

Modeling temporal trends in bedload transport in gravel-bed streams using hierarchical mixed-effects models



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ABSTRACT

In this paper, we used a bedload transport data set collected at North Fork Caspar Creek, California, to examine temporal variation in sediment transport rate over a 7-year period. Using a hierarchical mixed-effects model, we examined across and within-event variation to determine whether the bedload–shear stress relation trends over time. The relation between bedload transport and shear stress was modeled using $\log(Q_b) = \alpha + \beta \log(\tau) + \varepsilon$, where α and β are constants and ε is an error term. Depending on the length of observation, α and β can vary over several orders of magnitude, making modeling of transport based on flow challenging and highly inaccurate. We found a higher order yearly relation between bedload and shear stress, indicating systematic changes to the system over time. In the absence of significant additions to the system, α decreases roughly linearly over time, while β does not show any trend. From the systematic decline in α , we infer changes to sediment availability in the stream over time. Mixed-effects models have the potential to be a useful predictive tool in fluvial geomorphology, as they are more powerful at detecting trends in sediment transport rates than individual linear regressions.

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1. Introduction

Over the last three decades increased attention has been devoted to the study of erosion, sediment transport, and deposition in gravel-bed streams. The interest in the topic has been motivated by the importance of sediment transport and channel stability for river management, aquatic habitat, and river restoration. In spite of a large body of research on sediment transport, however, there is no agreement on an appropriate governing equation for the estimation of sediment movement in stream channels. In part, this is caused by temporally and spatially variable fluid forces, bed surface structures and armoring, and episodic changes in sediment supply and storage, all of which play important roles in controlling channel stability and regulating sediment transport (e.g., Hassan et al., 2008; Pryor et al., 2011).

In gravel-bed streams, the fluid shear stress on the bed is mostly near the threshold for sediment entrainment and rarely exceeds twice the critical value for mobilizing sediment during sediment transport events (Parker et al., 1982). Small variations in the flow and bed surface composition may cause large fluctuations in sediment transport rates (e.g., Paintal, 1971; Church et al., 1991; Recking, 2013). Furthermore, field and flume data under steady flow show a wide scatter in the relation between hydraulic parameters and sediment transport rates

(e.g., Gomez and Church, 1989). Within individual storm events, marked fluctuations in sediment transport rates have been observed (e.g., Hayward, 1980; Hoey, 1992; Warburton, 1992; Moog and Whiting, 1998; Whiting et al., 1999). In most cases, peak bedload transport rates do not correspond directly with peak discharge (Reid et al., 1985; Adenlof and Wohl, 1994). Temporal and spatial variation in sediment transport rates have been attributed to heterogeneity in sediment (e.g., texture, sorting, and shape), bed surface structures (e.g., pebble clusters, cells), bed morphology, flow turbulence (fluctuations in near bed velocity), and interactions among these factors (e.g., Ashworth and Ferguson, 1987).

The traditional approach to sediment transport has focused on the relations between sediment transport rate and hydraulic variables such as discharge, however, field observations suggest that sediment transport rate may not always be directly related to flow discharge because of the major role of sediment supply (e.g., Hassan et al., 2008). During low flows, scour and fill in streams are localized (e.g., Carling and Hurley, 1987), most of the bed remains intact, and mainly sand and fine gravel are transported (e.g., Jackson and Beschta, 1982; Ashworth and Ferguson, 1987; Hassan and Church, 2000). At intermediate flows, larger particles are entrained but a large proportion of the coarse fractions are partially mobile (i.e., less than their proportion in the bed) (Ashworth and Ferguson, 1987; Wilcock and McArdeil, 1993, 1997). At the highest flows, full mobility conditions may prevail and almost all sizes are mobile in its proportion in the bed (Parker et al., 1982; Wilcock and McArdeil, 1993, 1997).

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A common method for prediction of sediment transport rates uses hydraulically based functional relations that are based on theoretical principles and/or empirical correlations. The most frequently used deterministic approach is based on fixed values for model parameters yielding a single outcome (Schmelter and Stevens, 2013). This approach can result in more than an order of magnitude difference between the measured and calculated sediment transport in streams (Gomez and Church, 1989). These discrepancies have been explained by the dynamic nature of bed surface structures and armoring (Church et al., 1998), sediment supply and storage differences between entrainment and disentrainment thresholds, and sediment mobility. Bed state (Hassan and Church, 2000; Pryor et al., 2011) and sediment storage play an important role in regulating sediment transport and channel stability and are hence the focus of the current research (Lisle and Church, 2002; Church, 2006).

The stochastic method is another approach that is based on assigning probability density distributions to model parameters (Schmelter and Stevens, 2013). The advantage in using such a model is the ability to represent the uncertainty of the underlying fixed parameter assumptions (Schmelter and Stevens, 2013). An example of the stochastic approach is Einstein's (1937) pioneering work on probability of particle dispersion in streams. Similar approaches were developed by Hamamori (1962) and Turowski (2010). Bayesian statistics is another approach that has been used to model channel morphology (Griffiths, 1982), sediment entrainment (Wu and Chen, 2009), sediment transport (Schmelter et al., 2011), and sediment budgeting (Schmelter et al., 2011). In a recent paper, Schmelter and Stevens (2013) discussed the use of a Bayesian approach to model sediment transport in streams highlighting advantages of using such an approach.

Bedload data often are limited to few events, and the lack of long-term measurements limits our ability to study temporal variation in the bedload–shear stress relations. The 7-year data set collected at North Fork Caspar Creek, California, provides an opportunity to study temporal variations in bedload transport. In this paper, we use a hierarchical mixed-effects model to determine whether bedload–shear stress relations show trends over time. We use this class of linear model to show that event-to-event variation in the bedload–shear stress relation can be modeled over a period of observation.

2. Study area

The study was conducted in Caspar Creek in Jackson Demonstration State Forest near Fort Bragg, California (Fig. 1). Established in 1961, the watershed is a research site for the evaluation of the impacts of timber harvesting on erosion, streamflow, and sedimentation. The watershed is unique in providing long-term hydrological and geomorphological

data on channel dynamics in response to changes in land use and climate variability (Ziemer, 1998).

Caspar Creek drains an 8.97 km² (4.73 km² North Fork and 4.24 km² South Fork) watershed dominated by Mediterranean climate typical of low-elevation terrains of the Pacific Northwest (Ziemer, 1998). The watershed is carved into uplifted marine terraces underlain by a coastal belt of the Franciscan Formation of Cretaceous age, consisting locally of interbedded sandstone and shale. The soils in the watershed are clay loam and 1–2 m in depth. The watershed is dominated by second-growth coast redwood (*Sequoia sempervirens* (D. Don) Endl.), Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco), western hemlock (*Tsuga heterophylla* (Raf.) Sarg.), and grand fir (*Abies grandis* (Dougl. ex D. Don) Lindl.) (Ziemer, 1998). Over 90% of the precipitation falls between October and April during low-intensity rain storms. The summer is typically dry. Annual precipitation ranges between 300 and 2000 mm with a mean value of 1190 mm for the period 1962–1997. Flow data are available since 1962. Annual floods in the North Fork Caspar Creek for the period 1962–1995 ranged in magnitude between 0.76 and 8.61 m³ s^{−1} with a mean annual flood of about 4 m³ s^{−1}. Flows with recurrence interval of 10 and 50 years have a peak discharge of 7 and 10 m³ s^{−1}, respectively. Near the headwaters, channels are incised into bedrock and saprolite in steeper valleys. Channel longitudinal profiles alternate between steep and low gradient reaches. Sand, silt, and gravel dominate alluvial fill in confined valley bottoms (Ziemer, 1998).

The North Fork Caspar Creek channel at the study reach is single-thread with plane-bed and riffle-pool segments (Lisle and Napolitano, 1998). The channel is strongly affected by large wood, which creates jams, scour holes, and alluvial deposits. The bankfull channel has an average width of 5.2 m and depth of 0.48 m; channel gradient is 0.0123. The bed of the channel typically consists of a thin (<0.5 m) layer of cobbles, gravel, and finer material overlying bedrock. Much of the bed consists of a framework of subangular lag material of coarse cobbles that is derived from streamside landslides and debris flows. Alluvial bars of rounded pebble gravel and sand overlie the lag material upstream and downstream of logjams and in reaches widened by bank erosion. Bars furnish the bulk of bedload material during peak flows. Fine bedload (sand and fine gravel) is stored in pools and mobilized during lower winter flows. Otherwise, the channel is generally armored; the median size of the surface and subsurface material are 12 and 7 mm, respectively (Fig. 2). Most of the mobile material is delivered from upstream

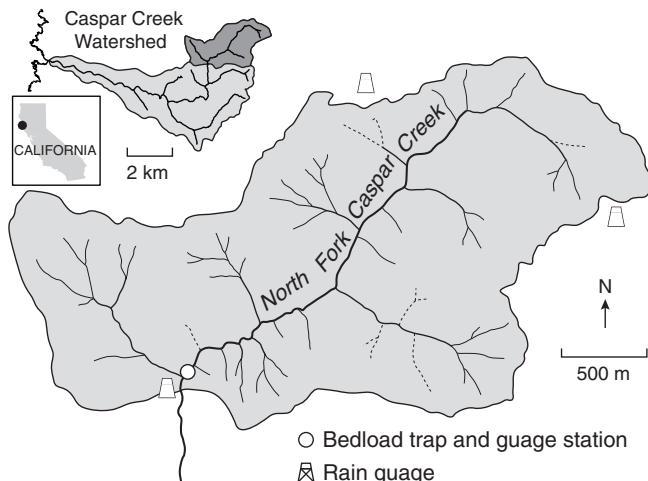


Fig. 1. Caspar Creek Experimental Watersheds, study site and gauge locations.

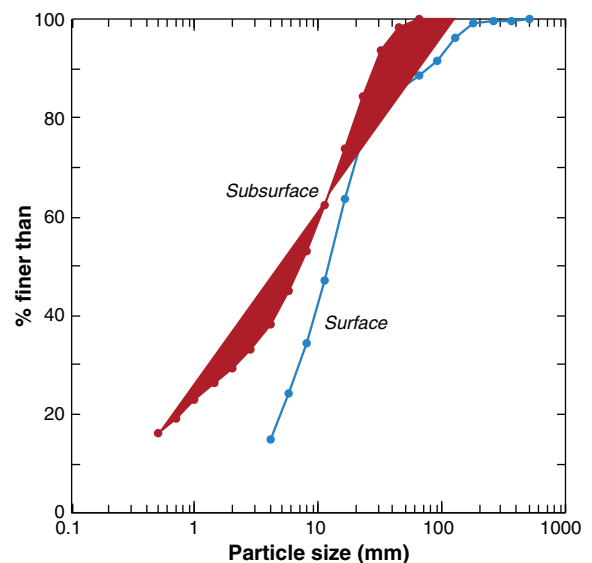


Fig. 2. Size distributions of surface and subsurface bed material in the study site of North Fork Caspar Creek.

reaches, bank erosion, and log jams. Local recruitment of sediment occurs during relatively large flows.

Thirty-seven percent of the North Fork basin was experimentally clearcut between 1989 and 1992 (Ziemer, 1998). As a result, peak flows and suspended sediment yield in logged tributaries increased several-fold after 1992 and approximately recovered in the following decade (Lewis et al., 2001). However, bedload yields as measured by annual surveys of filling of the reservoir at the outlet of the experimental watershed do not show significant increases during this period when adjusted for annual effective discharges (Lisle and Napolitano, 1998).

Sediment transport and storage are strongly affected by in-channel wood. Annual incidences of bank erosion and deposition and erosion of alluvial deposits are commonly associated with shifts in wood, including formation and breakup of wood jams. The release and capture of bed material by wood induce strong temporal and spatial variations in bedload transport (Lisle and Napolitano, 1998).

3. Data collection

Bedload data are available for the North Fork at the Arfstein gauge for the period 1988–1995 (www.fs.fed.us/psw/topics/water/caspar/; Figs. 1 and 3). Bedload samples were collected in pit traps similar to the original Birkbeck design (Reid et al., 1980). Four traps were installed across a 4.4-m-wide channel at the gauge section, where wooden baffles have been built to locally maintain a straight, uniform channel (Figs. 1 and 3). The traps allow for the continuous measurements of bedload across the channel. Sediment falls through a slotted plate covering a box that rests within a concrete pit in the streambed (Lewis, 1991). Sediment transport rates were initially monitored by a hydraulic system using a pressure transducer to sense pressure in a fluid-filled pillow and a data logger to record the pressure (Lewis, 1991). In the second and third year of observations, each pressure pillow and transducer were replaced by a load cell that produces a voltage that,

when interrogated by the data logger, is proportional to the submerged weight of the box (Lewis, 1991). The volume of the collecting box was 0.125 m³ and the opening of the slot in the pit cover was 0.1 m (cross stream) and 0.4 m (downstream) through which sediment can fall. The trap openings are evenly spaced 0.71 m across the channel and comprise about 9% of its width. Collected material was removed after each storm by a sump pump.

Sterling and Church (2002) compared the efficiency of the Helley-Smith sampler versus pit traps. Their analysis showed that at low discharges, the traps seemed to be reliable for material coarser than 0.25 mm. At high flows, they appeared to be reliable for material coarser than 1 mm. We assume that North Fork Caspar Creek traps behave in a similar way. For further comparison on the efficiency of the Helley-Smith sampler and pit traps see Bunte et al. (2004, 2008).

Data were recorded at 2- or 5-minute intervals and later aggregated to longer periods of 10 or 30 min; this procedure avoids short-term, random fluctuations in transport rate (as identified, for example, by Klingeman and Emmett, 1982). For large events, data are available for only part of the flood because of trap filling. The bedload samples were dried and sieved.

4. Modeling

The relationship between shear stress and bedload transport is usually best described by a power law, which becomes linear in log-log space (e.g., Barry et al., 2004). The small catchment size of the North Fork of Caspar Creek (~473 ha) has a fast hydrologic response to significant storms that quickly subsides to lower flow conditions shortly after the event. Because significant storm events at Caspar Creek were separated by a minimum of four days, it is reasonable to consider each flood event independent over this data set. Preliminary examination of independent event-level regressions indicates that there may be temporal trends of the model parameters (Table 1, Fig. 4). Here we describe two general approaches to analyzing parameter trends in linear models: (i) simple linear regressions of each independent event to obtain separate sets of intercept and slope parameter estimates, and (ii) a hierarchical mixed-effects model where parameters in the higher order model are assumed to have a dependence structure, defined by lower order models, on when bulk sediment transport took place (Raudenbush and Bryk, 2002). We show how each modeling approach gives us similar results when evaluating model parameter trends. However, because each independent regression has an associated false positive rate (type I error) that compounds with each regression, we show how using multiple simple regressions across several events results in a much higher overall type I error than those for hierarchical mixed-effects models. We show how a hierarchical mixed-effects model is more efficient as it is a single model containing all the events that can be used to directly evaluate trends simultaneously.

4.1. Parameter trends of independent linear models

Because this investigation focuses on independent trends of intercept and slope parameters from linear models across multiple events, it is necessary to remove any correlation between them in order to avoid bias in parameter estimates. To remove correlation, we centered the predictor variable on its mean following the transformation to log space (Cronbach, 1987; Hofmann and Gavin, 1998). Correlation of parameter estimates prior to mean centering was $-.98$, and post-mean centering was $-.030$. Model errors for all event-level linear models and the linear model used to obtain each trend were found to be normally distributed around zero, and the variance was constant.

When performing multiple hypothesis testing, the probability of having at least one false positive (falsely rejecting a true null hypothesis) is modeled by a binomial distribution given by $\alpha_F = 1 - (1 - \alpha_C)^q$, where α_F is the family-wise error rate that is dependent on the number of independent hypothesis tests, α_C is the comparison-wise error rate,



Fig. 3. View of the study reach in North Fork Caspar Creek showing the Arfstein gauge and bedload traps site during low (A) and high flows (B).

Table 1Independent event-level simple regressions (single parameters evaluated using Student's *t*-test).

Event	Date	Intercept	Std error	<i>t</i> -value	Pr(> <i>t</i>)	Slope	Std error	<i>t</i> -value	Pr(> <i>t</i>)
1	1988-11-22	3.561	0.1256	28.35	<0.0001	33.282	3.6525	9.11	<0.0001
2	1989-03-08	2.845	0.0499	57.01	<0.0001	11.366	1.9093	5.95	<0.0001
3	1989-03-18	2.83	0.0366	77.32	<0.0001	6.187	1.3487	4.59	<0.0001
4	1989-03-25	2.051	0.1229	16.69	<0.0001	7.228	2.8051	2.58	0.0101
5	1990-01-07	2.347	0.0382	61.44	<0.0001	17.232	1.0384	16.59	<0.0001
6	1990-05-26	2.727	0.1163	23.45	<0.0001	10.999	1.491	7.38	<0.0001
7	1992-02-19	2.214	0.0417	53.09	<0.0001	17.144	2.4477	7.00	<0.0001
8	1992-03-16	1.973	0.0406	48.60	<0.0001	19.601	3.5111	5.58	<0.0001
9	1992-12-10	1.875	0.0829	22.62	<0.0001	12.512	1.7241	7.26	<0.0001
10	1992-12-31	2.094	0.1108	18.90	<0.0001	13.593	1.734	7.84	<0.0001
11	1993-01-13	1.735	0.0353	49.15	<0.0001	14.746	1.2398	11.89	<0.0001
12	1993-01-19	1.947	0.0931	20.91	<0.0001	13.669	1.1622	11.76	<0.0001
13	1995-01-13	2.641	0.1103	23.94	<0.0001	5.948	1.6939	3.51	0.0005

and q is the number of hypotheses tested in the modeling process (Sidak, 1967; Games, 1977). Hypothesis testing any single parameter across all event-level models plus an additional hypothesis test for the trend results in the number of false positive type I errors to be as high as the number of events plus one. To simplify model evaluations as well as to illustrate the compounding effects of multiple hypotheses testing of parameters, we assume a posteriori that the significance level across all event-level models plus the trend model is the final p -value for the slope trend. Hence, the comparison-wise error rate will be the final p -value used to compute the family-wise error rate.

4.2. Hierarchical linear mixed modeling

Some hierarchical mixed-effects models are specified for longitudinal data, where characteristics of individuals are repeatedly measured over time, such that parameters are being studied for their temporal evolution (Laird and Ware, 1982). If we assume each flood event is an individual, then the repeated measures are aggregate bedload and shear stress (measured at 10- or 30-minute intervals) and introduce serial auto-correlation into the model. In this study, however, rather than examine bedload evolution across individual events, we study whether

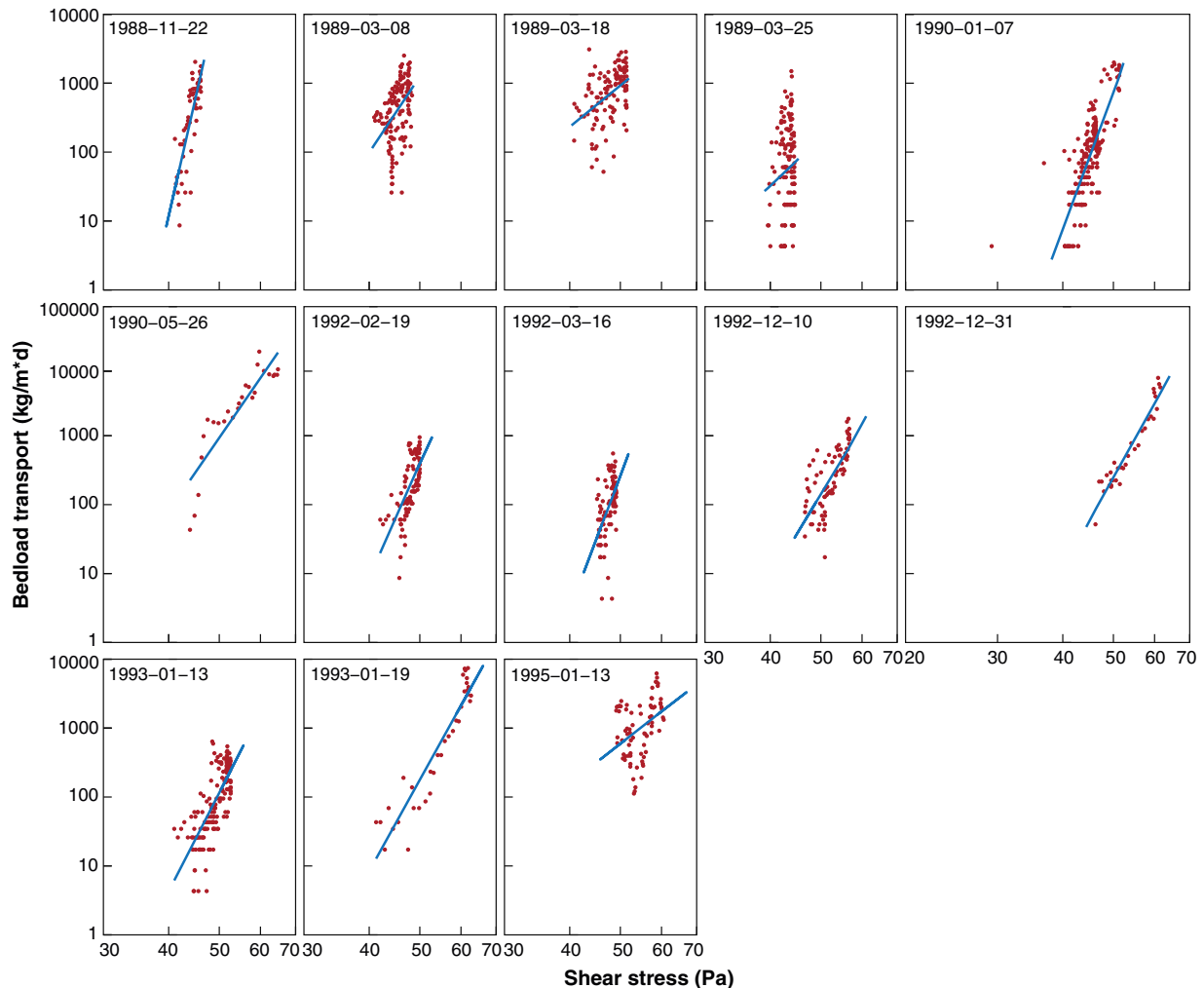


Fig. 4. Event-based scatter plots and regressions of sediment transport rate versus bed shear stress. For more information see Table 1.

there are temporal trends in the parameter estimates of linear models for bedload–shear stress over a multiyear period of record. Hence, we can use a longitudinal form of the hierarchical mixed model (e.g., example 2 in Laird and Ware, 1982; Verbeke and Molenberghs, 2009) to examine parameter trends without resorting to a model structure with a potential serial auto correlation.

In this study, we followed the general case shown by Laird and Ware (1982). First, we define the higher-order, event-level model for the log-linear relation between bedload and shear stress on each measured pair

$$\log(Q_b)_{ij} = \gamma_{0i} + \gamma_{1i} \log(\tau)_{ij} + \varepsilon_{ij}, \quad (1)$$

where $\log(Q_b)_{ij}$ is the $\log_{10}$ of the bedload response variable for the j^{th} observation out of n_i observations within the i^{th} event, $\log(\tau)_{ij}$ is $\log_{10}$ of the shear stress covariate for the j^{th} observation for the i^{th} event, γ_{0i}, γ_{1i} are linear model parameters for the i^{th} event, and ε_{ij} is the model error of unexplained variation for the higher-order model. Mean boundary shear stress exerted on bed particles was calculated by partitioning total boundary shear stress using bed particle size and hydraulic variables from gauging data.

Next, we assume the lower order model for intercepts and slopes, γ_{0i}, γ_{1i} , in model (1) are dependent on the year in which each event occurs:

$$\begin{aligned} \gamma_{0i} &= \beta_{00} + \beta_{01} \text{Year} + \alpha_{0i} \\ \gamma_{1i} &= \beta_{10} + \beta_{11} \text{Year} + \alpha_{1i} \end{aligned} \quad (2)$$

where β 's are fixed effects and α 's are the random effects, and Year is the number of years in which the event occurs since the start of the study. Substituting Eq. (2) into Eq. (1), the event-level hierarchical model with model assumptions are given by

$$\begin{aligned} \log(Q_b)_{ij} &= \beta_0 + \beta_1 \text{Year} + \beta_2 \log(\tau)_{ij} \\ &\quad + \beta_3 \text{Year} \times \log(\tau)_{ij} \\ &\quad + \alpha_{0i} + \alpha_{1i} \log(\tau)_{ij} + \varepsilon_{ij} \\ \alpha_{ki} &\sim N(0, \psi_k^2), \text{Cov}(\alpha_k, \alpha_{k'}) = \psi_{kk'} \text{ for } k \neq k' \\ \varepsilon_{ij} &\sim N(0, \sigma^2), \text{Cov}(\varepsilon_{ij}, \varepsilon_{ij'}) = 0 \text{ for } j \neq j' \\ k &= 0, 1 \quad j = 1, \dots, n_i, \quad i = 1, \dots, 13. \end{aligned} \quad (3)$$

Here the fixed effects are model coefficients of the first two lines of model terms above (with model interactions in the second line), and the random effects are model coefficients in the third line, which are α_{0i}, α_{1i} . Each random effect is assumed to be normally distributed with zero mean, constant variance, and α 's are correlated. The final term is model errors ε_{ij} , which are assumed to be independent, identically distributed, normal random variables with mean zero and constant variance.

Using the top-down approach recommended by Diggle et al. (2002), we selected the optimal structure for random effects followed by the optimal structure for fixed effects. Because of the hierarchical structure of the model, standard F -tests cannot be used to evaluate reduced forms of the model, so we used χ^2 tests of maximum likelihood (Verbeke and Molenberghs, 2009). Random effect terms were tested, and the model form was reduced based on significant differences in likelihood, starting with slope term (α_1) in the third line of Eq. (3), followed by the fixed effects in the first line until the optimal model structure is found. Finally, assumptions for model error and random effects were checked for homoscedasticity, zero mean, and normality.

5. Results

5.1. Observations

We will first describe the flow hydrographs, bedload textures, and sediment mobility in North Fork Caspar Creek. Secondly, we will examine relations between sediment transport rate and shear stress. Finally,

we will present the hierarchical linear mixed model of sediment transport.

Storm peak flow for the bedload measurements over the study period (1988–1995) ranged between 0.76 and 6.81 $\text{m}^3 \text{s}^{-1}$. In 1993, a peak discharge of 6.81 $\text{m}^3 \text{s}^{-1}$ was recorded with an estimated 10-year return period. The flow responsible for moving bedload in the creek can be described by cumulative duration curves. For each event, the total duration of movement was calculated as a percentage of the total flow duration that exceeded the flow required to initiate sediment movement in the creek. The threshold for movement was determined by first motion for each, and on event-specific value was applied. Duration of movement ranged between 17 and 363 h/y with a mean value of 185 h/y.

Examples of event-based size distributions of bedload collected in the traps are presented in Fig. 5. For comparison, the particle size of the bed surface material and subsurface material are plotted. At low flow ($\sim 0.02 \text{ m}^3 \text{s}^{-1}$), the sediment transport is dominated by fine material ($< 2 \text{ mm}$) and small gravels implying that the fine material moves over an intact bed. At higher flows the size distribution shifts as larger particles are mobile, and in some cases the size distributions became slightly bimodal. As shown in Fig. 5, for all cases the size distribution of the bedload remained finer than that of the bed surface.

A pattern of hysteresis in sediment transport rate is revealed when we compare samples collected during the rising stage and those collected during the falling stage. Sediment transport rate during the rising stage was generally higher than that during the falling stage indicating a clockwise hysteresis in the transport rate (Fig. 6). For example, at a shear stress of 50 Pa, the sediment transport rate was about four times higher during the rising limb than the falling limb (233 kg/m d at the rising limb and 102 kg/m d at the falling limb) (Fig. 6A). In some cases, however, differences in sediment transport between rising and falling limbs were relatively small.

To quantify the hysteresis patterns and magnitude we adopted the method suggested by Langlois et al. (2005). For each event, the rising and the falling limbs of each flow event were plotted and a best fit line was fitted through each limb separately, and the hysteresis index H was computed as the ratio between the areas under the rising and the falling limbs. Langlois et al. (2005) suggested that $H \sim 1$ indicates weak hysteresis, $H > 1$ indicates clockwise hysteresis, and $H < 1$ indicates counterclockwise hysteresis. Of 13 storms, we had sufficient data for eight events. For all examined cases, H was larger than one, indicating clockwise hysteresis. We found that H did not depend on the flow parameters (e.g., discharge, shear stress), but that there is a slight

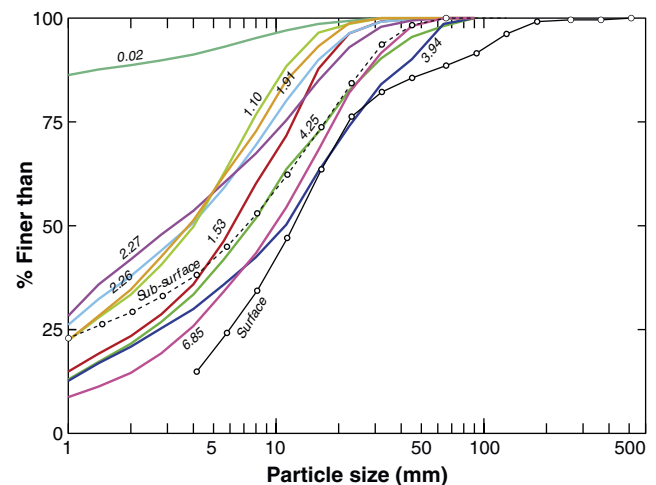


Fig. 5. Changes in bedload texture during several events in North Fork Caspar Creek. The particle size distribution represents the whole event. Numbers indicate the event peak flow.

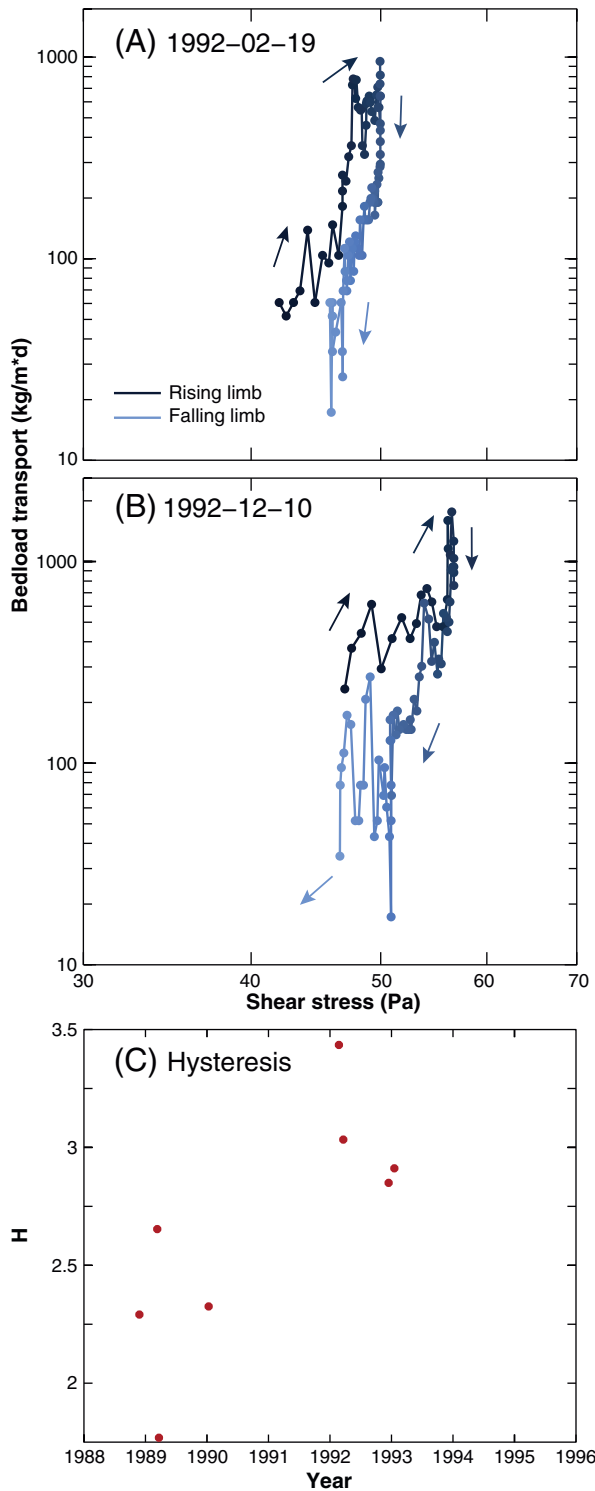


Fig. 6. (A) and (B) Two examples of bedload transport rate versus shear stress (note the differences between the rising and falling limbs of the hydrograph). (C) Relation between hysteresis index H and year.

increase in H over time, indicating an increase in the magnitude of hysteresis with time (Fig. 6C).

To gain a better understanding of the sediment dynamics, we examined the fractional mobility of bed material over a range of flow events. Wilcock and Southard (1988) defined the fractional mobility as the ratio of P_i/F_i , in which P_i is the fraction of the bedload in the i th size class and F_i is the fraction of the subsurface bed material of the class. As argued by Church and Hassan (2002), the use of the surface material is appropriate

for spatially uniform transport. However, in natural streams, sediment may be entrained from various surfaces including upstream reaches, tributaries, and banks, but this does not necessarily reflect local bed exchange of sediment between bed and flow (Church and Hassan, 2002). Furthermore, our fractional analysis is event-based, and there is no guarantee that the bed surface represents surface conditions at the exact time of the transport measurements. Under these circumstances, Church and Hassan (2002) argued that the subsurface material deposit in the bed accumulated over time by the full range of competent flows in the stream is the most relevant reference material. Our bed subsurface sample was taken from a bar located upstream of the gauge station and likely represents the whole study period. A ratio of about one indicates full mobility for grains of that particle size. If $P_i/F_i < 1$, then grains in that size class are partially mobile. The variation of the fractional transport rate with particle size was determined for whole flow events (in some cases until the sediment traps filled) because we collected sediment in the trap for the entire flood duration. Therefore, the results presented in Fig. 7 should be considered general trends in sediment mobility only. As shown in Fig. 7 for some fully mobile classes, $P_i/F_i > 1$ indicating that the mobile classes are overrepresented because some of the other classes are partially mobile (i.e., $P_i/F_i < 1$) (Oldmeadow and Church, 2006).

During the smallest floods ($< 1 \text{ m}^3 \text{ s}^{-1}$), mostly sand and small gravels are mobilized, causing those fractions to be overrepresented in the transport; larger sizes are greatly underrepresented or absent (Fig. 7). At the intermediate floods ($2\text{--}4 \text{ m}^3 \text{ s}^{-1}$), larger material is mobile but the sand and small gravels are still overrepresented in the transport. For large floods ($> 4 \text{ m}^3 \text{ s}^{-1}$), coarser material is mobile and the $P_i/F_i < 2$. For these floods, material as large as 16 mm is fully mobile in proportion to its presence in the bed ($P_i/F_i \sim 1$).

5.2. Modeling

Plotting all event-level regressions of $\log(Q_b)$ versus $\log(\tau)$ shows a wide variation in the transport rate (Fig. 8, Table 1). For a narrow range of shear stresses, sediment transport rate varies widely (Fig. 8) for the same flow condition ranging over three orders of magnitude. Two levels of variation in the sediment transport are evident; between events and between years (Fig. 8). Also, there is a systematic shift in the sediment transport rate between years; for a given flow condition, sediment transport rates in the early years (1988) are higher than

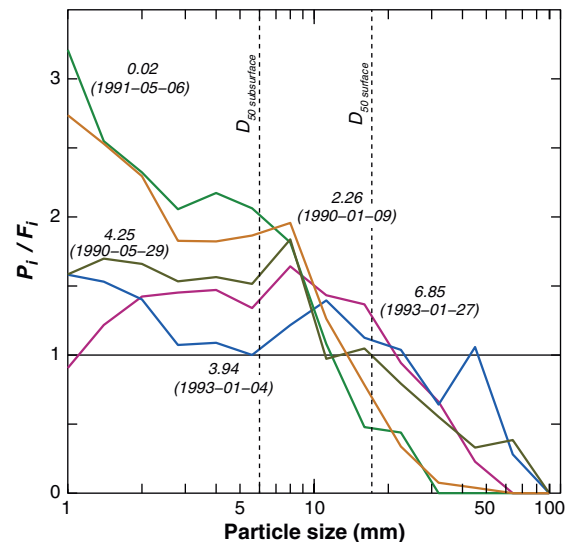


Fig. 7. Transport ratio as a function of grain size for selected floods. The transport ratio P_i/F_i , where P_i is the proportion of each size fraction i present in the transported material, and F_i is the proportion of each size fraction in the subsurface bed sediment. Numbers indicate the event peak flow.

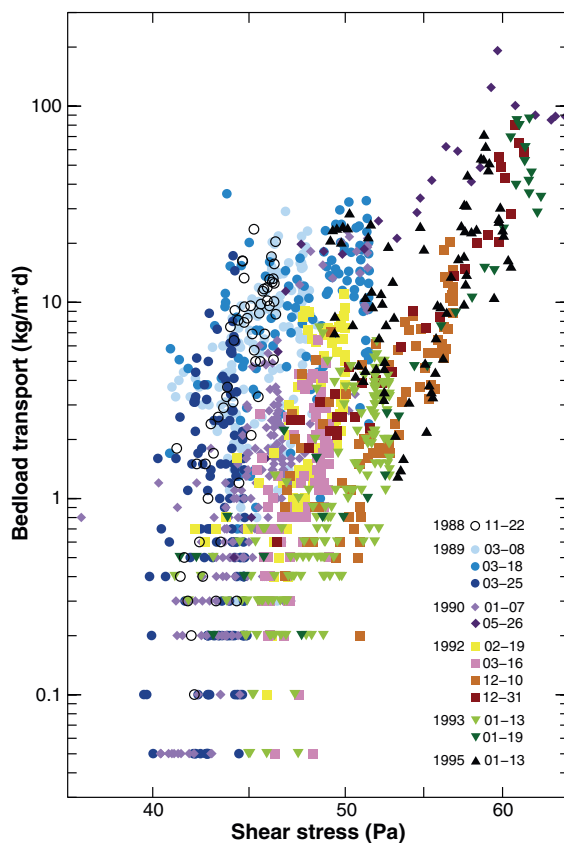


Fig. 8. Scatter plot of sediment transport rate versus bed shear stress for flood events for the years 1988–1995.

later years (1995). This difference can also be seen in Table 1, which summarizes the results of the individual regressions.

The results of independent linear models (event-level regressions) are shown in Table 1. Final p -values for the trend in intercept and slope parameter estimates were 0.0365 and 0.337, respectively. At first glance, there appears to be a significant trend for the intercept parameter across all 13 events. However, using the final p -value as the comparisonwise error rate, we find the familywise error rate is $1 - (1 - 0.0365)^{14} = 0.406$, creating a 40.6% chance that the trend for intercepts is insignificant across all 13 event-level models.

There is a significant change in the intercept of the transport function over time, but not the slope (Table 2; Fig. 9). The reduced form of the model is given by

$$\log(Q_b) = \hat{\beta}_0 + \hat{\beta}_1 \text{Year} + \hat{\beta}_2 \log(\tau) + \hat{\alpha}_0 + \hat{\alpha}_1 \log(\tau) \quad (4)$$

Table 2
Parameter estimates for the reduced hierarchical mixed-effect model.^a

Event	Date	Year	$\beta_0 + \alpha_{0i}$	$\beta_1 * \text{year}$	$\beta_2 + \alpha_{1i}$
1	1988-11-22	0	3.33	0	26.31
2	1989-03-08	1	2.99	−0.142	11.54
3	1989-03-18	1	2.97	−0.142	6.63
4	1989-03-25	1	2.29	−0.142	9.42
5	1990-01-07	2	2.63	−0.285	17.14
6	1990-05-26	2	2.98	−0.285	11.35
7	1992-02-19	4	2.79	−0.569	16.57
8	1992-03-16	4	2.55	−0.569	17.85
9	1992-12-10	4	2.46	−0.569	12.36
10	1992-12-31	4	2.67	−0.569	13.49
11	1993-01-13	5	2.45	−0.711	14.64
12	1993-01-19	5	2.67	−0.711	13.61
13	1995-01-13	7	3.54	−0.996	7.46

^a The transport function intercept is given by $\beta_0 + \alpha_{0i} + \beta_1 * \text{Year}$ while the slope is given by $\beta_2 + \alpha_{1i}$.

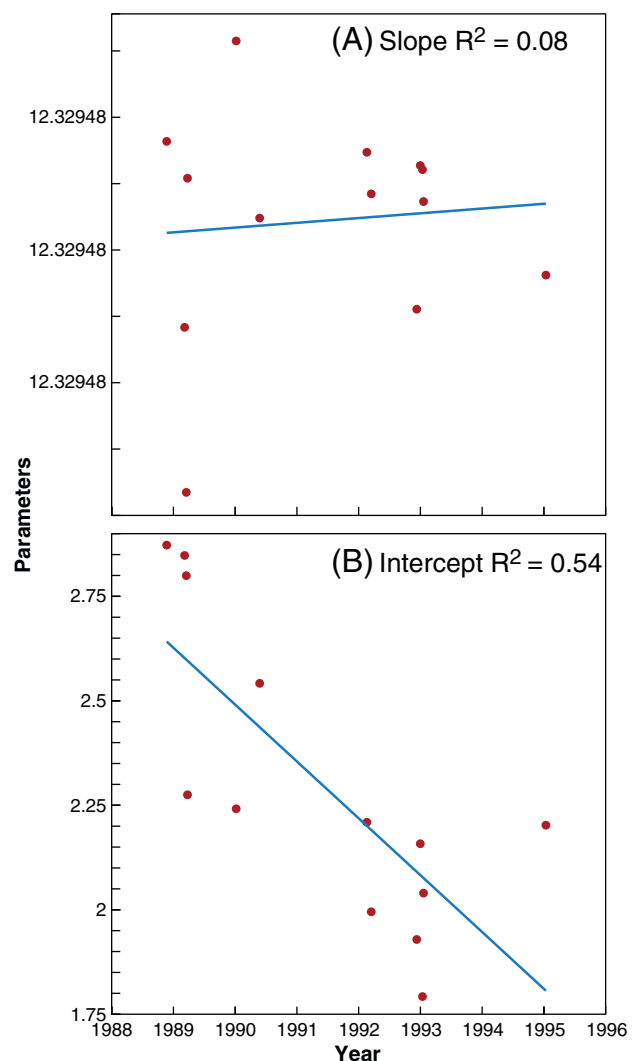


Fig. 9. Temporal changes in the intercept and the slope of the model. The intercept of the transport function is given by $\beta_0 + \alpha_{0i}$, while the slope is $\beta_2 \log(\tau) + \alpha_{1i}$ (see Table 2).

where $\hat{\beta}_0 = 2.79$, $\hat{\beta}_1 = -0.142$, and $\hat{\beta}_2 = 13.72$ are estimates of the fixed parameters in model (3), while $\hat{\alpha}_0$ and $\hat{\alpha}_1$ are estimates of the random parameters across individual events. Combining fixed and random estimates, the intercept coefficients for individual bedload models are given by $\hat{\beta}_0 + \hat{\alpha}_0 + \hat{\beta}_1 \text{Year}$, and slope coefficients for individual bedload models are given by $\hat{\beta}_2 + \hat{\alpha}_1$ (Table 2). Table 2 shows a clear linear trend on the intercept term across all events.

Because p -values are not properly defined for fixed-effect parameter tests in hierarchical models (Baayen et al., 2008), we used a Markov-Chain Monte Carlo (MCMC) technique to obtain p -values for the fixed effects. The p -value for β_1 was found to be 0.038, making the familywise error rate $1 - (1 - 0.038)^2 = 0.075$ (7.5%). This is much lower than the familywise error rate for the independent linear models (40.6%) simply because not as many hypotheses had to be tested. This shows that hierarchical mixed-effect modeling is much more sensitive to parameter trends across many transport events.

6. Discussion

We have examined variation in sediment transport in a gravel-bed stream for material collected in pit traps. The relation between sediment transport rate and shear stress changed between 1989 and 1995, with much higher transport being experienced in early years. In other

words, the relations are not stable and may be unreliable to predict sediment flux in other years. In this study we observed within-event and between-event changes in the sediment transport rate that are likely the result of local changes in bed composition and channel morphology, with large woody debris playing a role in periodic releases of sediment (e.g., Lisle and Napolitano, 1998). A clockwise hysteresis was observed for all events implying that within-channel sediment storage particularly in pools (e.g., Lisle, 1995; Lisle and Napolitano, 1998) controls sediment mobility in the stream (Fig. 6). Furthermore, the systematic decline in the sediment transport rate from 1988 to 1995 appears to be due to the decline in sediment supply, sediment trapping behind wood upstream of the study reach, and the development of an armored surface.

Sediment transport in North Fork Caspar Creek, even during relatively high flow, remains very modest in comparison with published field data (e.g., Klingeman and Emmett, 1982). During low events, fine sediment dominates sediment transport. As the flow increases, larger particles are entrained and transported. However, over the range of competent flows the texture of the transported material is finer than that of the bed surface and the subsurface (Lisle, 1995; Lisle and Napolitano, 1998). Bed material is too coarse to be transported by most floods. Cobbles and large pebbles that are commonly contributed by streamside slope failures and debris flows help to stabilize the bed surface and reduce scour depth; whereas, sand and fine gravels stored in pools dominate sediment transport during moderate peak flows (Lisle, 1995; Lisle and Napolitano, 1998). The movement of fractions larger than the median size of the bed surface material is rare and occurs only at high flows. Such flows may occur every few years and the movement might not last more than a few hours. Apparently in the case of the fine material, the sediment supply (and storage in pools) is the most important variable causing a decrease in the transport rates after peak discharge. After the mobilization of most of the available fine fractions, further increase in the sediment transport depends mainly on the local sources from the bed, which are controlled by the movement of coarse material. These circumstances explain the hysteresis in the sediment transport rates. These observations indicate that large parts of the bed surface remained intact during most of the observed flows over the study period. Our study describes the sediment transport regime in the presence of a persistently well-armored bed.

We used hierarchical mixed-effects modeling to determine whether a trend in model parameters exists using a model structure with serial correlation. There is an annual trend for only the intercept parameter in log-log space (Figs. 4 and 8), implying that the slope of the relationship between bedload transport and shear stress is constant over time but that the intercept term decreases (Fig. 9). The lack of a random slope term does not imply that the slope is static; on the contrary, for the independent linear regressions we found that the slope varied between 6.6 and 26.3 (in log-log space) but that it did not change systematically over the monitored time period. However, the decreasing trend of the intercept parameter seems to indicate either a decreasing sediment supply to the channel upstream of the sediment trap or changes in the composition of the bed surface or the trapping of sediment by large woody debris. It also bears mentioning that the data for this system were collected over a 7-year period and that these trends are probably not representative of longer scale temporal trends in slope and intercept, which require a long period of study.

Daily and monthly time scales were also used in the lower-order models to check for hierarchical model dependence on temporal scaling. Although these differences were not statistically tested, structurally similar model estimates across different time scales (days, weeks, months) suggests that the model for North Fork Caspar Creek works across a variety of time scales implying that North Fork Caspar Creek has seasonal invariance to bedload transport rate because trends were similar across all time scales. Because these yield similar results, annual time scales were ultimately used as they simplify the model estimates and analysis. Time scales were all zeroed to correspond with the first

bedload measurements so that numerical scaling across all model covariates was similar.

Random effects significantly improved the model, indicating highly variable bedload transport in the channel driven by changes in sediment supply and hydrologic conditions. The hierarchical structure of the model is sensitive to overall parameter trends, making it an ideal modeling tool to examine trends over time or to test for effects of other environmental variables (e.g., landslide sediment input, forest fire, storm intensity) within a given fluvial system. Event-level individual regression models can be used but are less sensitive to these temporal trends. Indeed, the annual trend for the intercept may not have been detected using the individual linear regressions, indicating that a simple linear model of bedload transport and shear stress is *not* sufficient to represent sediment transport in a channel over time.

Therefore, the purpose of using hierarchical mixed models for examining bedload relationships in channels is twofold. First, they allow for individual variation of bedload–shear stress power law relationships through the use of random effects, which yield independent power law relationships for each event. Second, they can easily detect subtle trends in model parameters over time, which indicate supply changes in sediment to the channel. For independent linear models, assessing whether parameters change over time requires a separate step in the modeling process, but these steps can be automatically incorporated into a mixed-effects model. Finally, hierarchical mixed-effects models are more sensitive than individual linear regressions, as they do not require controlling for multiple tests.

The decline in the intercept parameter over the study period could be attributed to changes in sediment availability in the study reach. Field studies of hillslopes and channels and long-term measurements of flow and sediment transport in Caspar Creek Experimental Watersheds show that sediment transport rates are significantly influenced by logging, gullying, and landsliding (Lisle and Napolitano, 1998; Ziemer, 1998; Reid et al., 2010). Infrequent landslides have delivered the largest amounts of sediment to the channel since 1962 and most notably in 1974, but over the study period, no major landslides or debris flows were recorded. Gullying of tributary channels is a major source of sediment and is increased by logging-related increases in runoff (Reid et al., 2010). Bank erosion is triggered by shifts in wood but is a minor source of sediment. Because increased suspended sediment yields and tributary channel erosion span the period of the bedload measurements, the decline in bedload transport rates cannot be attributed to variations in sediment supply from the available observations in the watershed. A remaining factor is increased storage capacity afforded by inputs of wood to the mainstem channel. Wood inputs have increased as the second-growth forest has matured after logging and splash-damming in the early twentieth century, and measured wood loading increased from 1994 to 1996, particularly from buffer strips left along the channel during recent logging (Reid and Hilton, 1998). Surveys of channel cross sections in 1991 and 1997 show punctuated increases in sediment storage that can be attributed to wood and wood jams (Lisle and Napolitano, 1998). Although these observations are not precise enough to explain the decrease in bedload transport in the later part of the study period, they do suggest an increase in sediment storage at the expense of throughput of bed material load, thus a general decrease in transport rate in the mainstem channel during this period. Partial sediment starvation may also have caused bed surfaces to coarsen, shifting the sediment entrainment conditions and hence sediment transport capacity of the flow (e.g., Lisle and Church, 2002), but temporal variations in surface texture were not measured.

7. Conclusions

The hierarchical mixed-effects modeling approach to sediment transport was found to be useful for modeling the temporal fluctuations in sediment transport. We found a systematic temporal decrease in the intercept of the bedload–shear stress relation, indicating that while

there was no systematic change in the slope, the amount of bedload transport that occurs was reduced over time. We attribute this to decreased availability of sediment in the study reach, drawing on several supporting factors. Firstly, the primary source of sediment input into this system is gully erosion, and no major inputs from bank collapse or landslides were noted during this time at the field site, leaving only what supply remains within the bed. Secondly, all of the events displayed a clockwise pattern of hysteresis in the sediment transport relationship, indicating that sediment was being removed during the rising limb of the hydrograph (Fig. 6). During a clockwise hysteresis event, fine material is entrained during the rising limb of the hydrograph from highly mobile storage zones in the pools where it had been collecting before the flood (Lisle and Napolitano, 1998). Over the course of the observed event, we infer that these repeated events reduce sediment storage within the channel over time, resulting in the decrease in the bedload–shear stress relationship seen in our mixed-effects model. During lower-magnitude events, the grain size distribution (GSD) of the bedload was much finer than the GSD of the bed, indicating that much of the bed remained intact during these events. However, full mobility was achieved in the finer fractions during these events, implying that this sediment was being fractionally removed from the bed surface and deep pools, further increasing the armouring present in the bed. Finally, during larger-magnitude events the GSD of the bedload was approximately equal to the GSD of the bed subsurface material, implying that coarse (>45 mm) fractions were either partially mobile or remained in place.

While this model is admittedly simple, our results show that the relation between bedload and flow can be modeled relatively accurately over time. Mixed-effects models are a useful tool in examining this relation and could be extended further to be used in predictive modeling of future sediment transport events.

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