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Copula-based nonlinear modeling of the law of one price for lumber products

Barry K. Goodwin[1] · **Matthew T. Holt**[2] · **Gülcan Önel**[3] · **Jeffrey P. Prestemon**[4]

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Abstract This paper proposes an alternative and potentially novel approach to analyzing the law of one price in a nonlinear fashion. Copula-based models that consider the joint distribution of prices separated by space are developed and applied to weekly prices for lumber products. The copulas capture nonlinearities that arise in the extremes of the joint distributions of price differentials and suggest faster equilibrating adjustments when deviations from parity are extreme.

Keywords Law of one price · Copulas · Nonlinear time series models

JEL Classification F-020 · C-580

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✉ Barry K. Goodwin
barry_goodwin@ncsu.edu

[1] Department of Economics, North Carolina State University, Box 8109, Raleigh, NC 27695, USA

[2] Department of Economics, Finance, and Legal Studies, University of Alabama, Box 870224, Tuscaloosa, AL 35487, USA

[3] Food and Resource Economics Department, University of Florida, Gainesville, FL 32611-0240, USA

[4] Forestry Sciences Laboratory, USDA Forest Service, 3041 East Cornwallis Road, Research Triangle Park, NC 27709, USA

1 Introduction

Notions of price parity, spatial arbitrage, and price transmission characterize many basic principles and relationships in economics. At the core, markets should efficiently function so as to eliminate any potential for riskless profits through arbitrage and trade. This fundamental condition is often called the "Law of One Price" (LOP). There has been considerable interest in and debate about the empirical validity of the Law of One Price (LOP), especially as it pertains to markets for tradable goods. On the one hand, economists take it as being nearly axiomatic that freely functioning markets for traded, homogeneous products should ensure that prices are efficiently linked across regional markets, the implication being that no persistent opportunities for spatial arbitrage profits exist.[1] The general implication underlying these basic concepts is that prices for homogeneous products at different geographic locations in otherwise freely functioning markets should differ by no more than transport and transactions costs. A large body of research often finds that the adjustment lags required to restore arbitrage equilibria are far longer than would seem natural based upon any reasonable understanding of the mechanics of physical trade as it pertains to the markets in question.

Recent research on spatial price linkages has turned to nonlinear models in an attempt to better capture the effects of unobservable transactions costs. In this paper, we propose an extension to these nonlinear models in the form of copulas. We use various goodness-of-fit criteria to compare copula model estimates to the conventional error correction models typically used to evaluate price parity. Our empirical analysis provides strong evidence that copula models better capture the nonlinearities inherent in spatial price linkages than is the case for more conventional approaches. Our application is to regional North American markets for oriented strand board (OSB), an important construction material that is widely traded among regional markets. We find that alternative copula specifications are generally favored over a Gaussian copula as well as over linear and nonlinear error correction models of the type typically applied in empirical studies of spatial market integration. An evaluation of the copula estimates suggests that patterns of adjustment to large deviations from parity conditions differ from adjustments to smaller price shocks and thus confirm the presence of nonlinear patterns of adjustment.

Early empirical studies generally fail to support the LOP. Isard (1977) found rather conclusive evidence against the LOP using disaggregate data for traded goods. Isard's conclusions were subsequently confirmed for a variety of commodities in a wide array of market settings by, among others, Richardson (1978), Thursby et al. (1986), Benninga and Protopapadakis (1988), and Giovannini (1988). Goodwin et al. (1990), however, found some support for the LOP when it is specified in terms of price expectations as opposed to observed prices. Following Engle and Granger's seminal paper (1987), cointegration techniques have been widely used to rationalize the LOP as a long-run concept. By adopting this view of the LOP, economists have been able to find more compelling evidence in favor of the LOP, including, for example, Buongiorno

[1] Distinctions between tests of LOP and spatial market integration are not especially meaningful. In both cases, the economic phenomena being evaluated (spatial market arbitrage) is identical. A survey of both strands of literature can be found in Fackler and Goodwin (2001).

and Uusivuori (1992) (US pulp and paper exports), Michael et al. (1994) (international wheat prices), Bessler and Fuller (1993) (US regional wheat markets), and Jung and Doroodian (1994) (softwood lumber markets).

The most recent literature in this area applies nonlinear refinements of autoregressive or vector error correction models in analyzing price relationships. The underlying motivation is that adjustments to equilibrium may not be linear and that this nonlinearity may, in turn, be associated with hard-to-observe transactions costs associated with arbitrage. The theoretical underpinnings for transactions costs-induced nonlinearity in the LOP are put forward by Dumas (1992), although the basic idea dates back at least to the work of Heckscher (1916), who notes that transactions costs may define "commodity points" within which prices are not directly linked because the price differences are less than the costs of trade.

The recent turn in the literature toward models that allow for nonlinearity in price linkages includes applications of the smooth transition autoregressive (STAR) models of Teräsvirta (1994) and the discrete threshold error correction models of Balke and Fomby (1997). A recent example includes an analysis of manufactured lumber products (oriented strand board or OSB) in the USA undertaken by Goodwin et al. (2011). Their analysis applies STAR models to consider price relationships among spatially distinct North American markets for manufactured OSB. The application is notable in light of the extreme volatility of markets for construction materials and recent litigation that charged that OSB manufacturers had practiced discriminatory and noncompetitive pricing during the latter part of the decade. The analysis reveals that nonlinearity is an important feature of price relationships in these markets and that the price parity relationships implied by economic theory and efficient arbitrage are generally supported by the STAR models. We extend the work of Goodwin et al. (2011) to consider a wider range of models for evaluating price parity relationships. In particular, we undertake a comparison of linear and STAR error correction models to the alternative specifications inherent in a variety of copula models. We find that copula specifications are uniformly favored for evaluating price linkages in OSB markets.

Although copula models have been extensively used in financial economics and risk management studies, they have not been extensively applied in modeling nonlinear, spatial arbitrage relationships.[2] In our application, copula models are used to capture differences in patterns of adjustment that may arise when price differentials are very large (i.e., large enough to exceed a transactions cost band). Copulas allow for varying degrees of tail dependence and thus are able to capture patterns of adjustment that may arise when price differentials are extreme.

Our approach is a natural extension of the existing (and abundant) time series evaluations of spatial price linkages. It involves direct examination of the joint probability

[2] Patton (2006) allowed for time variation in the conditional joint distribution of the returns on the Deutsche mark/US dollar and Japanese Yen/US dollar exchange rates by allowing the parameter(s) of a given copula to vary through time. Smith et al. (2011) examined linkages among logarithmic prices in regional Australian electricity markets but do not explicitly model the nonlinear error correction adjustment process that is standard in spatial arbitrage models and that we consider here. Reboredo (2011) also examined the comovement of crude oil prices using copulas, though his approach is also fundamentally different from that considered here in that it does not directly consider the error correction process commonly applied in models of spatial price linkages.

distribution of the key economic variables of interest. In this way, the approach is really no different than standard maximum likelihood methods applied to structural or nonstructural econometric models. However, we give particular attention to the nature of the jointness or correlation between these key variables. In particular, we allow this correlation to be "state dependent" and therefore to depend upon market conditions at any particular point in time. This is accomplished by considering a number of different copula specifications that allow for varying dependency structures. Our empirical application is to a set of 6 spatially distinct North American OSB markets.

We find that the adjustment to large departures from parity tends to provoke more substantial equilibrating adjustments than is the case for smaller deviations. The copulas allow for significant differences in the strength of bivariate relationships among prices in the tails of the marginal distributions of price differentials. That is, very large differences in prices at spatially distinct locations (i.e., extreme values in the tails of the distribution of price differentials) lead to substantially faster and more significant adjustments toward equilibrium parity relationships. This result is revealed through estimates of parametric copula models of the joint distribution characterizing price linkages. This finding is consistent with the results found in other nonlinear, autoregressive models that typically reveal stronger and faster adjustments to larger deviations from parity. We compare our results to the standard vector autoregressive and smooth transition models commonly used to evaluate price parity. Goodness-of-fit criteria are used to select the optimal copula specification and demonstrate that the nonlinear tail behavior inherent in specific parametric copulas provides a better fit to the data than is true for more conventional models. The results provide empirical support for the integration of these markets but also highlight the importance of nonlinearities in the price adjustment process.

2 Econometric models of spatial price relationships

As we have noted, a vast empirical literature has considered a wide array of empirical models of price relationships across space, time, and market form. This literature has evolved from a simple consideration of correlation coefficients and linear regression models to regime switching, time series models that allow for a form of "state dependence" when characterizing price linkages.

Consider a homogeneous commodity traded in a common currency in two regional or international markets represented, respectively, by location indices i and j. The individual market prices are denoted by P_i and P_j. The per-unit revenue to arbitragers selling in region j is therefore $(1-\kappa)P_j$, where κ denotes the per-unit proportional loss in value for the commodity due to transactions (or transport) costs, $0 < \kappa < 1$. In general, the greater the distance between locations i and j, the closer is κ to one. A simple model of the law of one price that incorporates the effects of transactions costs (and possibly other frictions) can be written as:

$$1/(1-\kappa) \geq P_i/P_j \geq (1-\kappa), \tag{1}$$

or, after taking natural logarithms and denoting $p_i = ln(P_i)$ and $p_j = ln(P_j)$,

$$-ln(1-\kappa) \geq (p_i - p_j) \geq ln(1-\kappa). \tag{2}$$

The implication from Eq. 2 is that there is a band, $[-ln(1-\kappa), ln(1-\kappa)]$, within which no profitable arbitrage activity will occur; arbitrage is, however, profitable when log price differentials, $p_i - p_j$, fall outside of the limits of the band. Over time, we would expect that log price differentials within the limits of the band may follow something very close to a unit root process, likely without drift.[3] However, log price differences that fall outside of the limits of the band should be mean reverting. The relationship in Eq. 2 implies a transactions cost band, which has often been assumed in the literature, and which typically yields an empirical model consistent with the threshold or smooth transition autoregressive models described above (see, for example, Goodwin and Piggott 2001 and Balcombe et al. 2007). As noted, these models typically find that market price adjustments to shocks in parity conditions tend to be faster or more apparent when the shocks are large. Threshold models typically allow the speed or degree of adjustment to vary in accordance with the size of the disequilibrium implied in parity relationships. In particular, threshold models usually recognize the fact that small price differences may not exceed the costs of conducting spatial trade and arbitrage, whereas large differences may imply arbitrage opportunities that are more quickly eliminated. In this analysis, we use copula-based models as an alternative nonlinear model that captures much of the same behavior addressed in nonlinear threshold and smooth transition models.

The most recent literature is usually based upon a standard autoregressive model of the form:

$$\Delta(p_t^i - p_t^j) = \alpha + \beta(p_{t-1}^i - p_{t-1}^j) + \epsilon_t, \tag{3}$$

where p^i and p^j are logarithmic prices in regions i and j, respectively, α and β are parameters that reflect the degree of market integration, and ϵ_t is a random, white noise error term.[4] In particular, β represents the degree of "error correction" that characterizes departures from price parity, which are reflected in large values of $p_{t-1}^i - p_{t-1}^j$. The closer β is to zero, the slower will be the adjustment to shocks.[5] The "error" term represents proportional deviations from market equilibrium. In some cases, α is taken to represent a proportional price difference that reflects transactions costs.[6] If one is confident that the price differentials $(p_t^i - p_t^j)$ are stationary, an equivalent representation can be expressed as:

$$(p_t^i - p_t^j) = a + (1+\beta)(p_{t-1}^i - p_{t-1}^j) + \alpha t + \epsilon_t, \tag{4}$$

[3] We are agnostic as to the specific linkage among prices within any transactions cost band and allow this relationship to be determined within the context of the overall joint distribution function. Alternative nonlinear dependencies across the span of the marginals can be accommodated within the range of copula functions that we consider. That said, we expect to see a stronger "error correction" type of relationship for bigger deviations from parity.

[4] See, for example, Taylor (2001), who applies regime switching, time series models of this form to empirical tests of purchasing power parity—an aggregate version of the LOP.

[5] A note of explanation regarding our nomenclature may be helpful in avoiding confusion. Here and below, we refer to values of $(p_t^i - p_t^j)$ as the "price differential" and values of $\Delta(p_t^i - p_t^j)$ as the "differenced price differential.".

[6] A specification that is often referred to as an "iceberg" model, reflecting the fact that the value of the commodity melts away via a proportionally lower price as it is shipped.

where a is a constant that is absent from the first-differenced relationship implied in (3) and t represents a linear time trend.

Recent empirical evaluations of spatial price linkages and the law of one price have recognized that the presence of transactions costs, which are notoriously difficult to measure but, nonetheless, are likely to be relevant in any empirical consideration of spatial commodity trade, may result in nonlinearities in the estimates of Eq. 3. Two specific avenues have been adopted to account for such nonlinearities. In the first, a "threshold" parameter that reflects the presence of transactions costs is estimated. The linkage between prices varies depending upon whether the departure from equilibrium represented by $p^i_{t-1} - p^j_{t-1}$ is large enough to evoke spatial arbitrage. In this case, a discrete break occurs between regimes where one regime may represent a case of no trade, while another represents conditions of profitable trade and arbitrage. These models are typically referred to as "threshold autoregressive" (TAR) or "threshold vector error correction" (TVEC) models. Alternatives to this simple model permit the switching between regimes to occur at a gradual and smooth pace.[7] The speed and degree of adjustment are implied by parameters of a "transition" function that incorporates nonlinear patterns of adjustment. A number of different specifications of such "smooth transition autoregressive" (STAR) models have been developed in the literature. These models have provided considerable flexibility in modeling spatial and vertical price linkages. However, in empirical practice, they often suffer from complications resulting from parameters that may be unidentified under certain null hypotheses and a resulting need to rely upon nonstandard inferential techniques.

Our approach involves a simple extension or recharacterization of the fundamental relationship expressed in Eq. 3. We make use of the widely recognized correspondence between β in Eq. 3 (or the equivalent $1 + \beta$ in 4) and the standard, linear Pearson correlation coefficient:

$$\hat{\beta} = \hat{\rho}\frac{\hat{\sigma_y}}{\hat{\sigma_x}}, \tag{5}$$

where y and x correspond to the random variables $\Delta(p^i_t - p^j_t)$ and $p^i_{t-1} - p^j_{t-1}$, ρ is the Pearson correlation coefficient, and σ_y (σ_x) represents the standard deviation of random variable y (x). The "error correction" relationship that characterizes the linkage between markets i and j is represented in the sample correlation coefficient ρ. To the extent that β realizes regime switching, the coefficient ρ will also reflect switching. If such switching is dependent upon market conditions (i.e., as reflected in the price differential), the correlation coefficient ρ may exhibit state dependence.[8]

The empirical approach adopted here involves considering the joint distribution functions of $\Delta(p^i_t - p^j_t)$ and $p^i_{t-1} - p^j_{t-1}$ and, alternatively, of $p^i_t - p^j_t$ and $p^i_{t-1} - p^j_{t-1}$. We make use of a widely recognized, fundamental result known as Sklar's

[7] The behavior underlying spatial price linkages is likely to be discrete—representing the two states of trade/no trade. However, in that empirical evaluations of such models almost always involve some degree of aggregation, the patterns of adjustment may be of a more smooth nature and therefore may favor the STAR-type models.

[8] As we discuss in greater detail below, the interpretations of the relationships in 3 and 4 are not fully analogous within the context of joint distribution functions since temporal correlation may have important implications for the interpretation of the implied density.

(1959) Theorem, which implies that any continuous $p-$variate cumulative probability function F can be represented using the marginals and a unique copula function $C(\cdot)$ for which

$$F(x_1, x_2, ..., x_p) = C(F_1(x_1), ..., F_p(x_p); \xi), \tag{6}$$

where $F_i(\cdot)$ are marginal distributions and ξ is a set of parameters that characterize dependence. Copula models have recently realized widespread application in empirical models of joint probability distributions. The models essentially use a copula function to tie together marginal probability functions that may be related.[9]

There are a large number of parametric families of copulas applied in the literature. Two of the most commonly used copula families are elliptical copulas and Archimedean copulas. Gaussian and t copulas are examples of elliptical copulas, while the Clayton and Gumbel are among Archimedean copulas. In our analysis, we consider a range of bivariate copula specifications that permit considerable flexibility in representing the relationships and implied patterns of adjustments to prices in spatially distinct markets. As we discuss below, we use goodness-of-fit criteria to choose the optimal specification for each comparison of prices from pairs of OSB markets.[10] Of course, as is true in any model specification problem, an infinite number of specifications exist and our consideration is intended to allow for a wide range of adjustment processes. The copula functions that we consider come from a number of different families of specifications, each of which has its own specific properties. Elliptical copulas have the property of radial symmetry and thus possess identical tail dependencies in extremes of the distribution. A second class of copulas—the Archimedean copulas—permits a straightforward approach to modeling dependencies with a single parameter. We also consider a class of two-parameter copulas that provide a greater degree of flexibility and allow for a wider range of asymmetries than is possible with single-parameter copulas. The elliptical copulas that we consider include the Gaussian and t copulas. Among Archimedean copulas, we evaluate the Clayton, Gumbel, Frank, and Joe specifications. Two-parameter mixtures of Archimedean copulas considered in our analysis include the Clayton–Gumbel, the Joe–Gumbel, the Joe–Clayton, and the Joe–Frank copulas. For all asymmetric copulas, we also consider rotated versions of the functions.[11] Finally, we considered the asymmetric, skewed t copula of Demarta and McNeil (2005). This four-parameter copula nests both the Gaussian and standard t copulas as special cases.

[9] For details on construction and properties of copulas, see, among others, Joe (1997) and Nelsen (2006). Much of the work on copulas has been motivated by their applicability to the issues in risk management, insurance and financial economics (see, among others, Rodriguez (2003), Cherubini et al. (2004), Hu (2006), Patton (2006), and Jondeau and Rockinger (2006)). Extensive surveys on the use of copulas in economics and finance are provided by Patton (2009, 2012) and Patton and Fan (2014).

[10] Specifically, we consider goodness of fit for the following eighteen parametric copulas: the Gaussian, t, Clayton, Gumbel, Frank, Joe, Clayton–Gumbel, Joe–Gumbel, Joe–Clayton, and the Joe–Frank copulas. We also consider 180-degree rotations of the asymmetric copulas, which include all of the preceding except for the Gaussian and t copulas.

[11] A copula is rotated 180° by using $1 - u_i^x$ in place of u_i^x, where u_i^x is the quantile corresponding to the marginal distribution for x_i. Such rotations (by 90° or 270°) can accommodate inverse correlations in copulas that can only represent positive correlation in the unrotated form. Likewise, tail dependencies can be reversed by rotating the copula 180°.

In the case of spatial price relationships and nonlinear patterns of adjustment, the degree of dependence that characterizes relationships in the tails of the distribution is of particular relevance. The notion of smooth or discrete regime switching models that are commonly used in empirical studies of spatial arbitrage is that the relationships among price differentials may be different when such differentials are large. Large price differentials correspond to large deviations from equilibrium arbitrage conditions and should therefore correspond to faster rates of adjustment to return markets to equilibrium conditions. The fact that such dependence may be different for large price differentials reflects the influences of unobservable transactions costs. In particular, price differentials that do not exceed transactions costs do not imply profitable arbitrage opportunities, while large differentials in excess of transactions costs should be eliminated quickly through arbitrage behavior. Copula models are especially well suited to considering tail behavior in that they allow for more flexible characterizations of tail dependence. The coefficients of upper tail dependence, λ_U, and lower tail dependence, λ_L, are defined and expressed as a function of a copula as:

$$\lambda_U = \lim_{u \to 1^-} P(X_2 > F_{X_2}^{-1}(u) | X_1 > F_{X_1}^{-1}(u)) = \frac{1 - 2u + C(u, u)}{1 - u}, \tag{7}$$

and

$$\lambda_L = \lim_{u \to 0^+} P(X_2 \leq F_{X_2}^{-1}(u) | X_1 \leq F_{X_1}^{-1}(u)) = \frac{C(u, u)}{u}. \tag{8}$$

Estimation of copula models proceeds along the lines of general maximum likelihood estimation techniques. Parameters are chosen by maximizing a joint likelihood function. Model fitting criteria such as the optimized log-likelihood function value or Akaike or Schwartz–Bayesian information criteria (AIC and BIC) can also be used. Of course, these criteria are equivalent among single-parameter copulas. In the case of copulas with two or more parameters, the AIC and BIC criteria apply penalties for each parameter estimated, with a stronger penalty being applied by the BIC. In our application, we select from among the set of 18 possible copulas by selecting that which minimizes the value of the BIC. Our results were not particularly sensitive to using the AIC or BIC, though exceptions in terms of the optimal specification selected did occur.[12]

An additional estimation issue pertains to the representation of the marginals. One may choose to estimate parameters describing the marginal densities prior to estimating the copula model in a two-step procedure or, alternatively, estimate the marginals and the copula parameters jointly in maximizing the joint likelihood function. The latter approach will naturally be more efficient but may also present difficult numerical optimization problems. If such a parametric approach to representing the marginals is to be followed, one must have *a priori* knowledge of the appropriate parametric family or must entertain a wider specification evaluation that includes both marginals and the copula. This approach may be preferred if one is interested in properties of the

[12] As a reviewer has noted, comparisons of goodness-of-fit criteria across alternative specifications provide a heuristic basis for comparing alternative specifications. Our approach is reinforced by a consideration of Cramér von Mises (CvM) and Kolmogorov–Smirnov (KS) tests of the specifications.

individual marginals as well as the implied dependency relationships. Alternatively, one can use nonparametric representations of the marginal densities by considering the quantiles of the distributions and focus on a parametric specification of the copula model representing dependence. We adopt this latter approach and use ranks of each individual random variable to determine nonparametric quantile functions.

Our empirical approach involves the use of conventional model fitting criteria (the Bayesian information criterion) to determine the optimal copula specification for each pair of spatially distinct prices from among the 18 different copula functions that we consider. In the case of single-parameter asymmetric copulas, a parametric structure that only allows tail dependence in one direction may be implied. This is justified if trade tends to mostly be unidirectional, as is typical in most regional commodity market relationships. In such cases, depending on the direction of trade flows and which price is usually higher, the sign of these parameters could be negative or positive. This reflects the fact that an increase in the higher price or a decrease in the lower price will trigger a tighter relationship between the two prices (in first-differenced form) in the subsequent period and assumes that markets display a relatively stable basis relationship, such that one price is generally above another (a characteristic that exists in most regional markets, where one market is usually "upstream" and the other is "downstream"). We first model the marginals nonparametrically and then use the quantiles to define and estimate the copulas. We estimate the parameters of the copula models using maximum likelihood techniques.[13]

3 Empirical application

For the empirical analyses, we consider regional North American markets for a prominent traded commodity—oriented strand board (OSB). OSB is a manufactured wood product that was first introduced in 1978 (the forerunner to oriented strand board was waferboard).[14] The Structural Board Association (SBA) reports that in 1980, OSB panel production in North America was 751 million square feet (on a 3/8th's inch basis) and that by as early as 2005, this number had grown to 25 billion square feet. The SBA also reports that by 2000, OSB production exceeded that of plywood, and that by 2006 OSB production enjoyed a sixty-percent market share among all panel products in North America. OSB now accounts for the largest share of the overall panel wood products market.

Spatial linkages in the OSB market are of particular interest because it is a good that is widely traded across considerable distances within the North American continent. Consumption is widespread and spatially dispersed, while production

[13] Estimation and inferences were accomplished using the "COPULA" and "NLP" procedures of SAS and the "copula" and "VineCopula" packages of the *R* language. Details are available in Chvosta et al. (2011), Schepsmeier et al. (2015), and Yan (2012). Excellent overviews of the *R* packages and implementation issues are presented by Yan (2007) and Czado (2011).

[14] OSB is engineered by using waterproof and heat-cured resins and waxes and consists of rectangular-shaped wood strands that are arranged in oriented layers. OSB is produced in long, continuous mats which are then cut into panels of varying sizes. In this regard, OSB is similar to plywood, although OSB is generally considered to have more uniformity than plywood and is, moreover, cheaper to produce.

tends to be concentrated in particular regions such as the USA, South and Eastern Canada. Depletion of old-growth timber stocks that traditionally served as a source for panel wood products brought about tremendous growth in the use of engineered wood products such as OSB. A burgeoning housing market (and its more recent contraction) has brought about a number of significant shocks to this rapidly expanding industry. Construction market responses to large hurricanes such as Andrew in 1992 and Katrina in 2005 are another source of OSB market price volatility that merits clearer understanding for better quantifying the economic impacts of these catastrophic events. These and related factors underscore the importance of understanding and quantitatively measuring linkages among regional OSB markets.

The data set consists of OSB in six regional North American markets. Specifically, the regions examined are (1) Eastern Canada (production deriving from plants in Ontario, Quebec, and New Brunswick); (2) western Canada (production from British Columbia, Manitoba, Saskatchewan, and Alberta); (3) mid-Atlantic USA (production from North Carolina, Virginia, and West Virginia); (4) North Central USA (production deriving from plants in Wisconsin, Michigan, and Minnesota); (5) Southeast USA (production deriving from plants in Georgia, Alabama, Mississippi, South Carolina, and Tennessee); and (6) Southwest USA (production deriving from plants in Texas, Louisiana, Arkansas, and Oklahoma). The result is that there are fifteen pairwise spatial price relationships that may be examined. The price data are for panels of 7/16th's inch thickness and are expressed in US dollars per thousand square feet. All price data are observed on a weekly basis and were obtained from the industry source *Random Lengths*.[15] The data span the period from October 2, 1998, through July 20, 2012, which yields 720 weekly observations. The basic unit of analysis used throughout the analysis is the natural logarithm of the price ratio, that is, $ln(P_t^i/P_t^j)$, where i and j indicate regional locations (i.e., $i, j = 1, \ldots, 6$) and t is a time index such that $t = 1, \ldots, T$, where $T = 720$. The price data are illustrated (in logarithmic and log-differential form) in Fig. 1. Histograms of the marginal distributions of the logarithmic price differentials $ln(P_t^i/P_t^j)$ and the first-differenced log price differentials $\Delta ln(P_t^i/P_t^j)$ are presented in Figs. 2 and 3. Nonparametric kernel density estimates and a normal density are also presented with each histogram. In most cases, the log differentials show a degree of skewness and the first-differenced values appear to be leptokurtic relative to a normal density.

We first consider the time series properties of individual prices and price differentials. The individual logarithmic prices are all nonstationary. Table 1 presents augmented Dickey–Fuller tests of the stationarity of the price differential and the stationarity of the residual of a regression of the first price on the second price of the pair.

[15] *Random Lengths* is an independent, privately owned price reporting service, providing information on commonly produced and consumed wood products in the USA, Canada, and other countries since 1944. Reported open-market sales prices are based on hundreds of weekly telephone interviews with producers, wholesalers, distributors, secondary manufacturers, buying groups, treaters, and some large retailers. The regional OSB price data used are FOB mill price averages

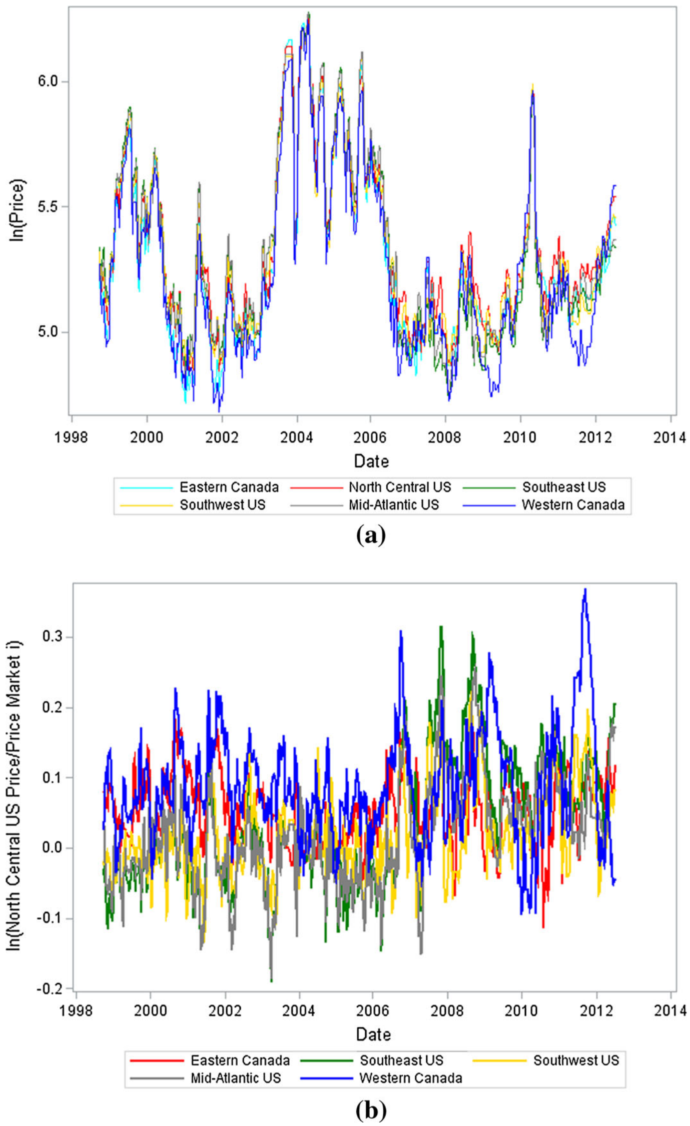


Fig. 1 OSB price series (in log terms). **a** Price series **b** log differences relative to North Central USA

In every case, the tests indicate that the differentials are stationary and the individual prices are cointegrated.[16]

We next consider the relationship between the first difference of the price differential, $\Delta(p_t^i - p_t^j)$, and the lagged price differential, $p_{t-1}^i - p_{t-1}^j$. A standard error

[16] Nonstationarity tests for the individual prices are not presented here but are available on request. The cointegration relationship is evaluated using the residuals from $ln(P_t^i) = \alpha + \beta ln(P_t^j) + \epsilon_t$. Note that a test of stationarity of the price differentials is equivalent to restricting $\alpha = 0$ and $\beta = 1$.

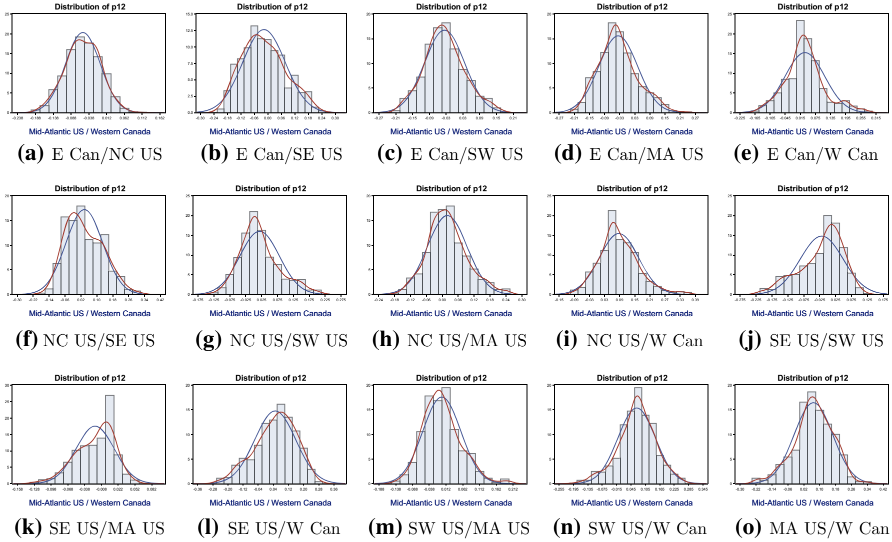


Fig. 2 Histograms and kernel density estimates of logarithmic price differentials ($\Delta(p_i - p_j)$)

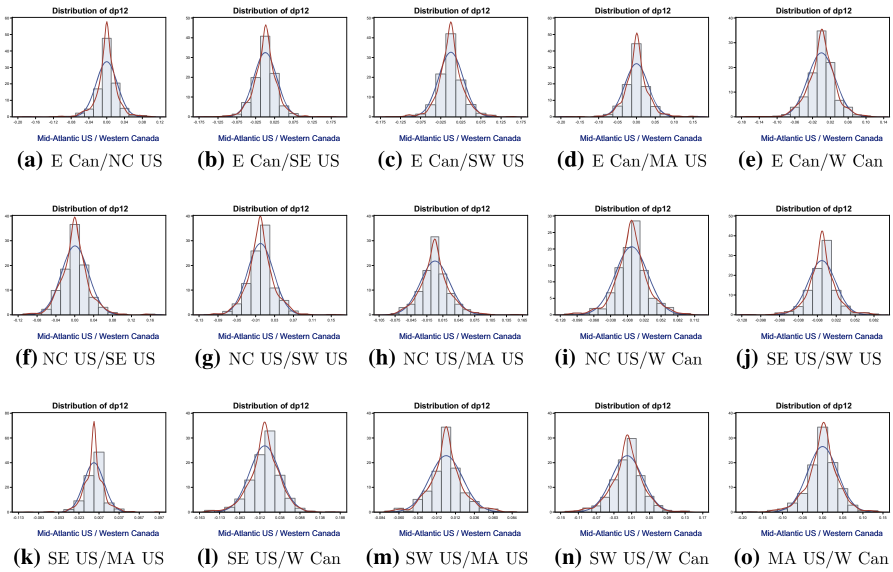


Fig. 3 Histograms and kernel density estimates of logarithmic price differentials ($\Delta(p_i - p_j)$)

correction type of model of the form typically used to evaluate the law of one price (such as in Eq. 3) was estimated using OLS methods. The time series properties of the price differentials were also evaluated using augmented Dickey–Fuller tests. A finding of nonstationarity in the price differentials would suggest a lack of price parity in that individual market prices are allowed to wander arbitrarily far apart. The

Table 1 OLS estimates of autoregressive error correction price parity model $\Delta(p_t^i - p_t^j) = \alpha + \beta(p_{t-1}^j - p_{t-1}^i)$

Markets	$\hat{\alpha}$	SE($\hat{\alpha}$)	$\hat{\beta}$	SE($\hat{\beta}$)	R^2	Half-Life[a]	Cointegration tests[b]	
							Difference	Residual
Eastern Canada/North Central US	−0.0064	0.0013	−0.1174	0.0176	0.0582	5.55	−6.48	−7.24
Eastern Canada/Southeast US	−0.0006	0.0012	−0.0524	0.0119	0.0262	12.88	−3.88	−4.47
Eastern Canada/Southwest US	−0.0032	0.0012	−0.0918	0.0156	0.0462	7.20	−6.37	−5.93
Eastern Canada/Mid-Atlantic US	−0.0027	0.0013	−0.0802	0.0147	0.0397	8.29	−5.01	−5.42
Eastern Canada/Western Canada	0.0023	0.0012	−0.0741	0.0145	0.0354	9.00	−4.75	−5.17
North Central US/Southeast US	0.0020	0.0011	−0.0460	0.0114	0.0221	14.74	−3.63	−4.44
North Central US/Southwest US	0.0020	0.0011	−0.0980	0.0162	0.0488	6.72	−7.20	−6.13
North Central US/Mid-Atlantic US	0.0014	0.0010	−0.0650	0.0136	0.0311	10.31	−4.66	−5.05
North Central US/Western Canada	0.0059	0.0016	−0.0683	0.0137	0.0334	9.79	−5.15	−5.47
Southeast US/Southwest US	−0.0011	0.0008	−0.0510	0.0120	0.0248	13.24	−4.74	−4.65
Southeast US/Mid-Atlantic US	−0.0020	0.0006	−0.0993	0.0159	0.0517	6.63	−4.92	−6.87
Southeast US/Western Canada	0.0023	0.0015	−0.0545	0.0128	0.0248	12.37	−4.45	−4.22
Southwest US/Mid-Atlantic US	0.0000	0.0008	−0.0673	0.0136	0.0331	9.95	−6.47	−5.07
Southwest US/Western Canada	0.0064	0.0017	−0.0971	0.0164	0.0464	6.79	−6.27	−6.15
Mid-Atlantic US/Western Canada	0.0045	0.0017	−0.0701	0.0143	0.0324	9.54	−5.26	−4.88

[a] Deviation half-lives represent the weeks required to eliminate one-half of the deviation from equilibrium and are given by $ln(0.5)/ln(1+\beta)$

[b] ADF tests consider the stationarity of price difference and cointegration of p_1 and p_2. The tests reject nonstationarity and a lack of cointegration in every case

estimation results and nonstationarity testing results are presented in Table 1. In every case, the tests strongly reject the null hypothesis of nonstationarity of the price differentials. Table 1 also presents the half-lives associated with each of the error correction model estimates. Half-lives represent the period of time (in weeks) required for one-half of a deviation from parity conditions to be eliminated. The half-lives range from 6-15 weeks and thus indicate a reasonably strong degree of integration among the regional OSB markets. In fact, the half-lives of deviations from equilibrium conditions implied by these estimates are very similar to those presented by Goodwin et al. (2011) in an application of STAR models to a similar set of OSB data.[17] The linear relationships among the individual markets imposes a constant pattern of adjustment with no differences in adjustments to large versus small shocks. As we have noted, the presence of unobservable transactions costs suggests the potential for nonlinearities in adjustments. In particular, models of the LOP and of parity conditions often reveal nonlinear patterns of adjustments with larger deviations from parity provoking faster adjustment (and shorter half-lives) than smaller deviations. Small deviations from parity may not provoke any adjustment, suggesting that the error correction coefficient β in Eq. 3 is zero, which corresponds to an infinitely long half-life and equivalently, locally nonstationary behavior of the price differentials. Smooth and discrete threshold models relax this assumption by permitting two (or more) regimes that correspond to different patterns of adjustment. Such models provide additional flexibility relative to the standard error correction model but still are limited by the number of regimes that are parameterized and by the manner in which switching among regimes is modeled.

In addition to the conventional error correction specification of price parity, we also considered a smooth transition autoregressive (STAR) error correction model. We allowed for two regimes that are linked by a logistic transition function of the form:

$$\Delta(p_t^i - p_t^j) = (\alpha + \beta(p_{t-1}^j - p_{t-1}^i)) + F(\tau, \gamma)(\alpha + \beta(p_{t-1}^j - p_{t-1}^i)) + \epsilon_t, \quad (9)$$

where $F(\cdot)$ is the logistic transition function with a threshold parameter of τ and a speed of adjustment parameter of γ:

$$F((p_{t-1}^j - p_{t-1}^i), \tau, \gamma) = \frac{1}{1 + e^{\gamma((p_{t-1}^j - p_{t-1}^i) - \tau)}}. \quad (10)$$

This specification suggests that regime switching will be triggered by the lagged price differential (i.e., the forcing variable).

In addition to the linear and STAR error correction models, we consider three alternative specifications as a benchmark for evaluating the copula model estimates. This includes a Gaussian copula, which is similar in spirit to the linear models, a t copula, which allows for positive but symmetric tail dependence, and a skewed t copula, which allows for asymmetric tail dependence. The t copula nests the Gaussian, and the skewed t copula nests the t and Gaussian copulas. It should be noted that the goodness-of-fit statistics for the conventional error correction models cannot be directly compared to

[17] Deviation half-lives are given by $ln(0.5)/ln(1-\beta)$.

those for the copula models since the copula models do not parameterize the marginal distributions. However, a Gaussian copula (Table 3) provides an analogous benchmark for evaluating the fit of the copula models.

Table 2 presents log-likelihood function and BIC values for the linear and STAR error correction specifications. In every case, the linear model specification is strongly preferred by the BIC. The STAR models involve an eight-dimensional set of parameters and the penalty associated with the loss in degrees of freedom significantly offsets the modestly higher log-likelihood function values associated with the STAR model. The nonlinearity tests of Lukkonen, Saikkonen, and Teräsvirta are included in the table and indicate the presence of nonlinearity in 5 of the 15 cases. STAR model estimates are only presented in those cases where linearity is rejected. Teräsvirta (1994) had shown that STAR models may suffer from an identification problem when a linear specification is supported. The transition parameters indicate very swift switching between regimes in every case.[18]

Table 3 presents BIC criteria for the optimal copula specification (from the 18 models considered) and for the t, skewed t, and Gaussian copulas. The error correction specification of price differentials usually favor alternatives to the t and Gaussian specifications. However, the Gaussian model is favored in 4 of the 15 cases. In most cases, a survival Gumbel or Clayton specification receives the most support. In the case of the copula model in levels, a t-specification is favored in 6 of the 15 cases. In both model specifications, significant tail dependence is suggested.

Table 4 presents maximum likelihood estimation results for a copula representation of the standard error correction model of the LOP. In particular, the joint distribution of $\Delta(p_t^i - p_t^j)$ and $(p_{t-1}^j - p_{t-1}^i)$ is represented by copula model estimates, with the parametric form of the copula being chosen by minimizing the Bayesian information criterion (BIC) among the 18 different copula specifications considered. The marginal distributions of each variable are represented using nonparametric, empirical cdf's. Note again that the interpretation of the copula estimates as representing the joint probability distribution function may be complicated by the temporal correlation that is inherent in the specification, which relates the first difference in the price differential to the lagged value of the differential. Note also that we switch the sign of the price differential (essentially rotating the relationship) in order to represent the relationship in terms of positive dependence rather than the negative dependence associated with the error correction parameters in Table 1. We undertake this simple transformation to accommodate the fact that many Archimedean copulas are only capable of representing positive dependency relationships. In the case of the Gaussian and t copulas, $\hat{\theta}_1$ is the correlation parameter, and for the t parameter, $\hat{\theta}_2$ is the degrees of freedom parameter.

A variety of different copula specifications are found to be optimal in representing the linkages among the 15 different pairs of market prices. In most cases, the optimal specification tends to be asymmetric with zero tail dependence in one of the tails. Interpretation of tail dependence in cases where such dependence is only allowed in one tail can be aided by a consideration of the typical basis relationships among markets. In particular, to the extent that one market tends to export to another, we

[18] In the interest of space and in light of the BIC values, we do not present estimates of the STAR model. These estimates are available from the authors on request.

Table 2 Summary statistics for linear and smooth transition error correction models

Markets	Linear		Smooth Transition			
	Log-likelihood	BIC	Linearity test	p-value	Log-likelihood	BIC
E Can/NC US	1,688.48	−3357.23	8.91	0.1787	NA	NA
E Can/SE US	1,495.37	−2971.00	22.15	0.0011	1,505.81	−4.12
E Can/SW US	1,505.83	−2991.92	5.19	0.5194	NA	NA
E Can/MA US	1,494.51	−2969.29	9.78	0.1344	NA	NA
E Can/W Can	1,495.03	−2970.33	4.27	0.6406	NA	NA
NC US/SE US	1,543.91	−3068.08	27.16	0.0001	1,557.74	−4.27
NC US/SW US	1,578.52	−3137.30	24.85	0.0004	1,590.96	−4.36
NC US/MA US	1,575.70	−3131.67	10.66	0.0995	1,580.99	−4.33
NC US/W Can	1,539.45	−3059.17	5.30	0.5059	NA	NA
SE US/SW US	1,739.72	−3459.70	8.87	0.1812	NA	NA
SE US/MA US	2,019.87	−4020.01	20.70	0.0021	2,030.27	−5.58
SE US/W Can	1,355.48	−2691.22	4.81	0.5680	NA	NA
SW US/MA US	1,770.99	−3522.25	0.62	0.9960	NA	NA
SW US/W Can	1,408.85	−2797.96	0.50	0.9979	NA	NA
MA US/W Can	1,350.76	−2681.78	0.44	0.9985	NA	NA

[a] The linear error correction model is given by $\Delta(p_t^i - p_t^j) = \alpha + \beta(p_{t-1}^j - p_{t-1}^i) + \epsilon_t$ and the smooth transition error correction model is given by $\Delta(p_t^i - p_t^j) = (\alpha + \beta(p_{t-1}^j - p_{t-1}^i)) + F(\tau, \gamma)(\alpha + \beta(p_{t-1}^j - p_{t-1}^i)) + \epsilon_t$, where $F(\cdot)$ is the logistic transition function with a threshold parameter of τ and a speed of adjustment parameter of γ. The smooth transition model is only estimated when the linearity tests support this specification. Teräsvirta (1994) has shown that an identification problem exists when the linear specification is supported

Table 3 Bayesian information criteria (BIC) for alternative copula specifications

Markets	Optimal copula	BIC values				Optimal copula	BIC values			
		Optimal	t	Skew *t*	Gaussian		Optimal	*t*	Skew *t*	Gaussian
E Can/NC US	Surv. Gumbel	−31.42	−25.13	−11.97	−31.20	*t*	−1207.19	−1207.19	−1196.28	−1061.25
E Can/SE US	Surv. Clayton	−21.24	−12.78	0.08	−14.60	Frank	−1583.56	−1501.09	−1488.42	−1432.96
E Can/SW US	Gumbel	−31.40	−25.19	−12.06	−28.12	t	−1243.29	−1243.29	−1230.92	−1193.59
E Can/MA US	*t*	−25.52	−25.52	−12.56	−24.26	*t*	−1327.93	−1327.93	−1314.92	−1264.63
E Can/W Can	Gaussian	−19.13	−11.92	1.24	−19.13	CG	−1320.08	−1307.51	−1295.30	−1258.76
NC US/SE US	Surv. Clayton	−32.21	−7.55	4.10	−10.95	Gumbel	−1650.63	−1532.38	−1523.08	−1445.34
NC US/SW US	Gumbel	−50.38	−33.74	−20.99	−34.20	Gumbel	−1159.29	−1077.89	−1066.63	−1028.43
NC US/MA US	Surv. Clayton	−28.52	−14.34	−1.72	−15.96	Gumbel	−1441.45	−1389.74	−1376.90	−1363.95
NC US/W Can	Gaussian	−22.54	−14.38	−1.24	−22.54	BB1	−1412.60	−1393.18	−1380.32	−1328.15
SE US/SW US	Surv. Gumbel	−16.79	−8.32	4.79	−16.74	R. Gumbel	−1583.49	−1456.12	−1443.63	−1404.79
SE US/MA US	Surv. Gumbel	−52.40	−35.72	−23.08	−43.48	R. Gumbel	−1215.37	−1181.86	−1169.34	−1065.36
SE US/W Can	Gaussian	−14.93	−5.56	7.53	−14.93	R. CG	−1493.17	−1486.05	−1473.32	−1443.52
SW US/MA US	Clayton	−24.00	−17.43	−4.62	−22.48	*t*	−1451.54	−1451.54	−1438.41	−1413.80
SW US/W Can	Gumbel	−30.29	−21.85	−8.77	−27.81	*t*	−1185.73	−1185.73	−1173.32	−1165.55
MA US/W Can	Gaussian	−19.40	−10.78	2.29	−19.40	*t*	−1395.86	−1395.86	−1383.21	−1377.85

[a] The optimal copula specification is given by the minimized value of the BIC. CG is the Clayton–Gumbel copula

Table 4 Copula parameter estimates (with empirical marginals)[a] $C(\Delta(p_t^i - p_t^j), (p_{t-1}^j - p_{t-1}^i))$

Markets	Optimal copula	$\hat{\theta}_1$	SE($\hat{\theta}_1$)	$\hat{\theta}_2$	SE($\hat{\theta}_2$)	CvM Stat	CvM p value	KS Stat	KS p value	Tail dependence Lower	Tail dependence Upper	Kendall's τ
E Can/NC US	R. Gumbel	1.1429	0.0292	–	–	0.2560	0.0040	1.0359	0.0320	0.1661	0.0000	0.1250
E Can/SE US	R. Clayton	0.2286	0.0495	–	–	0.0736	0.4600	0.6138	0.6680	0.0000	0.0482	0.1026
E Can/SW US	Gumbel	1.1546	0.0305	–	–	0.0795	0.3800	0.7476	0.3320	0.0000	0.1773	0.1339
E Can/MA US	t	0.1924	0.0399	8.0495	3.1148	0.0625	0.6480	0.6413	0.6640	0.0351	0.0351	0.1233
E Can/W Can	Gaussian	0.1902	0.0358	–	–	0.0884	0.4360	0.6510	0.6680	0.0000	0.0000	0.1219
NC US/SE US	R. Clayton	0.2748	0.0510	–	–	0.2358	0.0040	1.0818	0.0120	0.0000	0.0802	0.1208
NC US/SW US	Gumbel	1.1835	0.0312	–	–	0.0748	0.4200	0.7113	0.4400	0.0000	0.2038	0.1551
NC US/MA US	R. Clayton	0.2657	0.0517	–	–	0.0344	0.8840	0.5130	0.8720	0.0000	0.0736	0.1173
NC US/W Can	Gaussian	0.2022	0.0356	–	–	0.0511	0.7800	0.5933	0.8000	0.0000	0.0000	0.1296
SE US/SW US	R. Gumbel	1.1157	0.0283	–	–	0.0718	0.4360	0.7254	0.3680	0.1388	0.0000	0.1037
SE US/MA US	R. Gumbel	1.1892	0.0320	–	–	0.0962	0.2600	0.7243	0.3640	0.2088	0.0000	0.1591
SE US/W Can	Gaussian	0.1743	0.0361	–	–	0.0471	0.7720	0.7464	0.4480	0.0000	0.0000	0.1115
SW US/MA US	Clayton	0.2569	0.0522	–	–	0.1660	0.0400	1.0890	0.0160	0.0673	0.0000	0.1138
SW US/W Can	Gumbel	1.1517	0.0302	–	–	0.0600	0.5760	0.5910	0.7280	0.0000	0.1745	0.1317
MA US/W Can	Gaussian	0.1912	0.0358	–	–	0.0733	0.5680	0.7529	0.4560	0.0000	0.0000	0.1225

[a] CvM is the Cramér von Mises (CvM) specification test and KS is the Kolmogorov–Smirnov specification test. Tail dependence is calculated using Eqs. 7 and 8 above

generally expect to see price differences tending to be either positive or negative, but not both. This reflects the presence of transactions costs which are a component of basis price differences. The asymmetry in commodity flows is a relatively common feature in most basic commodity markets, including manufactured wood products. Figure 2 presents nonparametric densities for the price differentials for all six pairs of markets. In nearly every case, definite patterns of basis, reflecting a relationship where one market price is generally above another, are indicated. Further, skewness is apparent in all of the nonparametric densities illustrated in Fig. 2, indicating that large departures from price equality are more common in one direction than another. This suggests that the asymmetric tail dependence associated with the Clayton and Gumbel copulas (and their rotated versions) may be appropriate. The parameter values defining the copula relationships are all highly statistically significant.

Deheuvels (1979) introduced the idea of rank-based, nonparametric empirical copula functions and demonstrated that they converge uniformly to the underlying true parametric copula. Genest and Rivest (1993), Wang and Wells (2000), and Genest et al. (2009) discussed a number of goodness-of-fit statistics based upon general Cramér von Mises (CvM) and Kolmogorov–Smirnov (KS) types of statistics. These tests are based on a comparison of the nonparametric empirical copula function and the proposed parametric copula. In the case of rotated copulas, the uniform variates are transformed and the goodness-of-fit procedure for the corresponding nonrotated copula is used. Berg and Bakken (2006) noted that the KS statistic tends to be sensitive around the median of the distribution and less sensitive to deviations in the tails. In contrast, they point out that the CvM statistic tends to be stable across the distribution, including deviations in the tails. We apply both the CvM and KS specification tests to evaluate the goodness-of-fit for each fitted copula model. Genest et al. (2009) noted that the limiting distribution of the statistic may be sensitive to sample size and to the family of copulas included under the composite null hypothesis. They recommend a double bootstrap procedure as an alternative to tabular critical values. We pursue this approach and utilize 1,000 bootstrap replications in our evaluation of each specification.

The test statistics support the chosen specification in 12 of the 15 cases. The alternative versions (CvM and KS) of the goodness-of-fit statistics lead to identical conclusions at the $\alpha = 0.05$ level, though differences arise at smaller α levels. In the cases of the Eastern Canada/North Central US, North Central US/Southeast US, and the Southwest US/Mid-Atlantic US price comparisons, the specification tests do not support the chosen specification at the 0.05 level. However, the specification is supported by at least one of the specification tests at the $\alpha = 0.01$ in every case. Specifically, the KS test supports the specifications in every case though the CvM test rejects the specification at the 0.01 level for Eastern Canada/North Central USA and North Central USA/Southeast USA

Table 4 also presents the values of Kendall's τ statistic. Noting that the sign of the implied dependency is reversed, these values suggest a reasonable degree of positive dependency and thus support a general notion of market integration. Upper and lower tail dependencies are also presented for the estimated copula for each pair of markets. In most cases, significant dependency in one or the other tail (again reflecting the fact that the Archimedean copulas impose zero dependency in one tail) is revealed. Only in the cases of the Eastern/Western Canada, Mid-Atlantic US/Western

Canada, North Central US/Western Canada, and Southeast US/Western Canada price comparisons did the estimates suggest zero dependency in both tails (i.e., a Gaussian copula specification). Results from these four cases are consistent with constant linear correlation and thus are analogous to the OLS regression estimates of the standard error correction model represented by Eq. 3. The nature of the price linkages implied by the copula models can be illustrated through a consideration of the joint probability density function. A constant linear relationship is implied in the elliptical shape imposed by a Gaussian copula. Positive tail dependencies that correspond to nonlinear patterns of adjustment to deviations from parity are implied for joint distribution functions that deviate from an elliptical shape. In particular, the types of adjustments implied by nonlinear error correction models such as threshold autoregressive models are represented by joint densities that are "pinched" in one or both tails.

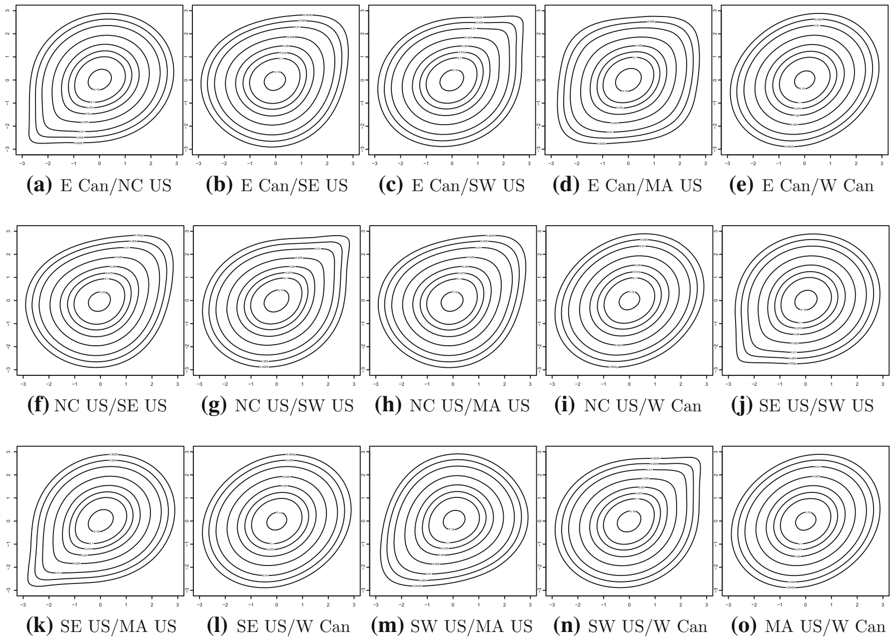
An alternative specification that involves reparameterizing the relationship implied by Eq. 3 into that of 4 involves a copula model of the joint distribution of $(p_t^i - p_t^j)$ and $(p_{t-1}^i - p_{t-1}^j)$. Patton (2009) discussed the application of copulas for autocorrelated, univariate time series of the form $C(y_t, y_{t-1})$, which is analogous to our second specification where $y_t = p_t^i - p_t^j$. In a fashion identical to that used in the preceding application, maximum likelihood estimation methods were used to estimate the specification that minimized the BIC across the 18 different copulas considered. The resulting estimates and summary statistics are presented in Table 5. Several differences in terms of the form of copula and the implied extent of tail dependence are apparent when compared to the results in Table 4. Of course, the specification of the data generating process is different and one would not anticipate any direct correspondence in terms of the optimal copula or degree of tail dependence. The CvM and KS goodness-of-fit statistics support the specification for 12 of the 15 cases at the $\alpha = 0.01$ level and 11 of the 15 cases at the $\alpha = 0.05$ level. Although the alternative specification tests generally agree and the p-values tend to be very similar, some differences do exist. An interesting finding is that a much greater degree of dependency in opposite tails is apparent and in no case do the results support a standard Gaussian specification. Dependencies are generally large and Kendall's τ values tend to be around 0.7–0.8. This reflects a high degree of temporal dependence in the price differentials though, as was indicated by the ADF testing results presented in Table 1, the roots of the process appear to be considerably away from 1.0. An implicit test of the optimal copula specifications against a conventional linear model (fit by maximum likelihood methods) can be derived from a consideration of the goodness-of-fit criteria and tests for a standard Gaussian copula model. A Gaussian specification is preferred by the BIC criterion in only 4 of the 15 cases for the error correction specification and in none of the comparisons of the price differentials in levels.

Figures 4 and 5 present the joint probability distribution contours for the 15 different price comparisons. Figure 4 presents the distribution for changes in the price differentials, while Fig. 5 presents the analogous distributions for differentials. A variety of different patterns of tail dependence are notable. In most cases, the dependence is tighter in one extreme, indicating more adjustment for extreme values of the price

Table 5 Copula parameter estimates (with empirical marginals)[a] $C((p_t^i - p_t^j), (p_{t-1}^i - p_{t-1}^j))$

Markets	Optimal copula	$\hat{\theta}_1$	SE($\hat{\theta}_1$)	$\hat{\theta}_2$	SE($\hat{\theta}_2$)	CvM Stat	CvM p value	KS Stat	KS p value	Tail dependence Lower	Tail dependence Upper	Kendall's τ
E Can/NC US	t	0.9038	0.0077	2.4923	0.3661	0.2107	0.0440	1.0696	0.0280	0.6989	0.6989	0.7184
E Can/SE US	Frank	19.1074	0.6780	–	–	0.1268	0.0000	0.9726	0.0000	0.0000	0.0000	0.8087
E Can/SW US	t	0.9125	0.0062	4.1788	0.8501	0.0731	0.1960	0.6061	0.4600	0.6464	0.6464	0.7317
E Can/MA US	t	0.9237	0.0054	4.0838	0.7846	0.0954	0.0800	0.7630	0.1400	0.6718	0.6718	0.7496
E Can/W Can	CG	0.4419	0.0882	3.1147	0.1487	0.0439	0.2880	0.6231	0.2520	0.6044	0.7508	0.7370
NC US/SE US	Gumbel	4.8742	0.1540	–	–	0.1196	0.0000	0.8366	0.0080	0.0000	0.8472	0.7948
NC US/SW US	Gumbel	3.3700	0.1061	–	–	0.0741	0.0520	0.6980	0.1200	0.0000	0.7716	0.7033
NC US/MA US	Gumbel	4.1491	0.1305	–	–	0.0341	0.4680	0.4913	0.6600	0.0000	0.8182	0.7590
NC US/W Can	CG	0.2313	0.0783	3.6721	0.1694	0.0306	0.5760	0.4990	0.5960	0.4421	0.7922	0.7559
SE US/SW US	R. Gumbel	4.4982	0.1427	–	–	0.0458	0.1480	0.5732	0.2880	0.8334	0.0000	0.7777
SE US/MA US	R. Gumbel	3.5677	0.1135	–	–	0.1106	0.0080	0.8440	0.0240	0.7856	0.0000	0.7197
SE US/W Can	R. CG	0.2435	0.0765	3.8784	0.1767	0.0655	0.0360	0.6560	0.1240	0.8043	0.8363	0.7702
SW US/MA US	t	0.9326	0.0048	4.1052	0.8375	0.0713	0.1240	0.8099	0.0640	0.6903	0.6903	0.7650
SW US/W Can	t	0.8996	0.0071	4.5793	1.1338	0.0324	0.8600	0.5469	0.7200	0.6082	0.6082	0.7124
MA US/W Can	t	0.9271	0.0050	5.0042	1.2873	0.0470	0.4720	0.6515	0.3120	0.6504	0.6504	0.7554

[a] CvM is the Cramér von Mises (CvM) specification test and KS is the Kolmogorov–Smirnov specification test. CG is the Clayton–Gumbel copula. Tail dependence is calculated using Eqs. 7 and 8 above

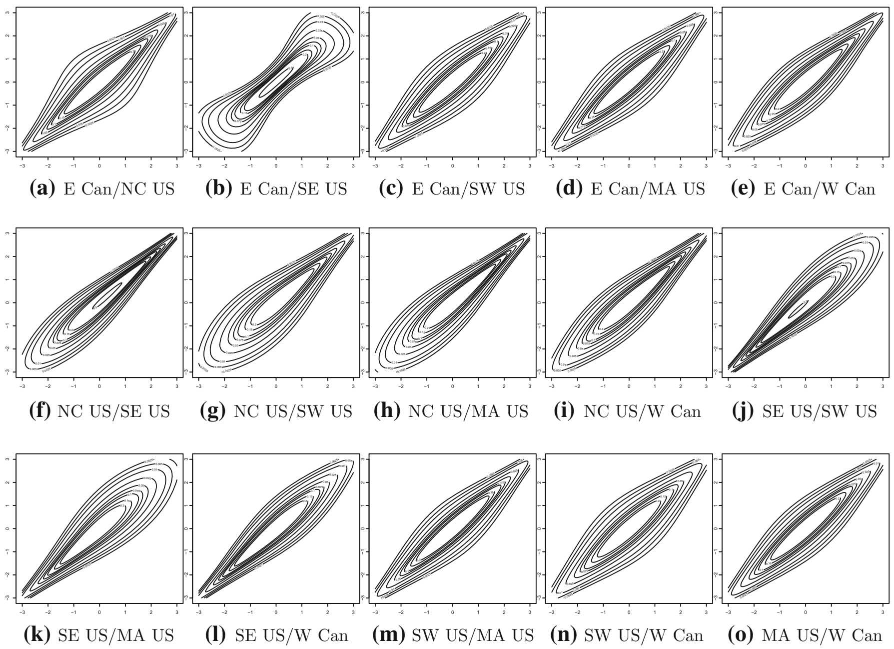


(a) E Can/NC US (b) E Can/SE US (c) E Can/SW US (d) E Can/MA US (e) E Can/W Can

(f) NC US/SE US (g) NC US/SW US (h) NC US/MA US (i) NC US/W Can (j) SE US/SW US

(k) SE US/MA US (l) SE US/W Can (m) SW US/MA US (n) SW US/W Can (o) MA US/W Can

Fig. 4 Joint probability density function from estimated copulas (simulated using standard normal marginals): $C(\Delta(p_t^i - p_t^j), (p_{t-1}^i - p_{t-1}^j))$

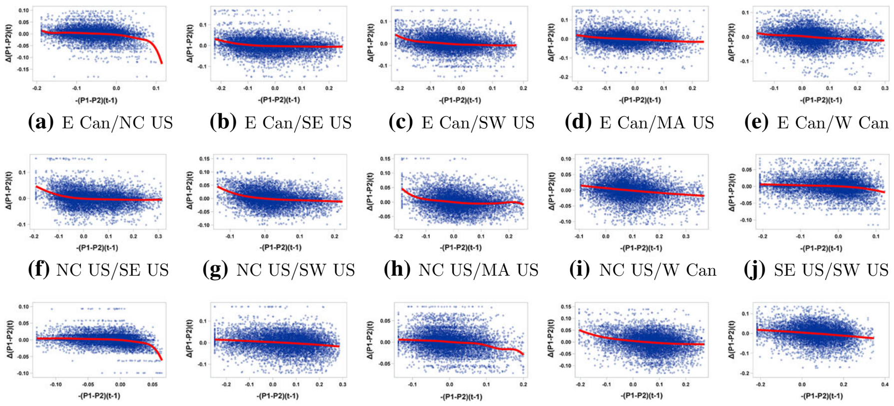


(a) E Can/NC US (b) E Can/SE US (c) E Can/SW US (d) E Can/MA US (e) E Can/W Can

(f) NC US/SE US (g) NC US/SW US (h) NC US/MA US (i) NC US/W Can (j) SE US/SW US

(k) SE US/MA US (l) SE US/W Can (m) SW US/MA US (n) SW US/W Can (o) MA US/W Can

Fig. 5 Joint probability density function from estimated copulas (simulated using standard normal marginals): $C((p_t^i - p_t^j), (p_{t-1}^i - p_{t-1}^j))$



(a) E Can/NC US **(b)** E Can/SE US **(c)** E Can/SW US **(d)** E Can/MA US **(e)** E Can/W Can

(f) NC US/SE US **(g)** NC US/SW US **(h)** NC US/MA US **(i)** NC US/W Can **(j)** SE US/SW US

(k) SE US/MA US **(l)** SE US/W Can **(m)** SW US/MA US **(n)** SW US/W Can **(o)** MA US/W Can

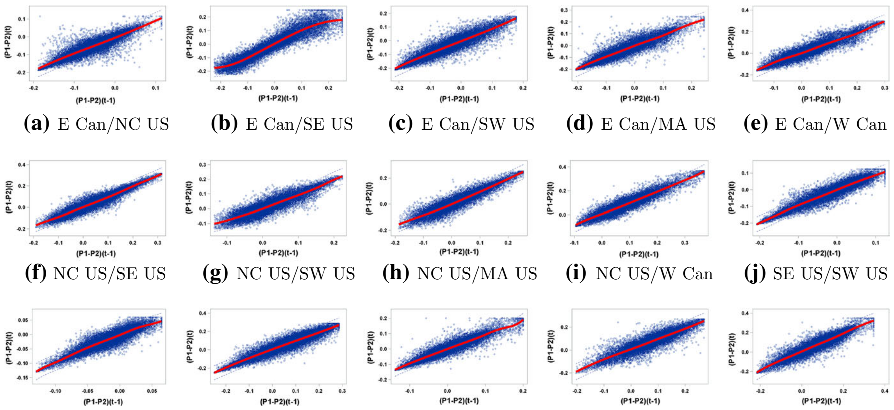
Fig. 6 Estimated copula function mean relationships (nonparametric marginals): $C(\Delta(p_t^i - p_t^j), (p_{t-1}^i - p_{t-1}^j))$. Mean relationship (*red line*) estimated from simulated copula models with nonparametric marginal densities. (Color figure online)

differential. The copulas are illustrated using random variates drawn from a standard normal distribution for the marginals.[19] The positive degree of tail dependencies represented by the estimates in Tables 4 and 5 is obvious in the joint densities. For example, the joint distributions for price pairs from Eastern Canada/North Central USA (panel (a)) and Eastern Canada and the Southwest USA (panel (c)) exhibit obvious tail dependence. Likewise, the elliptical, symmetric distributions with zero tail dependencies are apparent for the price pairs generated from a Gaussian copula. This finding is consistent with the typical nonlinear patterns of adjustment that are often revealed in nonlinear models of price parity. In economic terms, the results suggest that market linkages may not be as strong when price differentials are small, but that adjustment occurs rapidly when large deviations from parity arise. This is consistent with the presence of a transactions cost band that inhibits price adjustments to small deviations from parity but that reflect significant adjustments in response to large shocks to price differentials.

Perhaps of greatest interest is a consideration of how the price adjustment process varies over the different quantiles of the marginal distributions. We used the nonparametric density estimates of the marginals and the estimated copula function parameters to illustrate the price relationships.[20] Fig. 6 represents the relationships between first-

[19] Standard normal marginals are used only to illustrate the dependencies in the joint distributions. We use the empirical marginals to evaluate the nature of price adjustment below.

[20] Mean relationships were estimated by simulating data from the estimated copula and the nonparametric marginal densities. Splines and high-ordered polynomial expressions were then fit to the simulated data. Note that simulation of the joint distributions using nonparametric density estimates involved generating random, dependent draws from the univariate values and then using a nonparametric estimate of the inverse cdf. A fine grid was defined for the kernel density estimates of the density, and then, quantiles were matched to the randomly drawn uniform variates to generate values of the two dependent variables $(\Delta(p_t^i - p_t^j), (p_{t-1}^j - p_{t-1}^i))$.



(a) E Can/NC US **(b)** E Can/SE US **(c)** E Can/SW US **(d)** E Can/MA US **(e)** E Can/W Can

(f) NC US/SE US **(g)** NC US/SW US **(h)** NC US/MA US **(i)** NC US/W Can **(j)** SE US/SW US

(k) SE US/MA US **(l)** SE US/W Can **(m)** SW US/MA US **(n)** SW US/W Can **(o)** MA US/W Can

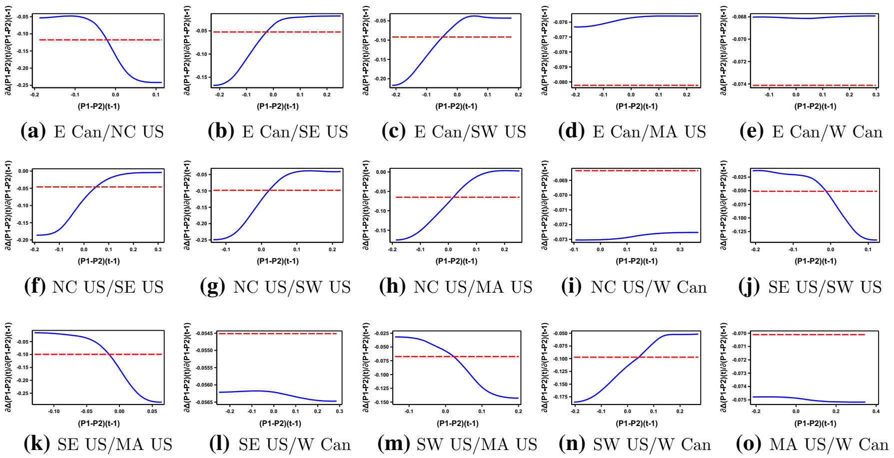
Fig. 7 Estimated copula function mean relationships (nonparametric marginals): $C((p_t^i - p_t^j), (p_{t-1}^i - p_{t-1}^j))$. Mean relationship (*red line*) estimated from simulated copula models with nonparametric marginal densities. (Color figure online)

differenced price differentials (on the vertical axis) and the lagged price differentials (on the horizontal axis). The mean relationship implied by the copula estimates is represented by the line. The parametric relationships that underlie the error correcting price adjustment processes for the regional/international OSB markets imply important nonlinearities. In particular, large deviations from parity typically tend to invoke a stronger adjustment among prices. Figure 7 presents the same mean relationships among the current and lagged price differentials. Again, nonlinearities in the error correcting price adjustment processes appear to characterize market linkages, though the degree of nonlinearity that is apparent in the figures is less obvious than is the case when the relationship is expressed in first-differenced form.

A more straightforward approach to illustrating the price adjustment process can be obtained by considering the derivative of the relationship between the expected value of the first-differenced price differential in time t and the value of the price differential in time $t-1$. That is, we consider the adjustment processes that are implied by the copula estimates by differentiating the expected first-differenced differential with respect to the lagged value of the price differential:

$$\frac{\partial E \Delta (p_t^i - p_t^j)}{\partial E(p_{t-1}^j - p_{t-1}^i)}, \tag{11}$$

where E represents the expectations operator. In the case of the simple, linear error correction model, this derivative is constant and is given by β in Eq. 3. In the case of the nonlinear, copula models (of which a simple linear relationship is a special case that corresponds to an elliptical Gaussian copula), the derivative may vary across the domains of the marginals. The relationship is determined by the optimal copula function and the estimates of the parameters that characterize this function. Figure 8 presents these relationships for the standard error correction model, and Fig. 9 presents

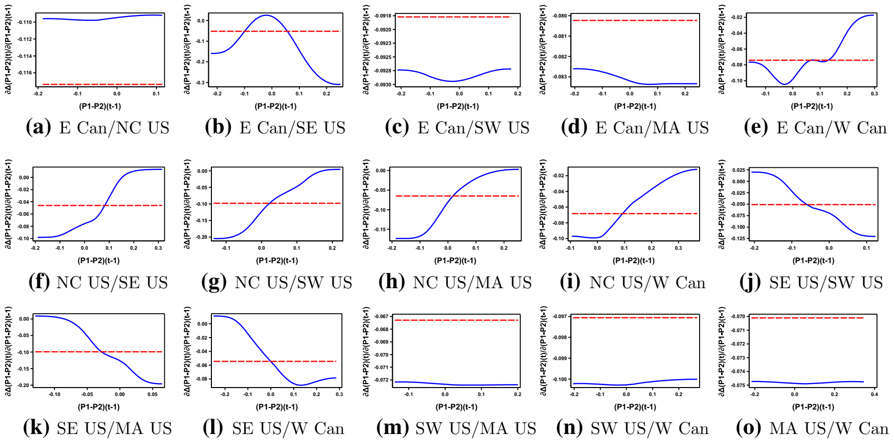


(a) E Can/NC US **(b)** E Can/SE US **(c)** E Can/SW US **(d)** E Can/MA US **(e)** E Can/W Can

(f) NC US/SE US **(g)** NC US/SW US **(h)** NC US/MA US **(i)** NC US/W Can **(j)** SE US/SW US

(k) SE US/MA US **(l)** SE US/W Can **(m)** SW US/MA US **(n)** SW US/W Can **(o)** MA US/W Can

Fig. 8 Implied adjustment process from error correction copula model (Specification 1) $\partial(\Delta(p_t^i - p_t^j)/\partial(p_{t-1}^i - p_{t-1}^j))$. *Dashed red line* corresponds to slope parameter from conventional linear model and *blue curve* represents copula model patterns of adjustment. (Color figure online)



(a) E Can/NC US **(b)** E Can/SE US **(c)** E Can/SW US **(d)** E Can/MA US **(e)** E Can/W Can

(f) NC US/SE US **(g)** NC US/SW US **(h)** NC US/MA US **(i)** NC US/W Can **(j)** SE US/SW US

(k) SE US/MA US **(l)** SE US/W Can **(m)** SW US/MA US **(n)** SW US/W Can **(o)** MA US/W Can

Fig. 9 Implied adjustment from Markov process copula model (Specification 2) $\partial(p_t^i - p_t^j)/\partial(p_{t-1}^i - p_{t-1}^j))$. *Dashed red line* corresponds to slope parameter from conventional linear model and *blue curve* represents copula model patterns of adjustment. (Color figure online)

the analogous relationships for the reparametrization of this relationship implied by the joint distribution of the current and lagged values of the differentials. In each case, the figures include the estimated values of β in Eq. 3.[21]

[21] Derivatives are derived by solving the bivariate relationship suggested by the copula for one variable in terms of the other. Splines and high-ordered polynomials were then used to parameterize this relationship, and derivatives are obtained by differentiating this polynomial. This provides an accurate approximation of the patterns of price responses that can be compared to conventional linear models.

Several points are apparent from the figures. First, important differences in price adjustments are apparent for extreme values of price differences. In most cases, the adjustments appear to be stronger in one or the other tail of the distribution of the lagged differentials. As noted above, this corresponds to the fact that most markets for commodity trade tend to be downstream, such that important exporting regions tend to ship to importing regions but the opposite flow of commodities may be rare. The price adjustment processes are generally similar for the two alternative copula specifications though a number of differences do exist. Recall that a negative value of the derivative is expected and the larger is the derivative in magnitude, the faster is the adjustment, and the smaller is the half-life of deviations. In most cases, the adjustment process implied by the linear model, which is restricted to be constant across the distribution of price differentials, has a value that is similar to the nonlinear adjustments in the middle quantiles of the differentials but differs substantially in the tails. Again, this is consistent with nonlinear adjustments that occur at a faster speed when prices are far apart. Such adjustment, as is true in most nonlinear time series models of the law of one price, corresponds to the actions of spatial arbitragers and the frictions associated with unobservable transactions costs. In those cases where a Gaussian is implied, the linear pattern of adjustment that is implied by the copulas is apparent and the derivatives are quite similar in numerical values to the constant values implied by the standard error correction model. More nonlinearities appear to arise for comparisons involving the Eastern Canada market. Likewise, a greater degree of nonlinearity appears to exist for market linkages involving the North Central USA and the Southeastern USA. The degree of adjustment, reflected by the magnitude of the coefficients, tends to be smaller with less nonlinearities when markets are significantly separated by distance (e.g., eastern and western Canada, the southeast USA and western Canada). Recall that the parameters in the error correction specification correspond to the half-lives of deviations from parity conditions. A much longer period of time is required to eliminate significant departures from the equilibrium relationships that are expected to correspond to efficient parity conditions. For example, a linear error correction model implies that a shock to the parity condition between the southeast and southwest USA requires over 12 weeks to eliminate one-half of the deviation, whereas an analogous shock to the southwest US and western Canadian markets requires only 7 weeks for one-half of the shock to dissipate. The copula models demonstrate that such patterns of adjustment usually depend upon the magnitude of the shock, with larger price shocks corresponding to faster adjustments.

In summary, our results uniformly support the Law of One Price for regional and international markets for a standardized product—oriented strand board. The product is a leading construction material and is commonly traded among US and Canadian markets. The markets appear to be strongly linked in most cases. Nonlinear adjustments are confirmed in most cases. These results are to be expected in light of the homogenous nature of the commodity and the fluid regional trade that exists. In accordance with existing research, the results indicate that market adjustments are generally larger in response to large price differences which reflect more substantial disequilibrium conditions (and therefore larger arbitrage opportunities). The implications are similar to those provided in other estimation approaches that allow for nonlinearities. In particular, regime switching and threshold models generally imply that price linkages and adjustment patterns are stronger and quicker when deviations from equilibrium are large. This suggests the presence of transactions costs and Heckscher's "commodity points." It is also notable that the existence of a national border between the USA and Canada does not appear to significantly inhibit price transmission and thus that efficient trade and integration exist between US and Canadian OSB markets.

4 Summary and concluding remarks

We evaluate the adherence to the economic conditions typically required for efficiently linked markets by considering the degree and nature of dependence and the strength of market linkages implied by copula models of joint distributions of spatially related prices. Our application is to a set of 6 important US and Canadian markets for oriented strand board (OSB), which is an important building material that is widely traded across the North American continent. To allow and model nonlinear behavior that might be caused by transactions costs, we adopt specific classes of copula functions that allow for "state-dependent" correlation and dependence, where the state is defined by the degree of market disequilibrium represented by spatial price differences at any point in time. Our general approach involves the use of model fitting criteria to select the optimal copula specification and maximum likelihood estimation of the parameters of the copula. We evaluate the specifications using goodness-of-fit tests. We evaluate the price adjustment process and the integration of markets by considering derivatives of the functional relationships among price differences and differentials, which can be compared directly to standard linear error correction models. In most cases, important nonlinearities in price adjustment are revealed.

We find that transactions costs bands are implied by certain nonlinear patterns of correlation and that several market linkages appear to reflect unidirectional trade. We consider two alternative specifications of the joint density of price linkages and obtain similar results in each case, though differences do arise. The results provide strong support for the Law of One Price in North American OSB markets. In every case, the results correspond to an error correction type of process for regional price differentials whereby deviations from parity conditions are of a temporary nature. In some cases, the responses tend to be highly nonlinear, likely reflecting transactions costs. In other cases, results are similar to those obtained for standard linear error correction models.

We view our approach as one more generalization of the movement toward increasingly flexible, nonlinear models of price linkages. Our approach has advantages over other approaches involving thresholds in that standard maximum likelihood inferences are possible and the results can be interpreted within the context of a joint probability distribution function. More general approaches including those that allow for the parameters of the joint distribution to be state dependent or time varying remain as an extension to this work. Oh and Patton (2013) and De Lira Salvatierra and Patton (2015) had considered time-varying, dynamic copula models of asset price spreads and returns. A similar approach to evaluating structural changes in OSB prices that may have arisen in response to the housing market crisis is an appealing avenue for additional inquiry into market relationships for building materials.

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