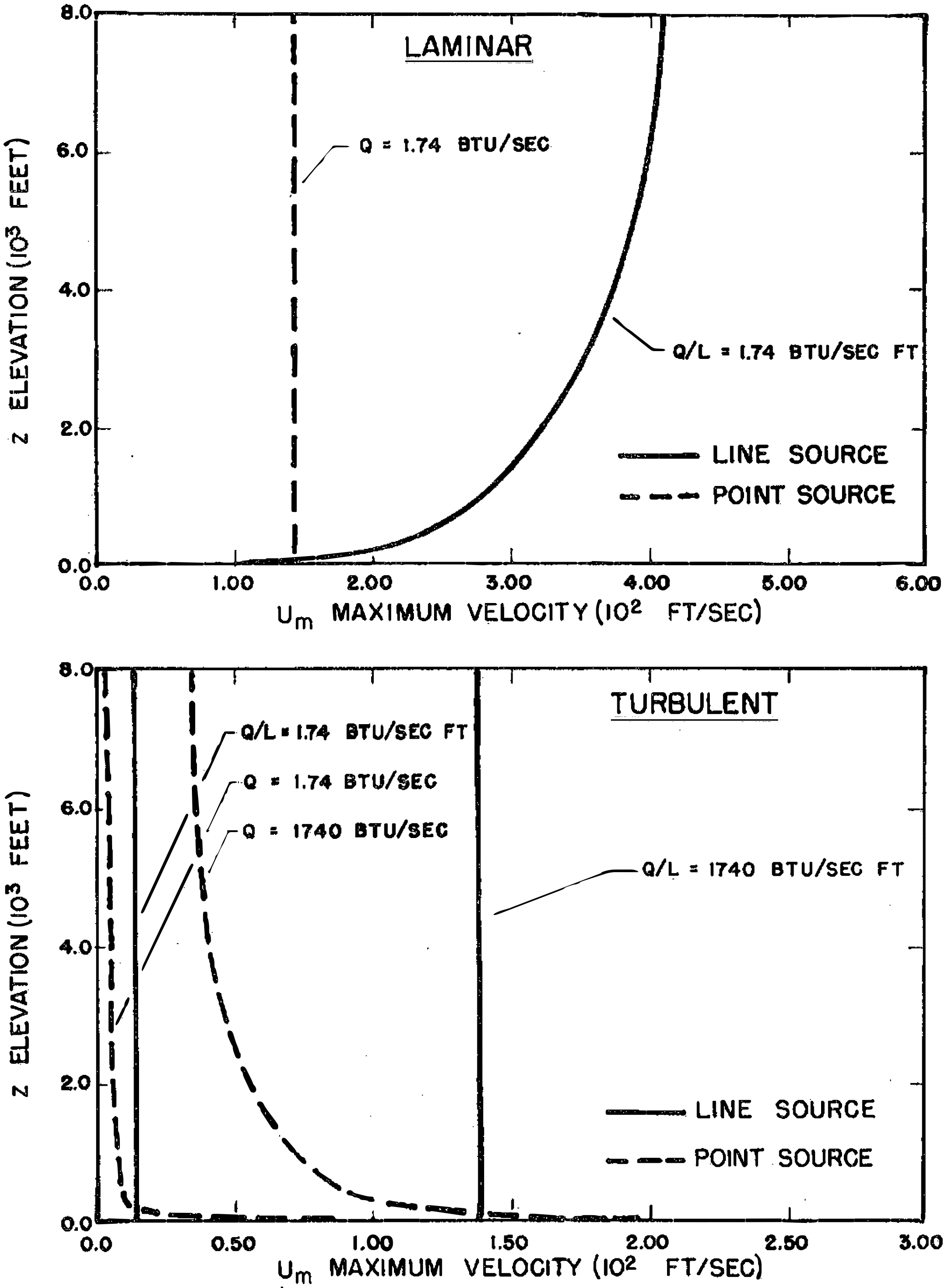


Figure 3.--Theoretical maximum velocity variation for laminar and turbulent line and point sources.

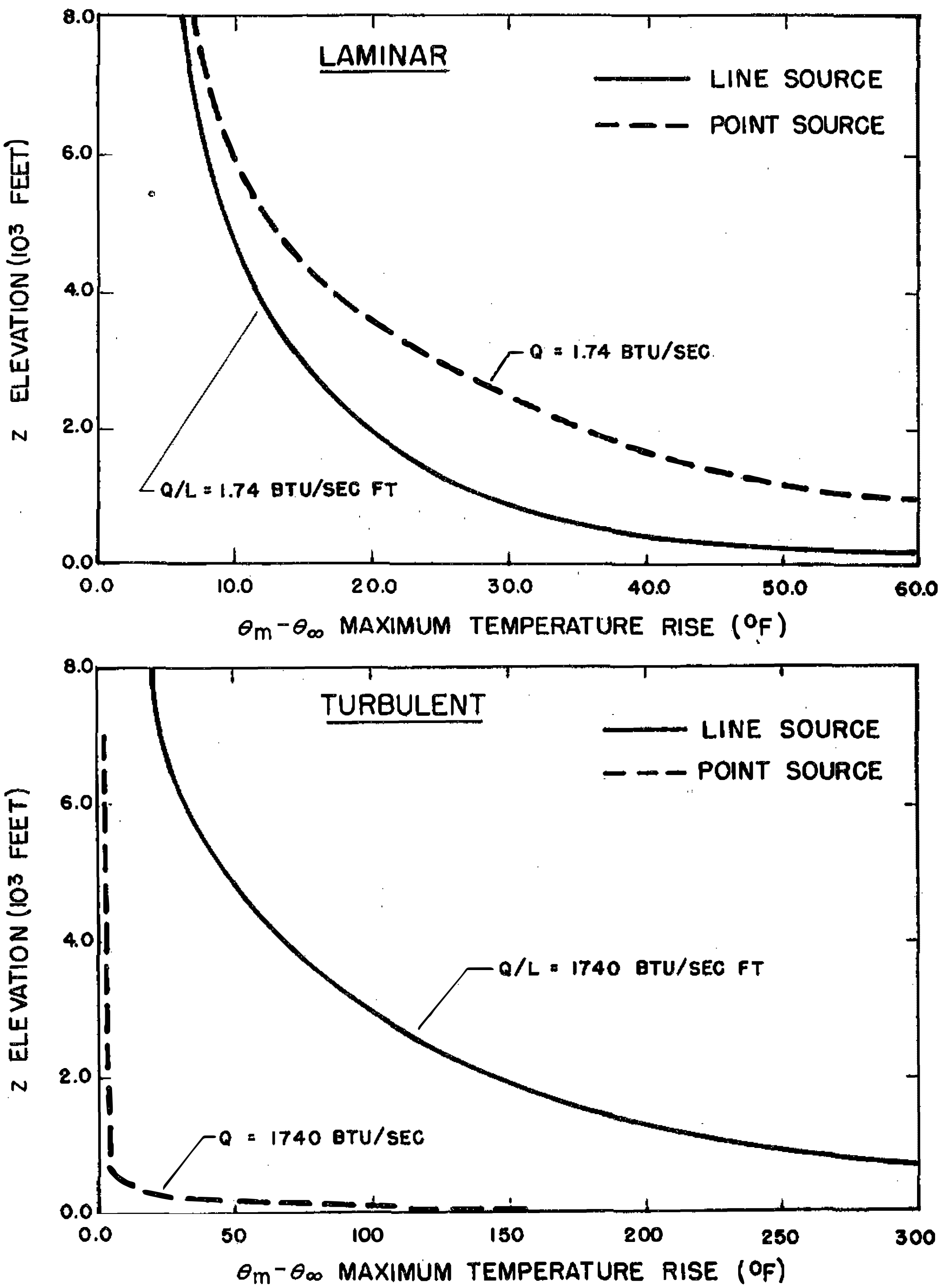
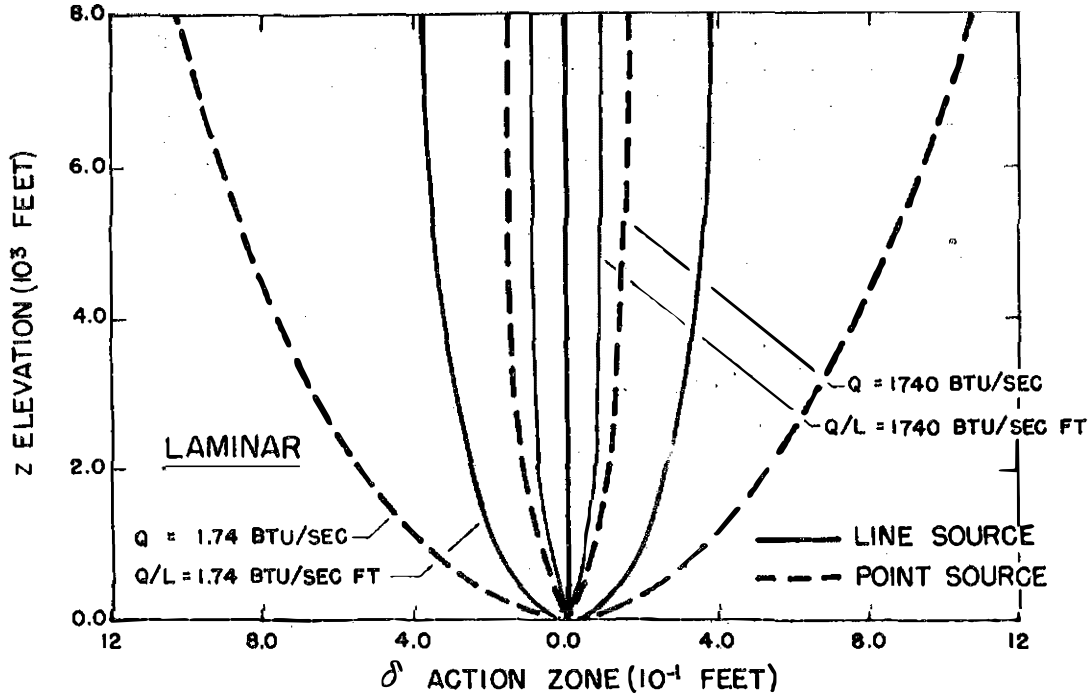


Figure 4.--Theoretical maximum temperature variation for laminar and turbulent line and point sources.



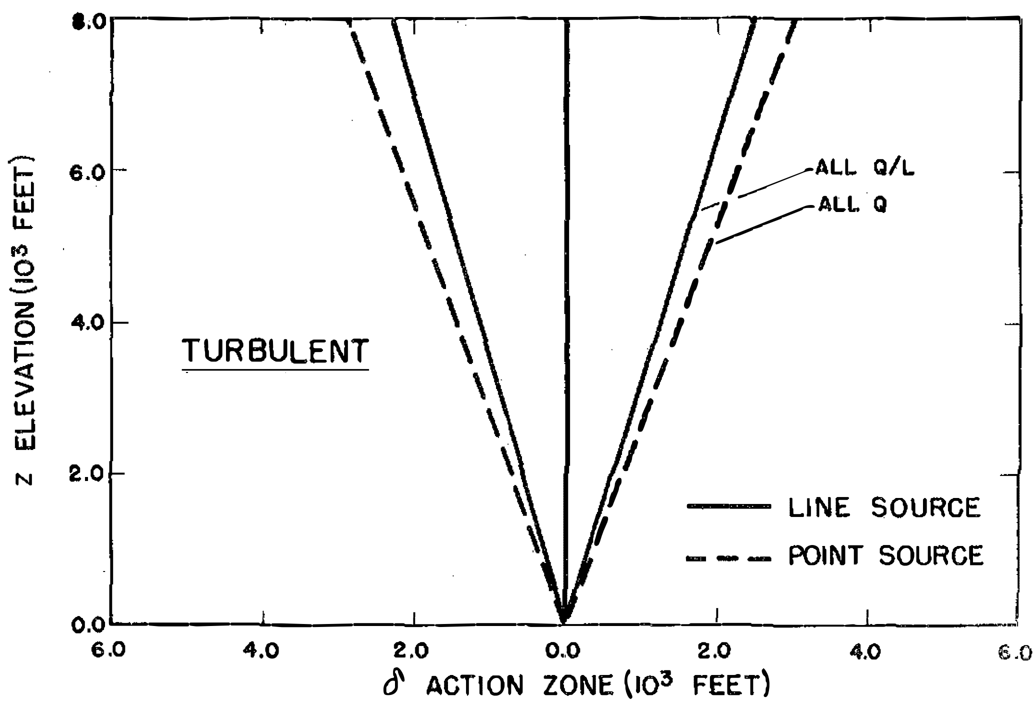


Figure 5.--Theoretical variation of action zone for laminar and turbulent line and point sources.

DISCUSSION

Although these investigations of the laminar hypothesis and the Prandtl mixing-length hypothesis are idealized, they are readily amenable to mathematical solution. The approach has been to initially retain mathematical simplicity, thereby avoiding complex differential equations. After the idealized investigations provide a feeling for the problem, modification of the system is necessary to conform with the physical system, provided that data are available. Application of the results to actual fires must be done by keeping in mind that the theoretical development has been formulated according to the following conditions:

1.s Steady states

2.s Constant propertiess

3. Omission of combustion zones

4. Omission of radiation heat transfers

5.s Dry air (perfect gas)s

6.s Zero velocity and uniform potential temperature ins ambient regions

Certain of these conditions present serious limitations when considering application of the theory to the real problem of fire behavior. The combustion process and the transient nature of the actual problem are important. Combustion of a forest fuel may occur over a range of combustion efficiency which is believed to be controlled by chemical reactivity and the diffusion of oxygen. The slower process will determine the combustion rate. A change in the combustion process gives rise to the transient problem. As a consequence the rate of combustion attains a new equilibrium level.

The analysis has considered the vertical pressure gradient in terms of the density of the exterior region. For the energy equation the vertical pressure gradient has been accounted for by the potential temperature. The potential temperature is then utilized to eliminate the density difference in the momentum equation. This potential temperature difference generates the velocities. What is implied by variable properties is the influence of the variation of the absolute viscosity and density from the exterior region to the center of the convection column. The analysis of this condition, that is, one considering compressibility, remains mathematically intractable at the present time.

In actual combustion of a solid fuel, water vapor arises from: the moisture driven from the fuel, the moisture resulting from the oxidation of the fuel, and the moisture existing in the ambient air. The concentration of water vapor in the convection column could be determined if the oxidation reaction for the solid fuel is known. In the convection column the water vapor provides a region of nonuniformly distributed heat sources which induce a velocity field similar to a thunderstorm cell in the cumulus stage of development.

Considering conservation of mass for constant properties, the rate of change of velocity into the column with elevation is--

$$2L\rho_\infty\left[\int_0^1 \frac{u}{u_m}\, d\left(\frac{y}{\delta}\right)\right]\frac{d(u_m\delta)}{dz} = 2L\rho_\infty v_\delta \qquad \text{line source,}$$

$$2\pi\rho_\infty\left[\int_0^1 \frac{y}{\delta}\frac{u}{u_m}\, d\left(\frac{y}{\delta}\right)\right]\frac{d(u_m\delta^2)}{dz} = 2\pi\rho_\infty \delta v_\delta \qquad \text{point source.}$$

Considering conservation of energy which is produced by the heat source for constant properties yields--

$$\frac{Q}{L} = 2\rho_\infty C_p g\left[\int_0^1 \frac{u}{u_m}\left(\frac{\theta-\theta_\infty}{\theta_m-\theta_\infty}\right) d\left(\frac{y}{\delta}\right)\right] u_m\delta(\theta_m-\theta_\infty) \qquad \text{line source,}$$

$$Q = 2\pi\rho_\infty C_p g\left[\int_0^1 \frac{y}{\delta}\frac{u}{u_m}\left(\frac{\theta-\theta_\infty}{\theta_m-\theta_\infty}\right) d\left(\frac{y}{\delta}\right)\right] u_m\delta^2(\theta_m-\theta_\infty) \qquad \text{point source.}$$

The foregoing relations are valid for finite sources provided that velocity and temperatures profiles are universal in character. If the equations for the line and point sources are utilized in the conservation of mass relations, the integrals are constants. For the conservation of energy relations, substitution of the line and point source equations for the turbulent cases yields constants which are dependent on the mixing length proportionality factor.

Comparison of these cases of laminar and turbulent flow indicates diverse characteristic behavior. Figure 1 shows the vertical variation of the stream function from which the indraft rate may be obtained. If the velocity and temperature distributions are universal, the mass and energy balance relations indicate that the rate of change of the inverse of the maximum temperature difference with elevation controls the indraft velocity. For instance, in the line source case, if the maximum temperature difference decays as the inverse of the elevation, then the indraft velocity is constant. From these relations several combinations of thee dependent variables are possible. Only experimental evidence concerning the universal nature of the velocity and temperature profiles and the behavior of the independent variables can provide the final answer.

Comparison of the entraining velocities is presented in figure 2, which shows a decrease in velocity with an increase in elevation, except for the turbulent line source, which is constant. Laminar velocities are smaller in magnitude than the turbulent velocities for the same source strength. Turbulent velocities are more sensitive to the source strength than laminar velocities.

Maximum vertical velocities which occur at the perpendicular axis of the sources are presented in figure 3. The higher heat output for the laminar case has not been shown since the magnitudes are above the velocity of sound. These results indicate greater velocities for the laminar case than the turbulent for a constant source strength. In addition, the laminar source velocities are more sensitive to source strength.

Axis potential temperature variations are presented in figure 4. Results for the larger source output for the laminar cases have been omitted since the temperatures are excessive. In contrast, the smaller heat output for the turbulent cases has been omitted due to the small resulting temperatures. In all cases the potential temperatures decrease with height which is in agreement with the physical system.

Figure 5 shows the shape of the zone of influence due to the sources investigated. The laminar sources indicate a very narrow spread becoming more concentrated as the source output is increased. In direct contrast the turbulent cases expand very rapidly in a linear behavior, and are independent of source strength on the basis that the mixing length proportionality factor c is a true constant. Since this factor does appear in the majority of the variables of the turbulent cases, there remains the possibility that this proportionality factor may depend on other parameters. Only further investigation of the atmospheric diffusive mechanism can determine this behavior.

Recent inquiries have been directed to the effect of lapse rate on the convection column which would thereby affect the heat source. In Appendix C an approximate investigation has been undertaken for the effect of nonuniform potential temperature for the laminar line source. This analysis indicates that the gradients of potential temperature must be very large to be contributing factors for heat source outputs considered in this report. This conclusion was also obtained by Byram (1) in his thermodynamic analysis. For small heat source strengths the lapse rate is undoubtedly of importance as, for instance, in whirlwinds or cumulus cloud development. The foregoing does not imply that the lapse rate is unimportant in the problem of atmospheric fire behavior since the eddy viscosity and conductivity are greatly affected at least near the surface.

CONCLUSIONS

The present investigation shows that the laminar sources yield narrow convection columns which consequently require large vertical velocities and temperatures, extending the column to higher elevations. Since the turbulent sources spread in a horizontal direction, the vertical velocities and temperatures are smaller in magnitude, and the column does not attain comparable heights.

From the results which indicate the magnitudes of the variables, it may be concluded that the laminar process is not compatible with the physical system. Therefore, the results of the turbulent investigation should be used for practical application.

A source of heat induces a velocity field wherein momentum and heat must be diffused. For the condition of negligible free wind speeds, the mode of diffusion plays an important role in the behavior of the variables. This may be understood in that the region of influence of the heat source controls the velocity and temperature profiles and, by virtue of conservation of mass and energy, controls the indraft requirement. Characteristics of the heat and momentum diffusion must be determined to ultimately understand the problem.

For large heat outputs the potential temperature gradients do not influence the results of the basic theory for an established transfer process, but are important in effecting the diffusion mechanism.

RECOMMENDATION

An experimental investigation to determine the effect of radius and heat rate on the indraft velocities and velocity and temperature distributions in the convection zone is recommended.2/

This experiment should be conducted in the laboratory so that the eddy viscosity and eddy conductivity would not be affected by the lapse conditions. Then the condition of linear mixing length with height could be fulfilled so that the mixing length proportionality constant c would be a true constant. The system should be designed for actual flame sources whereby the heat production rate could be controlled for a specific range

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2/ Application of the turbulent point source to the case of a finite area source of heat is presented in Appendix D.

of values. The sizes of the flame front would be determined by the maximum energy release which is feasible such that heat rates of sufficient magnitude produce measurable velocities.

Also the system should be flexible so that electrical heaters could replace the flame sources in order to find the effect of sources with and without combustion. Shielded thermocouples would be designed for temperature traverses, and the heated-thermocouple type anemometer and hot wire anemometer for velocity measurements. A traversing mechanism would allow the two measuring circuits to be located in the vertical and radial directions.

Damping screens located at ample distances surrounding the source would provide the necessary damping so that any exterior disturbances would decay thereby not influencing the convection column. This would also allow the indraft makeup to feed the convection column. The height of the damping section would be determined by the growth of the convection zone.

APPENDIX A

BASIC ANALYSIS

Laminar Line Source

The continuity, momentum, and energy equations for the two-dimensional flow due to a line source for a fluid of constant Prandtl Number, specific heat, and viscosity are

$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\rho u \frac{\partial u}{\partial z} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 u}{\partial y^2} - \rho g \qquad (2)$$

$$\rho g J c_p u \frac{\partial T}{\partial z} + \rho g J c_p v \frac{\partial T}{\partial y} - u \frac{\partial p}{\partial z} = Jk \frac{\partial^2 T}{\partial y^{2n}} \qquad (3)$$

wherein the condition that $\frac{\partial p}{\partial y}$ is of order zero has been obtained from an non-dimensional order of magnitude investigation of the Navier-Stokes equations. Applying equation (2) in the region exterior to the region of influence there results:

$$-\frac{\partial p_\infty}{\partial z} = \rho_\infty g = -\frac{\partial p}{\partial z}$$

since $p = p_\infty$. The foregoing equations become

$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} = 0 \qquad (4)$$

$$\rho u \frac{\partial u}{\partial z} + \rho v \frac{\partial u}{\partial y} = g(\rho_\infty - \rho) + \mu \frac{\partial^2 u}{\partial y^2} \qquad (5)$$

$$\rho g J c_p u \frac{\partial T}{\partial z} + \rho g J c_p v \frac{\partial T}{\partial y} - u \frac{\partial p}{\partial z} = Jk \frac{\partial^2 T}{\partial y^2} \qquad (6)$$

From the perfect gas law at constant pressure

$$\frac{\rho_\infty}{\rho} = \frac{T}{T_\infty}$$

which gives

$$\rho_\infty - \rho = \rho\left(\frac{T - T_\infty}{T_\infty}\right)$$

Using the definition of potential temperature

$$\frac{1}{\theta}\frac{\partial \theta}{\partial z} = \frac{1}{T}\left(\frac{\partial T}{\partial z} - \frac{1}{J c_p \rho g}\frac{\partial p}{\partial z}\right)$$

and

$$\frac{1}{\theta}\frac{\partial \theta}{\partial y} = \frac{1}{T}\frac{\partial T}{\partial y} \qquad\qquad \frac{1}{\theta}\frac{\partial^2 \theta}{\partial y^2} = \frac{1}{T}\frac{\partial^2 T}{\partial y^2}$$

the foregoing equations are

$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial y} = \nu_\infty \frac{\partial^2 u}{\partial y^2} + g\left(\frac{\theta - \theta_\infty}{\theta_\infty}\right) \tag{8}$$

$$u\frac{\partial(\theta - \theta_\infty)}{\partial z} + v\frac{\partial(\theta - \theta_\infty)}{\partial y} = \frac{\nu_\infty}{\sigma}\frac{\partial^2(\theta - \theta_\infty)}{\partial y^2} \tag{9}$$

with the boundary conditions

$$y = 0 \qquad v = 0 \qquad \frac{\partial u}{\partial y} = 0 \qquad \frac{\partial(\theta - \theta_\infty)}{\partial y} = 0$$

$$y \rightarrow \pm\infty \qquad u = 0 \qquad \theta - \theta_\infty = 0$$

The equations have been derived on the basis that the density difference is the important factor compared to the magnitude of the density itself. Thus the variation of the density and absolute viscosity with temperature has not been considered.

Solution of equations (8) and (9) proceeds as follows

$$\eta = \left(\frac{Q}{L c_p \theta_\infty \rho_\infty \nu_\infty^3}\right)^{1/5} \frac{y}{z^{2/5}} \tag{10}$$

$$\psi = \nu_\infty \left(\frac{Q}{L c_p \theta_\infty \rho_\infty \nu_\infty^3} \right)^{1/5} z^{3/5} f(\eta) \tag{11}$$

so that

$$u = \frac{\partial \psi}{\partial y} = \nu_\infty \left(\frac{Q}{L c_p \theta_\infty \rho_\infty \nu_\infty^3} \right)^{2/5} z^{1/5} \frac{df}{d\eta} \tag{12}$$

$$v = -\frac{\partial \psi}{\partial z} = \nu_\infty \left(\frac{Q}{L c_p \theta_\infty \rho_\infty \nu_\infty^3} \right)^{1/5} z^{-2/5} \left(\frac{3}{5} f - \frac{2}{5} \eta \frac{df}{d\eta} \right) \tag{13}$$

$$\theta - \theta_\infty = \frac{\nu_\infty^2 \theta_\infty}{g} \left(\frac{Q}{L c_p \theta_\infty \rho_\infty \nu_\infty^3} \right)^{4/5} z^{-3/5} h(\eta) \tag{14}$$

where u and v as so chosen satisfy the continuity equation (7).

In addition, the condition that the heat output of the source is equal to the heat flow at any cross-section of the region of influence yields

$$\frac{Q}{L} = \int_{-\infty}^{+\infty} \rho_\infty g c_p u (\theta - \theta_\infty) \, dy \tag{15}$$

With the foregoing transformations, equations (8), (9) and (15) are transformed into a system of ordinary differential equations as

$$5 \frac{d^3 f}{d\eta^3} + 3f \frac{d^2 f}{d\eta^2} - \left(\frac{df}{d\eta} \right)^2 + 5h = 0 \tag{16}$$

$$\frac{5}{3\sigma} \frac{d^2 h}{d\eta^2} + \frac{d(fh)}{d\eta} = 0 \tag{17}$$

$$\int_{-\infty}^{+\infty} h \frac{df}{d\eta} \, d\eta = 1 \tag{18}$$

with the corresponding boundary conditions

$$f(0) = f''(0) = h'(0) = 0$$

$$f'(\pm\infty) = h(\pm\infty) = 0$$

For a Prandtl Number of 2/3 the functions satisfying the foregoing equations and boundary conditions are

$$f(\eta) \frac{\sqrt{5}}{2} \tanh \frac{5\sqrt{5}}{2} \eta \tag{19}$$

$$h(\eta) = \frac{3}{2\sqrt{5}} \operatorname{sech}^2 \frac{5\sqrt{5}}{2} \eta \tag{20}$$

Turbulent Line Source

The equations for this case are written by utilization of the Prandtl mixing length hypothesis wherein the eddy diffusivity for heat and momentum are considered equivalent and as $\ell^2 \frac{\partial u}{\partial y}$ thereby yielding

$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} = 0 \tag{21}$$

$$u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial y} = -\frac{\partial\left(\ell^2 \frac{\partial u}{\partial y}\frac{\partial u}{\partial y}\right)}{\partial y} + g\left(\frac{\theta - \theta_\infty}{\theta_\infty}\right) \tag{22}$$

$$u\frac{\partial(\theta - \theta_\infty)}{\partial z} + v\frac{\partial(\theta - \theta_\infty)}{\partial y} = -\frac{\partial\left(\ell^2 \frac{\partial u}{\partial y}\frac{\partial(\theta - \theta_\infty)}{\partial y}\right)}{\partial y} \tag{23}$$

where $\ell = cz$

It should be pointed out that the dependent variables used herein represent mean quantities.

By use of

$$\eta = \frac{y}{z} \tag{24}$$

$$\psi = \nu_\infty \left(\frac{Q}{L\, c_p \theta_\infty \rho_\infty \nu_\infty^3}\right)^{1/3} z f(\eta) \tag{25}$$

so that

$$u = \frac{\partial\psi}{\partial y} = \nu_\infty\left(\frac{Q}{Lc_p\theta_\infty\rho_\infty\nu_\infty^3}\right)^{1/3}\frac{df}{d\eta} \tag{26}$$

$$v = -\frac{\partial\psi}{\partial z} = -\nu_\infty\left(\frac{Q}{Lc_p\theta_\infty\rho_\infty\nu_\infty^3}\right)^{1/3}\left(f - \eta\frac{df}{d\eta}\right) \tag{27}$$

$$\theta - \theta_\infty = \frac{\nu_\infty^2\theta_\infty}{g}\left(\frac{Q}{Lc_p\theta_\infty\rho_\infty\nu_\infty^3}\right)^{2/3} z^{-1}\, h(\eta) \tag{28}$$

yields the transformed equations and condition as

$$c^2\,\frac{d\left(\frac{d^2f}{d\eta^2}\right)^2}{d\eta^2} - f\frac{d^2f}{d\eta^2} - h = 0 \tag{29}$$

$$c^2\,\frac{d\left(\frac{d^2f}{d\eta^2}\frac{dh}{d\eta}\right)}{d\eta} - \frac{d(hf)}{d\eta} = 0 \tag{30}$$

$$\int_{-\infty}^{+\infty} h\frac{df}{d\eta}\, d\eta = 1 \tag{31}$$

with boundary conditions

$$f(0) = f''(0) = h'(0) = 0$$

$$f'(\pm\infty) = h(\pm\infty) = 0$$

Letting $\eta = (2c^2)^{1/3}\eta_1$ and $h = \frac{H}{(2c^2)^{2/3}}$ equations (29) and (30) become

$$\frac{d^2f}{d\eta_1^2}\,\frac{d^3f}{d\eta_1^3} - f\frac{d^2f}{d\eta_1^2} - H = 0 \tag{32}$$

$$\frac{d^2f}{d\eta_1^2}\frac{dH}{d\eta_1} - 2Hf = 0 \qquad (33)$$

The functions f and H have been obtained by Schmidt (4) who utilized a series solution approach wherein the first set for f and H applied near the region $y = 0$ and the second set near the edge of the region affected by the source.

The set near the central axis $(y = 0)$ is

$$f(\eta_1) = 1.1398\eta_1 - 0.3771\eta_1^{5/2} + 0.0633\eta_1^4 - 0.0051\eta_1^{11/2} + \ldots\ldots\ldots$$

$$H(\eta_1) = 1 - 1.0746\eta_1^{3/2} + 0.6108\eta_1^3 - 0.1154\eta_1^{5/4} + \ldots\ldots\ldots\ldots \qquad (34)$$

This approach is utilized since the foregoing series do not converge adequately for large values of η_1.

The variables become

$$(2c^2)^{1/3}\eta_1 = \frac{y}{z}$$

$$\psi = \nu_\infty\left(\frac{Q}{L c_p \theta_\infty \rho_\infty \nu_\infty^3}\right)^{1/3} z\, f(\eta_1)n$$

$$u = \nu_\infty\left(\frac{Q}{L c_p \theta_\infty \rho_\infty \nu_\infty^3}\right)^{1/3} \frac{1}{(2c^2)^{1/3}}\frac{df}{d\eta_1} \qquad (35)$$

$$v = -\nu_\infty\left(\frac{Q}{L c_p \theta_\infty \rho_\infty \nu_\infty^3}\right)^{1/3}\left(f - \eta_1\frac{df}{d\eta_1}\right)$$

$$\theta - \theta_\infty = \frac{\nu_\infty^2\theta_\infty}{g}\left(\frac{Q}{L c_p \theta_\infty \rho_\infty \nu_\infty^3}\right)^{2/3} \frac{H}{(2c^2)^{2/3}} z^{-1}$$

Laminar Point Source

For this case the equations for the axisymmetrical flow are

$$\frac{\partial(yu)}{\partial z} + \frac{\partial(yv)}{\partial y} = 0n \qquad (36)$$

$$u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial y} = \frac{\nu_\infty}{y}\,\frac{\partial\left(y\frac{\partial u}{\partial y}\right)}{\partial y}+g\left(\frac{\theta-\theta_\infty}{\theta_\infty}\right) \tag{37}$$

$$u\,\frac{\partial(\theta-\theta_\infty)}{\partial z}+v\,\frac{\partial(\theta-\theta_\infty)}{\partial y} = \frac{\nu_\infty}{\sigma y}\,\frac{\partial\left[y\frac{\partial(\theta-\theta_\infty)}{\partial y}\right]}{\partial y} \tag{38}$$

By use of

$$\eta = \left(\frac{Q}{c_p\theta_\infty\rho_\infty\nu_\infty^3}\right)^{1/4}\frac{y}{z^{1/2}} \tag{39}$$

$$\psi = 4\nu_\infty z\, f(\eta) \tag{40}$$

so that

$$u = \frac{1}{y}\frac{\partial\psi}{\partial y} = 4\nu_\infty\left(\frac{Q}{c_p\theta_\infty\rho_\infty\nu_\infty^3}\right)^{1/2}\frac{1}{\eta}\,\frac{df}{d\eta} \tag{41}$$

$$v = -\frac{1}{y}\,\frac{\partial\psi}{\partial y} = -2\nu_\infty\left(\frac{Q}{c_p\theta_\infty\rho_\infty\nu_\infty^3}\right)^{1/4} z^{-1/2}\left(2\frac{f}{\eta}-\frac{df}{d\eta}\right) \tag{42}$$

$$\theta-\theta_\infty = 4\frac{\nu_\infty^2\theta_\infty}{g}\left(\frac{Q}{c_p\theta_\infty\rho_\infty\nu_\infty^3}\right) z^{-1}\,h(\eta) \tag{43}$$

yields the transformed equations and conditions as

$$\frac{d^3f}{d\eta^3}+(4f-1)\frac{d\left(\frac{1}{\eta}\frac{df}{d\eta}\right)}{d\eta}+\eta h = 0 \tag{44}$$

$$\frac{d(hf)}{d\eta} = -\frac{1}{4\sigma}\,\frac{d\left(\eta\frac{dh}{d\eta}\right)}{d\eta} \tag{45}$$

$$\int_{-\infty}^{+\infty} h \frac{df}{d\eta} \, d\eta = \frac{1}{16\pi} \qquad (46)$$

with boundary conditions

$$f(0) = f''(0) = h'(0) = 0$$

$$f'(\pm\infty) = h(\pm\infty) = 0$$

For a Prandtl Number of unity the functions satisfying the foregoing equations and boundary conditions are

$$f(\eta) = \frac{3}{2}\left(1 - \frac{1}{1 + \frac{1}{6\sqrt{2\pi}}\eta^2}\right) \qquad (47)$$

$$h(\eta) = \frac{1}{3\pi} \frac{1}{\left(1 + \frac{1}{6\sqrt{2\pi}}\eta^2\right)^3} \qquad (48)$$

Turbulent Point Source

The axisymmetrical equations utilizing the Prandtl mixing length are

$$\frac{\partial(yu)}{\partial z} + \frac{\partial(yv)}{\partial y} = 0 \qquad (49)$$

$$u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial y} = -\frac{1}{y} \frac{\partial\left(l^2 y \frac{\partial u}{\partial y} \frac{\partial u}{\partial y}\right)}{\partial y} + g\left(\frac{\theta - \theta_\infty}{\theta_\infty}\right) \qquad (50)$$

$$u\frac{\partial(\theta - \theta_\infty)}{\partial z} + v\frac{\partial(\theta - \theta_\infty)}{\partial y} = -\frac{1}{y} \frac{\partial\left[l^2 y \frac{\partial u}{\partial y} \frac{\partial(\theta - \theta_\infty)}{\partial y}\right]}{\partial y} \qquad (51)$$

where $\mathcal{J} = cz$

By use of

$$\eta = \frac{y}{z} \tag{52}$$

$$\psi = \nu_\infty \left(\frac{Q}{c_p \theta_\infty \rho_\infty \nu_\infty^3} \right)^{1/3} z^{5/3} f(\eta) \tag{53}$$

so that

$$u = \frac{1}{y} \frac{\partial \psi}{\partial y} = \nu_\infty \left(\frac{Q}{c_p \theta_\infty \rho_\infty \nu_\infty^3} \right)^{1/3} \frac{1}{\eta} \frac{df}{d\eta} z^{-1/3} \tag{54}$$

$$v = -\frac{1}{y} \frac{\partial \psi}{\partial z} = -\nu_\infty \left(\frac{Q}{c_p \theta_\infty \rho_\infty \nu_\infty^3} \right)^{1/3} z^{-1/3} \left(\frac{5}{3} \frac{f}{\eta} - \frac{df}{d\eta} \right) \tag{55}$$

$$\theta - \theta_\infty = \frac{\nu_\infty^2 \theta_\infty}{g} \left(\frac{Q}{c_p \theta_\infty \rho_\infty \nu_\infty^3} \right)^{2/3} z^{-5/3} h(\eta) \tag{56}$$

yield the transformed equations and condition as

$$c^2 \frac{d\left[\eta \left(\frac{d \frac{1}{\eta} \frac{df}{d\eta}}{d\eta} \right)^2 \right]}{d\eta} - \frac{1}{3} \frac{1}{\eta} \left(\frac{df}{d\eta} \right)^2 - \frac{5}{3} f \frac{d\left(\frac{1}{\eta} \frac{df}{d\eta} \right)}{d\eta} - \eta h = 0 \tag{57}$$

$$c^2 \frac{d\left[\eta \frac{d\left(\frac{1}{\eta} \frac{df}{d\eta} \right)}{d\eta} \frac{dh}{d\eta} \right]}{d\eta} - \frac{5}{3} \frac{d(fh)}{d\eta} = 0 \tag{58}$$

$$\int_{-\infty}^{+\infty} h \frac{df}{d\eta} d\eta = 1 \tag{59}$$

with the boundary conditions

$$f(0) = f''(0) = h'(0) = 0$$

$$f'(\pm\infty) = g(\pm\infty) = 0$$

Letting $\eta = (3c^2)^{1/3}\eta_1$ and $h = \dfrac{H}{(3c^2)^{2/3}}$ equations (57) and (58) are the same as those derived by Schmidt (5). Again using the series development as performed by Schmidt there obtains for the axis region

$$\begin{aligned} f(\eta_1) &= 0.5405\,\eta_1^2 - 0.2750\,\eta_1^{7/2} + 0.0642\,\eta_1^5 - 0.0086\,\eta_1^{13/2} + \ldots\ldots \\ H(\eta_1) &= 1 - 1.2480\,\eta_1^{3/2} + 0.6802\,\eta_1^3 - 0.2220\,\eta_1^{9/2} + 0.0500\,\eta_1^6 \ldots\ldots \end{aligned} \tag{60}$$

yielding

$$\begin{aligned} (3c^2)^{1/3}\eta_1 &= \frac{y}{z} \\ \psi &= \nu_\infty\left(\frac{Q}{c_p\,\theta_\infty\,\rho_\infty\,\nu_\infty^3}\right)^{1/3} z^{5/3}\, f(\eta_1) \\ u &= \nu_\infty\left(\frac{Q}{c_p\,\theta_\infty\,\rho_\infty\,\nu_\infty^3}\right)^{1/3} \frac{1}{(3c^2)^{2/3}}\, z^{-1/3}\, \frac{1}{\eta_1}\,\frac{df}{d\eta_1} \\ v &= -\nu_\infty\left(\frac{Q}{c_p\,\theta_\infty\,\rho_\infty\,\nu_\infty^3}\right)^{1/3} \frac{1}{(3c^2)^{1/3}}\, z^{-1/3}\left(\frac{5}{3}\,\frac{f}{\eta_1} - \frac{df}{d\eta_1}\right) \\ \theta - \theta_\infty &= \frac{\nu_\infty^2\,\theta_\infty}{g}\left(\frac{Q}{c_p\,\theta_\infty\,\rho_\infty\,\nu_\infty^3}\right)^{2/3} \frac{1}{(3c^2)^{2/3}}\, z^{-5/3}\, H(\eta_1) \end{aligned} \tag{61}$$

APPENDIX B

INVESTIGATION OF HORIZONTAL PRESSURE GRADIENT

Since the analysis presented herein is based upon the approximations of the boundary layer theory wherein the pressure gradient in the y direction is taken of zero order, an investigation of the validity of the approximation is presented for the line sources.

Taking the momentum equation in the y direction as

$$u\frac{\partial v}{\partial z}+v\frac{\partial v}{\partial y}-\nu_\infty\frac{\partial\left(\frac{\partial v}{\partial z}\right)}{\partial z}=\frac{1}{\rho_\infty}\frac{\partial p}{\partial y} \qquad \text{(Laminar)} \quad (62)$$

Transforming the equations from z,y to η there results

$$p_\delta-p_m=\frac{\rho_\infty\nu_\infty^2\phi^{2/5}}{z^{4/5}}\int_0^\delta\left(\frac{9\sqrt{5}}{8}\operatorname{sech}^2\frac{5\sqrt{5}}{2}\eta\tanh\frac{5\sqrt{5}}{2}\eta\right.$$

$$\left.-\frac{25}{4}\eta\operatorname{sech}^4\frac{5\sqrt{5}}{2}\eta-\frac{375}{4}\eta\operatorname{sech}^2\frac{5\sqrt{5}}{2}\eta\tanh\frac{5\sqrt{5}}{2}\eta\right)d\eta \quad (63)$$

$$+\frac{\rho_\infty\nu_\infty^2}{z^2}\int_0^\delta\left(\frac{21\sqrt{5}}{125}\tanh\frac{5\sqrt{5}}{2}\eta-\frac{5}{2}\eta\operatorname{sech}^2\frac{5\sqrt{5}}{2}\eta\right)d\eta$$

Using the largest value of the heat output function used in the previous results as $\phi=10^{15}$ and the property values as previously used of $\rho_\infty=2.34\times10^{-3}$ and $\nu_\infty=1.80\times10^{-4}$ there results

$$p_\delta-p_m=\frac{7.6\times10^{-5}}{z^{4/5}}I_1+\frac{7.6\times10^{-11}}{z^2}I_2 \quad (64)$$

Thus even for small values of z the pressure drop is of zero order.

For the turbulent line source the result given by Schmidt (4) is

$$p_\delta-p_m=0.39(2c^2)^{2/3}\rho_\infty\nu_\infty^2\phi^{2/3} \quad (65)$$

Using the previous values of the quantities and taking $c = 0.0417$ there results

$$p_{\delta} - p_m = 0.00675 \text{ lb./sq. ft.} \sim \text{zero order}$$

APPENDIX C

EFFECT OF NONUNIFORM POTENTIAL TEMPERATURE

To obtain an idea as to the effect of vertical potential temperature gradients in the atmosphere upon a heat source, an approximate approach was undertaken. The analysis shall be presented for the laminar line source.

Taking the continuity, momentum, and energy equations as previously derived

$$\frac{\partial(\rho u)}{\partial z} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{66}$$

$$\rho u \frac{\partial u}{\partial z} + \rho v \frac{\partial u}{\partial y} = \frac{\partial\left(\mu \frac{\partial u}{\partial y}\right)}{\partial y} + \rho g\left(\frac{\theta - \theta_\infty}{\theta_\infty}\right) \tag{67}$$

$$\rho u \frac{\partial \theta}{\partial z} + \rho v \frac{\partial \theta}{\partial y} = \frac{1}{\sigma} \frac{\partial\left(\mu \frac{\partial \theta}{\partial y}\right)}{\partial y} \tag{68}$$

and use of the Mises transformation,

$$\rho u = \rho_\infty \frac{\partial \psi}{\partial y} \qquad \rho v = -\rho_\infty \frac{\partial \psi}{\partial z}$$

which satisfies the continuity equation (66) if ρ_∞ = constant. Equations (67) and (68) become with the condition that $\mu\rho = \mu_\infty\rho_\infty$ 3/

$$\frac{\partial u}{\partial z} = \nu_\infty \frac{\partial\left(u \frac{\partial u}{\partial \psi}\right)}{\partial \psi} + \frac{g}{u}\left[\frac{\theta(z,\psi)}{\theta(z,\infty)} - 1\right] \tag{69}$$

$$\frac{\partial \theta}{\partial z} = \frac{\nu_\infty}{\sigma} \frac{\partial\left(u \frac{\partial \theta}{\partial \psi}\right)}{\partial \psi} \tag{70}$$

3/ This relation adequately describes the variation of viscosity of air with temperature as outlined by Chapman and Rubesin (2).

By assuming velocity and temperature profiles which satisfy the boundary conditions of the problem asd

$$u(z,\psi) = u_m(z)\, e^{-\frac{\psi^2}{d^2}} \qquad \text{(71)d}$$

where $u(z,\infty) = 0$d and $u'(z,0) = 0$d

andd

$$\frac{\theta(z,\psi) - \theta(z,\infty)}{\theta(z,0) - \theta(z,\infty)} = e^{-\frac{\psi^2}{d^2}} \qquad (72)$$

where $\theta(z,\infty) = \theta(z,\infty)$d $\quad \theta'(z,0) = 0$d

Upon substitution of these distributions into equations (69) andd (70) at the condition $\psi = 0$ by reason that the central axis is the region wherein the greatest variations in velocity and temperature occur, the final equations ared

$$\frac{dU}{dz} = -2\frac{U^2}{D^2} + \frac{1}{U}\,\frac{\theta(0,\infty)}{\theta(z,\infty)}\,\Theta \qquad (73)$$

$$\frac{dB}{dz} + \frac{d\Theta}{dz} = -\frac{2}{\sigma}\,\frac{U}{D^2}\,\Theta \qquad (74)$$

andd

$$\Theta = \frac{1}{\sqrt{\pi}}\,\frac{\phi}{D} \qquad (75)$$

from the condition thatd

$$\frac{Q}{L} = \int_{-\infty}^{+\infty} \rho\, g\, c_p\, u \left[\theta(z,\psi) - \theta(z,\infty)\right] dy$$

whered

$$U = \frac{u_m}{\nu_\infty}$$

$$D = \frac{d}{\nu_\infty}$$

$$\Theta = \frac{\theta(z,0)-\theta(z,\infty)}{\dfrac{\theta(0,\infty)\,\nu_\infty^2}{g}}$$

$$B = \frac{\theta(z,\infty)}{\dfrac{\theta(0,\infty)\,\nu_\infty^2}{g}}$$

$$\phi = \frac{Q}{L c_p\,\theta(0,\overline{\infty})\,\rho_\infty \nu_\infty^3}$$

By eliminating Θ and B from the three equations there results a second order nonlinear differential equation for the unknown function D

$$D^3 \frac{dD}{dz}\frac{d^2D}{dz^2} + 2D^2\left(\frac{dD}{dz}\right)^3 - \frac{4}{\sqrt{\pi}}\,\frac{g}{\nu_\infty^2}\,\frac{\phi}{B} = 7\sqrt{\pi}\,D^4\left(\frac{dD}{dz}\right)^2 \frac{d\left(\frac{B}{\phi}\right)}{dz}$$

$$+\sqrt{\pi}\,D^5\,\frac{dD}{dz}\,\frac{d^2\left(\frac{B}{\phi}\right)}{dz^2} + \sqrt{\pi}\,D^5\,\frac{d^2D}{dz^2}\,\frac{d\left(\frac{B}{\phi}\right)}{dz} - 6\pi D^6\,\frac{dD}{dz}\left[\frac{d\left(\frac{B}{\phi}\right)}{dz}\right]^2 \qquad (76)$$

$$-\pi D^7\,\frac{d\left(\frac{B}{\phi}\right)}{dz}\,\frac{d^2\left(\frac{B}{\phi}\right)}{dz^2} + \sqrt{\pi}\,\pi D^8\left[\frac{d\left(\frac{B}{\phi}\right)}{dz}\right]^3$$

with the boundary conditions

$z = 0 \quad D = 0 \quad$ since $\; u(0,\phi) = 0 \;$ and $\; \theta(0,\phi) = (0,\infty)$

$z \to \infty \quad D \to \infty \quad$ since $\; u(\infty,\phi) = u_m(\infty) \;$ and $\; \theta(\infty,\phi) = \theta(\infty,0)$

As a preliminary verification of equation (76) a check for the constant potential temperature case can be obtained. For this case $B = \text{Const} = g/\nu_\infty^2$. Thus equation (76) is

$$D^3\,\frac{dD}{dz}\,\frac{d^2D}{dz^2} + 2D^2\left(\frac{dD}{dz}\right)^3 - \frac{4}{\sqrt{\pi}}\,\phi = 0$$

which has the solution satisfying the boundary conditions

$$D = \left(\frac{125}{9\sqrt{\pi}}\right)^{1/5} \phi^{1/5} z^{3/5}$$

thereby yielding

$$U = \frac{3}{10}\left(\frac{125}{9\sqrt{\pi}}\right)^{2/5} \phi^{2/5} z^{1/5}$$

$$\Theta = \frac{1}{\sqrt{\pi}}\left(\frac{9\sqrt{\pi}}{125}\right)^{1/5} \phi^{4/5} z^{-3/5} \tag{77}$$

These results are in the same form as obtained by the rigorous solution for the laminar line source as given in table 1. The constants do not compare favorably.

The nonlinear equation (76), which does not have a formal solution available, could be solved by a numerical method or series development of D in terms of z for the non-dimensional parameter B/ϕ. This approach was not deemed necessary since an investigation of the order of magnitude of the factor B/ϕ discloses a magnitude of

$$\frac{1.73 \times 10^{-3}}{\frac{Q}{L}} \; \frac{\theta(z,\infty)}{\theta(0,\infty)}$$

For the heat outputs considered in this report, gradients of potential temperature would necessarily be very large such that the terms of the right-hand side of equation (76) become comparable to the terms of the left-hand side. Thus, this order of magnitude argument indicates that the potential temperature gradient is a higher order effect. This does not imply that the lapse rate is unimportant in the problem of atmospheric fire behavior.

APPENDIX D

APPLICATION OF FINITE SOURCES

Investigation of the finite source dependent variables from an exact mathematical approach presents a system of extreme complexity. In the point source investigation, the boundary layer assumptions provide a single momentum equation which cannot be realized in the finite source case. To obtain an order of magnitude of the variables of interest, the point source analysis has been applied to the finite area source by making definite assumptions.

The postulate is made that the finite heat source of total heat strength of Q BTU/sec. is equivalent to the point source of identical heat rate functioning at a fictitious distance z_0 below the actual finite source wherein the equations for v_δ, $\theta_m - \theta_\infty$ and u_m are applicable. The foregoing statement implies that there exists an equivalent point source which yields the identical behavior of the dependent variables as the actual finite source. The vertical distance is modified to make the equality. Such a condition could be reasonable for the region exterior to the immediate vicinity of the finite area source. Furthermore, if these results are to be applied to actual fire behavior, the reader must realize that the combustion zone and flame front have not been considered in the free convection investigation. In addition, actual fire behavior is a transient phenomena from build-up to decay which must be the subject for final consideration.

Since data on finite sources of heat in free convection are virtually nonexistent, the determination of the parameters which influence z_0 must remain simple at this stage. The quantity is obtained from the boundary layer growth of the point source which is

$$z_0 = \frac{R}{2.1(3c^2)^{1/3}} \qquad (78)$$

thereby indicating that the fictitious distance is a function of the radius of the finite source and the mixing length proportionality constant. It is fully realized that other parameters do influence this determination. Until experimental evidence is obtained, no refinements are considered warranted.

By the reasoning outlined the dependent variables become

$$v_\delta = -\frac{0.40}{(3c^2)^{1/3}} \left(\frac{Q}{c_p \theta_\infty \rho_\infty}\right)^{1/3} \frac{1}{(z' + z_0)^{1/3}} \qquad (79)$$

$$\theta_m - \theta_\infty = \frac{\theta_\infty}{(3c^2)^{2/3} g} \left(\frac{Q}{c_p \theta_\infty \rho_\infty}\right)^{2/3} \frac{1}{(z' + z_0)^{5/3}} \tag{80}$$

$$u_m = \frac{1.08}{(3c^2)^{2/3}} \left(\frac{Q}{c_p \theta_\infty \rho_\infty}\right)^{1/3} \frac{1}{(z' + z_0)^{1/3}} \tag{81}$$

Substitution for z_0 the condition that $Q = \pi R^2 q$ and using the property values previously indicated there results

$$v_\delta = -\frac{0.87}{\left(1 + \frac{z'}{z_0}\right)^{1/3n}} \frac{q^{1/3} R^{1/3}}{c^{4/9}} \tag{82}$$

$$\theta_m - \theta_\infty = \frac{235}{\left(1 + \frac{z'}{z_0}\right)^{5/3}} \frac{q^{2/3}}{R^{1/3} c^{2/9}} \tag{83}$$

$$u_m = \frac{1.63}{\left(1 + \frac{z'}{z_0}\right)^{1/3}} \frac{q^{1/3} R^{1/3}}{c^{10/9}} \tag{84}$$

The foregoing equations relate the dependent variables to the heat rate per unit area, size of heat source, and mixing length proportionalityn constant. At this stage the indraft velocity has not been corrected for surface shear. Investigation of the mixing length proportionality constant has been conducted by Sheppard (6). This study indicates that the mixing length is approximately linear with height only for the neutral case (adiabatic lapse rate). Further research concerning the turbulent mixing in the atmosphere or the effect of turbulence on controlled fires need ben undertaken.

The relations derived herein were applied for the values of q = 20 BTU/sec ft.^{2} and a radius of 100 ft. for values of c equal to 0.50n and 0.30. The heat rate taken corresponds to a forest fuel rate of 3 tons/acre-min. The results indicate that the indraft velocities are not very sensitive to the mixing length proportionality constant. The temperature distribution given by the relation is for a convection column not accounting for the flame region. Temperatures in the flame region are greater in magnitude than the values given by the relation. Temperature distributions in flames is another experimental study of considerable interest. The vertical velocities are influenced considerably by the mixing length proportionality constant. These results have been plotted in figures 6 and 7 respectively.

A model for the convective column dependent variables generated by a finite area steady state heat source has been established by utilizing a basic theoretical approach. In order to obtain the reliability and apply refinements to the model advanced herein, an experimental investigation must be conducted with the mixing length proportionality constant invariant but with the variation of the heat rate and size of the flame front. Instrumentation is required to obtain the dependent variables. In this manner the logical foundation of a complete model for ultimate fire behavior may become realized. Continued progress from the theoretical approach must be aided and guided by controlled experiments with the influence of the heat rate and size and source providing the basic foundation.

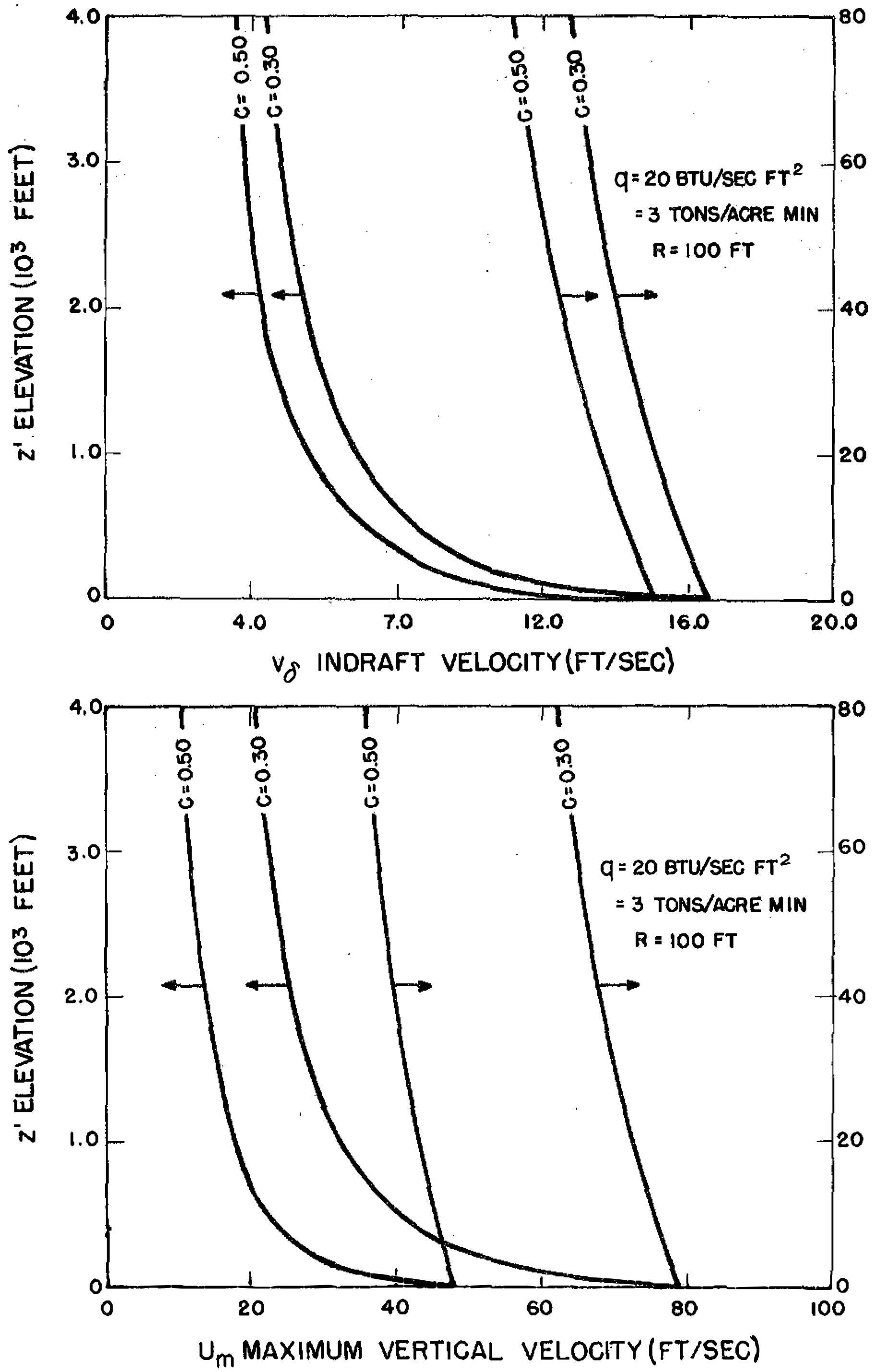

Figure 6.--Theoretical finite area source velocity variations.

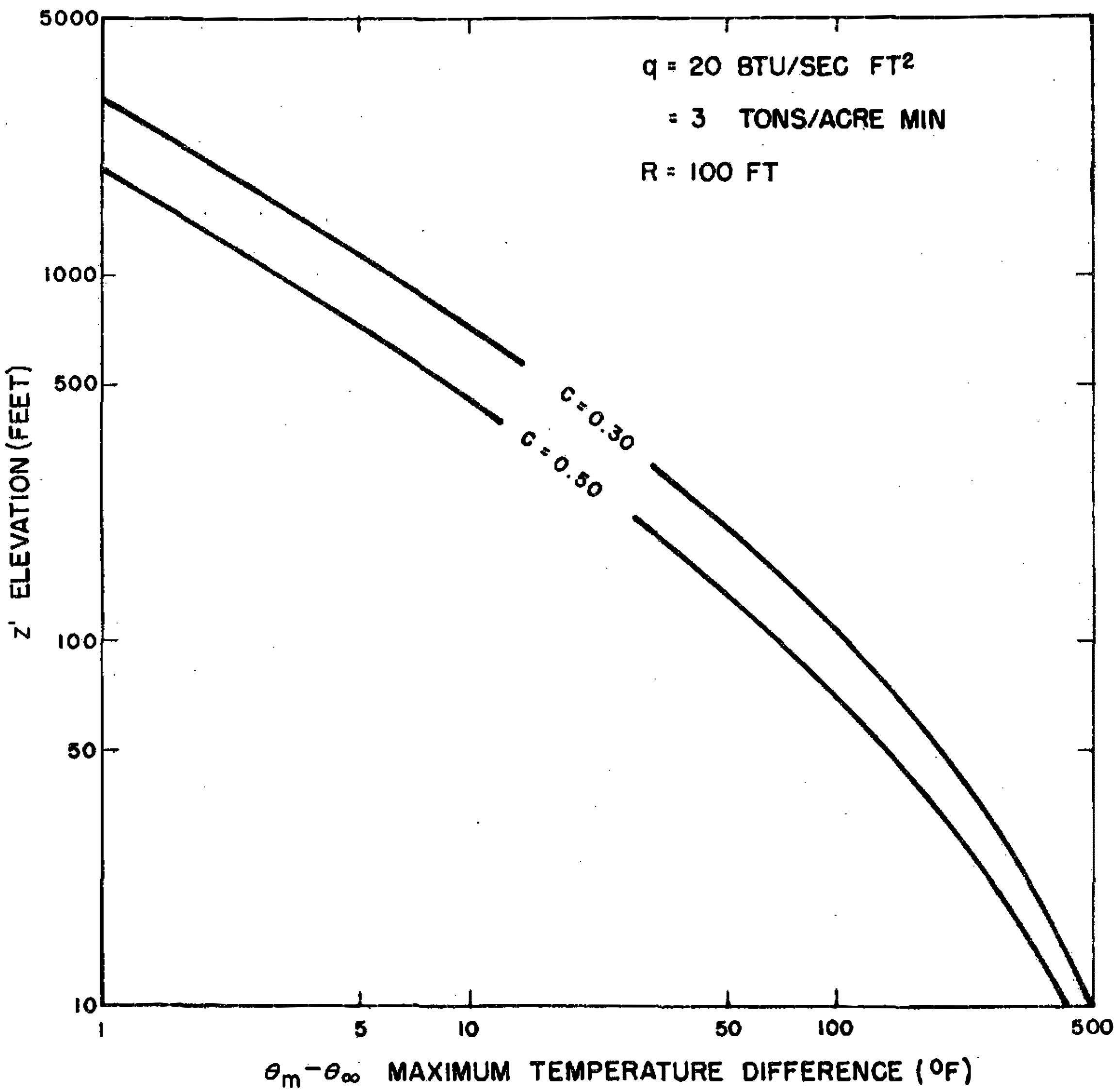

Figure 7.--Theoretical finite area source maximum temperature variation

NOMENCLATURE

The following nomenclature is used in the paper:

B = Function of equation (74)

c = Mixing length proportionality constant

c_p = Specific heat at constant pressure; BTU/lb. °F

d = Function of equations (71) and (72), ft.^{2}/sec.i

D = Non-dimensional function of equations (73), (74) and (75)

$f(\eta)$ = Function of the independent variable

g = Gravitational constant, ft./sec.^{2}

$h(\eta)$ = Function of the independent variable

$H(\eta_1)$ = Function of the independent variable

J = Mechanical heat equivalent, ft.lb./BTU

k = Thermal conductivity, BTU/sec.-sq.ft. °F/ft.

ℓ = Mixing length, ft.

L = Line source length, ft.i

p = Pressure in source zone, psf

P_∞ = Atmospheric pressure, psf

Q = Heat output of source, BTU/sec.

q = Heat output per unit area, BTU/sec.-ft.^{2}i

R = Radius of finite source, ft.

T = Temperature in source zone, °R

T_∞ = Ambient temperature, °R

ui = Velocity component in vertical direction, fpsi

U_m = Vertical velocity at source axis, fps

U = Function of equations (73) and (74), 1/ft.t

v = Velocity component perpendicular to source axis, fpst

y = Distance normal to source axis, ft.

z = Distance vertical from point source, ft.

z' = Distance vertical from finite source, ft.

z_0 = Fictitious vertical distance for finite source, ft.

η = Independent variable

η_1 = Turbulent independent variable

$\theta(z,\phi)$ = Potential temperature for approximate method appearing in equation (69), $^{\circ}R$

(z,∞) = Atmospheric potential temperature for approximate method, $^{\circ}R$

$(0,\infty)$ = Surface potential temperature for approximate method, $^{\circ}R$

Θt = Function of equations (73), (74) and (75)

μ = Absolute viscosity, lb.sec./ft.^{2}

ν_∞ = Ambient kinematic viscosity, ft.^{2}/sec.

ρ = Mass density of source zone, lb.sec.^{2}/ft.^{4}

ρ_∞ = Ambient mass density, lb.sec.^{2}/ft.^{4}

σ = Prandtl Number, $\mu c_p/k$ non-dimensionalt

ϕ = Heat output function

ψ = Stream function

θ = Potential temperature in source zone, $^{\circ}R$

θ_∞ = Ambient potential temperature, $^{\circ}R$

θ_m = Maximum potential temperature in source zone, $^{\circ}R$

δ = Action zone, ft.

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