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Hybrid estimators for mean aboveground carbon per unit area

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ABSTRACT

Carbon accounting is at the heart of efforts to mitigate the effects of climate change. One approach for estimating population parameters for live tree stem carbon entails three primary steps: (1) construction of an individual tree, allometric carbon model, (2) application of the model to tree-level data for a probability sample of plots, and (3) use of a probability-based (design-based) estimator of mean carbon per unit area for a population of interest. Compliance with the IPCC good practice guidance requires satisfaction of two criteria, one related to minimizing bias and one related to minimizing uncertainty. For this carbon estimation procedure, the portion of uncertainty attributed to the variance of the probability-based estimator of the population mean using the plot-level predictions is usually correctly estimated, but the portion attributed to the variance of the allometric model estimator is usually ignored. The result is that the total variance of the population mean estimator cannot be asserted to comply with the IPCC good practice criteria because not only is it not minimized, it is not even correctly estimated.

Within the framework of what is coming to be characterized as hybrid inference, model-based inferential methods were used to estimate the variance of the tree-level allometric model estimator which was then propagated through to the variance of the probability-based estimator of mean carbon per unit area. This combined estimator, consisting of a model-based estimator used to predict a variable for a probability sample of a population followed by a probability-estimator of the population total or mean using the sample predictions, is characterized as a hybrid estimator. For this study, two probability-based estimators of the mean were considered, simple random sampling estimators and model-assisted regression estimators that used airborne laser scanning (ALS) data as auxiliary information. The variance of the allometric model estimator incorporated variances of distributions of diameter and height measurement errors, covariances of model parameter estimators, model residual variance, and variances of distributions of wood densities and carbon content proportions.

The novel features of the study included the hybrid inferential framework, consideration of six sources of uncertainty including the variances of distributions of wood densities and carbon content proportions, use of ALS data with model-assisted regression estimators of the population mean, and use of confidence intervals for the population mean as the basis for comparisons rather than intermediate products such as model prediction accuracy. The primary conclusions were that the variance of the allometric model estimator was negligible or marginally negligible relative to the variance of the probability estimator when using species-specific allometric models and simple random sampling estimators, but non-negligible when using species-specific models and model-assisted regression estimators and when using a non-specific model with either estimator.

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1. Introduction

1.1. Carbon estimation

Among the six economic sectors identified by the United Nations Framework Convention on Climate Change as sources of anthropogenic greenhouse gas (GHG) emissions, the Land Use, Land Use Change and Forestry sector is the only terrestrial sector with the potential to remove GHG emissions from the atmosphere. The contributions of this sector to carbon sequestration are further reflected in three of the five carbon pools identified by the Marrakesh Accords for the maintenance of existing carbon stocks: aboveground biomass, below ground biomass, and deadwood (Angelsen et al., 2009, p. 311; Penman et al., 2013, Table 3.1.2).

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One approach for estimating carbon population parameters for the aboveground biomass pool entails three steps (Jalkanen et al., 2005; Petersson et al., 2012). First, an individual tree, allometric volume model is constructed using a sample consisting of tree-level species observations, diameter at breast-height (dbh) and height (ht) measurements, and carefully measured volumes considered to be observations without measurement error. Second, the model is applied to predict individual tree volumes for a probability sample of field plots with tree-level species observations, dbh measurements, and either ht measurements or ht predictions as predictor variables. Species-specific wood densities and carbon content proportions are then used to convert the tree-level volume predictions to tree-level carbon predictions. The tree-level carbon predictions are then aggregated to produce plot-level carbon per unit area predictions. Third, a probability-based (design-based) estimator is used to estimate the population mean carbon per unit area using the plot-level carbon predictions. Simple random sampling estimators can be used to estimate the population parameter, but their precision may be insufficient because of small sample sizes and/or large variability among population unit carbon values. In these instances, stratified or model-assisted estimators using remotely sensed auxiliary information may have greater precision.

1.2. Individual tree, allometric volume models

Samples used as sources of data for constructing the allometric volume models are typically purposive and at least partially external to the populations to which they are applied. Further, volume observations for the trees to which the models are applied are typically not available, meaning that no direct comparisons of tree-level volume observations and corresponding allometric model predictions for the probability sample units are possible. Under these conditions, model-based rather than probability-based estimators must be used to estimate the variances of both the tree-level and plot-level estimators.

A crucial issue is the degree to which ignoring the variance of the allometric model estimator causes underestimates of the variance of the estimator of the population mean carbon per unit area. McRoberts and Westfall (in press) noted that the variance of an allometric volume model estimator can be attributed to four primary sources of uncertainty: (1) model misspecification, (2) covariances of the model parameter estimators, (3) allometric model residual variance, and (4) measurement errors for the predictor variables. For purposes of volume estimation, the adverse effects of allometric model misspecification are often negligible, because the models are usually quite accurate with pseudo-R^2 values greater than 0.90 (Brown et al., 1989; Mugasha et al., 2013) and often greater than 0.95 (Chave et al., 2005; McRoberts and Westfall, in press; McRoberts et al., 2015). Predictor variable measurement errors have been studied but few generalizations have been forthcoming (Westfall and Patterson, 2007; Westfall, 2008; Berger et al. 2014; Breidenbach et al., 2014; McRoberts et al., 2015; Chen et al., 2015; Shettles et al., 2015; McRoberts and Westfall, in press). The covariances of the model parameter estimators are often expressed by the model parameter covariance matrix, and for a correctly specified model, mean square error is typically used as the estimator of homogeneous model residual variance. In addition, wood densities and carbon content proportions vary due to multiple factors including site attributes, climate, tree age and size, stem location, and competition. The effects of the variances of the distributions of wood densities and carbon content proportions on the variance of the allometric model estimator are not known to have been rigorously addressed.

Multiple recent studies using a variety of methods have addressed estimation of the variance of estimators of population mean volume that rely on allometric models. Ståhl et al. (2014) and Chen et al. (2015) used sampling theory; Thurner et al. (2014) and Magnussen et al. (2014) used Taylor series approximations; and Breidenbach et al. (2014), McRoberts et al. (2015) and McRoberts and Westfall (in press) used Monte Carlo simulations. An important issue is that the variance of the estimator of the population mean could have large bias when the variance of the allometric model estimator is ignored. Subject to allometric model calibration dataset sizes and prediction accuracies, Breidenbach et al. (2014) and McRoberts and Westfall (2014) reported that estimates of the bias were negligible when using the simple random sampling estimator of the population mean for temperate forests; McRoberts et al. (2015) reported a similar result for a sub-tropical Brazilian dataset. However, Ståhl et al. (2014) reported a bias of approximately 10% when using simple random sampling estimators; McRoberts and Westfall (in press) reported non-negligible bias when using post-stratified estimators; and McRoberts et al. (2013) suggested greater bias when using model-assisted estimators than when using post-stratified estimators.

1.3. Hybrid inference

Corona et al. (2014) coined the term *hybrid inference* to describe situations whereby a population mean is estimated as the mean of model predictions for a probability sample of auxiliary information used as model predictor variables. The key features of hybrid inference are fourfold: (1) a probability sample of auxiliary information, (2) a model using the auxiliary information to predict the response variable for the sample units, (3) a probability-based estimator of the population mean using the sample unit predictions, and (4) both model-based and probability-based inferential methods to estimate the variance of the hybrid estimator of the population mean. Failure to recognize the requirement for both forms of inference leads to incorrect estimation of the variance of the estimator of the population mean. In particular, although correct methods are typically used for estimating the variance of the probability-based estimator of the mean, the variance of the model estimator is often ignored with the result that the variance of the hybrid estimator is systematically under-estimated.

In a comprehensive review of applications of models for large area inference, Ståhl et al. (2016) documented multiple applications of hybrid inference, although the term was not specifically used in any of the reports. For example, Ståhl et al. (2011) and Gobakken et al. (2012) obtained probability samples of strips of ALS data, used a model of the relationship between biomass and ALS metrics to predict biomass for cells that tessellated the strips, aggregated the cell predictions to obtain strip predictions, used probability-based cluster estimators to estimate the population mean biomass per unit area, and used model-based inference to estimate the variance of the cluster estimators and probability-based inference to estimate the variance of the estimator of the population mean.

The approach previously described in Section 1.1 using allometric models to estimate mean carbon per unit area constitutes another application of hybrid inference. The probability sample consists of the field plots; the auxiliary information consists of the plot-level, individual tree, species observations and dbh (1.37 m, 4.5 ft) and ht measurements; the allometric carbon model is used to predict carbon per unit area for the field plots; and probability-based estimators such as the simple random sampling or model-assisted regression estimators are used with the field plot predictions to estimate mean carbon per unit area. McRoberts et al. (2015) and McRoberts and Westfall (in press) reported examples of this application of hybrid inference, although as for the Ståhl et al. (2016) examples, the term itself was not used.

1.4. Objectives

The primary study objectives were to describe and illustrate the hybrid inferential framework, to estimate the variance of the hybrid estimator of the population mean of live tree stem carbon per unit area, and to assess the degree to which variances associated with particular information sources contribute to the variance of the hybrid estimator. The technical objectives included incorporating the effects on the variance of the individual tree, allometric model estimator of the covariances of the model parameter estimators, allometric model residual variance, variances of distributions of individual tree dbh and ht measurement errors, and variances of distributions of species-specific wood densities and carbon content proportions. Additional technical objectives were to compare the variance of the allometric model estimator to the variances of both simple random and model-assisted estimators of the population mean based on the allometric model predictions and to compare variances of the hybrid estimator when using species-specific and non-specific allometric models.

When estimating volume, biomass or carbon for large areas, models are not stand-alone or final products, but rather are intermediate products enroute to confidence intervals for the population mean. As a result, measures of model prediction accuracy such as root mean square error or pseudo-R^2 may contribute little to understanding the effects of the variance of the allometric model estimator on the variance of the hybrid estimator of the population mean. Therefore, model prediction accuracy should be assessed not only in absolute terms but also in the context of the model application, i.e., estimation of the population mean (Gregoire et al., 2016).

Similarly, the effects of the variances of distributions of wood densities and carbon content proportions should be assessed in the context of their application. Singh (1984), Baker et al. (2004), Chave et al. (2006) and Nock et al. (2009) all report relevant information on wood density including assessments of the degree to which species-specific wood densities vary with respect to site attributes, tree age and size, and stem location. However, the relative contributions to the variance of the hybrid estimator have not been fully addressed.

A Monte Carlo approach was used with nonlinear, individual-tree, allometric volume models constructed specifically for this study so that the covariances of the model parameter estimators and the allometric model residual variance could be rigorously estimated. The novel features of the study are characterization of the problem in the hybrid inferential framework, incorporation of the effects of variances in distributions of wood densities and carbon content proportions, use of the model-assisted regression estimators with ALS auxiliary information, and confidence intervals rather than model accuracies as the basis for assessments and comparisons.

2. Data

2.1. Study area

The study area was Itasca County in north central Minnesota in the United States of America (USA) (Fig. 1). The 7583-km^2 study area is characterized by approximately 80% forest land, 11% non-forest land, and 9% water. Tree species include upland deciduous mixtures, pines (*Pinus* spp.) spruce (*Picea* spp.) and balsam fir (*Abies balsamea*) and lowland species including spruce (*Picea* spp.), tamarack (*Larix laricina*), white cedar (*Thuja occidentalis*), and black ash (*Fraxinus nigra*).

2.2. Airborne laser scanning data

Wall-to-wall ALS data were acquired in April 2012 with a nominal pulse density of 0.67 pulses/m^2 using Leica ALS 60 and ALS 70 sensors. The average flying height above ground was 2100–2300 m, the field of view was 40 degrees, and the vertical accuracy was 11–15 cm. Ground returns were classified by the provider and used to construct a digital terrain model via interpolation using Tiffs (Toolbox for Lidar Data Filtering and Forest Studies) software (Chen, 2007).

Distributions of all first echo heights were constructed for the 168.3-m^2 plots and the 169-m^2 square cells that tessellated the study area. For each plot and cell, the mean, standard deviation, skewness, and kurtosis of the distributions were calculated as was quadratic mean height (QMH) (Lefsky et al., 1999; Chen et al., 2015). In addition, heights corresponding to the 10th, 20th, ..., 100th percentiles of the distributions were calculated, and canopy densities were calculated as the proportions of echoes with heights greater than 0%, 10%, ..., 90% of the range between 1.3 m above ground and the 95th height percentile (Gobakken and Næsset, 2008).

2.3. Allometric model calibration data

Data to calibrate the allometric volume model were originally acquired for a taper model study (Westfall and Scott, 2010) encompassing 24 northeastern states of the USA. For the current study, the model calibration data were restricted to the States of Michigan, Minnesota, and Wisconsin which span the ecological province that includes the study area. For the approximately 2400 individual trees whose data were used to calibrate the models, species was determined and dbh and total tree ht were measured. In addition, diameter measurements were obtained using a Barr and Stroud dendrometer at heights of 0.3, 0.6, 0.9, 1.4, and 1.8 m and at approximately 2.5-cm diameter taper intervals up to total tree ht. Volumes (m^3) of sections between height measurements were calculated using Smalian's formula (Avery and Burkhart, 2002, p. 101) as the product of mean cross-sectional area and section length, and total aboveground stem volumes (m^3) for individual trees were calculated by adding volumes for all sections. McRoberts and Westfall (in press) describe the sampling procedure and protocols for individual tree measurements in detail. For future reference, these data are assumed to be without error and are characterized as the *calibration dataset*.

2.4. Forest inventory ground data

Data were obtained for plots established by the Forest Inventory and Analysis (FIA) program of the U.S. Forest Service which conducts the national forest inventory (NFI) of the USA (McRoberts et al., 2010). Each FIA plot consists of four 7.32-m (24-ft) radius circular subplots that are configured as a central subplot and three peripheral subplots with centers located at 36.58 m (120 ft) and azimuths of 0°, 120°, and 240° from the center of the central subplot. Centers of forested, partially forested, or previously forested plots were determined using global positioning system receivers with sub-meter accuracy, whereas centers of non-forested plots were verified using aerial imagery and digitization methods. Field crews observe species and measure dbh (cm) and ht (m) for all trees with dbh of at least 12.7 cm (5 in) where the measurements are subject to error. Data were used for 86 forest plots and 29 non-forest plots measured in 2014. Further, data for only the central subplot of each plot were used to avoid issues of spatial correlation among subplot observations. For future reference, the term *plot* is used to refer to the central subplot and the corresponding data are characterized as the *estimation dataset*.

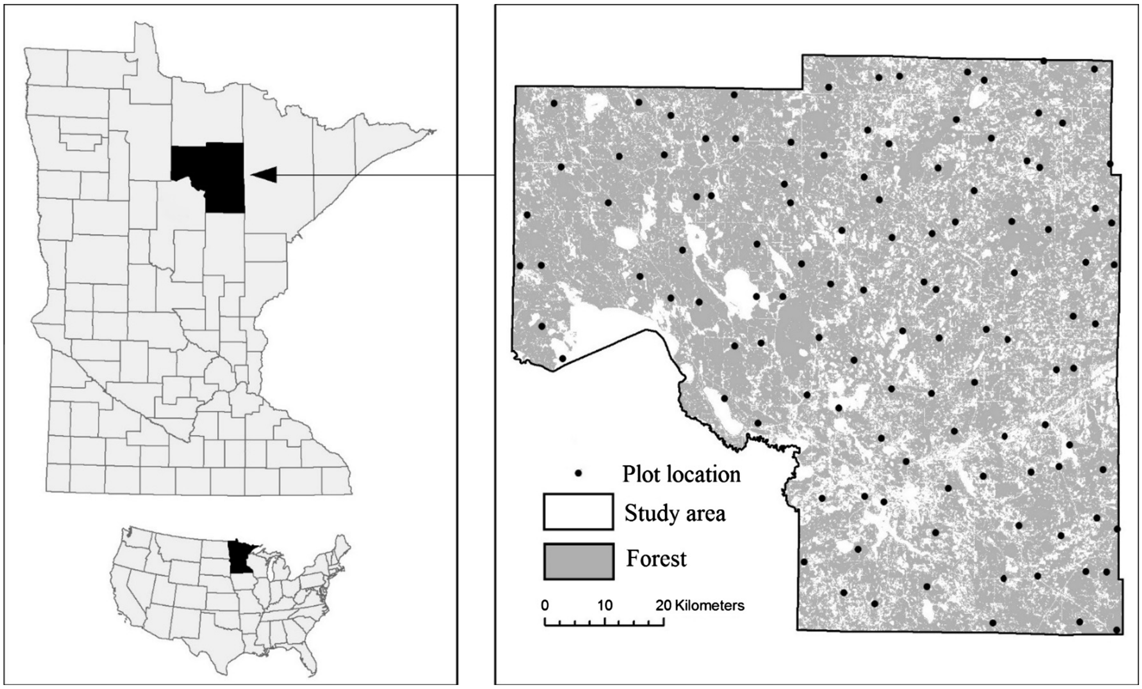

Fig. 1. Itasca study area in Minnesota, USA.

3. Methods

3.1. Hybrid inference

As described in Section 1.3, hybrid inference characterizes situations for which models are used to predict the response variable for a probability sample of auxiliary data and probability-based estimators are used to estimate population parameters based on the probability sample predictions (Corona et al., 2014; Ståhl et al., 2016). For the current study, an allometric model was used to predict individual tree stem volume using individual tree observations of species and measurements of dbh and ht as predictor variables for a probability sample of field plots. Species-specific biomass and carbon conversion factors were used to convert tree-level volume predictions to biomass predictions and then to carbon predictions which were then aggregated at plot-level and converted to a per unit area basis (kg/ha). Two forms of probability inference were used. First, simple random sampling estimators were used with the plot-level carbon predictions to estimate mean carbon per unit area. Second, ALS metrics were calculated for each plot and for regular polygons that tessellated the study area, and a model of the relationship between plot-level carbon and ALS metrics was used to predict carbon for the plots and the polygons. Mean carbon per unit area was then estimated using model-assisted regression estimators.

3.2. Model-based inference

Species-specific allometric models of the relationship between tree-level stem volume, V, as the response variable and dbh and ht as the predictor variables were formulated as,

$$v_i = \beta_0 \cdot dbh_i^{\beta_1} \cdot ht_i^{\beta_2} + \varepsilon_i, \tag{1}$$

where i indexes individual trees, ε_i is a random residual, and the βs are parameters to be estimated. McRoberts and Westfall (2014) documented the global popularity of this model form. The quality of model fit to the calibration data was assessed using pseudo-R^2,

$$R^{2*} = \frac{SS_{mean} - SS_{dev}}{SS_{mean}}, \tag{2}$$

where SS_{mean} is the sum of squared deviations of the observations from their mean, and SS_{dev} is the sum of squared deviations of observations from their predictions. A similar procedure was used to construct a non-specific, individual tree allometric model.

The individual tree, allometric models were constructed using data for the larger ecological province, nearly all of which is external to Itasca County (Section 2.3). Therefore, because a probability sample of individual tree carbon observations was not available, model-based inference was used to estimate the variance of the tree-level allometric model estimators. Although parametric model-based variance estimators could be used, Monte Carlo simulation approaches are often used when model predictor variables are subject to measurement error (Gertner and Dzialowy, 1984; McRoberts et al., 1994). For this study, the tree- and plot-level carbon predictions were affected by uncertainty from six sources: the covariances of the allometric model parameter estimators, the allometric model residual variance, the variances of distributions of dbh and height measurement errors, and the variances of distributions of wood densities and carbon content proportions. The analyses focused on estimating the variances associated with the six sources and using Monte Carlo procedures to propagate them through the variance of the allometric model estimator to the variance of the hybrid estimator of the population mean.

3.2.1. Allometric model parameter covariances

The covariances of the estimators of the allometric volume model parameters were estimated using a Monte Carlo approach: (i) the calibration dataset (Section 2.2) consisting of approximately 2400 observations was ordered by dbh and grouped into the same six dbh classes (originally inches, here cm) used to acquire the data: [0.00–12.70), [12.70–22.86), [22.86–33.02), [33.02–43.18), [43.18–60.96), [60.96+]; (ii) each dbh class was resampled with replacement until the original class sample size was achieved; and (iii) the model was fit to the resampled data and the parameters were estimated. Steps (i)–(iii) were replicated a large number

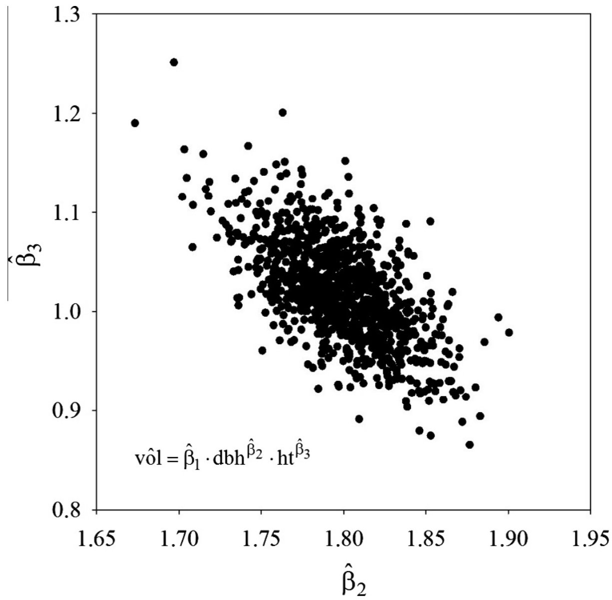

Fig. 2. Distribution of allometric model parameter estimates.

of times. The resulting multivariate distribution of parameter estimates represented the covariances of the allometric model parameter estimators (Fig. 2).

3.2.2. Allometric model residual variance

The heterogeneous allometric model residual variance was estimated using a 4-step procedure that accommodated heteroskedasticity: (i) from the fit of the model of Eq. (1) to the calibration data, the pairs $(v_i, \hat{v}_i)$ were ordered with respect to the model prediction, $\hat{v}_i$; (ii) the pairs were aggregated into groups of size 25; (iii) within each group, g, the mean of the predictions, $\bar{\hat{v}}_g$, and the standard deviation, $\hat{\sigma}_g$, of the residuals, $\varepsilon_i = v_i - \hat{v}_i$, were calculated; and (iv) the relationship between the group standard deviations, $\hat{\sigma}_g$, and the group prediction means, $\bar{\hat{v}}_g$, was represented using the model,

$$\hat{\sigma}_g = \lambda \cdot \bar{\hat{v}}_g + \varepsilon_g, \tag{3}$$

where λ is a model parameter to be estimated (Fig. 3).

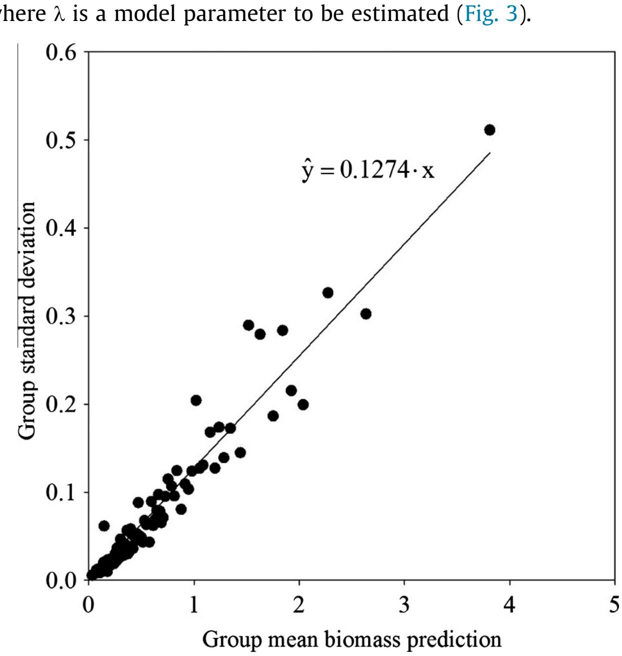

Fig. 3. Heteroscedastic residual standard deviation.

3.2.3. Diameter and height measurement error

The tolerance for dbh measurement errors specified by the FIA protocols is that 95% of measurements are to be within 0.5% of the true dbh (US Forest Service, 2015, Appendix 7). Assuming that dbh measurement errors follow a Gaussian distribution with mean 0, the standard deviation of the distribution is,

$$\sigma_{dbh} = \frac{0.005 \cdot \text{dbh}}{1.96} \approx 0.00255 \cdot \text{dbh}. \tag{4}$$

The tolerance for ht measurement errors specified by the FIA protocols is that 90% of measurements are to be within 10% of the true ht (US Forest Service, 2015, Appendix 7). Assuming that the ht measurement errors follow a Gaussian distribution with mean 0, the standard deviation of the distribution is,

$$\sigma_{ht} = \frac{0.10 \cdot \text{ht}}{1.645} \approx 0.06079 \cdot \text{ht} \tag{5}$$

3.2.4. Wood density

Wood densities are necessary to convert individual tree volume predictions to biomass predictions. For the Canadian provinces of Alberta, Manitoba, and Saskatchewan which are in close proximity to the study are, Singh (1984) reported wood density means (g/cm^3) for 10 prairie species, eight of which were also observed in Itasca County (Table 1). These means were based on 10–13 measurements per tree for 60 trees per species with 15 trees for each of four diameter classes. Further, the trees were sampled equally from across the Canadian provinces of Manitoba, Saskatchewan, and Alberta. For eastern Canada, Blouin and Cormier (2012) reported wood density means for 17 of the 20 species found in Itasca County. These wood densities were taken mostly from Alemdag (1984) who state that their data were obtained from "a full range of sites in Ontario" and represented different heights along the stem for an average of 59 trees per species. The two sets of Canadian wood density means were similar to each other but deviated, on average, by approximately 0.05 g/cm^3 from the corresponding species means for the USA reported by Miles and Smith (2009). A simple linear regression model with the combined sets of Canadian means as the response variable and the American means reported by Miles and Smith (2009) as the predictor variable was used to characterize the relationship between the Canadian and American means. An F-test of significance (Ratkowsky, 1983, p. 135) indicated that the estimated slope was not statistically significantly different from 1 ($\alpha = 0.05$). Therefore, a reduced model was fit with the slope fixed to 1; the resulting estimate of the intercept was 0.0507 (Fig. 4). For all species in Itasca County, the means of species-specific wood density distributions were denoted $\delta^{spe,0}$ and were estimated as the American means plus 0.0507, the estimate of the regression model intercept. The same mean was assumed for all trees of the same species within the study area regardless of factors such as site attributes, tree age and size, and stem location. For use with the non-specific allometric volume model, the mean of the pooled species-specific distributions of wood densities was estimated as the mean of the means of the species-specific distributions

Information on the variances of distributions of wood densities is sparse. For a tropical uncertainty assessment, Chave et al. (2004) assumed an overall ratio of the standard deviation to the mean for wood densities of 0.10. Chave et al. (2009) later reported mean wood density (g/cm^3) for North American species of 0.540 with standard deviation of 0.153. For a study of northern boreal and temperate forests in Russia, Europe and the USA, Thurner et al. (2014) reported mean wood density of 0.570 and standard deviation of 0.150 for broadleaved species; mean of 0.464 and standard deviation of 0.057 for needleleaf deciduous species; and mean of 0.411 and standard deviation of 0.066 for needleleaf conifer

Table 1
Species-specific sample sizes, wood densities, and carbon content proportions.

Species	Sample size		Wood density (g/cm^3)					Carbon content proportion	
	Calibration dataset	Estimation dataset	Mean				Std Dev	Mean	Range
			Miles and Smith (2009)	Blouin and Cormier (2012)	Singh (1984)	Predicted	Singh (1984)		
Balsam fir	65	64	0.33	0.341	0.372	0.3807	0.037	0.5008	0.0045
Tamarack	93	38	0.49	0.494	0.530	0.5407	0.041	0.4721	0.0035
White spruce	53	17	0.37	0.383	0.404	0.4207	0.037	0.5039	0.0045
Black spruce	90	13	0.38	–	0.457	0.4307	0.034	0.5040[a]	–
Jack pine	104	8	0.40	0.418	0.451	0.4507	0.036	0.5040	0.0043
Red pine	79	65	0.41	0.372	–	0.4607	–	0.5328	0.0033
White pine	78	1	0.34	0.342		0.3907	–	0.4974	0.0016
No. white cedar	89	9	0.29	0.311	–	0.3407	–	0.5172	0.0017
Red maple	112	12	0.49	0.588	–	0.5407	–	0.4864	0.0052
Sugar maple	132	22	0.56	0.616	–	0.6107	–	0.4932	0.0019
Yellow birch	73	2	0.55	0.596	–	0.6007	–	0.4627	0.0033
Paper birch	83	47	0.48	0.539	0.607	0.5307	0.045	0.4837	0.0021
Black ash	89	50	0.45	0.545	–	0.5007	–	0.4780	0.0048
Green ash	56	6	0.53	–	–	0.5807	–	0.4800[c]	–
Balsam poplar	102	41	0.31	0.354	0.409	0.3607	0.040	0.4779[b]	–
Quaking aspen	156	192	0.35	0.387	0.424	0.4007	0.033	0.4709	0.0075
Bur oak	14	3	0.58	–	–	0.6307	–	0.4779[b]	–
No. red oak	97	10	0.56	0.590	–	0.6107	–	0.4963	0.0032
Am. basswood	64	3	0.32	0.428	–	0.3707	–	0.4643	0.0017
Am. Elm	67	8	0.46	0.580	–	0.5107	–	0.4632	0.0027

[a] Conifer mean.
[b] Deciduous mean.
[c] Johnson et al. (2009, p. 457).

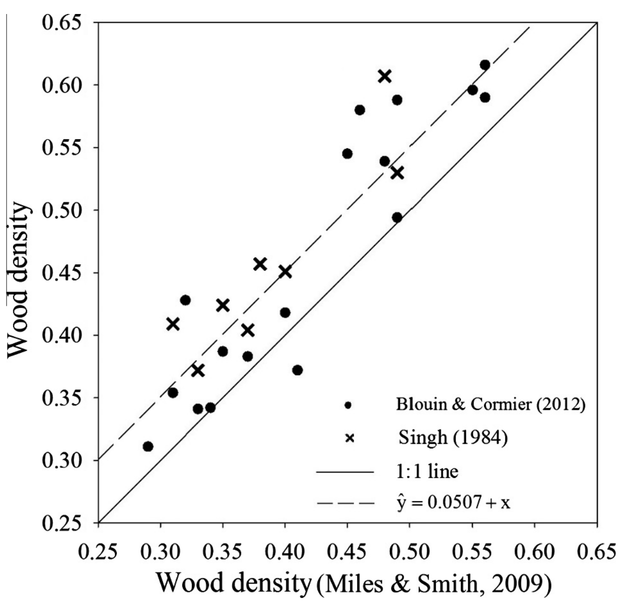

Fig. 4. Mean wood densities (g/cm^3) by species for eastern Canada (Blouin and Cormier, 2012) and the Canadian prairie provinces (Singh, 1984) versus national means for the USA (Miles and Smith, 2009).

species. For the previously reported Canadian study, Singh (1984) also reported species-specific standard deviations that ranged from 0.033 to 0.045. Because of the close proximity of the Canadian prairie provinces to the study area, the standard deviations reported by Singh (1984) were used to estimate a common species-specific wood density standard deviation of $\sigma_\delta = 0.0407$. The ratio of this value and the mean predicted wood density (Table 1) corresponds well with the overall ratio of the standard deviation to the mean of 0.10 assumed by Chave et al. (2004). For use with the non-specific allometric volume model, the variance of the pooled species-specific distributions of wood densities was estimated as the variance among the species-specific means plus the species-specific common variance of $\sigma^2_\delta = (0.0407)^2$.

3.2.5. Carbon content proportion

Lamlom and Savidge (2003; IPCC, 2003, Vol. 4., Table 4.3) reported carbon content proportions and ranges for 41 North American boreal and temperate species which included 16 of the 20 species observed in Itasca County (Table 1). These 16 species-specific proportions, denoted $\gamma^{spe,0}$, were used for this study. Carbon content proportion for green ash was obtained from Johnson et al. (2009), and conifer or deciduous means, as appropriate, were used for the remaining species. Carbon content proportions vary with respect to the same factors as wood densities, but for this study the same proportions, representing the means of the species-specific distributions, were used for all trees of the same species within the study area. The 16 ranges reported by Lamlom and Savidge (2003) were each assumed to represent four standard deviations and served as the basis for estimation of a common species-specific standard deviation, $\sigma_\gamma = 0.04$. For use with the non-specific allometric volume model, the mean of the pooled species-specific distributions of carbon content proportion was estimated as the mean of the means of the species-specific distributions. The variance of this pooled distribution was estimated as the variance among the species-specific means plus the common species-specific variance of $\sigma^2_\gamma = (0.04)^2$.

3.3. Probability-based inference

The validity of probability-based inference relies on the crucial assumptions that each population unit has a positive probability of selection for the sample and that a probability sampling design is used to select sample units (McRoberts et al., 2013). Hansen et al. (1983) used the term *probability-based* as an alternative to the more familiar term *design-based*, because the requirement is not just a sampling design, but rather a probability sampling design. Familiar estimators associated with probability-based inference include the simple random sampling, stratified, post-stratified, and model-assisted regression estimators.

3.3.1. Simple random sampling estimators

For equal probability samples, the simplest approach for estimating population parameters is to use the familiar simple random sampling (SRS) estimators,

$$\hat{\mu}_{SRS} = \frac{1}{n}\sum_{j=1}^{n} c_j \tag{6a}$$

and

$$\hat{Var}(\hat{\mu}_{SRS}) = \frac{\sum_{j=1}^{n}(c_j - \hat{\mu}_{SRS})^2}{n(n-1)}, \tag{6b}$$

where c_j is the carbon value for the jth population unit selected for the sample. The primary advantages of the SRS estimators are that they are intuitive, simple, and unbiased when used with an SRS design; the disadvantage is that variances are frequently large, particularly for large within-population variability and/or small sample sizes. Although $\hat{Var}(\hat{\mu}_{SRS})$ from Eq. (6b) may be biased when used with systematic sampling, it is usually conservative in the sense that it over-estimates the variance (Särndal et al., 1992, p. 83). For this study, finite population correction factors were ignored because of the small sampling intensity of approximately one 168.1-m^2 plot per 6600 ha of study area.

3.3.2. Model-assisted estimators

Model-assisted regression estimators use models based on auxiliary data to enhance inferences but rely on the probability sample for validity (Särndal et al., 1992). After considering multiple alternatives, an area-based model of the relationship between plot-level carbon and the ALS metrics was formulated as,

$$c_j = \beta_1 + \beta_2 \cdot \exp(\beta_3 \cdot QMH_j) + \varepsilon_j, \tag{7}$$

where j indexes plots, QMH_j is quadratic mean height, c_j is a plot-level carbon prediction, ε_j is a random residual, and the βs are parameters to be estimated. The β_1 parameter compensates for the fact that an exponential function is always positive, even when the argument is 0, and therefore cannot accurately predict biomass for non-forest plots with no carbon. For future reference, the model of Eq. (7) is characterized as the *area-based model* to distinguish it from the individual tree-level allometric volume model of Eq. (1)

A *synthetic* estimator of the population mean is,

$$\hat{\mu}_{Syn} = \frac{1}{N}\sum_{j=1}^{N}\hat{c}_j, \tag{8a}$$

where N is the population size and $\hat{c}_i$ is the model prediction of carbon for the ith population unit from Eq. (7). Hansen et al. (1983) note that models that do not "represent the state of nature" induce bias into this estimator which, for equal probability samples, can be estimated as,

$$\hat{Bias}(\hat{\mu}_{Syn}) = \frac{1}{n}\sum_{j=1}^{n}\hat{\varepsilon}_j, \tag{8b}$$

where $\hat{\varepsilon}_j = \hat{c}_j - c_j$. The *model-assisted*, *generalized regression* (GREG) estimator is then defined as,

$$\begin{aligned}\hat{\mu}_{GREG} &= \hat{\mu}_{Syn} - \hat{Bias}(\hat{\mu}_{Syn}) \\ &= \tfrac{1}{N}\sum_{j=1}^{N}\hat{c}_j - \tfrac{1}{n}\sum_{j=1}^{n}(\hat{c}_j - c_j)\end{aligned} \tag{8c}$$

When least squares parameter estimation techniques are used, the bias estimate will be zero for linear models and generally small for nonlinear models; nevertheless, the form of the estimator expressed by Eq. (8c) with the adjustment for estimated bias is still used. The corresponding variance estimator is,

$$\hat{Var}(\hat{\mu}_{GREG}) = \frac{1}{n(n-p)}\sum_{j=1}^{n}(\varepsilon_j - \bar{\varepsilon})^2, \tag{8d}$$

where p is the number of model parameters and $\bar{\varepsilon} = \frac{1}{n}\sum_{j=1}^{n}\varepsilon_i$ (Särndal et al., 1992; Särndal, 2011). Särndal et al. (1992, p. 326) suggests the *g*-weighted variance estimator for which the *g*-weights compensate for using a model estimated from the same sample as is used for calculating the residuals, ε_i. For this study the *g*-weights were ignored with the possible consequences of a slight underestimation of the variance (Mandallaz, 2013).

The primary advantage of the GREG estimators is that they capitalize on the relationship between the sample observations and their corresponding model predictions to reduce the effects of within-population variability and therefore reduce the variance of the estimator of the population mean. The degree to which the auxiliary information increases precision and thereby shortens the confidence interval is calculated using relative efficiency,

$$RE = \frac{\hat{Var}(\hat{\mu}_{SRS})}{\hat{Var}(\hat{\mu}_{GREG})}. \tag{9}$$

3.4. The simulation procedure

3.4.1. Model-based inference

A replicated Monte Carlo approach similar to that described by McRoberts and Westfall (in press) was used to generate n_{rep} sets of simulated plot-level carbon observations where each set represents a different set of allometric volume model parameter estimates, measurement errors, wood densities, and carbon content proportions. For replications, k = 1, …, n_{rep}:

(1a) Select a set of allometric volume model parameter estimates from the distribution constructed in Section 3.2.1.

(1b) For the ith tree on the jth plot in the estimation dataset, draw a random number, ε, from a Gaussian (0,1) distribution and simulate a dbh observation as,

$$dbh_{ij} = dbh_{ij}^0 + \varepsilon \cdot \sigma_{dbh},$$

where dbh_{ij}^0 is the observation from the estimation dataset, and σ_{dbh} is as described in Section 3.2.3.

(1c) For the ith tree on the jth plot in the estimation dataset, draw a random number, ε, from a Gaussian (0,1) distribution and simulate a ht observation as,

$$ht_{ij} = ht_{ij}^0 + \varepsilon \cdot \sigma_{ht},$$

where ht_{ij}^0 is the observation from the estimation dataset, and σ_{ht} is as described in Section 3.2.3.

(1d) For the ith tree on the jth plot in the estimation dataset, use the parameter estimates from Step (1a) and the simulated dbh and ht observations from Steps (1b) and (1c) to calculate an initial tree volume observation as,

$$v_{ij}^{k,0} = \hat{\beta}_1^k \cdot dbh_{ij}^{\hat{\beta}_2^k} \cdot ht_{ij}^{\hat{\beta}_3^k}.$$

Draw a random number, ε, from a Gaussian (0,1) distribution and calculate the heteroskedastic allometric volume residual standard deviation, $\hat{\sigma}_{ij}$, using Eq. (3) from Section 3.2.2 with $v_{ij}^{k,0}$ as the value of the predictor variable and simulate the individual tree volume as,

$$v_{ij}^k = v_{ij}^{k,0} + \varepsilon \cdot \hat{\sigma}_{ij}.$$

(1e) For each species, draw a random number ε from a Gaussian (0,1) distribution and simulate species-specific wood density as $\delta^{spe} = \delta^{spe,0} + \varepsilon \cdot \sigma_\delta$ where $\delta^{spe,0}$ and σ_γ are from Section 3.2.4. Similarly, for each species, draw a random number ε from a Gaussian (0,1) distribution and simulate species-specific carbon content proportion as $\gamma^{spe} = \gamma^{spe,0} + \varepsilon \cdot \sigma_\gamma$ where $\gamma^{spe,0}$ and σ_γ are from Section 3.2.5. Use γ^{spe} and δ^{spe} to simulate a carbon (Mg/ha) observation for the ith tree on the jth plot as,

$$c_{ij}^k = \gamma^{spe} \cdot \delta^{spe} \cdot v_{ij}^k,$$

where v_{ij}^k is tree-level volume from Step (1d).

(1f) Simulate the plot-level carbon observation for the jth plot in the estimation dataset as $c_j^k = \sum_{i=1}^{n_j} c_{ij}^k$ where n_j is the number of trees on the jth plot.

3.4.2. Probability-based inference

A replicated Monte Carlo simulation procedure was used as the estimator of mean carbon per unit area using the simulated plot-level carbon observations from Step (1f) and both the SRS and GREG estimators. For replications, k = 1, . . ., n_{rep}:

(2a) Use the plot-level carbon predictions from Step (1f) and the SRS estimators, from Eqs. (8a and b) to estimate mean carbon per unit area, $\hat{\mu}_{SRS}^k$, and the variance of the estimator of the mean, $\hat{Var}(\hat{\mu}_{SRS}^k)$.

(2b) Fit the area-based model, Eq. (7), to the kth set of simulated plot-level carbon observations from Step (1f) using the ALS metric, QMH, as the predictor variable.

(2c) Use the area-based model, Eq. (7), with parameter estimates from Step (2b) to predict carbon for each plot and for each of the 13-m × 13-m grid cells that tessellate Itasca County.

(2d) Use the GREG estimators from Eqs. (8c and d) with the simulated plot-level carbon observations from Step (1f) and both the plot-level and grid cell carbon predictions from Step (2b) to estimate mean carbon per unit area, $\hat{\mu}_{GREG}^k$, and to estimate the variance of the estimator of the mean, $\hat{Var}(\hat{\mu}_{GREG}^k)$.

3.4.3. Population parameter estimation

Combine the n_{rep} SRS estimates of means and variances from Step (2c) as per Rubin (1987, pp. 76–77) to calculate overall population estimates,

$$\hat{\mu} = \frac{1}{n_{rep}} \sum_{k=1}^{n_{rep}} \hat{\mu}^k, \tag{10a}$$

and

$$\hat{Var}(\hat{\mu}) = \left(1 + \frac{1}{n_{rep}}\right) \cdot W_1 + W_2, \tag{10b}$$

where $W_1 = \frac{1}{n_{rep}-1}\sum_{k=1}^{n_{rep}} (\hat{\mu}^k - \hat{\mu})^2$ is the among-replications variance, and $W_2 = \frac{1}{n_{rep}}\sum_{k=1}^{n_{rep}} \hat{Var}(\hat{\mu}^k)$ is the mean within-replications variance. Replications continue until $\hat{\mu}$ and $SE(\hat{\mu}) = \sqrt{\hat{Var}(\hat{\mu})}$ stabilize. Similarly, combine the n_{rep} GREG estimates of means and variances from Step (2d).

Three issues merit minor clarification. First, in Steps (1b) - (1e), if $|\varepsilon| > 2.5$, then ε was redrawn. Second, the covariances of the allometric model parameter estimators depend on the model residual variance as is illustrated via the parametric form of the parameter covariance matrix for a linear model,

$$Var(\hat{\beta}) = \sigma^2 \cdot (X' \cdot X)^{-1}, \tag{11}$$

where X is the matrix of values of the predictor variables and σ^2 is the residual variance (Bates and Watts 1988, p. 5). Therefore, because the effects of the covariances of the model parameter estimators cannot be separated from the effects of the model residual variance, neither the covariances of the model parameter estimators nor the model residual variance was incorporated into the simulation procedure apart from the other. Third, in Eq. (10b), W_2 is the mean within-replications variance conditional on particular sets of imputed tree-level carbon values. As per Rubin (1987) these sets of imputed values produce "complete-data estimates." W_1 is the variance "among the complete-data estimates," and Eq. (10b) is the total variance estimator (Rubin, 1987, pp. 76–77).

3.5. Analyses

The procedure described in Sections 3.4.3 was implemented for combinations of the six sources of uncertainty: variances of distributions of dbh and ht measurement errors, covariances of allometric model parameter estimators, allometric model residual variance, and variances of distributions of wood densities and carbon content proportions. The primary technical objectives were to assess the relative effects of the six sources of uncertainty on the variance of the allometric model estimator and to assess the effects of the variance of the allometric model estimator on the variance of the hybrid estimator when using simple random sampling and model-assisted regression estimators in the probability-based component.

Additional analyses were conducted to assess the sensitivity of the estimates of mean carbon per unit area to multiple other factors: (i) the species-specific wood density means reported by Miles and Smith (2009) as reported in Section 3.2.4; (ii) the wood density standard deviations reported by Chave et al. (2009) and by Thurner et al. (2014) in Section 3.2.4; and (iii) the species-specific carbon content proportion means reported by Lamlom and Savidge (2003) in Section 3.2.5.

4. Results and discussion

4.1. Allometric volume model prediction accuracies

Accuracies for the individual tree, species-specific allometric volume models were uniformly large with $0.94 \leqslant R^{2*} \leqslant 0.99$ over all species with the single exception for Tamarack for which $R^{2*} = 0.83$. For the non-specific allometric volume model, $R^{2*} = 0.95$. R^{2*} values in this range are typical, not only for temperate forests, but also for tropical and sub-tropical forests (Brown et al., 1989; Chave et al., 2005; Mugasha et al., 2013; McRoberts et al., 2015).

4.2. Area-based model prediction accuracies

Depending on the estimates of β_1 and β_2 for the area-based model of Eq. (7), some model predictions could be negative, although such was not the case for this study, regardless of whether uncertainty from the six sources were or were not incorporated. In addition, because the model has no upper asymptote, some predictions may be unrealistically large, particularly if QMH is larger for a population unit than the greatest value in the sample. However, for this study, no prediction was larger than the largest of the more than 8500 subplot-level observations obtained for the approximately 630 plots measured at 5-year intervals between 1999 and 2015 for the study area.

Median R^{2*} values for the area-based model were in the range $0.818 \leqslant R^{2*} \leqslant 0.831$, regardless of whether the species-specific or non-specific allometric models were used to predict individual tree volumes (Table 2). These R^{2*} values were similar to $0.76 \leqslant R^{2*} \leqslant 0.89$ reported by Næsset et al. (2011) and $R^{2*} = 0.84$ reported by McRoberts et al. (2013) for two Norwegian study areas; greater than $R^{2*} = 0.74$ reported by Strunk et al. (2012) for a study area in Washington, USA; and greater than $0.59 \leqslant R^{2*} \leqslant 0.72$ reported by d'Oliveira et al. (2012) for a Brazilian study area.

4.3. Estimates of mean carbon per unit area

For each of the four combinations of species-specific or non-specific allometric model and SRS or GREG estimators, estimates of mean carbon per unit area were very similar (Table 3). Differences are attributed primarily to random effects and to the nonlinear nature of the models whereby equal negative and positive perturbations around a given value of a predictor variable do not necessarily produce equal perturbations of predictions around the prediction for that given value. For each allometric model type, differences among estimates of the mean for the SRS and GREG estimators reflect differences in the sample and population distributions of the ALS predictor variables. Model misspecification characterized by the failure of the predictions to describe the pattern of the observations was negligible, and any contributions to estimator bias were offset by the second term in the GREG estimator. For each estimator, differences among estimates of the mean for the species-specific and non-specific allometric models were relatively small and are attributed to differences in the models themselves. These results are consistent with McRoberts and Westfall (2014) who reported only small differences in estimates of mean volume for species-specific and non-specific models for a different Minnesota dataset, and McRoberts et al. (2015) who reported similar results for a sub-tropical Brazilian dataset.

4.4. Uncertainty analyses

4.4.1. General results

The smaller SEs for the GREG estimators than for the SRS estimators, regardless of allometric model type or sources of uncertainty considered, reflect the utility of the ALS data to increase the precision of estimators. Although this result has been frequently reported, failure to observe it would have undermined the utility of the GREG estimators for reducing variances.

Bias in the variance of the hybrid estimator resulting from ignoring the variance of the allometric model estimator was assessed by comparing SEs obtained when none of the six sources of uncertainty was incorporated into the variance of the allometric model estimator to SEs obtained when all six were incorporated. For the species-specific allometric models, the SE with the SRS estimator was approximately 5% greater and the SE with the GREG estimator was approximately 23% greater when uncertainty from all six sources was incorporated relative to SEs when no uncertainty from any of the sources was incorporated (Table 3). These percentage differences for SEs equate to an approximate 10% difference in variances for the SRS estimator and an approximate 51% difference in variances for the GREG estimator. The 10% increase in variance for the SRS estimator is approximately the same as the increase reported by Ståhl et al. (2014) who, however, had much larger allometric model calibration sample sizes but did not consider uncertainty associated with wood densities or carbon content proportions. The much greater increase for the GREG estimator confirms the suggestion of McRoberts et al. (2013). For the non-specific allometric model, the differences in SEs were even greater, approximately 21% for the SRS estimators and approximately 92% for the GREG estimators; these SE percentage increases equate to 46% and 368% increases in variances, respectively.

4.4.2. Results by source

For all four combinations of allometric model type and probability-based estimator, the combined effects of dbh and ht measurement errors were negligible. For the species-specific allometric models with both the SRS and GREG estimators, the effects of the covariances of the allometric model parameter estimators, the allometric model residual variance, and the wood density and carbon content variances were approximately equal and had the greatest effects. For the non-specific allometric models with both the SRS and GREG estimators, the variance of the pooled distribution of wood densities accounted for nearly all the vari-

Table 2
Area-based, carbon-lidar model prediction accuracy.

Source of uncertainty						R^{2*}		
Allometric volume model		Measurement error		Conversion factors		5th percentile	50th percentile	95th percentile
Parameter uncertainty	Residual variance	dbh	ht	Wood density	Carbon content			
Species-specific allometric volume model								
–	–	–	–	–	–	0.831	0.831	0.831
X	X	–	–	–	–	0.823	0.831	0.832
–	–	X	X	–	–	0.818	0.828	0.838
–	–	–	–	X	X	0.805	0.830	0.849
–	–	–	–	X	–	0.828	0.831	0.834
–	–	–	–	–	X	0.804	0.829	0.850
X	X	X	X	–	–	0.815	0.828	0.838
X	X	–	–	X	X	0.803	0.827	0.847
–	–	X	X	X	X	0.801	0.828	0.849
X	X	X	X	X	X	0.797	0.826	0.848
Non-specific allometric volume model								
–	–	–	–	–	–			
X	X	–	–	–	–	0.830	0.831	0.833
–	–	X	X	–	–	0.818	0.830	0.841
–	–	–	–	X	X	0.740	0.820	0.861
–	–	–	–	X	–	0.753	0.824	0.860
–	–	–	–	–	X	0.807	0.831	0.851
X	X	X	X	–	–	0.819	0.830	0.841
X	X	–	–	X	X	0.744	0.821	0.861
–	–	X	X	X	X	0.730	0.818	0.860
X	X	X	X	X	X	0.743	0.820	0.858

Table 3
Estimates.

Source of uncertainty						Mean carbon estimates (Mg/ha)					
Allometric volume model		Measurement error		Conversion factors		SRS[a]			GREG[b]		
Parameter covariances	Residual variance	dbh	ht	Wood density	Carbon content	$\hat{\mu}_{SRS}$ (Mg/ha)	$SE(\hat{\mu}_{SRS})$ (Mg/ha)	Confidence interval[c]	$\hat{\mu}_{GREG}$ (Mg/ha)	$SE(\hat{\mu}_{GREG})$ (Mg/ha)	Confidence interval[c]
Species-specific allometric volume model											
–	–	–	–	–	–	21.16	3.24	[14.68, 27.64]	19.79	1.32	[17.15, 22.43]
X	X	–	–	–	–	21.03	3.31	[14.41, 27.65]	19.63	1.47	[16.69, 22.57]
–	–	X	X	–	–	21.17	3.26	[14.65, 27.69]	19.78	1.34	[17.10, 22.46]
–	–	–	–	X	X	21.15	3.31	[14.53, 27.77]	19.76	1.45	[16.86, 22.66]
–	–	–	–	X	–	21.16	3.24	[14.68, 27.64]	19.79	1.32	[17.15, 22.43]
–	–	–	–	–	X	21.14	3.31	[14.52, 27.76]	19.76	1.46	[16.84, 22.68]
X	X	X	X	–	–	21.04	3.32	[14.40, 27.68]	19.64	1.48	[16.68, 22.60]
X	X	–	–	X	X	21.05	3.40	[14.25, 27.85]	19.64	1.61	[16.42, 22.86]
–	–	X	X	X	X	21.21	3.33	[14.55, 27.87]	19.82	1.48	[16.86, 22.78]
X	X	X	X	X	X	21.05	3.41	[14.23, 27.87]	19.64	1.62	[16.40, 22.88]
Non-specific allometric volume model											
–	–	–	–	–	–	23.17	3.51	[16.15, 30.19]	21.63	1.42	[18.79, 24.47]
X	X	–	–	–	–	23.14	3.52	[16.10, 30.18]	21.63	1.44	[18.75, 24.51]
–	–	X	X	–	–	23.20	3.52	[16.16, 30.24]	21.69	1.45	[18.79, 24.59]
–	–	–	–	X	X	23.18	4.21	[14.76, 31.60]	21.68	2.49	[16.70, 26.66]
–	–	–	–	X	–	23.19	4.15	[14.89, 31.49]	21.71	2.40	[16.91, 26.51]
–	–	–	–	–	X	23.16	3.61	[15.94, 30.38]	21.65	1.61	[18.43, 24.87]
X	X	X	X	–	–	23.17	3.53	[16.11, 30.23]	21.66	1.47	[18.72, 24.60]
X	X	–	–	X	X	23.04	4.13	[14.78, 31.30]	21.56	2.40	[16.76, 26.36]
–	–	X	X	X	X	23.26	4.21	[14.84, 31.68]	21.75	2.47	[16.81, 26.69]
X	X	X	X	X	X	23.19	4.24	[14.71, 31.67]	21.71	2.54	[16.63, 26.79]

[a] SRS: simple random sampling estimators.
[b] GREG: model-assisted, generalized regression estimators.
[c] 2-standard error confidence interval.

ances for both the allometric model estimator and the hybrid estimator.

The effects of the covariances of the allometric model parameter estimators and the allometric model residual variance were much greater for the species-specific models than for the non-specific models, whereas the effects of the wood density variance were much less for the species-specific models than for the non-specific models. These results are attributed primarily to the much larger calibration sample size for the non-specific models which resulted in much smaller covariances for the model parameter estimators. Simultaneously, the similarities in model prediction accuracies as reflected in R^{2*} for both types of allometric models meant there was no offsetting effect of larger residual variance. Second, for the non-specific model, the variance of the pooled distribution of wood densities incorporates both the common species-specific variance and the variance among the species-specific means, but for the species-specific models, the variance among the species-specific means is not incorporated. In particular, for the species-specific models, the common species-specific wood density standard deviation was 0.004, but for the non-specific model the standard deviation of the pooled distribution was estimated as 0.095. The latter value is comparable to the mean standard deviation of 0.091 reported by Thurner et al. (2014) for northern boreal and temperate forests and less than the standard deviation of 0.153 reported by Chave et al. (2009) for all North American species. Thus, the effects of the variance of the pooled distribution of wood densities used with the non-specific model for this study may be regarded as comparable or less than effects reported in the literature.

In general, uncertainties for individual sources that function at the tree- or plot-level had smaller effects than uncertainties that function at the population level. For example, dbh and ht measurement errors function at the tree-level and, therefore, measurement errors more readily compensate for each other at plot-level. Conversely, uncertainties that function at the population level such as the variance of the pooled distribution of wood densities have greater effects.

4.5. Comparative analyses

4.5.1. Comparisons for carbon conversion factors

Although species-specific carbon content proportions were used for this study, commonly a non-specific value of 0.50 g/cm^3 is used (Woodall et al., 2011, p. 3). For the four combinations of allometric model type and probability estimator, differences in estimates of the mean carbon per unit area resulting from using the common non-specific value of 0.50 g/cm^3 ranged from 0.33 to 0.53 Mg/ha and on a percentage basis from 1.6% to 2.5%. These negligible differences can be attributed to the similarity of the species-specific carbon proportions to the common non-specific value of 0.50 g/cm^3 and to the small standard deviations of the species-specific distributions (Table 1).

4.5.2. Comparisons for wood densities

The estimates of the population means were compared for the wood densities used for this study and the American wood densities reported by Miles and Smith (2009) (Table 1). For the four combinations of allometric model type and probability-based estimator, the differences in estimates of the means ranged from −1.69 to −1.84 Mg/ha and on a percentage basis from −7% to to 8%. These marginally negligible differences were all less than two SEs and are consistent with the average difference of approximately 0.05 g/cm^3 between the wood density means used and the American wood densities (Fig. 4).

Estimates of means and SEs obtained for the wood density means and their standard deviations selected in Section 3.2.4 were compared to estimates obtained using the North American wood density means and standard deviations reported by Chave et al. (2009). For the non-specific allometric models and a common carbon content proportion, the difference in the means for the SRS estimators was 5.35 Mg/ha or 23%, and the difference in SEs was 1.19 Mg/ha or 29%; for the GREG estimators, the difference in the means was 5.10 Mg/ha or 23%, and the difference in SEs was 40%. These differences are certainly not negligible and can at least partially be attributed to the effects of using means over the entirety of North America rather than means more closely targeted to the study area.

Estimates of means and SEs were similarly compared for the broadleaf, needleleaf deciduous, and needleleaf evergreen wood density means and corresponding standard deviations for boreal and temperate regions reported by Thurner et al. (2014). To avoid confounding between the effects of different species data used as the basis for the allometric volume models and different wood density means and standard deviations, models for the same species groups as reported by Thurner et al. (2014) were constructed specifically to facilitate these comparisons. Ignoring the effects of covariances for the allometric model parameter estimators and the allometric residual variance, estimates of mean carbon per unit area when using the Thurner et al. (2014) wood density means were larger by nearly 20% and the SE was larger by 45% for the SRS estimators and 105% for the GREG estimators. These differences in the estimates and particularly the SEs are even less negligible than the estimates obtained using the Chave wood density means.

5. Conclusions

Three primary conclusions were drawn from the study. First, regardless of whether species-specific or non-specific allometric volume models were used, and regardless of whether simple random sampling or generalized regression estimators were used, estimates of mean carbon per unit area were similar and within two standard errors of each other.

Second, the estimate of bias in the variance of the hybrid estimator resulting from ignoring the variance of the allometric model estimator was 10% when using the species-specific allometric model and the simple random sampling estimator, 51% when using the species-specific allometric model and the generalized regression estimator, 46% when using the non-specific allometric model and the simple random sampling estimator, and 368% when using the non-specific allometric model and the generalized regression estimator. Differences in results for the simple random and generalized regression estimators are mostly attributed to reduction in the variance of the hybrid estimator resulting from use of the auxiliary airborne laser scanning data. Differences in results for the species-specific and non-specific allometric models are mostly attributed to the much greater variance of the pooled species-specific distributions of wood densities relative to the variances of the individual species-specific distributions. Overall, the effects of ignoring the variance of the allometric model estimator were negligible only for the combination of the species-specific allometric model and the simple random sampling estimator. These results suggest that when non-specific allometric models are used, as is the case for many tropical inventories, ignoring the effects of the variance of the allometric model estimator may substantially under-estimate the SE of estimates of mean carbon per unit area.

Third, the effects of continental-level and/or ecological province-level wood density means on both estimates of mean carbon per unit area and the variances of the estimators were substantial.

Two additional issues merit comment. First, all results are based on allometric model calibration sample sizes in the range of 65–155 with mean of 85, and R^{2*} generally greater than 0.95. Smaller sample sizes and/or less accurate models could lead to somewhat different results. Second, any reduction in the variance of the hybrid estimator of mean carbon per unit area obtained from non-specific allometric models would require a smaller variance of the pooled distributions of the wood densities. Unfortunately, the greatest contributor to this variance is the variation among species, not the variance within species. For tropical inventories, with large numbers of species, reduction of the variance of the hybrid estimator will remain a challenge.

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