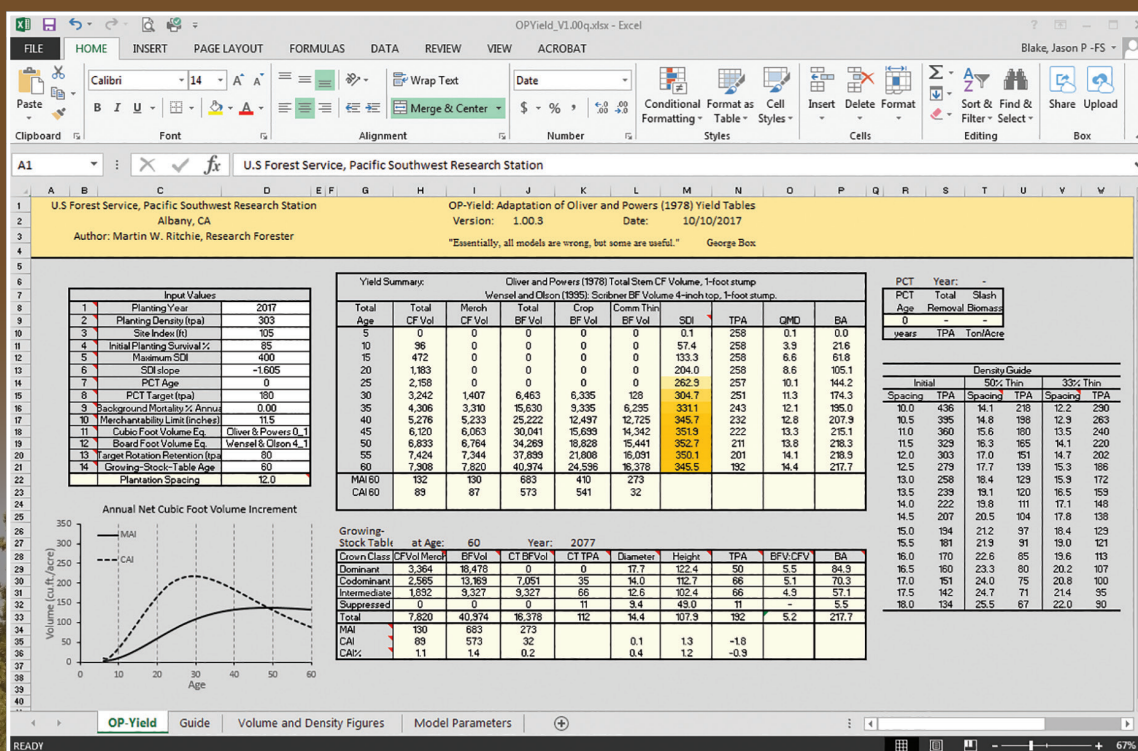




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# OP-Yield Version 1.00 User's Guide

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Cover photos: Screenshot of OP-Yield spreadsheet.

## Abstract

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OP-Yield is a Microsoft Excel™ spreadsheet with 14 specified user inputs to derive custom yield estimates using the original Oliver and Powers (1978) functions as the foundation. It presents yields for ponderosa pine (*Pinus ponderosa* Lawson & C. Lawson) plantations in northern California. The basic model forms for dominant- and subdominant-height (Powers and Oliver 1978), height-diameter relationships, and stand density index-driven mortality (Reineke 1933) were derived from the original publication in the first implementation of this model. However, after reviewing trends, the height-diameter relationships were revised by refitting the model with more current data. Thus, results do not match the original.

In the original Oliver and Powers (1978) model, many controlling parameters were fixed, significantly limiting the ability to interpolate. These fixed parameters also have relegated the original model obsolete in some respects. However, in OP-Yield several fixed parameters in the original system are allowed to flex. The other major difference between OP-Yield and the original presentation is the derivation of cubic-foot volume by age and site. In Oliver and Powers (1978), final yield is derived from a complex polynomial smoothing function of site and age. In OP-Yield, volume is instead an expression of these preliminary calculations. In this sense, OP-Yield volumes are internally consistent. With these changes, yields differ between the two systems, even for situations in which the settings in OP-Yield match the assumptions of Oliver and Powers (1978). In general, volume and basal area are substantially less in OP-Yield than in forecasts by Powers and Oliver (1978).

Keywords: Ponderosa pine, yield, growth model, density management.

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## Introduction

OP-Yield is a Microsoft Excel<sup>1</sup> spreadsheet with 14 specified user inputs to derive custom yield estimates using the original Oliver and Powers (1978) functions as the foundation. The only functional change is in the height-diameter relationship, which was updated with observed values from the initial spacing study at Challenge Experimental Forest (Zhang et al. 2006). Oliver and Powers (1978) presented a system of equations for yield of ponderosa pine (*Pinus ponderosa* Lawson & C. Lawson) plantations in northern California. Yields were presented for site index (Powers and Oliver 1978) values of 40, 60, 80, 100, and 120 ft (index of total age 50) for ages 10 to 50 by 10-year increments. Yields were presented in total stem cubic feet per acre. Densities were fixed in Oliver and Powers at 6-, 8-, 10-, and 12-ft spacing.

Yield tables are a very simplified form of a whole stand-growth model (Munro 1974) in that the inputs are stand-level inputs. No tree-level input is required to access yield in Oliver and Powers (1978). Instead, the user specifies age, initial spacing, and site index (Powers and Oliver 1978). Technically, Oliver and Powers (1978) is a form of a disaggregative model (Ritchie and Hann 1997) as, in addition to traditional yield summary information (e.g., volumes at particular ages), some information is disaggregated to crown class cohorts of the stand.

There are several problems with the application of Oliver and Powers (1978). First, spacing is fixed at only four levels. The user therefore cannot apply it to spacing wider than 12 ft, despite the application of plantation spacings below this level having largely fallen from common use. Also, the user cannot explore spacings between these fixed levels without some ad hoc method for interpolating results.

Second, yields in Oliver and Powers (1978) are only provided at 5-year intervals; the user needs to interpolate these values if others are desired for periods between these intervals.

Third, Oliver and Powers (1978) employed several assumptions that are overly restrictive and not necessarily in keeping with our current understanding of plantation dynamics. Among these assumptions are those associated with mortality rates, maximum density, and the specified cubic-foot-volume equation. No board-foot yields are presented in Oliver and Powers (1978).

Fourth, Oliver and Powers (1978) did not employ any merchantability limit. Total stem cubic-foot volume with a 1-ft stump is the only output for volume. These volumes include tops and volume for trees below a size (diameter) that is sufficient

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<sup>1</sup> The use of trade or firm names in this publication is for reader information and does not imply endorsement by the U.S. Department of Agriculture of any product or service.

for commercial thinning. This is an important restriction because currently, decision-makers need to develop a better understanding of commercial thinning thresholds. Managers are increasingly employing management strategies to minimize risk from disturbance, and, in the case of ponderosa pine, the primary concerns are with fire (Keyes and O'Hara 2002) and bark-beetle-induced mortality (Fettig et al. 2007).

Finally, Oliver and Powers (1978) used a complex multinomial smoothing function for final yields that excludes any possibility of extrapolation to wider spacings or intermediate spacings.

The OP-Yield adaptation of Oliver and Powers (1978) provides approximations of the original yield system using most of the foundational equations in the original publication. Among these is the assumption that competing vegetation is effectively treated such that trees experience little or no competition from hardwoods and shrubs. Stands with inadequate vegetation control will fail to reach the yield forecasts within the time displayed. Yields will be delayed and mortality will generally increase without vegetation control. In addition, rotation lengths will necessarily be extended and commercial harvest delayed. There is no means to account for competing vegetation impacts directly with OP-Yield.

## Methods

In this system of equations, the primary function is that for dominant height ( $H_D$ ) as a function of site ( $S$ ) and age ( $A$ ) (Powers and Oliver 1978):

$$H_D = (1.88 S - 7.178)[1 - \exp(-0.025A)]^{0.001S + 1.640} \quad (1)$$

## Proportion of Stems by Crown Class

The second element was a system to partition trees per acre by crown class (see fig. 4 in Oliver and Powers 1978). Because these relationships are only presented graphically, it is not possible to reproduce them with absolute numeric precision. In OP-Yield, this figure is closely approximated by the following system of equations for dominant, codominant, intermediate, and suppressed trees:

$$P_D = 0.7313[X - 1]^{-0.2428 - 0.003992(X - 1)} \quad (2)$$

$$P_C = 1 - P_D - P_I - P_S \quad (3)$$

$$P_I = 0.7279 + 0.02321(X - 1) - 0.008762[X - 1]^{1.2} - 0.7403[X - 1]^{-0.1} \quad (4)$$

$$P_S = 3.8721[1 - \exp(-0.002807(X - 1))]^{1.5023} \quad (5)$$

Where  $P_i$  proportion of stems in dominant ( $P_D$ ), codominant ( $P_C$ ), intermediate ( $P_I$ ), and suppressed ( $P_S$ ) crown class and,  $X = \text{trees acre}^{-1} \times H_D \div 1,000$  (fig. 1).

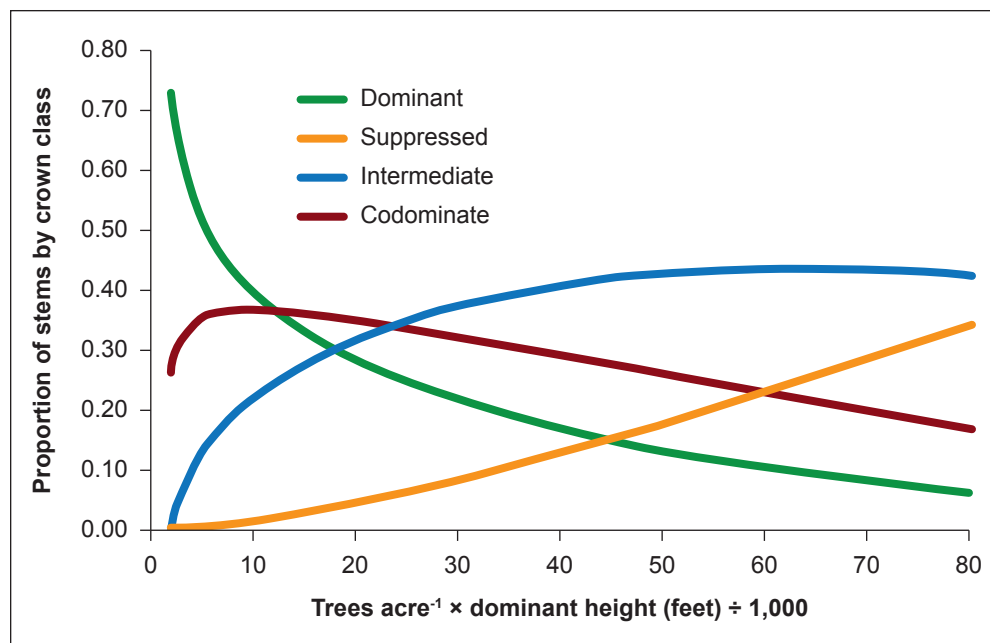


Figure 1—OP-Yield approximation of figure 4 from Oliver and Powers (1978).

## Heights by Crown Class

Heights for codominant ( $H_C$ ) and intermediate ( $H_I$ ) trees were derived from Oliver and Powers (1971):

$$H_C = 0.929 H_D - 1.009 \quad (6)$$

$$H_I = 0.848 H_D - 1.433 \quad (7)$$

In the absence of any empirical data, Oliver and Powers (1978) estimated mean height of suppressed trees as 40 percent of  $H_D$ .

### Diameter estimation—

Mean diameter by crown class ( $D_i$ ) was estimated by Oliver and Powers (1978) with spacing (ft) and height above breast height (ft):

$$\ln(D_D) = -1.6107 + 0.6625 \ln((H_D - 4.5) \times S) \quad 8$$

$$\ln(D_C) = -1.4217 + 0.6173 \ln((H_C - 4.5) \times S) \quad 9$$

$$\ln(D_I) = -1.4332 + 0.6008 \ln((H_I - 4.5) \times S) \quad 10$$

$$\ln(D_S) = -1.4 + 0.54 \ln((H_S - 4.5) \times S) \quad 11$$

Where  $S$  is tree spacing in feet.

However, on close examination of results in early versions of OP-Yield, several problems were evident. Yields were high and mortality also seemed high when

compared to the Challenge Initial Spacing Study data (Oliver 1979, 1990; Zhang et al. 2006). The most prominent driver of this appeared to be a misfit of the original equations (8–11). This may be related to an overestimate of the size of dominant and codominant trees.

The diameter-height relationship was refit for the current version of OP-Yield (see appendix), producing substantially lower yields.

## Mortality

First growing season mortality was fixed by Oliver and Powers (1978) at 15 percent. In OP-Yield, the user defines initial growing season survival as a proportion rather than a percentage. Survival may be specified as any value between 100 and 75 percent. The default is 85 percent, which matches the Oliver and Powers (1978) assumption of 15 percent mortality. Given high plantation survival rates frequently observed under current regeneration practice in California, this default value may be low for most applications.

Oliver and Powers (1978) assumed no annual background mortality after the first year; this is probably unrealistic. In OP-Yield, the user can define a value between 0 and 1 percent annual mortality. The default is zero, but a value around 0.2 to 0.4 is probably more reasonable.

The density-related mortality as stated by Oliver and Powers (1978) was initiated with a maximum stand density index (SDI) (Reineke 1933) of 488:

$$\ln(TPA) = 10.2687 - 1.7712 \ln(QMD) \quad (12)$$

Where  $TPA$  = trees acre<sup>-1</sup> and  $QMD$  = quadratic mean diameter (inches). Mortality ( $M_i$ ) is allocated among stems using the equation presented in Oliver and Powers (1978) as derived from Larson (1975):

$$M_i = 0.276 + 0.003(D_i - 260) \quad (13)$$

For  $D_i$  derived from equations 8 through 11.

By default, OP-Yield comes with the SDI set at 400, as 488 is high for this species. In the original Oliver and Powers formulation, the slope of the SDI relationship in log-log space was defined as -1.7712 (also see Oliver 1995), rather than the value -1.605 that Reineke (1933) presented. Another similar slope, very close to that used in Oliver and Powers (1978), is -1.7653 (Cochran 1992, DeMars and Barrett 1987). A fourth slope found in the literature is -1.661 that Edminster (1988) presented. In OP-Yield, the slope may be selected from these four values. The default is -1.605. In general, slightly greater mortality occurs with the steeper slopes.

Oliver and Powers (1978) did not allow any precommercial thinning. OP-Yield allows the user to specify a year (between 5 and 15) for precommercial thinning and a target density. It is not possible to allow precommercial thinning before this date because some elements of the system do not extrapolate to lower ages. For this reason, Oliver and Powers (1978) set a lower limit of age 10 for output. Because OP-Yield derives component volumes, it can initiate a yield stream at age 5 without any evident problems.

## Application

The OP-Yield spreadsheet has several tabs. The primary tab to be selected, and the only one with active cells which the user can modify is called "OP-Yield." There are currently several dynamic and static tables and a single figure in the "OP-Yield" tab. This tab displays four dynamic tables (1) the input values table, (2) a yield summary table, (3) a PCT (precommercial thinning) summary table, and (4) a single stock (crown class) table for the user-specified age. It also displays one dynamic figure: total cubic-foot volume, mean annual increment (MAI), and current annual increment (CAI). Last, there are two static tables presented as guides for the user.

Users are only allowed to change the values in the white input box of the input values table. Note that there are pop-up comments in the table, which activate when the user places the cursor over a cell with red triangles.

## OP-Yield Input

The input table (table 1) has 14 user-specified values and 1 derived value (spacing):

The input values are:

1. Planting year.
2. Planting density in trees acre<sup>-1</sup>, range 130 to 1,210 TPA.
3. Site index (feet) at a total age 50 (Powers and Oliver 1971), range 40 to 125.
4. First-year survival percentage, range 75 to 100.
5. Maximum SDI, acceptable range 365 to 500.
6. Dropdown box for the slope term from SDI (Reineke 1933), acceptable values of -1.7712 (Oliver and Powers 1978), 1.7653 (DeMars and Barrett 1987), -1.605 (Reineke 1933), and -1.661 (Edminster 1988).
7. Precommercial thin age, acceptable values, 0, 5 to 15; 0 implies no thinning. This is a drop-box selection.
8. PCT target retention in stems acre<sup>-1</sup>, acceptable range 80 to 300.
9. Background annual rate of mortality percentage, acceptable range 0 to 1.
10. Merchantability range 10 to 16 inch quadratic mean diameter.
11. Text string describing volume equation for cubic-foot volume. This is a drop-box selection.

12. Text string describing volume equation for Scribner board-foot volume.  
This is a drop-box selection.
13. Target for retention of trees to rotation. This is the number of stems acre<sup>-1</sup> that the user would anticipate should be retained to full rotation. This value is used to estimate how much volume is available for a commercial thin at various stages in the yield stream.
14. Age for display in the stock table.

**Table 1—Input table showing default settings in OP-Yield Version 1.00**

| Input values |                                 |                     |
|--------------|---------------------------------|---------------------|
| 1            | Planting year                   | 2017                |
| 2            | Planting density (TPA)          | 303                 |
| 3            | Site index (feet)               | 105                 |
| 4            | Initial planting survival (%)   | 85                  |
| 5            | Maximum SDI                     | 400                 |
| 6            | SDI slope                       | -1.605              |
| 7            | PCT age                         | 0                   |
| 8            | PCT target (TPA)                | 200                 |
| 9            | Background mortality (% annual) | 0                   |
| 10           | Merchantability limit (inches)  | 11.5                |
| 11           | Cubic-foot volume equation      | Oliver & Powers 0_1 |
| 12           | Board-foot volume equation      | Wensel & Olson 4_1  |
| 13           | Commercial thin retention (TPA) | 80                  |
| 14           | Growing-stock table age         | 40                  |
|              | Plantation spacing              | 12.0                |

TPA = tree acre<sup>-1</sup>; SDI = site density index; PCT = precommercial thinning.

The final value plantation spacing is a value in feet as derived from plantation spacing (table index item 1)  $Spacing = \sqrt{\frac{43560}{TPA}}$ . Note that the ranges of values acceptable for input are fixed (see “Model Parameters” tab). If a value is input outside the range, a box will pop up with the warning, “Value you entered is not valid.”

#### **Item 1: planting year—**

This input item is functionally irrelevant; it can be any value the user desires. The only impact is in the calculation of the year displayed in the growing stock table. Otherwise it has no effect on values in yield.

**Item 2: planting density—**

Planting density determines initial spacing, average spacing changes over time with mortality; it also impacts—the diameter-height relationships in the model and hence volume, and eventually the onset of density-related mortality, as well. The relationship to volume is both direct and indirect. Density directly influences volume: more trees equals more volume. However, it also has an indirect effect on volume because at higher densities, trees will have a smaller diameter for a given height, which translates to less volume.

**Item 3: site index—**

The site index used by OP-Yield is from Powers and Oliver (1978). The site index is the height of dominants at total age of 50 years, measured in feet. Use of OP-Yield assumes that the site index has been determined by means appropriate for the application of Powers and Oliver (1978). The site index influences the driving variable in the system, which is height. The height-over-age relationship from Powers and Oliver (1978) establishes the growth trajectory for trees in the dominant crown class. Subdominant crown-class height trajectory is derived from this as a modifier on height growth as described earlier.

**Item 4: first-year survival—**

This value determines the number of trees that will carry forward in the simulation. If one specifies a density of 303 trees acre<sup>-1</sup>, and a survival percentage of 90 percent, then the forecast will begin with 273 trees acre<sup>-1</sup>. Although Powers and Oliver (1978) employed a survival of 85 percent, this is probably a little low for most pine plantations in California given current regeneration practices.

**Item 5: maximum site density index—**

Density-induced mortality is driven by both the location of the SDI maximum defined by the user and the slope of the relationship, which is also selected by the user. In the original Oliver and Powers (1978) yield estimation, the maximum was set at 488 and the slope at -1.7712. There is some evidence that 488 value may be high for some applications, particularly where western pine beetle (*Dendroctonus brevicomis* LeCante) is present in west-side stands. The lower the value selected by the user, the earlier density-related mortality will take effect. Stands in OP-Yield will not exceed the chosen value for SDI in this input field.

**Item 6: stand density index slope term—**

The SDI slope is the slope of a line relating the log of trees acre<sup>-1</sup> on the y-axis and the quadratic mean diameter on the x-axis (Reineke 1933). Because this line was defined for fully stocked or “normal” stands, this line defines the upper limit of stand density. A review of the literature on ponderosa pine SDI reveals four different values that have been used: -1.605 (Reineke 1933), -1.7712 (Oliver and Powers 1978), -1.7653 (Cochran 1992, DeMars and Barrett 1987), and -1.661 (Edminster 1988). In some published references, these values have been rounded (e.g., Long and Shaw 2005 used -1.6), but in OP-Yield, all trailing digits are employed. The user selects this from a dropdown box.

**Item 7: precommercial thinning age—**

This age selection determines the point at which PCT will remove trees to the level specified for retention in item 8. This selection will influence density and have both direct and indirect influences on volume as with the planting density selection. Acceptable ages are from 5 to 15 or zero to specify no commercial thinning. Timing of PCT has no effect on the long term for OP-Yield; however, it will influence the estimates of slash load from PCT.

**Item 8: precommercial thinning target—**

This is the target density (trees acre<sup>-1</sup>) of trees retained after PCT. The acceptable range is 80 to 300. If a value is selected that is below the density of the stand at the time of PCT, then no thinning takes place. The choice of PCT will significantly affect the onset of density-related mortality. A light precommercial thin, or no commercial thin, will tend to result in the earlier onset of density-related mortality in the yield output. Thinning retention will also influence slash load estimates.

**Item 9: background mortality rate—**

Oliver and Powers (1978) assumed no mortality after the 15 percent first-year mortality, until the onset of density-related mortality. This seems somewhat generous, as even healthy plantations may see some small amount of mortality over time. The user may specify annual mortality at a rate between 0 and 1 percent as a decimal in the range of 0 to 1.00. A value of 0.10 will produce an annual mortality of 1/10<sup>th</sup> of a percent per year. Users should employ caution here because, although values within the acceptable range seem low, they can have a pronounced effect on yield over time. For example, a value of 0.5, although perhaps not unreasonable for natural stands, is probably quite excessive, and unlikely, in California pine plantations.

**Item 10: merchantability limit—**

This is the diameter below which one assumes there is no value in a tree. This value is what separates output of cubic-foot volume and merchantable cubic-foot volume. Thus total cubic-foot volume is the volume in cubic feet of all trees in the stand regardless of size. Merchantable cubic-foot volume is the volume in trees above this limit. Because all trees in a crown class are treated as the same size in OP-Yield, then all trees in a crown class will cross the merchantability threshold at the same time; this can sometimes produce a pulse of merchantable volume.

The merchantability limit also influences total board-foot volume. Because board-foot volume implies a merchantable product, total board-foot volume and its derivatives (crop volume and commercial thin volume) cannot be produced from trees below this limit. So if the diameter of a particular crown class is below the merchantability limit, then there is no board-foot volume for that crown class in OP-Yield.

**Item 11: cubic-foot volume equation—**

This is a dropdown box selection of published ponderosa pine volume equations. There are currently three in OP-Yield. Selection of the default “Oliver & Powers 0\_1” will employ the volume equation used in Oliver and Powers (1978). It is a total stem (0-inch top), 1-ft stump-volume equation. Selection of “Wensel & Olson 0\_1” will employ the total stem, 1-ft stump equation of Wensel and Olsen (1995). Last, selection of “MacLean & Berger 4\_1” will employ the MacLean and Berger (1978) cylindrical form factor equation with a 4-inch top and 1-ft stump.

**Item 12: board-foot (Scribner) volume equation—**

As with item 11, the user selects the board-foot equation. Oliver and Powers (1978) did not produce board-foot volumes as an output. The default selection and the next two in the dropdown list correspond to Scribner board-foot volumes at an 8-, 6-, or 4-inch top with a 1-ft stump from Wensel and Olsen (1995), and the remaining selection is from MacLean and Berger (1976) for an equation with a variable top diameter (between 8 and 10 inches) and a 1-ft stump.

**Item 13: target rotation retention—**

OP-Yield is not capable of simulating a commercial thin directly. However, output in the simulator does allow the user to evaluate when volume becomes available for commercial thinning by modifying this value. This value is the trees acre<sup>-1</sup> that the user would like to keep to rotation if a commercial thin were conducted. OP-Yield then uses this value to divide the volume between the trees that would be retained to rotation (referred to as crop volume in the output) and the remainder that is available for commercial thinning if the specified TPA were retained.

**Item 14: growing-stock-table age—**

This item simply selects the age to use in the growing-stock table, it has no effect on the yield projections. The growing-stock table takes the year selected in this field and expands on what is in the yield stream for that given year. It breaks down volume, basal area, etc. by crown class. The utility of the growing-stock table is in evaluating the condition of the growing stock as it pertains to commercial thinning potential at that point in stand development.

**Yield Summary**

The yield summary shows ages from 5 to 60 at 5-year intervals (table 2). Intermediate age values are displayed on a separate tab. The yield summary has columns for age, total cubic-foot volume, merchantable cubic-foot volume, total Scribner board-foot volume, crop volume, noncrop volume, SDI, TPA, and QMD. Merchantable cubic-foot volume is the volume for crown class trees above the merchantable limit (input item 10). Scribner board-foot volume equations are applied only for trees above 12 inches in diameter at breast height in the MacLean and Berger (1976) equations. MacLean and Berger (1976) used a cylindrical form factor equation that extrapolates very poorly and requires a lower diameter limit. Crop volume is the volume of trees in superior crown classes that meet the target rotation trees acre<sup>-1</sup>.

Units on cubic-foot volume are cubic-feet acre<sup>-1</sup>. Units on board-foot volume are board feet acre<sup>-1</sup>. Stand density index (Reineke 1933) units are trees acre<sup>-1</sup> at an index quadratic mean diameter of 10 inches. The SDI is calculated with the original (Reineke 1933) using the original formulation:

$$SDI = TPA(QMD/10)^{-rs}$$

Where *rs* is the slope term selected in item 6 in table 2.

QMD is the diameter associated with the tree of mean basal area (Curtis and Marshall 2000).

**Stock Table**

The stock table (table 3) is derived from the stock table age provided in input table item 14. The values in the stock table (by crown class) are largely self-explanatory. The table includes total basal area (square feet acre<sup>-1</sup>), average diameter (inches), height (feet), and the board-foot:cubic-foot ratio. The CT BFVol column shows the board-foot volume available at the selected age (input item 14) assuming a thin from below with retention density specified as “Commercial Thin Retention” (input item 13). In like manner, CT TPA is the number of trees associated with the potential commercial thin using input items 13 and 14.

**Table 2—Yield summary from default settings in OP-Yield Version 1.00**

| Yield summary: |                 | Oliver and Powers (1978): total stem cubic-foot (CF) volume, 1-ft stump<br>Wensel and Olson (1995): Scribner board-foot (BF) volume 4-inch top, 1-ft stump |                 |                |                           |       |     |      |       |
|----------------|-----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|----------------|---------------------------|-------|-----|------|-------|
| Total age      | Total CF volume | Merchantable CF volume                                                                                                                                     | Total BF volume | Crop BF volume | Commercial thin BF volume | SDI   | TPA | QMD  | BA    |
| 5              | 0               | 0                                                                                                                                                          | 0               | 0              | 0                         | 0.1   | 258 | 0.1  | 0     |
| 10             | 96              | 0                                                                                                                                                          | 0               | 0              | 0                         | 57.4  | 258 | 3.9  | 21.6  |
| 15             | 472             | 0                                                                                                                                                          | 0               | 0              | 0                         | 133.3 | 258 | 6.6  | 61.8  |
| 20             | 1,183           | 0                                                                                                                                                          | 0               | 0              | 0                         | 204.0 | 258 | 8.6  | 105.1 |
| 25             | 2,158           | 0                                                                                                                                                          | 0               | 0              | 0                         | 262.9 | 257 | 10.1 | 144.2 |
| 30             | 3,242           | 1,407                                                                                                                                                      | 6,463           | 6,335          | 128                       | 304.7 | 251 | 11.3 | 174.3 |
| 35             | 4,306           | 3,310                                                                                                                                                      | 15,630          | 9,335          | 6,295                     | 331.1 | 243 | 12.1 | 195.0 |
| 40             | 5,276           | 5,233                                                                                                                                                      | 25,222          | 12,497         | 12,725                    | 345.7 | 232 | 12.8 | 207.9 |
| 45             | 6,120           | 6,063                                                                                                                                                      | 30,041          | 15,699         | 14,342                    | 351.9 | 222 | 13.3 | 215.1 |
| 50             | 6,833           | 6,764                                                                                                                                                      | 34,269          | 18,828         | 15,441                    | 352.7 | 211 | 13.8 | 218.3 |
| 55             | 7,424           | 7,344                                                                                                                                                      | 37,899          | 21,808         | 16,091                    | 350.1 | 201 | 14.1 | 218.9 |
| 60             | 7,908           | 7,820                                                                                                                                                      | 40,974          | 24,596         | 16,378                    | 345.5 | 192 | 14.4 | 217.7 |
| MAI 60         | 131             | 130                                                                                                                                                        | 683             | 410            | 273                       | —     | —   | —    | —     |
| CAI 60         | 89              | 87                                                                                                                                                         | 573             | 541            | 32                        | —     | —   | —    | —     |

— = no value for this item.

Note: SDI = stand density index; TPA = trees acre<sup>-1</sup>; QMD = quadratic mean diameter (inches); BA = basal area (square feet acre<sup>-1</sup>).

An important caveat regarding table 3: the CAI and CAI% values for board-foot volume (BFvol) can be very misleading. Because the model tracks cohorts of trees, all trees in a given cohort achieve a board-foot volume at one point in time. This can create spikes in the annual increment for board-foot volume as shown in table 3. If the user specifies an age immediately above or below the default of 40 displayed, then these values drop.

The yields in OP-Yield are expressed as an emergent property of the model specifications and assumed volume equations. This is unlike Oliver and Powers (1978), wherein volume is ultimately smoothed with a cumbersome multinomial of site and age, with parameters varying by 2-ft spacing intervals:

$$\ln(Y) = \alpha_{0i} + \alpha_{1i}A + \alpha_{2i}S + \alpha_{3i}A^2 + \alpha_{4i}S^2 + \alpha_{5i}A^4S^2 + \alpha_{6i}A^2S^1 + \alpha_{7i}A^1S^2$$

Where Y = net cubic-foot yield, A = 1/age since planting, SI = site index, and  $\alpha_{si}$  = parameters for spacings (i) from 6 to 12 ft by 2-ft increments. This complex function defies extrapolation, and, for this reason, it was not included to smooth volume output in OP-Yield.

**Table 3—Stocking table from default settings in OP-Yield version 1.00 using default settings (age = 40)**

| Stock table  | At age: 40                           | Year: 2057           |                          |        |          |        |      |             |       |
|--------------|--------------------------------------|----------------------|--------------------------|--------|----------|--------|------|-------------|-------|
| Crown class  | Merchantable<br>cubic-foot<br>volume | Board-foot<br>volume | CT board-<br>foot volume | CT TPA | Diameter | Height | TPA  | BFV:<br>CFV | BA    |
| Dominant     | 2,210                                | 11,079               | 0                        | 0      | 14.8     | 85.4   | 66   | 5.0         | 79.9  |
| Codominant   | 1,796                                | 8,532                | 7,113                    | 68     | 12.6     | 78.4   | 82   | 4.8         | 70.8  |
| Intermediate | 1,227                                | 5,611                | 5,611                    | 74     | 11.5     | 71.0   | 74   | 4.6         | 53.3  |
| Suppressed   | 0                                    | 0                    | 0                        | 11     | 8.2      | 34.2   | 11   | —           | 3.9   |
| Total        | 5,233                                | 25,222               | 12,725                   | 152    | 12.8     | 76.0   | 232  | 4.8         | 207.9 |
| MAI          | 131                                  | 631                  | 318                      | —      | —        | —      | —    | —           | —     |
| CAI          | 184                                  | 6,382                | 674                      | —      | 0.1      | 1.9    | -2.1 | —           | —     |
| CAI%         | 3.5                                  | 25.3                 | 45.1                     | —      | 1.0      | 2.5    | -0.9 | —           | —     |

— = no value for this item.

Note: CT = commercial thin, TPA = trees acre<sup>-1</sup>, BFV:CFV = board-foot to cubic-foot volume ratio, BA = basal area (square feet acre<sup>-1</sup>), MAI = mean annual increment (cubic feet acre<sup>-1</sup>), CAI = current annual increment (cubic feet acre<sup>-1</sup>), CAI% = current annual increment expressed as percentage.

### Mean Annual Increment and Current Annual Increment Figure

The MAI/CAI figure displays total cubic foot volume (net) from the selected volume equation (the default selection is the equation presented by Oliver and Powers (1978). The figure stops at the maximum age for OP-Yield, 60 (fig. 2).

Cubic-foot volume equations in OP-Yield are user specified. The default equation for cubic-foot volume was presented by Oliver and Powers (1978):

$$V = 0.024846 + 0.0017645 D^2 H \quad (14)$$

This equation (14) is for total stem, excluding a 1-ft stump. It is referenced as “Oliver & Powers 0\_1” in the input list. Other equations presented for selection are the Wensel and Olson (1995) equations for total stem cubic-foot volume with a 1-ft stump (“Wensel & Olson 0\_1”) and the MacLean and Burger (1976) equations for cubic-foot volume at a 4-inch top and with a 1-ft-stump allowance (“Maclean & Berger 4\_1”).

All equations used by Wensel and Olson (1995) are of the form:

$$V = \beta_0 DBH^{\beta_1} H^{\beta_2} \beta_3^{DBH} \quad (15)$$

The MacLean and Berger (1976) equation is a cylindrical form factor derivation:

$$V = CV [\beta_0 + \beta_1 / DBH] \quad (16)$$

Where  $CV$  is the volume of a cylinder using  $DBH$  and  $H$ :  $CV = 0.005454154(DBH^2)H$ .

In the current version, the only volume presented in the MAI/CAI figure (fig. 2) is the total stem cubic-foot volume; board-foot volume is not an option for this figure.

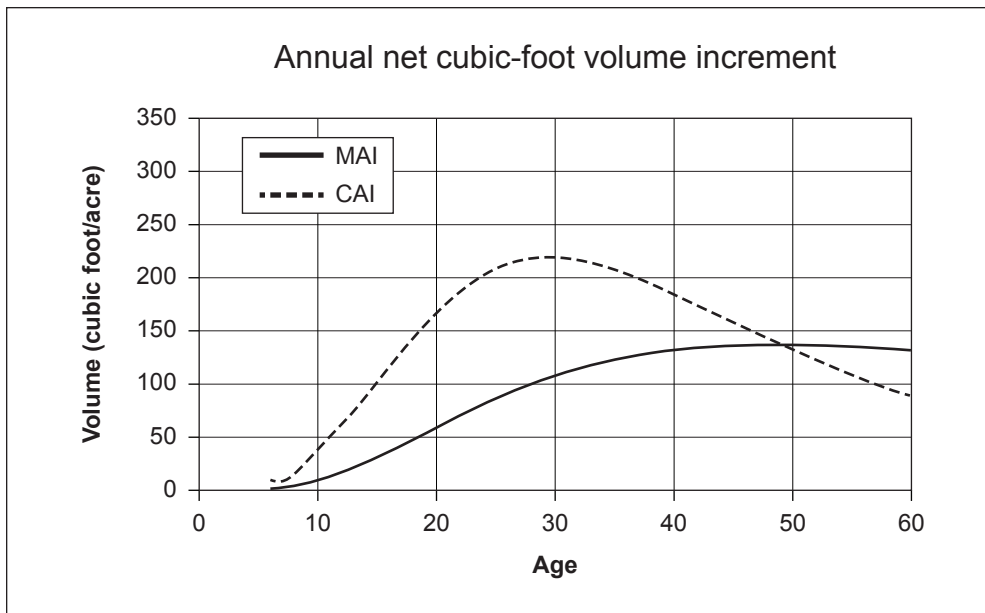


Figure 2—Mean annual increment (MAI) and current annual increment (CAI) display from OP-Yield using default input.

## Precommercial Thinning Table

The PCT table displays the year of PCT and three additional values (table 4). The first is the user-selected age for PCT (0 = no thinning; 5–15). The second item is the number of trees acre<sup>-1</sup> removed. The third is the slash load produced (tons acre<sup>-1</sup>) assuming felled trees are left untreated. Trees are removed from below. Slash load is calculated by first deriving a basal diameter from *DBH* in U.S. units. Then biomass was estimated using the equation of Powers et al. (2013) in metric units after converting basal diameter to centimeters. All values are converted to English units in application.

**Table 4—Precommercial thin (PCT) table using default values in OP-Yield 1.00, because default is no PCT (age = 0) the values for stems per acre and biomass are nil**

| PCT   | Year    | —         |
|-------|---------|-----------|
| PCT   | Total   | Slash     |
| Age   | removal | biomass   |
| 0     | —       | —         |
| Years | TPA     | Tons/acre |

— = ?

Note: TPA = trees acre<sup>-1</sup>.

## Static Density Table

There is one static table included on the OP-Yield tab and another in the “Volume and Density Figures” tab. The first is the static density guide (table 5). This table shows initial planting spacing along with spacing (density) associated with a 50 percent free thin and a 33 percent free thin. Spacing (in feet) in this context is the square root of average area (growing space) per tree. This table is presented as a quick guide for the user to relate spacing and density.

**Table 5—Static spacing (feet) guide in OP-Yield 1.00, where spacing is the square root of area tree<sup>-1</sup> derived from trees acre<sup>-1</sup> (TPA)**

|                  |     | Density guide |     |               |     |
|------------------|-----|---------------|-----|---------------|-----|
| Initial planting |     | 50% free thin |     | 33% free thin |     |
| Spacing          | TPA | Spacing       | TPA | Spacing       | TPA |
| 10.0             | 436 | 14.1          | 218 | 12.2          | 290 |
| 10.5             | 395 | 14.8          | 198 | 12.9          | 263 |
| 11.0             | 360 | 15.6          | 180 | 13.5          | 240 |
| 11.5             | 329 | 16.3          | 165 | 14.1          | 220 |
| 12.0             | 303 | 17.0          | 151 | 14.7          | 202 |
| 12.5             | 279 | 17.7          | 139 | 15.3          | 186 |
| 13.0             | 258 | 18.4          | 129 | 15.9          | 172 |
| 13.5             | 239 | 19.1          | 120 | 16.5          | 159 |
| 14.0             | 222 | 19.8          | 111 | 17.1          | 148 |
| 14.5             | 207 | 20.5          | 104 | 17.8          | 138 |
| 15.0             | 194 | 21.2          | 97  | 18.4          | 129 |
| 15.5             | 181 | 21.9          | 91  | 19.0          | 121 |
| 16.0             | 170 | 22.6          | 85  | 19.6          | 113 |
| 16.5             | 160 | 23.3          | 80  | 20.2          | 107 |
| 17.0             | 151 | 24.0          | 75  | 20.8          | 100 |
| 17.5             | 142 | 24.7          | 71  | 21.4          | 95  |
| 18.0             | 134 | 25.5          | 67  | 22.0          | 90  |

## Target Precommercial Thinning Density Table

A final dynamic static table provides density (trees acre<sup>-1</sup>) as a function of the upper level management zone (e.g., Edminster 1988, Long and Shaw 2002), the quadratic mean diameter at which the stand is commercially viable (table 6) and the SDI slope chosen. It is provided as a guide to decisions on PCT intensity. The upper limit of the management zone (UMZ) is considered the density at which probability of mortality crosses the critical threshold, expressed as a percent of the maximum. Long and Shaw (2005) placed this at SDI = 250. Edminster (1988) used SDI = 245, and Oliver (1995) used 230, so there is no universally accepted level for the UMZ, although they are generally in the same vicinity. The body of this table gives the maximum trees per acre after establishment (planting minus precommercial thin) that will keep the stand out of the zone of imminent mortality before reaching the merchantable quadratic mean diameter for a chosen pair of values: the UMZ and quadratic mean diameter anticipated for the stand at the time of a commercial thinning.

**Table 6—Trees acre<sup>-1</sup> for precommercial thinning (PCT) as a function of merchantable quadratic mean diameter (QMD) and the upper limit management zone (UMZ) stand density index (SDI), assuming SDI slope of -1.605**

| UMZ<br>SDI | Merchantable QMD          |     |     |     |     |     |     |     |     |     |     |
|------------|---------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|            | 8                         | 9   | 10  | 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  |
|            | <i>----- Inches -----</i> |     |     |     |     |     |     |     |     |     |     |
| 260        | 372                       | 308 | 260 | 223 | 194 | 171 | 152 | 136 | 122 | 111 | 101 |
| 250        | 358                       | 296 | 250 | 215 | 187 | 164 | 146 | 130 | 118 | 107 | 97  |
| 240        | 343                       | 284 | 240 | 206 | 179 | 158 | 140 | 125 | 113 | 102 | 93  |
| 230        | 329                       | 272 | 230 | 197 | 172 | 151 | 134 | 120 | 108 | 98  | 90  |
| 220        | 315                       | 261 | 220 | 189 | 164 | 144 | 128 | 115 | 103 | 94  | 86  |
| 210        | 300                       | 249 | 210 | 180 | 157 | 138 | 122 | 110 | 99  | 90  | 82  |
| 200        | 286                       | 237 | 200 | 172 | 149 | 131 | 117 | 104 | 94  | 85  | 78  |
| 190        | 272                       | 225 | 190 | 163 | 142 | 125 | 111 | 99  | 89  | 81  | 74  |
| 180        | 258                       | 213 | 180 | 154 | 134 | 118 | 105 | 94  | 85  | 77  | 70  |
| 170        | 243                       | 201 | 170 | 146 | 127 | 112 | 99  | 89  | 80  | 73  | 66  |
| 160        | 229                       | 189 | 160 | 137 | 119 | 105 | 93  | 83  | 75  | 68  | 62  |

### Other Tabs

Several tabs in the spreadsheet contain information on the derivation of the yield streams in OP-Yield. The “Volume and Density Graphs” tab displays the density by crown class from Oliver and Powers (1978) and the stand density over time.

The “Model Parameters” tab is a sheet that contains all the driving parameters and acceptable ranges for input to drive the yield stream. This worksheet is locked and will not accept input from the user. The “Crown Class Proportions” tab displays the original crown class allocation from Oliver and Powers (1978). The “Guide” tab holds the current user’s guide document in Adobe portable document format (pdf). Adobe Reader will be necessary to read the pdf file.

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### Metric Equivalents

| When you know:      | Multiply by: | To get:                  |
|---------------------|--------------|--------------------------|
| Inches              | 2.54         | Centimeters              |
| Feet (ft)           | 0.305        | Meters                   |
| Acres               | 0.405        | Hectares                 |
| Cubic feet per acre | 0.0700       | Cubic meters per hectare |
| Tons                | 907.185      | Kilograms                |

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## Appendix: Mixed-Model Analysis of Diameter-Height Relationship

The original models presented by Oliver and Powers (1978), when compared with long-term observations on height and diameter, appeared to overestimate diameter considerably. We opted to refit these equations with the hope that resulting behavior of the system of equations will provide more reasonable and more defensible growth projections. This is the only major reworking of the functionality of the system of equations.

The yield tables presented by Oliver and Powers (1978) employed a system of equations for the four crown classes (dominant, codominant, intermediate, and suppressed) with both mean tree height and spacing as predictors. The model form for fitting expressed tree breast height diameter ( $D$ ) as a function of total tree height ( $H$ ) and spacing ( $S$ ) was modified from the original so as to reduce Akaike information criterion (AIC). Fitting was done with a log transformation of  $D$ . The best dataset we could find for this effort came from the Challenge Initial Spacing Study (Oliver 1979), which has been observed with repeated measurements, over time, of trees established in 1966 across a range of spacings from 6 to 18 ft. The most recent measurement in this dataset was in 2012 at age 45. We used only plots free from shrub competition (Oliver 1979). Crown class was identified by a heuristic that specified dominant trees as those greater than or equal to 85 percent of the site-index-derived height of dominants predicted from Powers and Oliver (1978). Codominants were between 75 and 85 percent. Intermediates were between 60 and 75 percent. Suppressed trees were all those below 60 percent.

The Challenge dataset includes 117 ponderosa pine trees measured periodically from 1970 to 2012 across a range of spacings: 6-, 9-, 12-, 15-, and 18-ft for a total of 1,633 observed height-diameter pairs.

Here is the corresponding mixed-model notation with  $i, j, k$  and  $l$  representing the  $i$ -th crown class ( $i = 1$  for dominants,  $i = 2$  for codominants,  $i = 3$  for intermediates, and  $i = 4$  for suppressed), the  $j$ -th plot, the  $k$ -th tree, and the  $l$ -th observation on the  $k$ -th tree:

$$\log D_{ijkl} = a_i + \alpha_{ijk} + (b_i + \beta_{ijk}) \log(H_{ijkl} - 4.5) + (c_i + \gamma_{ijk}) \log S_{ijkl} + d_i \log(H_{ijkl} - 4.5) \times \log S_{ijkl} + e_i [\log(H_{ijkl} - 4.5) \times (H - 4.5)^{0.25}] + f/S_{ijkl} + \epsilon_{ijkl}$$

where the random effects  $\alpha$ ,  $\beta$ , and  $\gamma$  are assumed multivariate normal with expectation of zero and variance-covariance matrix  $\Sigma$ :

$$\begin{pmatrix} \alpha_{ijk} \\ \beta_{ijk} \\ \gamma_{ijk} \end{pmatrix} \sim \text{Normal} \left( \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_I^2 & \sigma_{IH} & \sigma_{IS} \\ \sigma_{IH} & \sigma_H^2 & \sigma_{HS} \\ \sigma_{IS} & \sigma_{HS} & \sigma_S^2 \end{pmatrix} \right)$$

and errors  $\epsilon_{ijkl} \sim N(0, \sigma^2)$ . The remaining terms in the model: ai, bi, ci, di, ei, and f are fixed-effects. Note that the last fixed-effect is not crown-class specific. The model was fit using SAS Proc Mixed with random statement subject = tree and type = unstructured. Our variance and covariance estimates from this fit were:

$$\hat{\sigma}^2 = 0.00406$$

$$\hat{\Sigma} = \begin{pmatrix} +0.224 & +0.00938 & -0.0933 \\ +0.00938 & +0.00282 & -0.0606 \\ -0.0933 & -0.00606 & +0.0421 \end{pmatrix}$$

the variance of  $\log D_{ijkl}$  is:  $Var(\alpha_{ijk} + \beta_{ijk} \log(H_{ijkl} - 4.5) + \gamma_{ijk} S_{ijkl} + \epsilon_{ijkl})$ :

$$Var(\log D_{ijkl}) = \sigma^2 + (1, \log(H_{ijkl} - 4.5), \log S_{ijkl}) \begin{pmatrix} \sigma_I^2 & \sigma_{IH} & \sigma_{IS} \\ \sigma_{IH} & \sigma_H^2 & \sigma_{HS} \\ \sigma_{IS} & \sigma_{HS} & \sigma_S^2 \end{pmatrix} \begin{pmatrix} 1 \\ \log(H_{ijkl} - 4.5) \\ \log S_{ijkl} \end{pmatrix}$$

$$= \sigma^2 + \sigma_I^2 + 2 \log(H_{ijkl} - 4.5) \sigma_{IH} + 2 \log S_{ijkl} \sigma_{IS} + (\log(H_{ijkl} - 4.5))^2 \sigma_H^2 +$$

$$2 \log(H_{ijkl} - 4.5) \log S_{ijkl} \sigma_{HS} + (\log S_{ijkl})^2 \sigma_S^2 \quad (A-1)$$

A naive estimate (Clifford et al. 2013) for a tree diameter of crown class  $i$  is then:

$$\hat{D}_i = \exp \{a_i + d_i[\log H_i - 4.5] \times \log(S)] + e_i[\log(H_i - 4.5) \times (H - 4.5)^{0.25}] + f/S\}$$

$$\times (H_i - 4.5)^{b_i} (S)^{c_i} \quad (A-2)$$

where  $H_i$  is the height associated with the  $i$ -th crown class, and fixed-effects-parameter estimates from table A-1 can be used for a median unbiased estimate of breast height diameter. However, this naive estimate is mean-biased (low) by somewhere between 1 and 3 percent in our analysis. A number of corrections have been suggested (see Clifford et al. 2013, Flewelling and Pienaar 1981) for this bias. We tested several of the corrections suggested by Clifford et al. (2013) and found all of these to be practically indistinguishable from each other in our analysis. The simplest of

**Table A-1—Fixed-effects parameter estimates (and standard errors) from mixed-model fit of equation 12**

| Crown class: | Parameter estimates for diameter-height relationship refit |                  |                  |                  |
|--------------|------------------------------------------------------------|------------------|------------------|------------------|
|              | Dominant                                                   | Codominant       | Intermediate     | Suppressed       |
| <i>a</i>     | 0.5099 (0.65)                                              | 0.7905 (0.65)    | 0.9514 (0.71)    | 0.1804 (0.73)    |
| <i>b</i>     | 0.4424 (0.036)                                             | 0.4108 (0.045)   | 0.3419 (0.095)   | 0.5938 (0.11)    |
| <i>c</i>     | -0.2442 (0.20)                                             | -0.3888 (0.20)   | -0.3981 (0.22)   | -0.05807 (0.24)  |
| <i>d</i>     | 0.1452 (0.015)                                             | 0.2178 (0.018)   | 0.2324 (0.034)   | 0.1333 (0.040)   |
| <i>e</i>     | -0.04849 (0.0029)                                          | -0.1023 (0.0058) | -0.1071 (0.015)  | -0.1234 (0.014)  |
| <i>f</i>     | -2.1059 (1.2989)                                           | -2.1059 (1.2989) | -2.1059 (1.2989) | -2.1059 (1.2989) |

these is analogous to the bias correction of Baskerville (1972) in which we insert our estimates  $\hat{\sigma}^2$  and  $\hat{\Sigma}$  into equation A-1 for correction factor  $C$ :

$$C = \exp\{0.5 \times \text{variance estimate}\}$$

$$= \exp\{0.5 \times [\hat{\sigma}^2 + \hat{\sigma}_I^2 + 2 \log(H - 4.5) \hat{\sigma}_{IH}^2 + 2 \log S \hat{\sigma}_{IS} + (\log(H - 4.5))^2 \hat{\sigma}_H^2 + 2 \log(H - 4.5) \log S \hat{\sigma}_{HS} + (\log S)^2 \hat{\sigma}_S^2]\}$$

Although for this analysis the correction factor,  $C$ , was often negligible across the range of heights and spacings observed, we did nonetheless apply this correction to the naive estimate (equation A-2). The back-transformed model fit, both corrected and uncorrected, with the data are displayed in figures (A-1 through A-4).

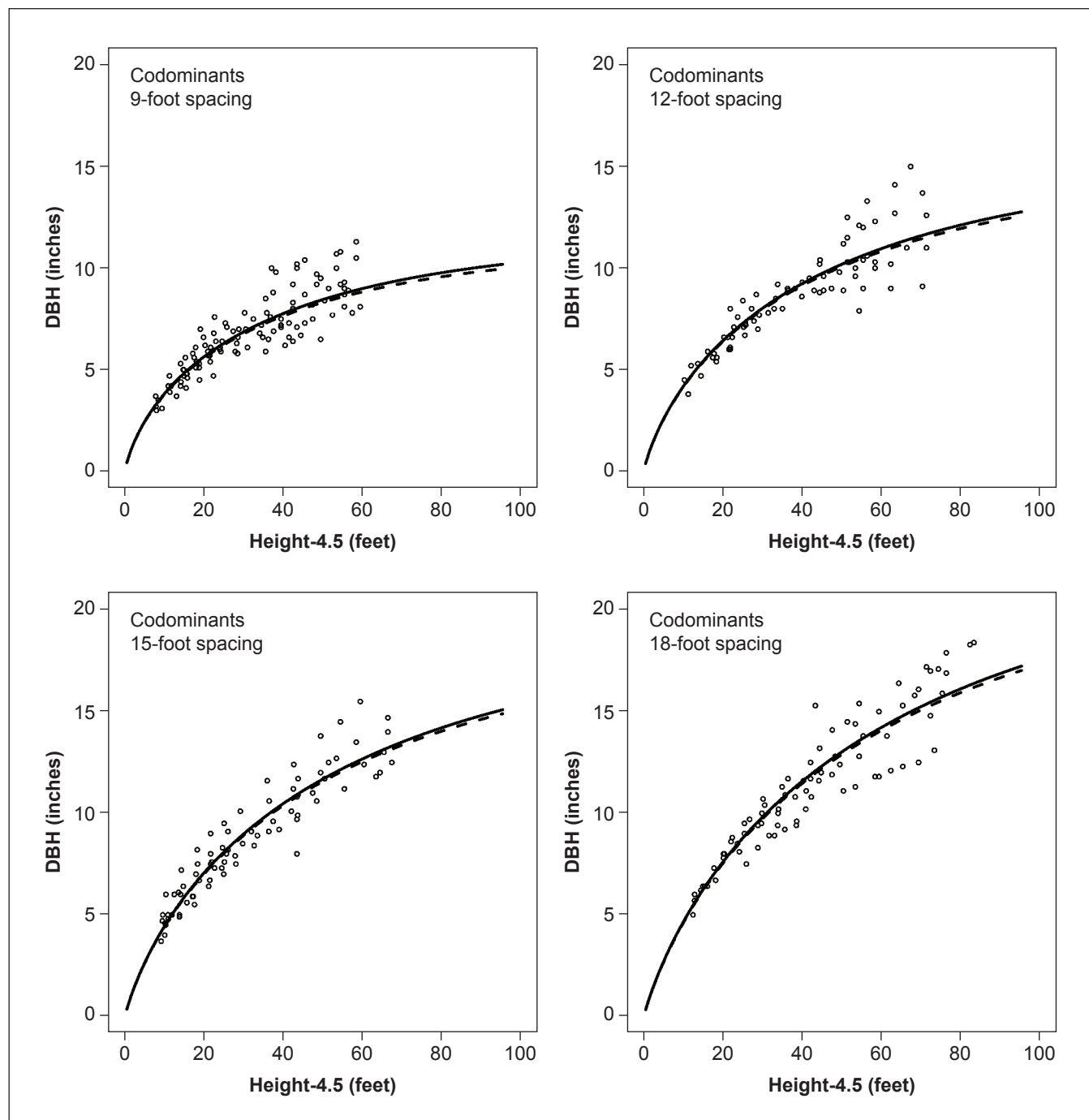


Figure A-2—Modeled diameter corrected (solid line) and uncorrected (dashed line) for codominant trees at four different plantation spacings. DBH = diameter at breast height.

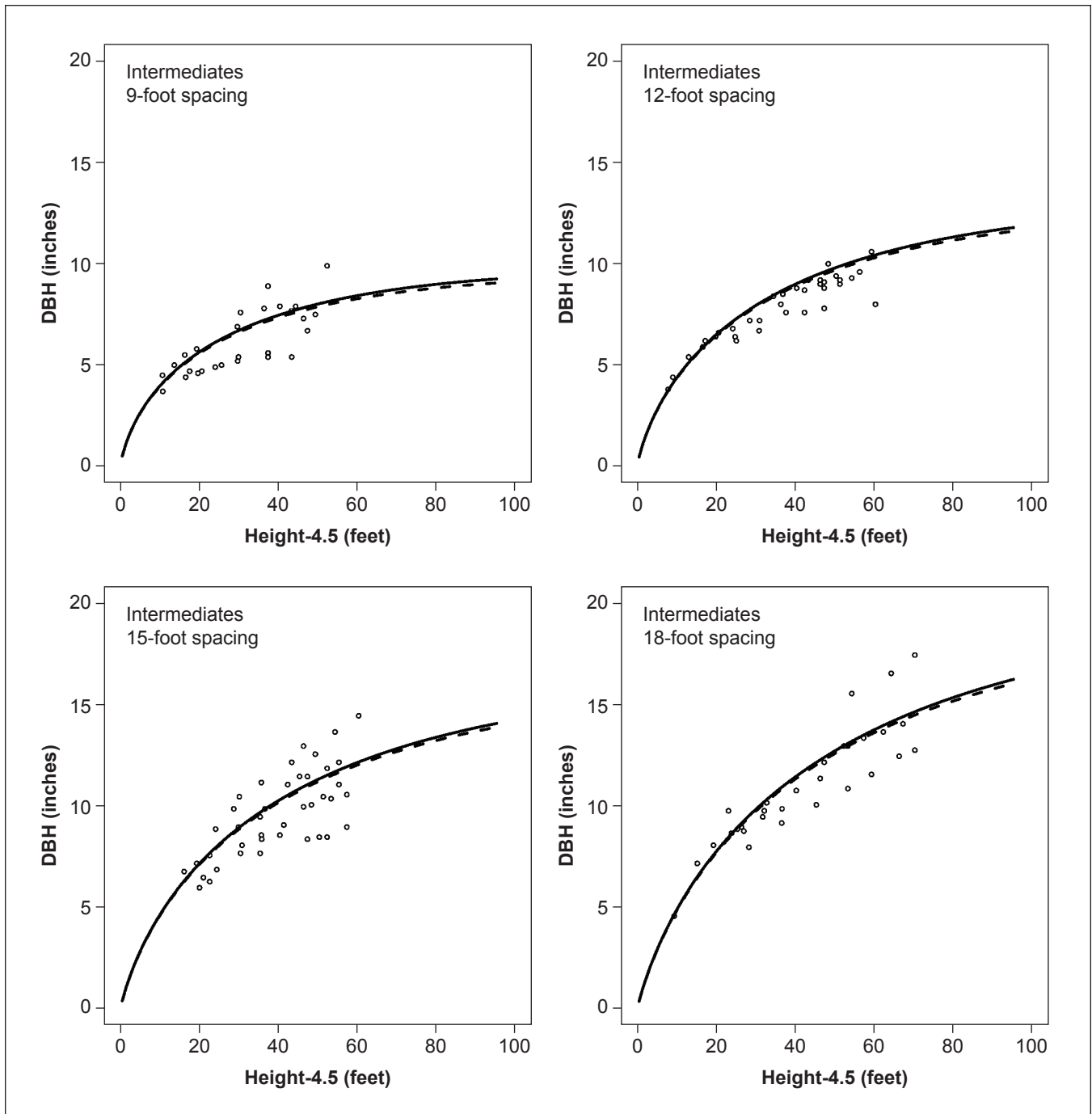


Figure A-3—Modeled diameter corrected (solid line) and uncorrected (dashed line) for intermediate trees at four different plantation spacings. DBH = diameter at breast height.

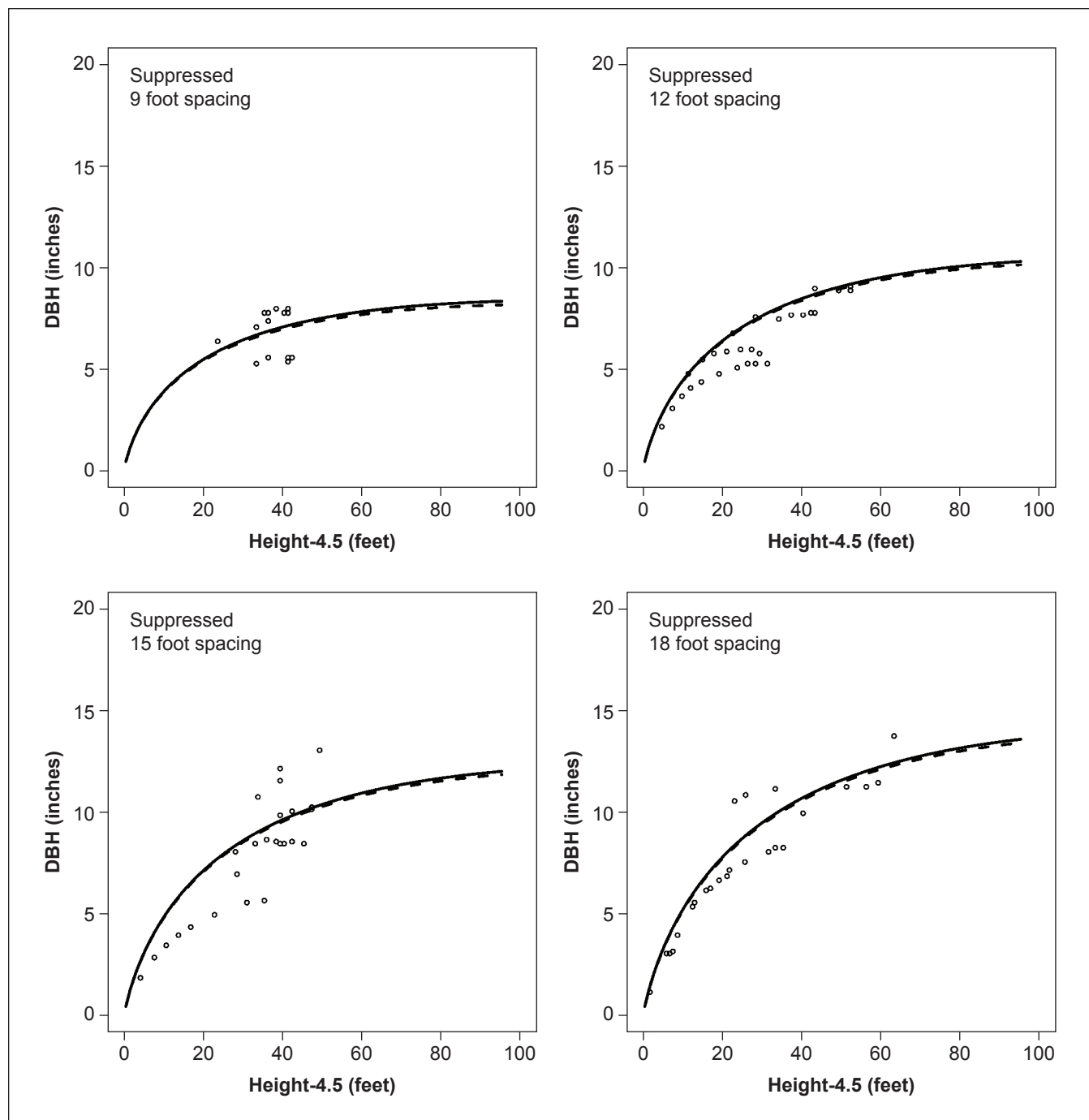


Figure A-4—Modeled diameter corrected (solid line) and uncorrected (dashed line) for suppressed trees at four different plantation spacings. DBH = diameter at breast height.







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