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ANALYSIS OF FIRE SPREAD IN LIGHT FOREST FUELS[1]

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INTRODUCTION

On the national forests there has been a trend during recent years toward placing on a systematic basis such fire-control practices as rating fire danger, determining the proper size of suppression crews and speed of attack, and planning fire-suppression strategy. This trend has resulted in greatly improving the techniques of planning and managing the fire-control organization. At the same time it has revealed a serious lack of essential information. The outstanding need is for specific data on the rate at which forest fires may be expected to spread under various conditions of forest cover, weather, and topography.

Earlier studies in California by Show (*13*)[4] and Curry and Fons (*6*, *7*) consisted of observations on the rate of fire spread under natural forest conditions of fuel, weather, and topography. The initial series of experiments by Curry and Fons (*6*) in one of the least complex fuel types, ponderosa pine needles, provided data on the empirical relationship between the rate of fire-perimeter increase, litter moisture content, wind velocity, slope, and time from origin of fire. Similar observations were made on other fuel types.

Analysis of these data made it apparent that without a basic understanding of the many physical processes involved, the results of the experiments could not be applied beyond the limits of the conditions under which the observations were made. Therefore, it became necessary to determine the fundamental laws governing the rate of fire spread in forest-type fuels. Under field conditions, however, none of the important factors, such as the attributes of the atmosphere, the arrangement of the fuel bed, and the physical properties of the fuel particles, remained sufficiently uniform throughout an experiment to allow exact description of the numerous variables influencing the rate of fire spread. In order to understand the influence of these variables it was decided to conduct experiments with model fires in beds of chosen homogeneous fuel particles. These were at first performed by Curry and Fons (*7*) in still air for different conditions and kinds of fuel, fuel-moisture content, and fuel-bed compactness, and later by Fons (*9*) in a wind tunnel in which air velocity was controlled.

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[2] Acknowledgment is made to C. C. Buck, of the California Forest and Range Experiment Station, and H. D. Bruce, of the Forest Products Laboratory. for criticism of portions of the paper: and to L. M. K. Boelter and Raymond Martiuelli, of the University of California, for valualbe suggestionson the mathemathical treatment.

[3] Maintained in cooperation with the University of California, Berkelep, Calif.

[4] Italic numbers in parentheses refer to Literature Cited, p. 121.

Concurrently with the latter experiments, an analytical treatment of the rate of spread of fire was being developed and utilized to direct the course and improve the techniques of fire control. This experiment and its results are described in this paper.[5]

GLOSSARY OF SYMBOLS

A=cross-sectional area of fuel particle, sq. in.
A_s=surface area of particle, sq. in.
$a, b, c, \ldots, n$=subscripts designating different fuels.
$B=\frac{W_a}{A_a\gamma_a}+\frac{W_b}{A_b\gamma_b}+\frac{W_c}{A_c\gamma_c}+\ldots+\frac{W_n}{A_n\gamma_n}$.
C=proportionality constant relating a particular kind of fuel bed to a standard chosen bed.
C_p=specific heat of dry fuel at constant pressure, B. t. u./lb. ° F.
$\overline{C}_p$=specific heat of moist fuel at constant pressure, B. t. u./lb. ° F.
C_t=temperature-difference ratio coefficient.
D=diameter of fuel particle, inches.
d=prefix indicating differential, also depth of fuel bed, inches.
E=fuel particle shape factor, $\sigma A^{1/2}$.
f=over-all transfer factor, also subscript indicating flame or film.
f_c=film conductance for convection, and conduction, B. t. u./sq. ft. hr. ° F.
f_r=heat transfer factor for radiation, B. t. u./sq ft. hr. ° F.
h=heat of adsorption of dry fuel, B. t. u./lb.
k_f=thermal conductivity of the air film, B. t. u./sq. ft. hr. ° F./ft.
L=spacing between particles, inches.
M=moisture content, weight of water/weight of dry fuel.
N=number of particles.
q=total rate of heat transferred, B. t. u./hr.
q_c=rate of heat transferred by convention and conduction, B. t. u./hr.
q_r=rate of heat transferred by radiation, B. t. u./hr.
R=rate of spread or $\frac{dr}{d\theta}$ in r-direction, ft./hr.
S=total fuel surface in volume V_1, sq. in.
T=temperature, degrees Rankine.
t=temperature of fuel particle at time θ, ° F.
t_0=temperature of fuel particle far away from flame, ° F.
t_1=temperature of fuel particle near flame, ° F.
t_f=temperature of flame, °F.
t_i=ignition temperature of fuel particle, ° F.
V=wind velocity at 1 foot above fuel-bed surface, m. p. h.
V_1=volume of fuel bed, cu. in.
V_2=volume of fuel in fuel bed volume V_1, cu. in.
v=volume of particle, cu. in.
W=weight of dry fuel, lbs.
γ= (gamma) density of dry fuel, lbs./cu. ft.
$\overline{\gamma}$= (gamma bar) density of moist fuel, lbs./cu. ft.
ϵ= (epsilon) flame emissivity.
θ= (theta) time in hours.
θ_i=ignition time of fuel; time in hours to raise a fuel particle from its initial temperature to ignition temperature.
λ= (lambda) volume of voids per unit of fuel surface, inches.
ν_f= (nu) kinematic viscosity of the film, sq. ft./hr.
σ= (sigma) surface-volume ratio, or fineness of fuel, inches^{-1} (1÷number of inches).
ϕ= (phi) slope of terrain, percent.

[5] A portion of the analysis in its earlier stages was submitted by the author in May 1940 as a thesis, entitled "Analytical Considerations of Model Forest Fires," in partial fulfillment of the requirements for the degree of master of science in the Department of Mechanical Engineering of the University of California.

DERIVATION OF EQUATIONS FOR RATE OF SPREAD IN AN IDEALIZED HOMOGENEOUS FUEL BED

Forest cover may be regarded as a heterogeneous mixture of dead and green fuel particles of varying size, shape, and density. Two layers may be distinguished: a lower layer of inflammable litter next to the ground surface, and an upper layer of dead and green standing vegetation. A fire may spread through either or both layers at a definite rate depending on the magnitude of many variables pertaining to (1) the atmosphere, (2) the fuel bed, (3) the fuel particles, and (4) the topography. The fire will spread quickest in that direction and through that layer where ignition is most favorable, or where resistance to flame propagation is at a minimum.

A forest fire characteristically travels more or less rapidly through the inflammable cover, singeing superficially only the finer, more ignitible particles, later slowly and thoroughly consuming the great mass of heavier fuels. In this theory of the rate of spread it is important to center our attention on the action at the head of the fire, where there is generally ample oxygen to support combustion. At

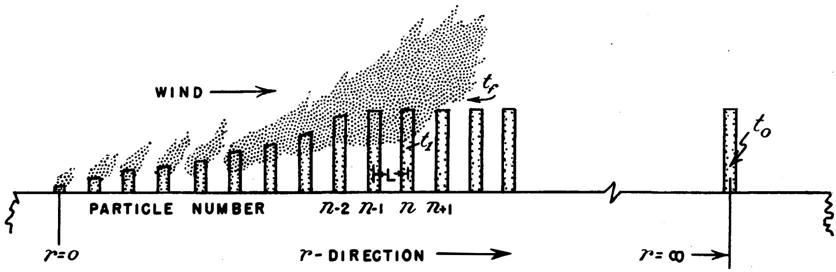

FIGURE 1.—Graphic representation of the rate of spread of fire in an idealized fuel bed composed of identical rodlike fuel particles.

this point only sufficient heat need be produced for a self-supporting fire to bring the immediate adjoining fuel to ignition temperature. Accordingly, flame propagation, or rate of fire spread in a fuel bed, may be visualized as proceeding by a series of successive ignitions, its magnitude controlled primarily by the ignition time of the particles and the distance between them.

As in other complex natural phenomena, the problem must be idealized in order to facilitate mathematical treatment of the theory. It is believed, however, that doubt as to the applicability to field conditions of the theoretical equations developed by this idealization will be removed when the predicted effects of the individual variables on rate of spread are compared with previous experimental results and with observations under actual field conditions.

RATE OF SPREAD AS A FUNCTION OF PARTICLE SPACING AND IGNITION TIME

For this derivation an idealized bed is assumed (fig. 1) composed of identical rodlike fuel particles, standing upright at equal distances from each other. If θ_i is the time for the n particle to burst into flame after the $(n-1)$ particle has ignited, and if L is the distance between the particles, it follows that the rate of spread R is

$$R=\frac{L}{\theta_i} \tag{1}$$

where $R=\frac{dr}{d\theta}$ in the r-direction down the row of fuel particles.

RATE OF SPREAD AS A FUNCTION OF FUNDAMENTAL VARIABLES

The ignition time θ_i in equation (1) depends on two types of variables: Those dealing with (a) the fuel particle and (b) the atmosphere surrounding the particle.

Again it is assumed that the bed is composed of idealized fuel particles (fig. 1) spaced at a uniform distance L and that immediately upon ignition of the $(n-1)$ particle its flame at temperature t_f extends far enough to be in contact with the n particle which is at a temperature t_1. When the latter particle is exposed to the gases of flame temperature, t_f, it will absorb heat by convection, conduction, and radiation.

In the theory of fluid flow (*11, 14*) it is generally accepted that when a gas or liquid moves over a solid a thin film of fluid forms on the surface. This becomes thinner as the velocity of the fluid increases, and the thinner the film the more rapid will be the transfer of heat to the solid by conduction, convection, and radiation. The film conductance or transfer factor, f, is used to express the net rate of heat transfer through the film per unit area per degree difference in temperature. Factor f is the sum of two transfer factors, f_c for conduction and convection, and f_r for radiation. These are determined independently because the heat exchange by radiation depends primarily on the temperatures of the surfaces, while that for conduction and convection depends on variables such as the physical properties of the fluid, the shape of the solid surface, and the velocity of the fluid past the solid boundary. The method of calculating the transfer factors is discussed below.

The heat transferred by convection and conduction in unit time from the atmosphere at t_f to the particle is

$$q_c=f_cA_s(t_f-t) \tag{2}$$

The heat transferred by radiation in unit time from the atmosphere at t_f to the particle is approximately:

$$q_r \approx f_rA_s(t_f-t) \tag{3}$$

The total heat absorbed in unit time is

$$q=q_c+q_r=(f_c+f_r)(t_f-t)A_s \tag{4}$$

where

A_s = surface area of the particle;

t_f = uniform temperature of flame surrounding the particle;

t = temperature of particle at any time θ, assumed to be uniform throughout the volume of the particle, since the temperatuce gradient is very low for fine fuel particles with large surfare-volume ratio;

f_c and f_r are the coefficients of heat transfer for convection and radiation energy, respectively.

The rate of heat absorption, q, will cause a rate of temperature change $\frac{dt}{d\theta}$, according to the density of the moist particle, $\bar{\gamma}$, the specific heat of the moist particle, C_p, and the volume of the fuel particle, v, as follows:

$$q=\bar{\gamma}\bar{C}_p v\frac{dt}{d\theta} \tag{5}$$

Substituting (5) in (4) yields:

$$\frac{dt}{(t_f-t)}=\frac{(f_c+f_r)A_s}{\bar{\gamma}\bar{C}_p v}d\theta \tag{6}$$

By integrating equation (6), with $\frac{(f_c+f_r)A_s}{\bar{\gamma}\bar{C}_p v}$ assumed constant, equation (7) is obtained:

$$-ln(t_f-t)=\frac{(f_c+f_r)A_s\theta}{\bar{\gamma}\bar{C}_p v}+lnC \tag{7}$$

where ln represents the Napierian logarithm.

The initial conditions to be satisfied are: when $\theta=0$, $t=t_1$. By substituting these values in (7) the constant of integration becomes

$$lnC=-ln(t_f-t_1) \tag{8}$$

Substituting (8) in (7) and rearranging

$$ln\left(\frac{t_f-t_1}{t_f-t}\right)=\frac{(f_c+f_r)\sigma\theta}{\bar{\gamma}\bar{C}_p} \tag{9}$$

where $\sigma=\frac{A_s}{v}$ = the surface-volume ratio of fuel particle.

The final conditions to be satisfied are: when $\theta=\theta_i$, $t=t_i$, where θ_i=ignition time, and t_i=ignition temperature.

Making these substitutions and rearranging for θ_i, equation (9) becomes

$$\theta_i=\frac{\bar{\gamma}C_p}{\sigma(f_c+f_r)}ln\left(\frac{t_f-t_1}{t_f-t_i}\right) \tag{10}$$

Equation (10) is an expression for the ignition time, θ_i, in terms of the physical properties of the light fuel particle,[6] the flame temperature, and the heat transfer factors.

The rate of spread for an idealized homogeneous fuel bed is derived directly by substituting the equivalent of θ_i from equation (10) in equation (1):

$$R=\frac{(f_c+f_r)\sigma L}{\bar{\gamma}\bar{C}_p\, ln\left(\frac{t_f-t_1}{t_f-t_i}\right)} \tag{11}$$

In this equation R is expressed in terms of certain fundamental variables that control the spread of a fire in light forest fuels. Such

[6] Derivation of an expression for heavier fuel particles, in which there is an appreciable temperature gradient through the particle, is possible by the application of an appropriate form of the conduction equations (*5*).

quantities as wind velocity, moisture content, time, and topography, which are ordinarily regarded as controlling spread, do not appear in the equation. They affect the rate of spread indirectly through their influence on the more fundamental variables.

Film Conductance for Convection

Experimental data for heat transfer from gases to single cylindrical rods, with the gas flowing at right angles to the axis, have been collected by numerous investigators. Data for cylinders from 0.0004 to 3.75 inches in diameter (*11, p. 218*), applicable to fine forest fuels such as needles and twigs, have been summarized and correlated in terms of dimensionless quantities related by the following empirical equation, which may be used as an approximation for calculating f_c:

$$\frac{f_c D}{k_f}=0.45+0.33\left(\frac{VD}{\nu_f}\right)^{0.56} \tag{12}$$

where f_c = film conductance for forced convection
D = diameter of cylinder
k_f = thermal conductivity of the air film
ν_f = kinematic viscosity of the air film
V = air velocity.

Thermal conductivity and kinematic viscosity depend on the arithmetic mean temperature of the film. Values of these properties for air are given in the International Critical Tables for a wide range of temperatures.

Forest fuels such as needles and twigs are nearly enough cylindrical in shape to allow replacing D in equation (12) with $\frac{4}{\sigma}$, where σ is the surface-volume ratio of the fuel particle.

Heat Transfer Coefficient for Radiation and Flame Temperature

The net rate of heat transfer by radiation, q_r, to a small particle surrounded by a flame (*14*) is obtained by the following equation:

$$q_r=0.172\epsilon A_s\left[\left(\frac{T_f}{100}\right)^4-\left(\frac{T_1}{100}\right)^4\right] \tag{13}$$

where ϵ = flame emissivity [7]
T_f = flame temperature
T_1 = particle temperature
A_s = surface area of the particle

An expression for f_r may be derived by substituting simplified equation (3) in general equation (13),

$$f_r=\frac{0.172\epsilon\left[\left(\frac{T_f}{100}\right)^4-\left(\frac{T_1}{100}\right)^4\right]}{t_f-t_1} \tag{14}$$

[7] Radiant emissivity is defined as the ratio of the total emissive power (total radiant energy emitted per unit time from unit surface) of a surface to the total emissive power of a black body at the same temperature.

Flame temperatures were measured by chromel-alumel thermocouples held at several points in the luminous flames of different kinds of burning forest fuel. The average temperature so measured was approximately 1500° F. Simultaneously, the emissive power of the flames was measured by a portable radiometer described by Boelter (*2*). From the measured flame temperature, the radiometer element temperature, and the flame emissive power, q_r, the emissivity, ϵ, was calculated by the radiation equation (13). The approximate value of ϵ so determined was 0.227.

Until more data are acquired, the values used for ϵ and t_f in calculating f_r are 0.23 and 1500° F., respectively. Since the particle is being heated until it reaches ignition temperature, an approximation must be made for the fuel temperature, t_1. On the assumption that the rate of change of the particle temperature is constant, the fuel temperature, t_1, may be approximated by using the arithmetic mean of the ignition temperature, t_1, and the fuel temperature, t_0. While t_1 may vary over a considerable range between t_i and t_0, its approximation at the mean value of these two will not result in an appreciable error in the calculation of f_r.

Ignition Temperature

There is no general agreement as to the ignition temperatures of solid fuels. The values determined by several investigators whose work was reviewed by Brown (*3*) show a large variation. For instance, values as high as 1200° and as low as 650° F. have been obtained for sawdust and as low as 410° for wood shavings. The principal causes of the variations are (1) the definition of ignition temperature, which may refer to the specimen temperature or to the external temperature of a bar, an ambient gas, or an oven, and (2) the criteria chosen by different investigators to indicate the ignition point. Criteria that have been used are the appearance of glow, the appearance of flame, or some critical point in a temperature- or pressure-time curve.

To be consistent with the preceding theory and the equations developed for the rate of fire spread in light forest fuels, the following definition is proposed: The ignition temperature of a light forest-fuel particle surrounded by ambient hot gases is the average specimen temperature at the instant a flame appears. For light forest fuels of known physical properties exposed to hot gases of known temperature, the average specimen temperature may be calculated by equation (10) from the ignition time, which is determined experimentally. Until such experimental data are available, an arbitrary chosen value of 550° F. is recommended for all light forest fuels irrespective of fuel size and moisture content. Jones and Scott[8] found the ignition temperature of dry wood to be in the range of 518° to 554° F.

Particle Spacing and Surface-volume Ratio

To calculate the rate of fire spread in a fuel bed by equation (11), *L*, the spacing, must be related to the measurable fuel and fuel-bed variables.

[8] Jones, G. W., and Scott, G. S. chemical considerations relating to fires in anthracite refuse. U. S. Bur. Mines Rpt. Invest. 3468, 13 pp., illus. 1939. [Processed.]

For a bed of fuel, the ratio of voids to fuel-surface area is

$$\lambda = \frac{V_1 - V_2}{S} \tag{15}$$

where λ = volume of voids per fuel-surface area
V_1 = volume of fuel bed
V_2 = volume occupied by the fuel in volume V_1
S = total surface area of fuel in volume V_1.

The reciprocal of λ may be thought of as the compactness of the fuel.

The volume of the idealized fuel bed is

$$V_1 = L^2 dN \tag{16}$$

where L = spacing of particles
d = depth of fuel bed
N = total number of particles.

The volume occupied by the fuel in the bed of volume V_1 is

$$V_2 = \frac{W}{\gamma} \tag{17}$$

where W is the total weight and γ is the density of the dry fuel. The total surface area of the fuel in bed of volume V_1 is

$$S = \frac{\sigma W}{\gamma} \tag{18}$$

Substituting equations (16), (17), and (18) in (15) gives

$$\lambda = \frac{L^2 dN - W/\gamma}{\sigma W/\gamma} \tag{19}$$

Solving equation (19) for L and letting the cross-sectional area $A = \frac{W}{N\gamma d}$ (since for an idealized bed the length of the particles is equal to the depth of the bed) (19) reduces to:

$$L = A^{1/2}(1 + \sigma\lambda)^{1/2} \tag{20}$$

Equation (20) expresses the particle spacing in terms of the cross-sectional area, the surface-volume ratio of the fuel particle, and the ratio of voids to fuel-surface area of the bed.

Multiplying both sides of equation (20) by σ and letting $E = \sigma A^{1/2}$ then

$$\sigma L = E(1 + \sigma\lambda)^{1/2} \tag{21}$$

The quantity E is independent of the size of the fuel particle and depends only on the shape of the cross section. For instance, for round sticks regardless of size, $E = 2\pi^{1/2}$; for square sticks, $E = 4.0$; for more irregular cross sections, $E > 4.0$. The cross-sectional area of an irregularly shaped particle may be determined from the relation $A = \frac{p}{\sigma}$, where p is the perimeter and σ the surface-volume ratio.

Specific Heat

Since the fuel particles nearly always carry adsorbed water, the specific heat of the moist fuel, $\overline{C}_p$, may be expressed in terms of specific heat of dry fuel, its moisture content, and the heat of adsorption.

The total heat required to raise a moist fuel particle at temperature t_0 to ignition temperature t_i is

$$H=(W+MW)\overline{C}_p(t_i-t_0) \tag{22}$$

where W=dry weight of fuel
M=moisture content, weight of water per unit weight of dry fuel
$\overline{C}_p$=specific heat of moist fuel.

The total heat, H, has three components: the heat required (1) to separate the water from the fuel, that is, the heat of adsorption; (2) to raise the dry fuel from temperature t_0 to ignition temperature, t_i; and (3) to vaporize the water and superheat the vapor to temperature t_i.

The heat required to raise the dry fuel to ignition temperature is

$$H_1=WC_p(t_i-t_0) \tag{23}$$

where C_p is specific heat of dry fuels (*8*)[9];

and to raise the water to temperature t_i is

$$H_2=MW(h_i-h_0)$$

where $h_i=1{,}057+0.46t_i$, the heat (*1, p. 309*) of the superheated steam at low pressure in B. t. u. per pound, and $h_0=t_0-32$, the heat content of water at temperature t_o, whence

$$H_2=MW(1{,}089+0.46t_i-t_0) \tag{24}$$

The heat of adsorption per unit dry weight of fuel is h, and heat required for weight of dry fuel W is

$$H_3=Wh \tag{25}$$

The heat of adsorption for wood at 32° F. is approximately 32 B. t. u. per pound of dry fuel (*10, p. 286*).

Adding equations (23), (24), and (25), the total heat, H, to bring W, weight of moist fuel, to ignition temperature t_i is

$$H=WC_p(t_i-t_0)+MW(1{,}089+0.46t_i-t_0)+Wh \tag{26}$$

[9] The equation for specific heat of dry wood from results considered as preliminary is

$$C_p=0.246+0.00063\left(\frac{t_i+t_0}{2}\right) \quad \text{B.t.u./lb. °F.}$$

Equating (26) to (22) and solving for $\overline{C}_p$, the specific heat of moist fuel is

$$\overline{C}_p=\frac{1}{1+M}\left[M\left(\frac{1{,}089+0.46t_i-t_0}{t_i-t_0}\right)+\frac{h}{t_i-t_0}+C_p\right] \quad (27)$$

DENSITY OF MOIST FUEL

The relation between the density of the moist fuel and dry fuel, γ, is $\overline{\gamma}=(1+M)\gamma$. Ordinarily the density of the dry fuel, γ, and

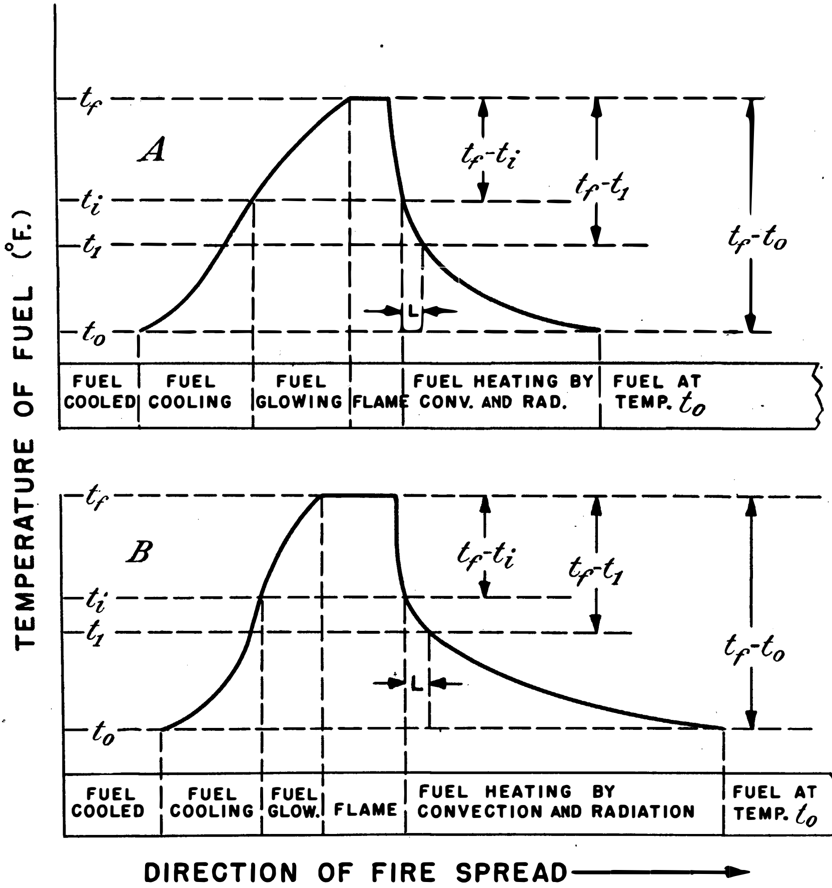

FIGURE 2.—Diagrammatic fuel temperature gradients at and near the flame front: B is for a fire burning with a higher wind velocity or on a steeper slope than A.

the moisture content, M, are obtained experimentally, and $\overline{\gamma}$ for the moist fuel is calculated.

FUEL TEMPERATURE

Temperature t_1 (fig. 2) will obviously vary with the following: (1) The flame angle from the horizontal, which is controlled by the wind velocity and slope of terrain; (2) the size of the flame front, which is controlled primarily by the time from the start or "age of fire" and the amount of fuel burning at the head of the fire. Temperature t_1 varies with the flame angle because, as the angle becomes smaller, more hot gases pass directly over the unburned fuel bed in the direction of the main flame front; thus the fuel particles take on more heat by convection and radiation. Temperature t_1 varies with the size of flame, which governs the amount of heat transferred to the unburned fuel particles by radiation. The time from start of fire also affects temperature t_1, because at zero time t_1 must equal t_0, but at subsequent intervals the fuel particles ahead of the fire have taken on heat, causing t_1 to become greater than t_0. Temperature t_1 is not easily measured even under ideal conditions. Temperature t_0 of fuel far removed from the advancing flame front is, however, readily determined. Temperature t_1 may be eliminated by substituting t_0 for it and correcting this new temperature-difference ratio in equation (11) by a coefficient, C_t, in the following manner:

Let
$$C_t\left(\frac{t_f-t_0}{t_f-t_i}\right)=\frac{t_f-t_1}{t_f-t_0} \tag{28}$$

wherefore
$$C_t=\frac{t_f-t_1}{t_f-t_0} \tag{29}$$

Figure 2 shows diagrammatically the fuel-temperature gradient and temperature differences at and near the flame front. For all practical forest conditions t_f may be assumed to be about 1500° F. The effect of increasing wind or slope makes t_1 numerically larger, which makes C_t smaller, according to equation (29). Since t_1 increases numerically with time from start of fire and size of flame, these also will make C_t smaller. Any independent change in t_0 will be nearly compensated by a change in the same direction in t_1 and will not therefore greatly change C_t. The temperature-difference ratio coefficient is primarily a function of wind velocity, slope, size of flame, and time from start of fire and must be evaluated experimentally. The approximate numerical range of C_t may be determined by substituting a reasonable value of t_0 and the limiting values of t_1 in equation (29). For the field conditions of $t_0=80°$ F. and $t_i=550°$ F., C_t will vary from 0.67 to more than 1.00. At the very start of a fire, when $t_1=t_0$, $C_t=1.00$. For the extreme conditions of zero flame angle, which occur at extremely high wind velocity or infinite slope, C_t will approach 0.67 when t_1 approaches t_i. For fires burning in still air in dead fuel exposed to full sunlight, C_t may become somewhat greater than 1.00 with time from start of fire; for under these conditions t_0 will be high and the fuel may actually be cooled by the indraft.

TRANSFORMATION OF EQUATIONS DERIVED FOR IDEALIZED HOMOGENEOUS BEDS TO NATURAL HOMOGENEOUS BEDS

In the idealized fuel bed all particles were assumed to be vertically oriented with equal spacing. In the natural homogenous bed, such as a mass of fallen pine needles, more or less horizontally oriented at random, the spacing is not equal but has instead voids of varying sizes. There are other differences between idealized and natural beds. In the latter the particles are in occasional contact with one another, and will not always be oriented with the flow of hot gases as in the idealized bed; this will influence the rate of heat conducted into the fuel. There may also be a fuel-moisture gradient in the natural fuel bed.

Granting that all the variables affecting the rate of spread have been included in the derivation, according to the theory of similitude (*12, p. 78*), if the processes of burning in the two beds follow the same laws and the respective variables stand in a constant ratio to each other over their whole range, the application of equation (11), derived for idealized beds, to fires in natural beds is permissible. On the assumption that the individual variables in the two processes are respectively proportional, as required by the theory of similitude, equations relating these variables may be written; for example, the relation between spacing L_1 of the idealized and L_2 of the natural bed may be expressed as

$$L_1 = C_L L_2$$

where C_L is a constant quantity for the whole range of spacing. Other variables such as specific heat, density, etc., may be dealt with in a similar manner.

Accordingly, the rate-of-spread equation will be identical in the natural and idealized beds, except that each variable will be multiplied by its corresponding coefficient. With $\frac{t_f - t}{t_f - t}$ replaced by $C_t\left(\frac{t_f - t_0}{t_f - t_i}\right)$, in which C_t serves as both the temperature-difference ratio coefficient and as a coefficient relating the natural to the idealized values of t_f and t_0, equation (11) for a natural fuel bed becomes

$$R = \frac{C_f(f_c + f_r)C_\sigma \sigma C_L L}{C_\gamma \bar{\gamma} C C_p \bar{C}_p ln\left[C_t\left(\frac{t_f - t_0}{t_f - t_i}\right)\right]} \tag{30}$$

Inasmuch as the coefficients C_f, C_L, etc., remain constant

let
$$C = \frac{C_f C_\sigma C_L}{C_\gamma C_{cp}} \tag{31}$$

then
$$R = \frac{C(f_c + f_r)\sigma L}{\bar{\gamma}\bar{C}_p ln\left[C_t\left(\frac{t_f - t_0}{t_f - t_i}\right)\right]} \tag{32}$$

where C represents a proportionality constant relating the rate in a natural bed with the rate in a standard bed under identical conditions.

The coefficients C_t and C in equation (32) are evaluated experimentally. As shown in the above discussion, C_t is primarily a function of wind velocity, slope, size of flame, and time from start of fire. The numerical value of C is dependent in part on the type of fuel bed, that is, whether it is a mass of fallen litter or standing vegetation, such as grass or brush, and in part on the technique used in measuring the variables, such as wind, fuel moisture, and fuel temperature. To determine the numerical value of C_t for any particular wind, time, and slope it is first necessary to evaluate C for the particular fuel used in the experiments. However, in order to obtain the value of C with reasonable accuracy we must have data for a wide range of fuel temperature, and this is experimentally impracticable. On the other hand, if C_t be evaluated with C taken as unity for a chosen standard natural fuel bed, for which a precise technique for measuring the wind, moisture content, and other variables has been developed (to a point where measurements of all variables can be duplicated), then C can be relatively evaluated for any other kind of natural bed. This is done by determining R experimentally for a particular kind of bed and computing a value for C by equation (32), using C_t as evaluated for the standard bed. The coefficient C determined in this manner would then be a constant, relating the particular kind of natural bed not with the idealized bed, but with the chosen natural standard.

DERIVATION OF EQUATIONS FOR RATE OF SPREAD IN NATURAL HETEROGENEOUS FUEL BEDS

Fuel beds of the homogeneous type are of rare occurrence in nature. Normally a natural bed is composed of particles varying in size, shape, density, and specific heat. Since the ignition time, ϕ_i is dependent on the material and dimensions of the particles, it follows that the rate of spread will be different for different particles.

Consider an idealized heterogeneous bed composed of particle types $a, b, c, \ldots, n$, where all particles are of the same length, equal to the depth of the fuel bed, and are evenly spaced at distance L. If $N_a, N_b, N_c, \ldots, N_n$ are the number of particles of respective fuel types per unit area, and N is the total number of particles, then in a given length of bed each kind will be present in amount proportional to $\frac{N_a}{N}, \frac{N_b}{N}, \frac{N_c}{N}, \ldots, \frac{N_n}{N}$. If these fractions are represented as $X_a, X_b, X_c, \ldots, X_n$ and if $R_a, R_b, R_c, \ldots, R_n$ represent the respective rates of spread through the different fuel types, the gross rate of spread of the mixture will be [10]

$$R=\frac{1}{\frac{X_a}{R_a}+\frac{X_b}{R_b}+\frac{X_c}{R_c}+\ \ldots\ +\frac{X_n}{R_n}} \tag{33}$$

[10] Equation (33) is analogous to one for total conductance of a series electrical circuit of length L composed of several electrical conductors of length l, each of different conductance but occurring n times.

For example, let $N_a, N_b, N_c, \ldots, N_n$ be the number of times conductors of length l, having conductances $C_a, C_b, C_c, \ldots, C_n$, respectively, occur. The total length of the circuit will be $L=N_al+N_bl+N_cl+\ \ldots\ +N_nl$,

and the total conductance

$$C=\frac{N_a+N_b+N_c+\ \ldots\ +N_n}{\frac{N_a}{C_a}+\frac{N_b}{C_b}+\frac{N_c}{C_c}+\ \ldots\ +\frac{N_n}{C_n}}$$

$$X_a=\frac{N_a}{N_a+N_b+N_c+\ \ldots\ +N_n},\ X_b=\frac{N_b}{N_a+N_b+N_c+\ \ldots\ +N_n},$$

where $X_a, X_b, X_c, \ldots, X_n$ are the fraction of the total conductance contributed by conductors type $a, b, c, \ldots, n$ respectively

Then

$$C=\frac{1}{\frac{X_a}{C_a}+\frac{X_b}{C_b}+\frac{X_c}{C_c}+\ \ldots\ +\frac{X_n}{C_n}}$$

Let $W_a, W_b, W_c, \ldots, W_n$ be the total dry weight of the several particles per unit area of the fuel bed. Then

$$N_a = \frac{W_a}{A_a \gamma_a d}, \; N_b = \frac{W_b}{A_b \gamma_b d}, \text{ etc.} \tag{34}$$

where A's and γ's are the cross-sectional areas and the densities of the dry particles, respectively, and d is their length.

By definition
$$X_a = \frac{N_a}{N}, \; X_b = \frac{N_b}{N}, \text{ etc.} \tag{35}$$

and
$$N = N_a + N_b + N_c + \ldots + N_n \tag{36}$$

Equation (35) becomes, after substituting (34) and (36):

$$X_a = \frac{W_a/A_a\gamma_a}{\dfrac{W_a}{A_a\gamma_a} + \dfrac{W_b}{A_b\gamma_b} + \dfrac{W_c}{A_c\gamma_c} + \ldots + \dfrac{W_n}{A_n\gamma_n}} \text{ etc.,} \tag{37}$$

for $X_b, X_c, \ldots, X_n$.

Substituting equation (37) in (33) and letting

$$B = \frac{W_a}{A_a\gamma_a} + \frac{W_b}{A_b\gamma_b} + \frac{W_c}{A_c\gamma_c} + \ldots + \frac{W_n}{A_n\gamma_n} \tag{38}$$

the following equation results:

$$R = \frac{B}{\dfrac{W_a}{A_a\gamma_a R_a} + \dfrac{W_b}{A_b\gamma_b R_b} + \dfrac{W_c}{A_c\gamma_c R_c} + \ldots + \dfrac{W_n}{A_n\gamma_n R_n}} \tag{39}$$

Equation (39) is an expression for the rate of spread of fire through a heterogeneous bed in terms of fuel composition and the rates of spread through each component fuel. The rate for each component is calculated by equation (32) with the value of the compactness, $1/\lambda$, equal to that of the mixture.

To calculate the rate of fire spread in a heterogeneous bed by equation (39) the spacing, L, which is not directly measurable, must be determined from the fuel and fuel-bed characteristics. The measurable factors from which L may be computed are V_1, V_2, W, γ, σ, and λ, where λ is determined by equation (15).

For a given bed, the ratio of volume of voids to fuel-surface area is expressed by equation (15); the volume of a fuel bed by equation (16).

The volume occupied by the fuel in the heterogeneous bed of volume V_1 is $V_2 = V_a + V_b + V_c + \cdots + V_n$ where $V_a, V_b, \cdots, V_n$ are volumes of the different kinds of particles. It follows that

$$V_2 = \frac{W_a}{\gamma_a} + \frac{W_b}{\gamma_b} + \frac{W_c}{\gamma_c} + \ldots + \frac{W_n}{\gamma_n} \tag{40}$$

The total surface area of the fuel in this bed is

$$S = \frac{\sigma_a W_a}{\gamma_a} + \frac{\sigma_b W_b}{\gamma_b} + \frac{\sigma_c W_c}{\gamma_c} + \ldots + \frac{\sigma_n W_n}{\gamma_n} \tag{41}$$

Substituting equations (16), (40), and (41), in (15) gives

$$\lambda=\frac{L^2dN-\left(\frac{W_a}{\gamma_a}+\frac{W_b}{\gamma_b}+\frac{W_c}{\gamma_c}+\cdots+\frac{W_n}{\gamma_n}\right)}{\frac{\sigma_a W_a}{\gamma_a}+\frac{\sigma_b W_b}{\gamma_b}+\frac{\sigma_c W_c}{\gamma_c}+\cdots+\frac{\sigma_n W_n}{\gamma_n}} \tag{42}$$

Solving equation (42) for L^2, then multiplying and dividing each term, the right side of the equation by the number of particles in that type of fuel, namely, N_a, N_b, N_c, $\cdots$, N_n, letting A_a, A_b, A_c, $\cdots$, A_n equal $\frac{W_a}{\gamma_a N_a d}$, $\frac{W_b}{\gamma_b N_b d}$, $\frac{W_c}{\gamma_c N_c d}$, $\cdots$, $\frac{W_n}{\gamma_n N_n d}$, and assuming a length of fuel particles equal to the depth of the fuel bed, the following equation results:

$$L=[\lambda(\sigma_a A_a X_a+\sigma_b A_b X_b+\sigma_c A_c X_c+\cdots+\sigma_n A_n X_n) + (A_a X_a+A_b X_b+A_c X_c+\cdots+A_n X_n)]^{1/2} \tag{43}$$

where X_a, X_b, X_c, $\cdots$, X_n are defined by equation (37).

By substituting the equations for X_a, X_b, X_c, $\cdots$, X_n and simplifying, equation (43) becomes:

$$L=\left[\frac{\lambda}{B}\left(\frac{\sigma_a W_a}{\gamma_a}+\frac{\sigma_b W_b}{\gamma_b}+\frac{\sigma_c W_c}{\gamma_c}+\cdots+\frac{\sigma_n W_n}{\gamma_n}\right) + \frac{1}{B}\left(\frac{W_a}{\gamma_a}+\frac{W_b}{\gamma_b}+\frac{W_c}{\gamma_c}+\cdots+\frac{W_n}{\gamma_n}\right)\right]^{1/2} \tag{44}$$

where B is defined by equation (38).

Equation (44) is an expression for the particle spacing L for a heterogeneous fuel bed in terms of its composition. The assumption of fuel-particle length equal to depth of the fuel bed requires, of course, use of the coefficient C_L in application of this measure to natural fuels. This coefficient is present in equation (32).

EXPERIMENTS IN NATURAL HOMOGENEOUS FUEL BEDS

EXPERIMENTAL PROCEDURE

For evaluating the coefficient C_t, ponderosa pine needles, being the most readily available uniform fuel in this region (California), were chosen as the natural fuel for the standard bed. In order to exercise control over all the variables, fires were studied on a model scale. Beds of needles 2 inches deep, 3 feet wide, and 8 or 12 feet long were prepared in trays with controlled compactness. These beds were placed in a wind tunnel (fig. 3) described elsewhere (*9*) and burned on the horizontal, i.e., at zero slope. Figure 4 shows the position of initial ignition of a fuel bed in the tunnel. Ignition was accomplished by an electrically heated coil placed on a match head inserted in the middle of a thin pyroxylin disk of 2-inch diameter. The fire traveling forward along the length of the bed was observed (fig. 5) from an overhead window. Each fire was allowed to burn

FIGURE 3.—Wind tunnel used in fire-behavior studies; test section is 30 feet long with a cross section 6 x 6 feet, over-all length 55 feet.

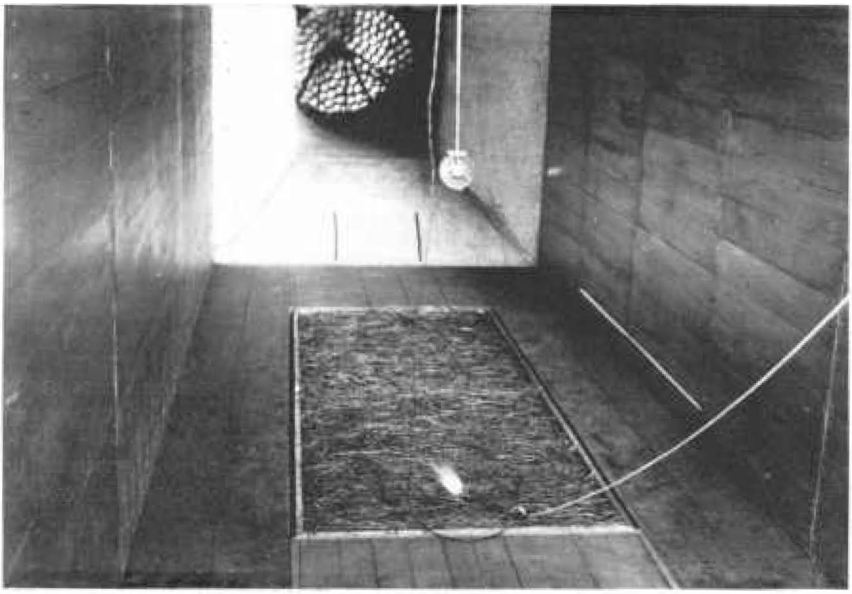

FIGURE 4.—Position of initial ignition of a fuel bed in the tunnel.

until it reached approximately 18 inches in width and was then instantly suppressed by water. A typical burned area is shown in figure 6.

The air velocity recorded for each fire was the velocity at 1 foot above the surface of the bed prior to ignition. A sample of fuel was collected at random from the surface of the bed just prior to burning and its moisture content was determined by xylene distillation (*4*). The fuel temperature was taken to be equal to the ambient air temperature, measured when the moisture sample was taken. Approximately 200 fires were burned in fuel beds of ponderosa pine needles ($\sigma = 128$ inches^{-1}) under varying conditions such as: wind velocity 2.0 to 12.0 m. p. h.; compactness, $1/\lambda$, 6.25 to 16.66 inches^{-1}; moisture content 4.0 to 15.0 percent; air temperature 50° to 85° F.

Figure 5.—Pine-needle fire 30 seconds after ignition, photographed from above; wind velocity 8 m. p. h.

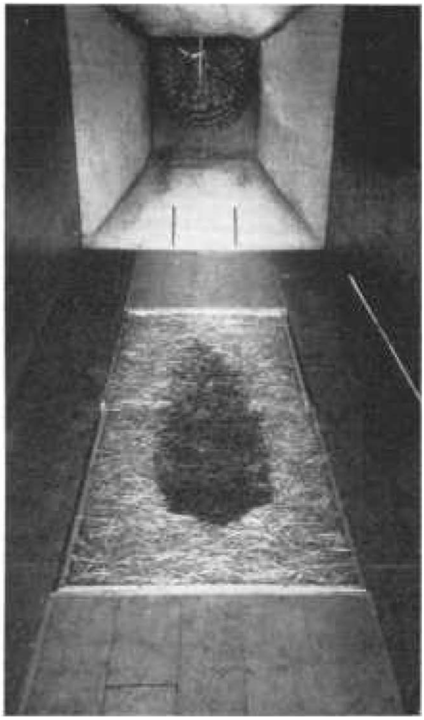

Figure 6.—Typical burned area of pine needles.

In order to simulate a standing type of fuel, such as brush, the beds were prepared by standing uniform twigs vertically and at equal spacing in sawdust treated with a fire-retardant chemical (fig. 7). The twigs were cut 7.5 inches in length from dead branches of ponderosa pine and segregated into three nominal size classes ⅛, $\frac{3}{16}$, and ¼ inch in diameter. The bed was set afire by a 30-gm. bundle of pine needles and to avoid the influence of the burning needles, the twigs were allowed to burn a distance of about 4 feet before zero time was called.

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The fire was not extinguished until it had reached the end of the bed, which was 12 feet in length (fig. 8). Observations were made and data taken only in that portion of the burn beyond the first narrow region influenced by the mode of ignition.

RESULTS

Numerical values of C_t were computed for each pine-needle fire by equation (32) using the observed rates of spread with C equal to unity.

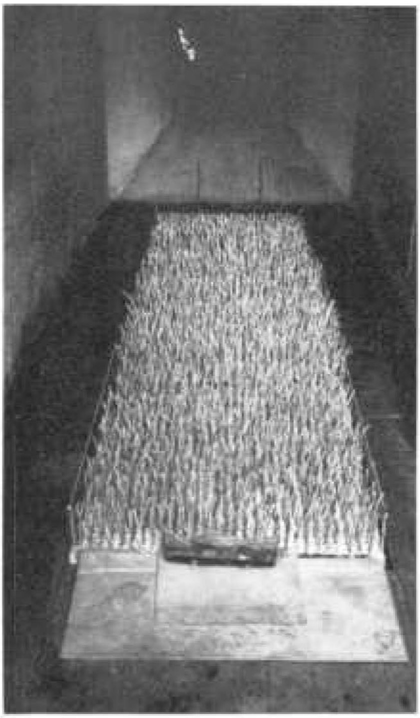

FIGURE 7.—Bed of ponderosa pine twigs $\frac{3}{16}$ inch in diameter with bundle of needles in the foreground used for ignition.

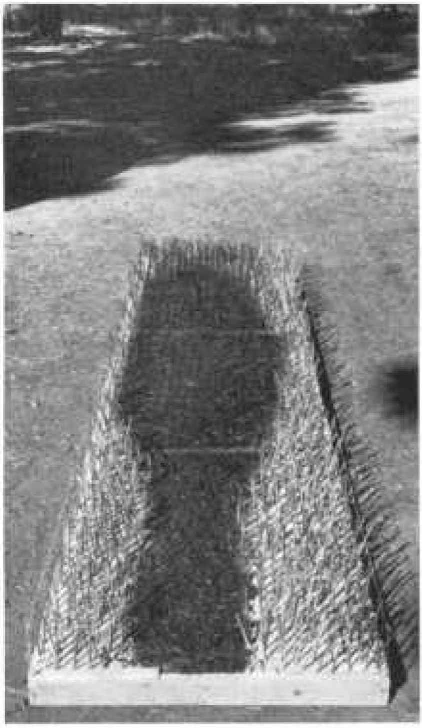

FIGURE 8. Typical burned area of twigs.

The average values of C_t were computed for several wind classes and are shown in figure 9. A value for the rate of spread of each fire was calculated by using appropriate values of C_t from the curve in figure 9. Figure 10 shows the computed versus the actual rates of spread plotted to reveal the agreement. It will be noted that the points fall close to and evenly about a 45° line which represents agreement oft calculated with experimental values.

The experimental data for the twig fires are presented in table 1, together with the calculated values for the coefficient C. The values of C for individual fires vary considerably from its mean of 1.61 owing mainly to the experimental technique used in igniting the twigs and to the error in judging the exact position of the fire at zero time and the time when the fire reached the end of the bed. Therefore, the value of 1.6 for C, for this type of bed, must be considered as only approximate until more data have been obtained. Since C was assumed to be unity for the pine-needle bed, the value of 1.6 is in the right order of magnitude for a fuel bed with twigs standing vertically, inasmuch as it was assumed that the flow of the gases passing the particles in the standard pine-needle bed was at right angles. It would have been more nearly correct, however, to assume that the gas flow is at right angles for only half the particles, and parallel or

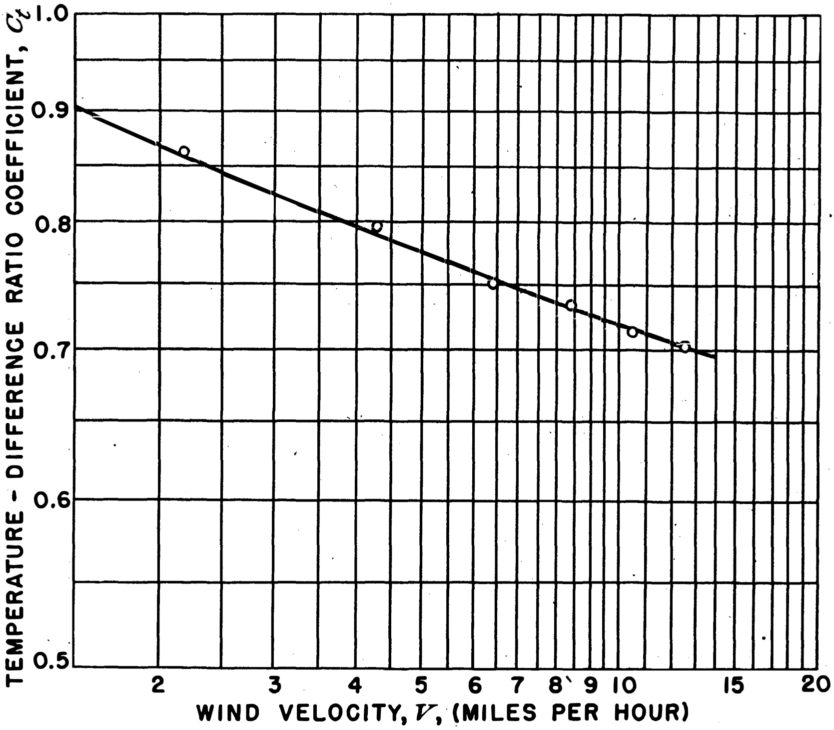

FIGURE 9.—Relation between temperature-ratio coefficient, C_t, and wind velocity, V (miles per hour) at 1 foot above fuel surface, zero slope, and 1 minute from start of fire. Fuel: Ponderosa pine needles.

longitudinal for the other half. Actually, in evaluating C_t by the type of bed chosen as standard, it might have been more proper to assume a value less than unity for C, in which case the numerical values of C_t would have been slightly higher. Then, with higher values of C_t for beds with twigs vertically oriented where cross flow is more nearly approached, C would have more nearly approached unity. Practically, however, since C may vary with the technique used in sampling fuel moisture and measurement of wind velocity, it would probably differ from unity for other types of fuel beds. Consequently, for the sake of simplicity it would be better to assume cross flow for all types of beds and let C, evaluated in that way, be a constant relating a particular kind of natural bed with a standard pine-needle bed, for which f_c was calculated on the basis that the gases flow at right angles to the needles.

TABLE 1.—*Experimental data and calculated values of C for model fires burning in fuel beds of ponderosa pine twigs placed vertically and spaced equally*

Fire (number)	Experimental data						Calculations [1]				
	Wind velocity (V)	Fuel temperature (t_o)	Fuel moisture content (M)	Fuel surface-volume ratio (σ)	Fuel spacing (L)	Rate of spread (R)	σL	f_c+f_r	$\overline{\gamma C_p}$	$ln\left[C_t\left(\frac{t_f-t_o}{t_f-t_i}\right)\right]$	C
	M.p.h.	°*F.*	*Pct.*	*Inches*$^{-1}$ (2)	*Inches*	*Ft./hr.*		*B.t.u./Sq.ft. hr. °F.*	*B.t.u./Cu.ft. °F.*		
22	4	80	12.4	20.7	1.50	195	31.0	14.7	24.6	0.180	1.89
3	4	86	9.1	20.7	1.50	223	31.0	14.7	22.2	.176	1.91
11	4	88	10.3	20.7	1.50	209	31.0	14.7	23.1	.174	1.84
51	6	69	11.9	15.7	1.50	225	23.6	15.5	23.9	.139	2.04
10	6	89	10.6	20.7	1.50	265	31.0	17.1	23.4	.125	1.46
21	6	79	12.1	20.7	1.50	285	31.0	17.1	24.3	.132	1.72
1	6	87	9.8	20.7	1.50	335	31.0	17.1	22.7	.126	1.81
37	6	68	11.4	27.8	1.25	367	34.8	19.0	23.5	.140	1.83
49	8	68	12.5	15.7	1.75	222	27.5	17.0	24.4	.105	1.22
56	8	73	10.9	15.7	1.75	264	27.5	17.0	23.3	.101	1.33
65	8	62	11.3	15.7	1.63	257	25.5	17.0	23.3	.109	1.51
40	8	71	10.7	15.7	1.50	294	23.6	17.0	23.1	.103	1.74
52	8	68	12.2	15.7	1.50	242	23.6	17.0	24.2	.105	1.53
39	8	71	10.6	15.7	1.38	306	21.6	17.0	23.0	.103	1.97
48	8	61	11.4	15.7	1.25	244	19.6	17.0	23.4	.110	1.88
64	8	62	9.7	20.7	1.25	360	25.9	19.0	22.1	.109	1.76
47	8	59	11.9	20.7	1.13	285	23.3	19.0	23.7	.111	1.69
2	8	87	9.5	20.7	1.50	492	31.0	19.0	22.5	.091	1.71
12	8	73	10.4	20.7	1.50	353	31.0	19.0	22.9	.101	1.39
20	8	79	12.3	20.7	1.50	395	31.0	19.0	24.5	.097	1.59
42	8	70	10.9	20.7	1.38	430	28.5	19.0	23.2	.103	1.90
41	8	71	11.1	20.7	1.25	339	25.9	19.0	23.4	.103	1.66
61	8	58	11.0	20.7	1.25	331	25.9	19.0	26.4	.112	1.99
43	8	74	11.6	27.8	1.38	457	38.2	21.2	23.8	.101	1.36
45	8	61	11.0	27.8	1.38	369	38.2	21.2	23.1	.110	1.16
63	8	62	8.7	27.8	1.38	497	38.2	21.2	21.3	.109	1.42
44	8	74	10.2	27.8	1.25	613	34.8	21.2	22.8	.101	1.91
46	8	60	11.4	27.8	1.13	497	31.3	21.2	23.3	.110	1.92
50	8	67	13.3	27.8	1.00	395	27.8	21.2	25.0	.106	1.78
57	8	73	11.0	27.8	1.00	465	27.8	21.2	23.3	.101	1.86
11a	5.8	71	11.6	20.7	1.38	222	28.5	16.7	23.8	.141	1.57
12a	5.8	71	11.7	20.7	1.25	233	25.9	16.7	23.8	.141	1.81
13a	5.8	71	11.0	15.7	1.25	171	19.6	15.1	23.3	.141	1.90
14a	5.8	71	11.1	15.7	1.50	149	23.6	15.1	23.4	.141	1.38
18a	5.8	88	4.6	27.8	1.25	457	34.8	18.7	18.6	.130	1.70
19a	5.8	89	5.3	27.8	1.13	436	31.3	18.7	19.2	.129	1.84
20a	5.8	89	5.5	20.7	1.50	320	31.0	16.7	19.4	.129	1.55
25a	5.8	94	5.0	15.7	1.75	236	27.5	15.1	19.1	.125	1.36
26a	5.8	92	4.6	20.7	1.13	297	23.3	16.7	18.7	.127	1.81
33a	5.8	91	5.7	27.8	1.38	395	38.2	18.7	19.5	.127	1.37
34a	5.8	89	5.4	15.7	1.38	222	21.6	15.1	19.3	.129	1.69
35a	5.8	89	5.0	15.7	1.63	242	25.5	15.1	19.0	.129	1.54
40a	5.8	84	7.1	27.8	1.00	347	27.8	18.7	20.6	.132	1.82
59a	5.8	81	7.2	20.7	1.38	277	28.5	16.7	20.6	.134	1.61
27a	7.8	92	5.1	15.7	1.75	391	27.5	16.7	19.1	.091	1.46
28a	7.8	91	5.4	20.7	1.38	395	28.5	18.7	19.3	.092	1.32
41a	7.8	84	6.4	15.7	1.38	250	21.6	16.7	20.0	.097	1.34
45a	7.8	85	6.6	15.7	1.63	244	25.5	16.7	20.2	.096	1.12
46a	7.8	85	7.0	15.7	1.50	256	23.6	16.7	20.5	.096	1.28

[1] According to equation (32) $C=\frac{R\overline{\gamma C_p}ln\left[C_t\left(\frac{t_f-t_o}{t_f-t_i}\right)\right]}{(f_c+f_r)\sigma L}$ where fuel density, γ, =26.1 lbs./cu. ft.; t_f= 1,500° F.; t_i=550° F.

[2] 1÷number of inches.

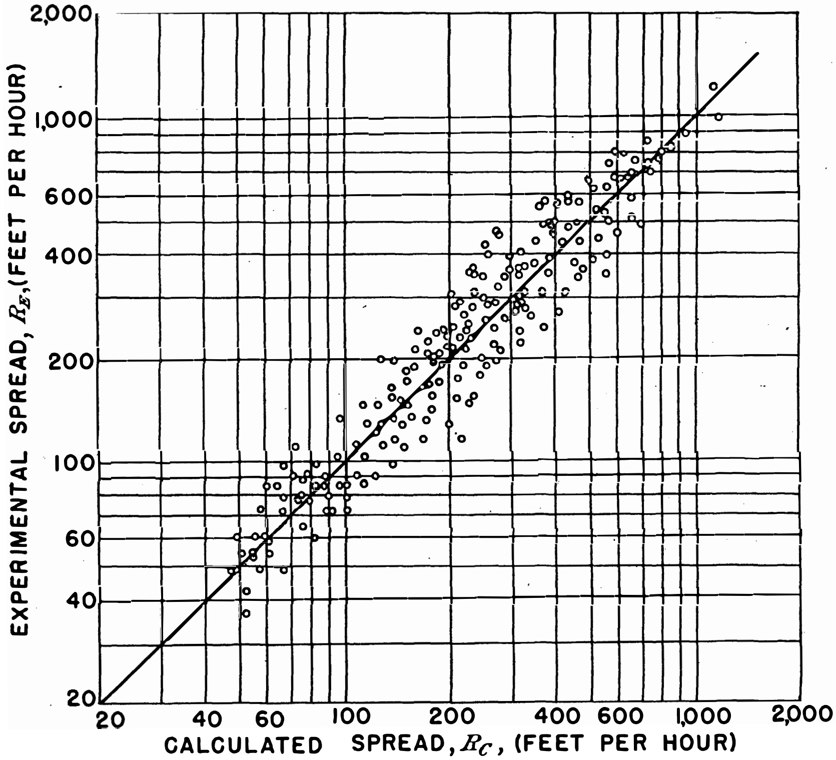

FIGURE 10.—Calculated versus experimental rates of forward spread for ponderosa pine-needle fires.

VARIABLES MEASURABLE IN FIELD PRACTICE

Equation (32) has been derived for the rate of spread in terms of the fundamental variables, which in practice are generally wind velocity, moisture content, time, and slope. Fuel-type or cover-type classifications also are used almost universally in recognizing the rate-of-spread differences attributable to fuel size and cover density, which the present study has revealed to be important and significant. Compactness, which incorporates fineness and crowdedness of the particles in the fuel bed, is used in this analysis to define cover density. Two other important measurable variables have been added, namely, fuel temperature and fuel-particle density, heretofore not generally recognized or applied in practice.

These measurable variables influence fire spread only as they produce changes in the fundamental variables. Some operate on as many as three or four fundamental variables. For example, the fuel temperature, t_0, affects film conductance for convection and conduction, f_c, heat-transfer factor, f_r, specific heat of moist fuel, $\overline{C}_p$, and in addition is itself a fundamental variable in the spread equation.

Figure 11 shows curves for rate of spread as a function of all the variables, except slope and time, as interpreted through application of the fundamental relationships expressed in equation (32).

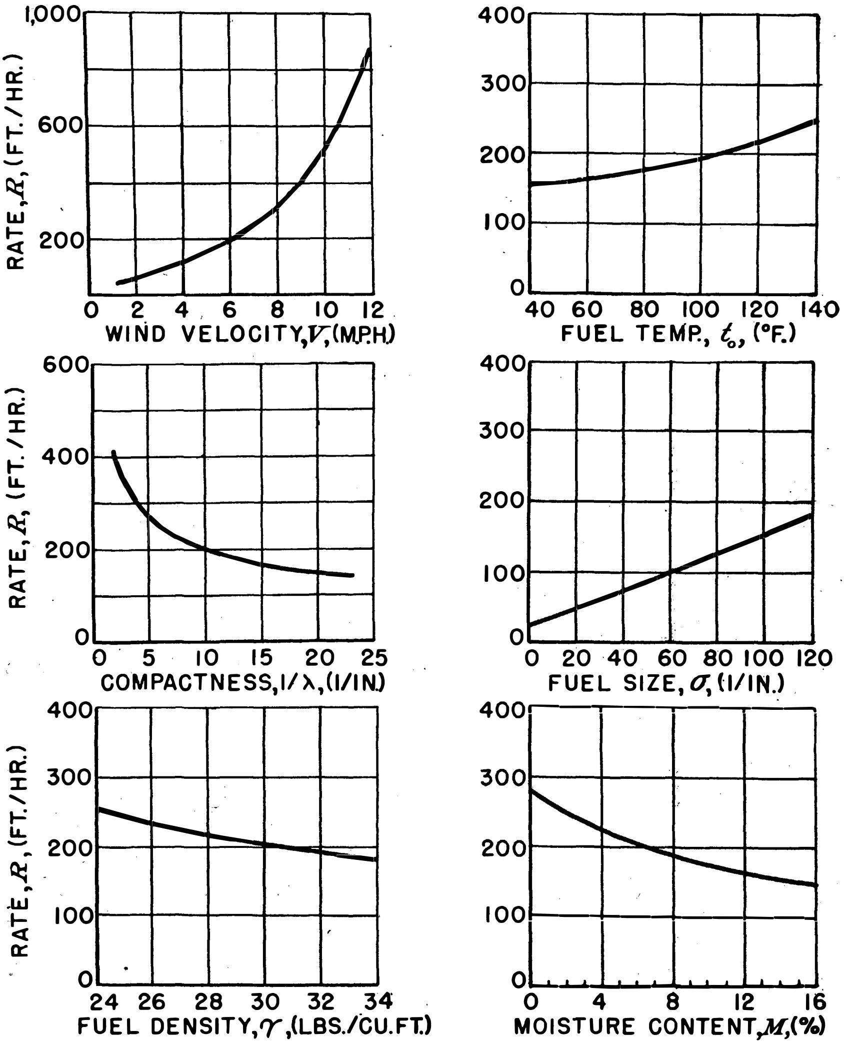

FIGURE 11.—Effect of measurable variables on rate of forward spread of fires. For each graph five variables are held constant while the sixth is varied over a practical range. When held constant, the variables have values as follows: V-6.0 m. p. h.; $1/\lambda=10$ inches $^{-1}$; $\gamma=31.6$ lbs./cu. ft.; $t_0=100°$ F.; $\sigma=128$ inches $^{-1}$; M=8 percent.

INFLUENCE OF MEASURABLE VARIABLES

Effect of Wind Velocity

Wind velocity influences fire spread by causing changes in the magnitude of film conductance, f_c, and temperature ratio coefficient, C_t. The film conductance varies approximately with the square root of the wind velocity (see equation 12). Since rate of spread is directly proportional to f_c, it will vary with the square root of the wind velocity. Most of the increase in fire spread that is due to the wind velocity, however, results from its effect on the temperature ratio coefficient, C_t. Increasing velocity decreases the flame angle, and this results in more effective preheating of the fuel; that is, fuel temperature, t_1, increases with wind velocity. From equation (29) it will be seen that higher temperature values t_1 of the fuel particle about to ignite will give smaller values of C_t; thus C_t is some inverse function of wind velocity. The way C_t varies with wind velocity is shown in figure 9. Since the rate of spread varies inversely as the logarithm of C_t, it will vary directly as the logarithm of the velocity. With wind measured at 1 foot above the fuel bed, the product of these separate functions indicates that rate of spread is approximately proportional to the first power of wind velocity for velocities less than 5 m. p. h. and to the 1.5 power of wind velocity for velocities from 5 to at least 12 m. p. h. The upper velocity limit (to which the 1.5 power holds) has not been definitely fixed. The first power for wind velocities less than 5.0 m. p. h. is confirmed by experimental results obtained by Curry and Fons (*6*) from test fires under field conditions.

Effect of Fuel Size

In this analysis, the surface-volume ratio, σ, of the fuel particle has been used to express the fuel size. The presence of σ in the numerator in equation (32) does not necessarily imply that rate of spread is directly proportional to σ. In fuel beds of constant compactness, variations in σ will affect the distance L between fuel particles (equation 21). In addition, σ affects the film conductance (equation 12). The effect of σ on ignition temperature, t_i, has not been considered in the analysis, but significant variations of t_i with σ are expected (equation 10). By holding all other factors constant, the expression for rate of spread in terms of σ becomes:

$$R=(a\sigma+b\sigma^{0.47}+c)(1+d\sigma)^{1/2}$$

where a, b, c, and d are the proper constants for the given values of wind velocity, initial fuel temperature, compactness, and moisture content. For fine fuels this expression very closely approximates a straight line having positive slope. Experimental results (*7*) show that the rate of spread for fine fuels is very nearly inversely proportional to the fuel size, i. e., it varies directly as the surface-volume ratio of the fuel particle.

Effect of Compactness

For want of a more descriptive word, the variable incorporating fineness and crowdedness of the fuel particles is here termed compactness. The reciprocal of the ration of volume of voids to the surface area of the fuel in the bed, denoted by λ, may be used as a direct measure of this element. By definition, the spacing of fuel particles, L, varies as the square root of λ. Since rate of spread is proportional to L, it too will vary as the square root of λ. For a fuel particle of given size it is necessary to increase the distance L between particles to obtain greater values of λ (equation 21); this would indicate that a unit increase at large values of λ would not increase the rate of spread as much as at the small values. There must be, of course, a limiting value of λ at which the fires would fail. Beyond a critical limiting point further separation of fuel particles decreases the volume of flame, and the hot gases produced in diminishing quantity are cooled and dissipated to such an extent that they are no longer effective in igniting the unburned fuel. Experimental results (7) have confirmed the square root of λ relationship.

Effect of Fuel Temperature

Changes in the initial temperature of the fuel, t_0, affect the four fundamental variables, film conductance, heat-transfer coefficient, specific heat of the moist fuel, and temperature ratio $\frac{t_f-t_o}{t_f-t_i}$. The film conductance changes with the fuel temperature because the kinematic viscosity and thermal conductivity of the film are affected (equation 12). An increase of 50° F. in fuel temperature, t_0, will, however, decrease the film conductance only 0.5 percent and may be disregarded. The heat transfer coefficient, f_r, is also negligible for large changes in fuel temperature, because flame temperature is large in comparison with t_o (equation 14). The specific heat of the moist fuel, $\overline{C}_p$, increases with a rise in fuel temperature (equation 27). Since rate of spread is inversely proportional to C_p (equation 32), an increase in $\overline{C}_p$ decreases the rate of spread. The important effect of fuel temperature on fire spread is that a change in temperature changes the ignition time, θ_i, as shown by equation (10), with $C_t\left(\frac{t_f-t_o}{t_f-t_i}\right)$ replacing $\frac{t_f-t_1}{t_f-t_i}$. Changes in fuel temperature affect the magnitude of the ratio $\frac{t_f-t_0}{t_f-t_i}$, and a small change in this ratio produces a comparatively large change in $ln\left[C_t\left(\frac{t_f-t_0}{t_f-t_i}\right)\right]$. The relative extent of the fuel temperature's effect on fire spread may be determined by calculating the magnitude of the factors $1/ln\left[C_t\left(\frac{t_f-t_0}{t_f-t_i}\right)\right]$ and $\frac{f_c+f_r}{\gamma \overline{C}_p}$ in equation (32). For conditions indicated in figure 11, an increase in fuel temperature from 70° to 120° F. causes the factor $1/ln\left[C_t\left(\frac{t_f-t_0}{t_f-t_i}\right)\right]$, and consequently rate of spread, to increase 35 percent; with the same change, the factor $\frac{f_c+f_r}{\gamma\overline{C}_p}$ causes a decrease of 5.5 percent. The total effect of the 50° F. increase in initial fuel temperature, under the conditions described, is thus a net increase of 29.5 percent in fire spread. Since C_t decreases with wind velocity, a larger increase in spread results with rising wind velocity for a given temperature change; i. e., for the same conditions as above except that with wind velocity at 10 m. p. h. a rise in temperature from 70° to 120° F. produces a 94-percent increase in the rate of spread. The tendency of a fire to slow down soon after a shadow from a cloud is cast on the fuel ahead of the fire has frequently been observed. The fuel temperature in the rate-of-spread equation offers a possible explanation of this phenomenon, since fine fuels in complete shade soon assume air-temperature conditions, while in direct sunlight a much higher fuel temperature generally prevails.

EFFECT OF FUEL DENSITY

The density of the moist fuel, $\bar{\gamma}$, appears in the form of an inverse proportion as one of the fundamental variables in the expression for rate of spread. Since the density of a body is its mass per unit volume, more heat is required to bring a body of higher density than one of lower density to a given temperature. Consequently, the ignition time for fuel particles of a given volume is directly proportional to density (equation 10), and by the same token fire spread is inversely proportional to density.

EFFECT OF MOISTURE CONTENT

The moisture content, M, appears in the factors $1+M$ and $\frac{M}{1+M}$ in the equations for density and specific heat of the moist fuel, respectively. The factor $1+M$ in the two expressions cancels, leaving M in the product $\bar{\gamma}\bar{C}_p$, which is an expression for specific heat based on volume or volumetric specific heat. Since fire spread varies inversely with $\bar{\gamma}\bar{C}_p$ (equation 32), it will also vary with the fuel-moisture content. Moisture content can be expected to have the same effect on the rate of spread as density has, in that the greater the moisture the more heat is required to bring the fuel to the ignition point and the slower it will burn. Since rate of spread must become zero at some given moisture content for a particular fuel size, it is doubtful whether the effect of absorbed water on rate of spread is accomplished entirely through its effect on $\bar{\gamma}$ and $\bar{C}_p$ for the complete range of moisture content. At high moisture contents it would seem probable that the excessive water vapor acts as a diluent for the inflammable fuel gases to the extent that rate of spread decreases more than $\bar{\gamma}$ and $\bar{C}_p$ might indicate. If the moisture is sufficiently high, the water vapor may dilute the gases to the point where combustion is entirely prevented; this would determine the upper moisture content limit. For the range of moisture content studied (4 to 15 percent) equation (32) is valid; otherwise values of C_t computed for the experimental fires for any given wind velocities would have varied with the moisture. This, however, was not found; thus the effect of absorbed water on rate of spread is apparently accounted for by $\bar{\gamma}$ and $\bar{C}_p$ for moisture contents up to 15 percent.

EFFECT OF SLOPE

Because hot gases or convection columns tend to rise vertically with respect to a horizontal surface, an increase in slope of terrain decreases the angle between the fuel bed and hot gases. Slope, therefore, has the same effect as wind in bringing the flame closer to the fuel. As such, it may be considered an added component of wind velocity. As the flame angle decreases, the fuel starts to absorb heat, both by radiation and convection, farther ahead of the actual burning front than with a flame perpendicular to the fuel surface. This results in increasing t_1, temperature of the fuel particle about to ignite, which in turn makes C_t smaller (equation 29). Since the rate of spread varies inversely as the logarithm of C_t, it will vary directly as the logarithm of the slope. The effect of slope on rate of spread indicated by this relationship is shown in figure 12, where the lines drawn for different wind velocities are broken to indicate that their true positions have not yet been experimentally established.

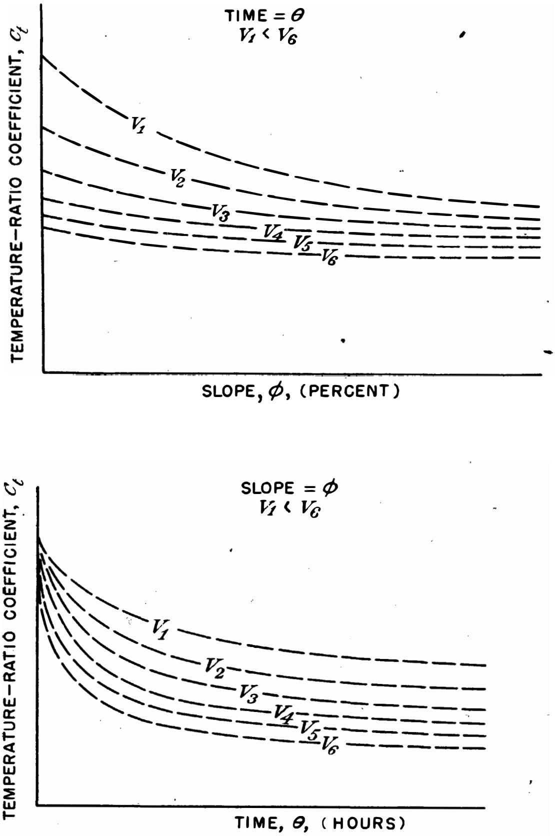

FIGURE 12.—Curves showing how the temperature-ratio coefficient may be expected to vary with slope and time for various wind velocities, *V*.

Effect of Time

Immediately after the start of a fire the volume of hot gases at its head increases progressively. As the volume of the flame rises the rate of heat transfer by radiation to the fuel particles is increased; thus t_1, increases with time until the volume of hot gases from the flame front has reached a more or less constant value. As in the case of slope, increasing values of t_1 result in decreasing values of C_t (equation 29); the rate of spread will therefore vary as the logarithm of time. This relationship was also indicated by earlier studies of field test fires (*6*). Model fires are of too short duration to permit studying the effect of time on C_t. Data from which to derive actual values of C_t at different time intervals must therefore be obtained from larger scale field experiments, which have not yet been undertaken. The way in which C_t is expected to vary with time for several wind velocities and given slope is illustrated in figure 12.

VALUES FOR GROUPS OF FUNDAMENTAL VARIABLES IN TERMS OF MEASURABLE VARIABLES

In table 2 are presented values of σ, γ, and E for different kinds of fuel. In tables 3 to 6, inclusive, are computed values of groups of fundamental variables that appear in equation (32), as functions of those measurable variables affecting the group.

TABLE 2.—*Surface-volume ratio, perimeter, density, and shape factor of several fuels*

Fuel type	Surface-volume ratio	Perimeter	Density	Shape factor
	σ	P	γ	E
Ponderosa pine:	[1] *Inches*$^{-1}$	*Inches*	*Pounds/cu. ft.*	
Dead twigs	$4/D$	[2] πD	29.1	3.6
Square sticks	$4/t$	[3] $4t$	26.7	4.0
Needles:				
Ponderosa pine	128	131	31.6	4.1
Sugar pine	165	.108	33.5	4.2
Lodgepole pine	140	.136	35.4	4.3
Fir	124	.165	34.3	4.5

[1] 1÷number of inches.
[2] Diameter of fuel.
[3] Thickness of fuel.

TABLE 3.—*Values*[1] *of f_c+f_r, (B. t. u./sq. ft. hr. °F.) as a function of wind velocity and surface-volume ratio of fuel*

Surface-volume ratio (inches^{-1})[2]	Wind velocity, in miles per hour indicated at 1 foot above fuel bed					
	2	4	6	8	10	12
10	9.7	11.8	13.5	14.8	16.1	17.2
20	11.5	14.5	16.8	18.9	20.7	22.2
40	14.3	18.5	21.8	24.5	26.9	29.1
70	17.6	22.6	26.8	30.7	33.8	36.8
100	20.3	26.3	31.3	35.2	39.0	42.3
130	22.7	29.4	34.9	39.2	43.6	47.2
160	25.1	32.2	38.4	43.5	47.9	51.8

[1] Values in table are within ±0.5 percent for a range of fuel temperature from 50° to 140° F.
[2] 1÷number of inches.

TABLE 4.—*Values* [1] *of $\frac{\sigma L}{E}$ as a function of surface-volume ratio of fuel and fuel bed compactness*

Surface-volume ratio (inches^{-1})[2]	Fuel bed compactness (inches^{-1})[2]							
	0.2	0.5	1	2	4	6	10	20
10	7.14	4.58	3.32	2.45	1.87	1.63	1.41	1.22
20	10.05	6.40	4.58	3.32	2.45	2.08	1.73	1.41
40	14.18	9.00	6.40	4.58	3.32	2.77	2.24	1.73
70	18.73	11.87	8.43	6.00	4.30	3.56	2.83	2.12
100	22.38	14.18	10.05	7.14	5.10	4.21	3.32	2.45
130	25.51	16.16	11.45	8.12	5.79	4.77	3.74	2.74
160	28.30	17.92	12.69	9.00	6.40	5.26	4.12	3.00

[1] To obtain σL for equation (32), multiply $\frac{\sigma L}{E}$ by an appropriate shape factor, E, of the fuel.
[2] 1÷number of inches.

TABLE 5.—*Values* [1] *of* $\frac{\overline{\gamma C_P}}{\gamma}$ *as a function of moisture content and temperature of fuel*

Fuel temperature (°F.)	Fuel moisture content (percent)							
	2	4	6	8	10	12	14	16
40	0. 546	0. 597	0. 648	0. 699	0. 750	0. 801	0. 852	0. 903
60	. 556	. 608	. 660	. 713	. 765	. 817	. 870	. 922
80	. 566	. 620	. 674	. 727	. 781	. 835	. 888	. 942
100	. 577	. 632	. 687	. 743	. 798	. 853	. 908	. 963
120	. 588	. 645	. 702	. 759	. 816	. 872	. 929	. 986
140	. 600	. 659	. 717	. 776	. 835	. 893	. 952	1. 010

[1] To obtain $\overline{\gamma C_P}$ for equation (32), multiply $\frac{\overline{\gamma C_P}}{\gamma}$ by the density of the fuel.

TABLE 6.—*Values* [1] *of* $ln\left[C_t\left(\frac{t_f-t_o}{t_f-t_i}\right)\right]$ *for surface fires as a function of wind velocity and fuel temperature,* t_o*, for time 1 minute from start of fire and zero slope*

Fuel temperature (°F.)	Wind velocity (in miles per hour indicated) at 1 foot above fuel bed					
	2	4	6	8	10	12
40	0. 2904	0. 2078	0. 1596	0. 1249	0. 0971	0. 0760
60	. 2769	. 1939	. 1458	. 1106	. 0834	. 0620
80	. 2631	. 1798	. 1319	. 0971	. 0695	. 0478
100	. 2484	. 1664	. 1178	. 0825	. 0554	. 0344
120	. 2343	. 1519	. 1035	. 0686	. 0411	. 0198
140	. 2199	. 1371	. 0889	. 0535	. 0266	. 0070

[1] Flame temperature, t_f, and ignition temperature, t_i, are considered constant at 1500° and 550° F., respectively.

SUMMARY AND CONCLUSIONS

The analysis of rate of fire spread in light forest fuels reported in this paper is based on the theory that the spread of a fire can be expressed as successive ignitions of adjacent fuel particles and that its rate is therefore governed by the time required to raise successive fuel particles to ignition temperature. Equations have been derived for rate of fire spread in homogeneous and heterogeneous fuel beds, taking into account the physical characteristics of the fuel particles, the arrangement of the bed, and the pertinent attributes of the atmosphere. The spread equations are expressed in terms of the following fundamental variables: (1) Film conductance; (2) heat transfer factor for radiation; (3) ignition temperature; (4) fuel particle spacing; (5) surface volume ratio of fuel; (6) specific heat; (7) density of fuel; and (8) fuel temperature. It has been shown that the variables ordinarily recognized in field practice—wind velocity, fuel moisture, fuel density, fuel size, fuel bed compactness, slope, and time—influence fire spread only as they produce changes in the fundamental variables.

The results of field and laboratory experiments were used to check the theory and analysis, as well as to provide quantitative information for determining the magnitude of the effects of variables measurable in field practice on the rate of spread.

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