

Evaluation of modulus of elasticity and modulus of rupture of full-size wood composite panels supported on two nodal-lines using a vibration technique

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HIGHLIGHTS

- A vibration testing method based on the principle of transverse free vibration.
- Full-size wood composite panel (WCP) supported on two nodal-lines.
- Modulus of elasticity (MOE) and modulus of rupture (MOR).
- Transverse free vibration technique can be used to determine MOE and predict MOR of full-size WCPs.

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ABSTRACT

The objective of this study was to determine the feasibility of using a transverse free vibration testing method to nondestructively evaluate the bending stiffness and strength of full-size (1220 × 2440 mm) wood composite panels (WCPs). Based on the principle of transverse free vibration of a thin plate, the dynamic modulus of elasticity (MOE) of a full-size WCP supported on two nodal-lines was derived using the Rayleigh method. Transverse free vibration test was conducted on 54 full-size medium density fiberboard (MDF) panels and 36 full-size particleboard (PB) panels under a two-nodal-line support. The modulus of elasticity (MOE) and modulus of rupture (MOR) of the panels were then determined by conducting static bending tests on the small samples cut from each panel. The results indicated that the transverse free vibration of a full-size panel supported on two nodal-lines was a valid testing approach for rapid determination of dynamic modulus of elasticity of full-size MDF and PB panels. Strong linear relationships were found between the dynamic MOE of full-size panels and static MOE of small samples, with the dynamic MOE approximately 8% higher than the static MOE. In addition, the two-variate models with dynamic MOE and thickness as predicting variables and the three-variate models with natural frequency, density and thickness as variables were found both effective in predicting the modulus of rupture of full-size MDF and PB panels.

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1. Introduction

Traditional wood composite panels (WCPs) such as medium density fibreboard (MDF) and particleboard (PB) are engineered wood products that are often used for cabinets, furniture, doors, and floor substrates. These panels are typically produced with a standard dimension of 1220 mm × 2440 mm (width × length) in

manufacturing facilities. To meet the certification requirements, full-size commercial WCPs must provide enough stiffness and strength under a variety of environmental conditions. Modulus of elasticity (MOE) and modulus of rupture (MOR) are two most important quality indices to characterize the mechanical performance of WCPs. To accurately evaluate MOE and MOR of full-size WCPs is therefore of great necessity.

The current quality control practice in WCP industry is to randomly select some panels from the production line, cut multiple small standard samples from the selected panels, test the samples to destruction in an off-line laboratory machine using a static

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bending method, and then obtain average MOE and MOR values for each panel [1,2]. The average MOE and MOR of the cut samples ought to meet the minimum requirements specified in the products specifications. While this destructive testing approach generally works well for quality control purpose in manufacturing facilities, it has the drawbacks of being time consuming, destructive, and not cost effective. In addition, studies have found that the testing results of the small samples cut from full-size panels often have a large variability existed in mechanical properties both within panels and among panels [3,4]. As a result, small-sample test does not fully reflect the average panel properties. An alternative to the small-sample test approach is to non-destructively test each individual panel in full-size so that the mechanical performance of each panel can be physically evaluated.

Non-destructive vibration testing techniques have been used to evaluate mechanical properties of solid wood and WCPs in scientific research for several decades [5,6]. In recent years, considerable advances have been made on property assessment of small beam-like wood composite specimens using a vibration method [7–11]. A dynamic cantilever-beam-apparatus was developed for determining elastic properties (static and dynamic MOE, dynamic viscoelasticity and shear modulus) of small and thin wood composite materials [9–11]. Although this method proved fast and convenience of use, it has the same shortcomings of cutting small samples from full-size WCPs as the standard static bending method does. Currently, the American standard test method ASTM D3043-00 [12] describes a pure moment method for testing full-size structural panels to acquire MOE of the panels. In addition, a continuous MOE tester that is based on the principle of mechanical static bending has been developed to non-destructively determine the MOE of full-size plywood panels in a production line [13]. However, free vibration technique has not been adopted for evaluating the mechanical performance of full-size WCPs.

Some laboratories studies have been done to evaluate MOE of full-size WCPs using the longitudinal vibration and modal testing methods [14–16]. However, these techniques are only applicable to laboratory settings and not suitable to production applications. Furthermore, studies have found that there existed a significant linear relationship between dynamic MOE obtained by vibration techniques and MOR tested by the standard static bending method for small WCPs specimens [17,18], and some statistical regression models have been developed to predict MOR of small wood composite specimens using dynamic MOE [19]. Hitherto there are no some statistical regression models that have been useful for predicting MOR of full-size WCPs in the light of dynamic MOE tested by vibration techniques.

In this study, a vibration test method that is based on the principle of transverse free vibration of a thin plate was employed for determining the bending stiffness and strength of full-size MDF and PB panels. The validity of this vibration method was tested in comparison with the results from the standard static bending test. This study was a part of an ongoing project to explore the use of free vibration testing technique to predict various elastic properties of full-size WCPs for on-line quality control [20–22].

2. Theoretical basis of transverse free vibration technique

In this study, full-size wood composite panels such as MDF and PB panels were assumed to be transversely isotropic thin plates with uniform distribution of mass and in-plane elastic properties. In essence, transversely isotropic material is a special form of orthotropic material. Without considering the effects of shear deformation and rotatory inertia, the governing differential equation for transverse free vibration of a thin rectangular orthotropic plate can be expressed as follows [23]:

$$\frac{E_d h^3}{12(1 - \nu_{xy} \nu_{yx})} \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} + 2(D_1 + 2D_{xy}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \quad (1)$$

where w represents $w(x, y, t)$; D_y , D_1 and D_{xy} are flexural rigidity in width direction of the panel, equivalent rigidity and torsional rigidity, respectively; E_d is dynamic MOE of the panel; ν_{xy} and ν_{yx} are the Poisson's ratios of the panel; and ρ and h are mass density and thickness of the panel, respectively.

The two-nodal-line support was adopted in this study because this support condition is easy to implement both in laboratory and manufacturing facilities. The two-nodal-line support refers to a full-size WCP supported on two nodal-lines of the first-order flexion mode, which are located at 22.4% and 77.6% of its length. The panel with two-nodal-line support can be treated as having a boundary condition with four edges completely free (FFFF).

In view of the complexity of FFFF boundary condition, there is no closed-form solution for the governing differential equation. Thus the numerical method such as Rayleigh method can be applied for solving such differential equation. The solution of the transverse free vibration of a thin plate can be assumed as:

$$w(x, y, t) = W(x, y) \sin \omega t \quad (2)$$

where $W(x, y)$ is mode shape function of the panel, ω is the natural free vibration angular frequency of the panel without damping.

Based on the law of energy conservation, the maximal potential energy $U_{\max}$ of the panel vibration system must be equal to its maximal kinetic energy $T_{\max}$, which can be expressed as:

$$U_{\max} = T_{\max} \quad (3)$$

For transverse free vibration of a thin plate, the maximal potential energy and kinetic energy of the panel can be expressed as [24]:

$$U_{\max} = \frac{1}{2} \int_0^b \int_0^a \left\{ \frac{E_d h^3}{12(1 - \nu_{xy} \nu_{yx})} \left(\frac{\partial^2 W}{\partial x^2} \right)^2 + D_y \left[\left(\frac{\partial^2 W}{\partial y^2} \right)^2 + 2 \nu_{xy} \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2} \right] + 4 D_{xy} \left(\frac{\partial^2 W}{\partial x \partial y} \right)^2 \right\} dx dy \quad (4)$$

$$T_{\max} = \frac{1}{2} \omega^2 \rho h \int_0^b \int_0^a W^2 dx dy \quad (5)$$

where W represents $W(x, y)$, a and b are length and width of the panel, respectively.

Through both experimental and theoretical modal analysis in previous studies, we have found that modal shape of the first-order vibration for a full-size WCP supported on two nodal lines was the first-order flexion along the length direction, namely mode (2, 0) [25,26]. For the mode (2, 0) of the panel, the mode shape function $W(x, y)$ of the panel can be given as [27]:

$$W(x, y) = \cos\left(\frac{\gamma x}{a} - \frac{\gamma}{2}\right) - \frac{\sin\left(\frac{\gamma}{2}\right) \cosh\left(\frac{\gamma x}{a} - \frac{\gamma}{2}\right)}{\sinh\left(\frac{1}{2}\gamma\right)} \quad (6)$$

where the value of γ is 4.73.

Based on Eq. (6), Eq. (4) and Eq. (5) can be substituted into Eq. (3), and the dynamic MOE of the panel supported on two nodal lines can be derived as:

$$E_d = \frac{12 \omega^2 m a^3 (1 - \nu_{xy} \nu_{yx})}{500.6 b h^3} = \frac{12 (2\pi f)^2 m a^3 (1 - \nu_{xy} \nu_{yx})}{500.6 b h^3} \quad (7)$$

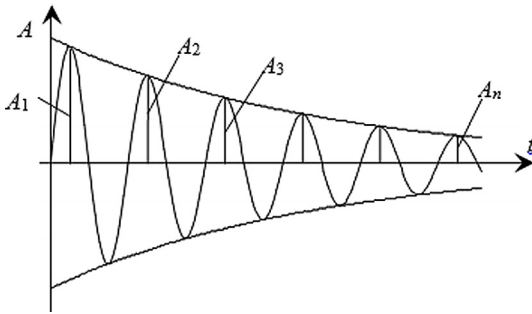


Fig. 1. Illustration of the waveform of free vibration of a full-size panel.

where f is the natural vibration frequency without damping at mode (2, 0) of the panel, m is mass of the panel. According to previous literatures [15,16,28], the value of $1 - \nu_{xy}\nu_{yx}$ for most wood materials was assumed to be 0.99.

Considering the existence of damping component during the vibration, free vibration of a panel supported on two nodal lines has a gradual damped amplitude (Fig. 1). As a consequence, the natural vibration frequency without damping is higher than the resonant frequency tested. Logarithmic decrement, as an index for measuring the damping component of a vibration, can be expressed as follows [9],

$$\Delta = \ln \frac{A_{n-1}}{A_n} = \frac{1}{n-1} \ln \frac{A_1}{A_n} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 2\pi\zeta \frac{f}{f_r} \quad (8)$$

where Δ is logarithmic decrement, A_1 , A_{n-1} and A_n are the first, the $(n-1)$ th and the n th amplitude of the damped sine wave, respectively, ζ is the damping ratio, f and f_r are the first natural vibration frequency of the panel without damping and the first resonant frequency tested, respectively.

Based on Eq. (8), the first natural vibration frequency (f) can be expressed using logarithmic decrement (Δ) and the measured first frequency (f_r) in Eq. (9):

$$f = \frac{f_r \sqrt{4\pi^2 + \Delta^2}}{2\pi} \quad (9)$$

Both the value of $1 - \nu_{xy}\nu_{yx}$ and the first natural vibration frequency (f) can be substituted into Eq. (7) and it can be written to acquire the dynamic MOE of the panel (E_d):

$$E_d = \frac{11.88ma^3f_r^2(4\pi^2 + \Delta^2)}{500.6bh^3} \quad (10)$$

When the panel geometry size (a , b , h) is known, and the first frequency (f_r), mass (m) and logarithmic decrement (Δ) are measured, MOE of a full-size WCP under two nodal lines support condition can be obtained by taking these parameters to Eq. (10). This is the theoretical basis for determining MOE of full-size WCPs supported on two nodal lines using the transverse free vibration testing method.

3. Materials and methods

3.1. Materials

Fifty-four MDF panels of three nominal thicknesses (12, 16 and 18 mm) and thirty-six PB panels of two nominal thicknesses (16 and 18 mm) were obtained from two different manufactures and used in this laboratory study. Based on our communication with the manufacturers, both types of panels were manufactured from mixed species (pine, poplar, Locust, and Eucalyptus) using urea-formaldehyde resin. However, layout patterns and detailed manufacturing process were not known. The average moisture content of the MDF and PB panels were about 4% and 5%, respectively. The average internal bond strength of the panels were about 0.9 MPa for MDF and 0.4 MPa for PB. The basic information of the full-size WCPs used in this study is shown in Table 1.

3.2. Transverse free vibration test of full-size panels

The transverse free vibration test was first performed on each full-size panel using a laboratory testing apparatus shown in Fig. 2. This apparatus was consisted of a supporting structure, two load sensors, one laser sensor, and a data acquisition and signal processing system. The supporting structure provided a two-nodal-line support to a test panel through two steel pipes located at the two nodal lines of the first-order flexion mode of the panel (located at 22.4% and 77.6% along the length direction). Two load sensors located under one steel pipe were used to measure the weight of the panel; the laser sensor located in the mid-span was used to obtain vibrational displacement of the panel with a sampling frequency of 1000 Hz. Data acquisition system included a data acquisition card (USB6008, NI) and a LabVIEW-based testing program. Both load and vibrational displacement signals were collected to the testing software through the data acquisition card and then processed to calculate MOE of the panel tested.

Vibration testing of each full size panel was conducted by manually applying a mechanical displacement on the short edge of a panel and then release such that a transverse free vibration in the first-order flexion mode was introduced to the test panel. The vibration displacement signal recorded by the laser sensor was transmitted to the LabVIEW-based testing software in the computer. As an example, Fig. 3 shows the typical time and frequency domain plots of the vibration signal obtained from a full-size MDF panel (MDF12). The first frequency (f_r) and logarithmic decrement (Δ) of the vibration signal were immediately determined through the software and the MOE (E_d) of the panel was subsequently calculated based on Eq. (10).

3.3. Static bending test of small samples

Following the transverse free vibration test of the full-size panels, standard small samples were cut from each panel for static bending test according to Chinese standards GB/T 11718 [29] and GB/T 4897 [30]. For each full-size panel, six small samples were cut along the length direction with a dimension of $(20h + 50) \times 50 \times h$ mm (length $\times$ width $\times$ thickness). The dimensions of the small samples cut from the MDF and PB panels are given in Table 2.

The static bending test was then conducted on all small samples using a mechanical testing machine (IB600, ILME) according to the Chinese GB standards GB/T 17657-2013 [2]. Each small sample was subject to a third-point loading and tested to destruction with a speed of 10 mm/min. The bending MOE (E_s) and MOR of each small sample were calculated the following equations:

$$E_s = \frac{1}{4} \frac{\Delta P}{\Delta f} \frac{l_s^3}{dt^3} \quad (11)$$

$$MOR = \frac{3P_{max}l_s}{2dt^2} \quad (12)$$

where E_s is static modulus of elasticity, MOR is modulus of rupture, l_s is the span, d and t are width and thickness of the sample, $\Delta P/\Delta f$ is the slope of the load-deflection curve from 10% to 40% maximum load P_{max} . Fig. 4 shows the load-deflection curve from the third-point bending test of a MDF or PB sample. The average values of static MOE and MOR of the six small samples from each panel represent the static MOE (E_s) and MOR (σ_s) of the full-size panel tested.

Table 1
Specimen basic information for full-size MDF and PB panels tested.

Panel code	Quantity (panels)	Average panel sizes (mm)			Average density (kg/m ³)
		Thickness	Width	Length	
MDF12	18	12.03	1221.4	2443.8	823
MDF16	18	16.04	1220.6	2443.9	738
MDF18	18	18.01	1221.3	2443.4	758
PB16	18	16.03	1223.1	2441.0	702
PB18	18	18.01	1220.2	2437.1	656

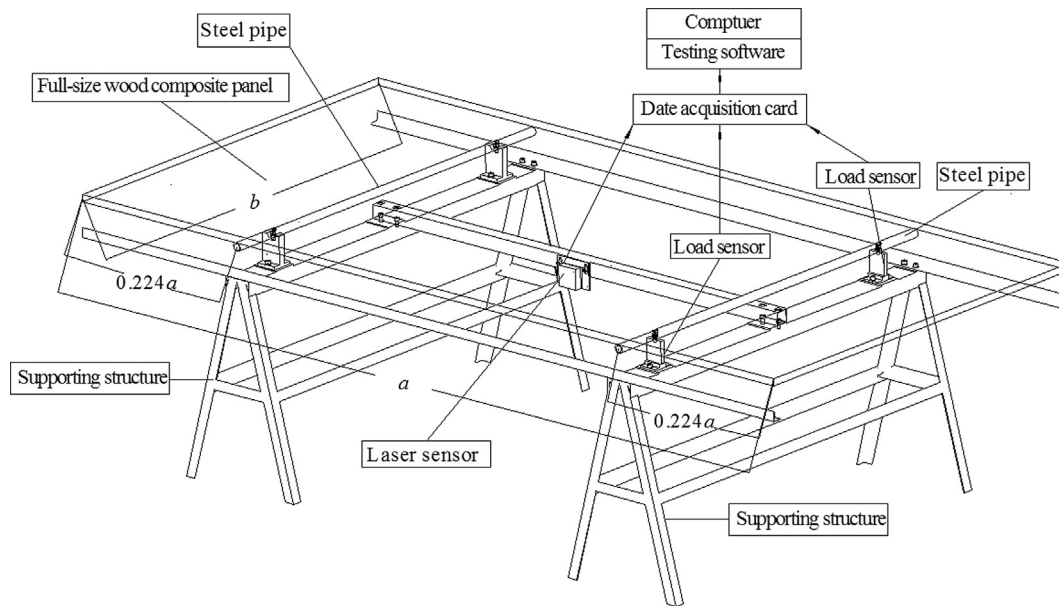


Fig. 2. Laboratory test apparatus for conducting transverse free vibration test of full-size panels.

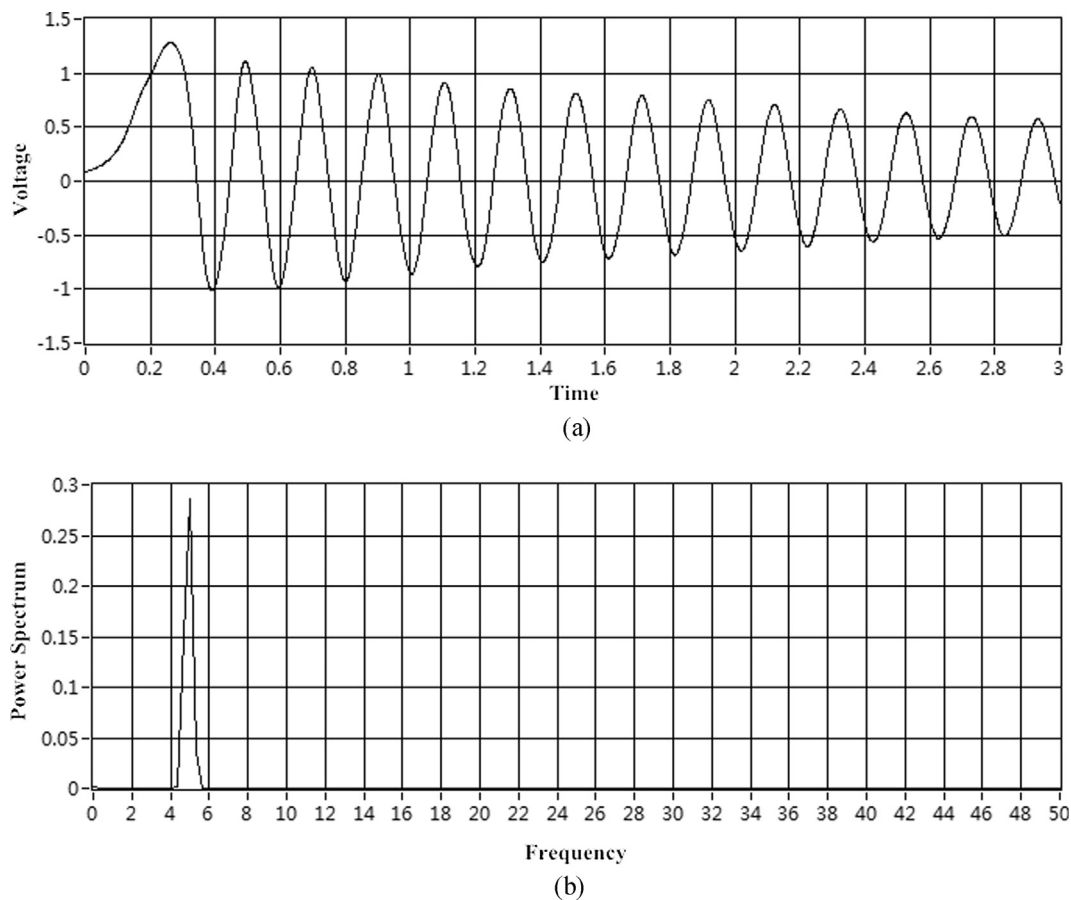


Fig. 3. A typical free vibration signal recorded by the laser sensor. (a) Time domain plot. (b) Frequency domain plot.

4. Results and discussion

4.1. Comparison of dynamic and static testing results

The results from the vibration test of full-size MDF and PB panels and the static bending test of small samples are summarized in

Table 3. The standard deviation (SD), coefficient of variation (COV) and ratio of E_d and E_s for the two types of panels are also listed in Table 3. The average values of dynamic MOE (E_d) determined by the vibration test method are slightly higher than the average values of static MOE (E_s) measured by the static bending test for both MDF and PB. These findings are consistent with the results in previous

Table 2
Sizes of small samples for static test.

Samples series	Length l (mm)	Width d (mm)	Span l_s (mm)
MDF12	290	50	240
MDF16	370	50	320
MDF18	410	50	360
PB16	370	50	320
PB18	410	50	360

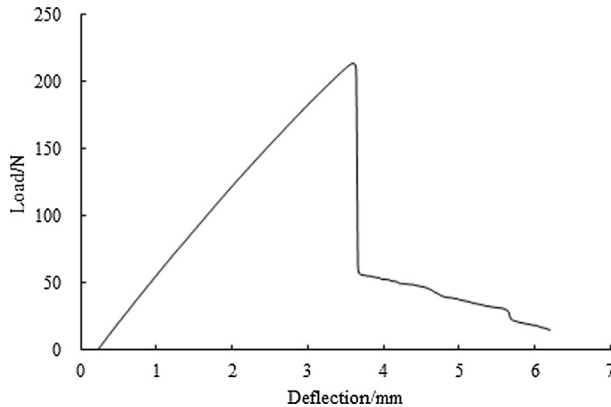


Fig. 4. A load-deflection curve from the third-point bending test on a MDF or PB sample.

studies [15,16,20]. The ratio of dynamic MOE to static MOE (E_d/E_s) was in the range of 1.02–1.15 for MDF and 1.07–1.14 for PB. It appears that the ratio of E_d/E_s increased with the increase of panel thickness. This could be because the increase in panel thickness resulted in an increase in transverse shear deflection such that the effect of transverse shear deflection on natural frequency of mode (2, 0) increased. In addition, for the same thickness of MDF or PB panels, the coefficient of variation (COV) of the modulus of elasticity measured by the two methods were close to each other, with the minimum COV of 2.96% and the maximum COV of 7.16%.

In contrast to the modulus of elasticity, the coefficient of variation (COV) values of the modulus of rupture (MOR) obtained from static bending test were slightly higher, with the minimum of 3.30% and the maximum of 7.59%. The variability found in the static MOE and MOR for the same batch and same thickness types of panels indicates the quality variations existed in the panel manufacturing process and therefore points to the need of online evaluation of full-size panels.

4.2. Relationships between dynamic MOE and static MOE

The experimental data plots in Fig. 5 show the relationships between the dynamic MOE of full-size MDF and PB panels and

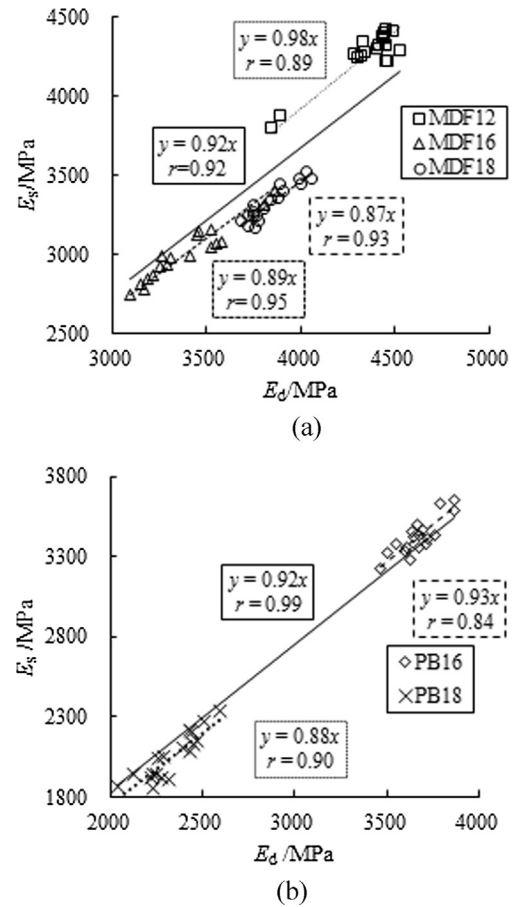


Fig. 5. Relationships between dynamic MOE of full-size panels and static bending MOE of small samples (a) MDF panels. (b) PB panels.

the static bending MOE of small samples cut from the panels. It can be observed from the plots that dynamic MOE obtained from vibration testing had strong linear correlations with the static MOE within each panel type and thickness type and for all panels combined. The coefficients of correlation (r) were in the range of 0.89–0.95 for MDF and 0.84–0.99 for PB. The slopes of the linear correlation models ranged from 0.87 to 0.98, indicating that the dynamic MOE of the panels measured by vibration method was approximately 8% higher than the static MOE. Despite the differences in fiber types, thicknesses and density for the tested panels, the strong linear relationships existed between the dynamic MOE and static MOE proved that the vibration testing based on the theory of transverse free vibration of a thin plate is feasible method to determine the modulus of elasticity of full-size WCPs.

Table 3
Testing results of full-size WCPs tested using vibration and static test methods.^a

Panel	Method	MOE(MPa)			Ratio of MOE(E_d/E_s)	MOR(MPa)		
		Average	SD	COV		Average	SD	COV
MDF12	Vibration	4346.71	187.09	4.30%	1.02	40.85	1.35	3.30%
	Static	4263.04	166.59	3.91%				
MDF16	Vibration	3397.15	218.62	6.44%	1.13	30.16	1.08	3.57%
	Static	3010.06	177.13	5.88%				
MDF18	Vibration	3837.48	117.64	3.07%	1.15	31.54	1.64	5.21%
	Static	3325.70	112.69	3.39%				
PB16	Vibration	3668.50	108.41	2.96%	1.07	13.42	0.90	6.70%
	Static	3424.19	114.28	3.34%				
PB18	Vibration	2333.36	142.85	6.12%	1.14	8.43	0.64	7.59%
	Static	2048.14	146.55	7.16%				

^a SD-standard deviation; COV- coefficient of variation; E_d -dynamic MOE; E_s -static MOE.

4.3. Relationship between dynamic MOE and MOR

The relationship between dynamic MOE of full-size panels and modulus of rupture (MOR) of small samples cut from the panels is shown in Fig. 6. Overall, there was a strong linear relationship between E_d and MOR for both MDF and PB panels when all thickness types were combined, with a coefficient of correlation of 0.91 for MDF and 0.99 for PB. For the test data of each thickness panel series, a simple linear regression analysis and the t -test were performed to establish the linear regression model and determine the regression coefficient using the R language [31]. Table 4 shows the linear regression equations and the related parameters between MOR and dynamic MOE for each thickness series of MDF and PB panels tested. A good linear correlation was found between MOR and E_d within each thickness series for both MDF and PB. The relationships were statistically significant at the 0.001 level with the correlation coefficients (r) being greater than 0.8.

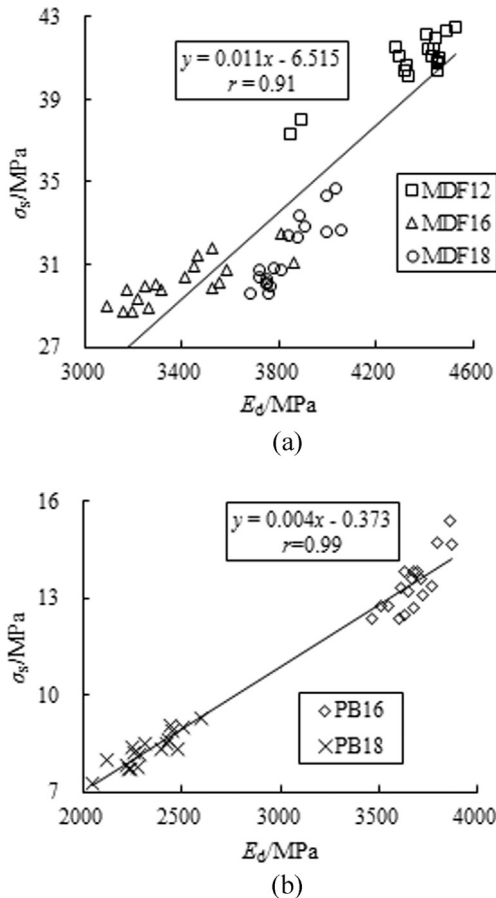


Fig. 6. Relationships between dynamic MOE of full-size panels and MOR of small samples. (a) MDF panels. (b) PB panels.

4.4. Relationship between density and MOR

The general relationships between density and MOR for MDF and PB panels are shown in Fig. 7. Our test results showed a positive linear relationship between MOR and density for all thickness types combined, with a coefficient of correlation of 0.91 for MDF and 0.96 for PB. Table 5 summarizes the statistical analysis results for the relationships between MOR and density of MDF and PB panels within each thickness type. The panel density was found to have a statistically significant linear relationship at 0.001 level with the modulus of rupture for all panels except 16-mm MDF. The correlation coefficients of the linear relationships were in the range of 0.67–0.87 for MDF and 0.78–0.86 for PB. In comparison with the overall data, a small data spread for each thickness series of these panels may lead to the slightly low correlation between

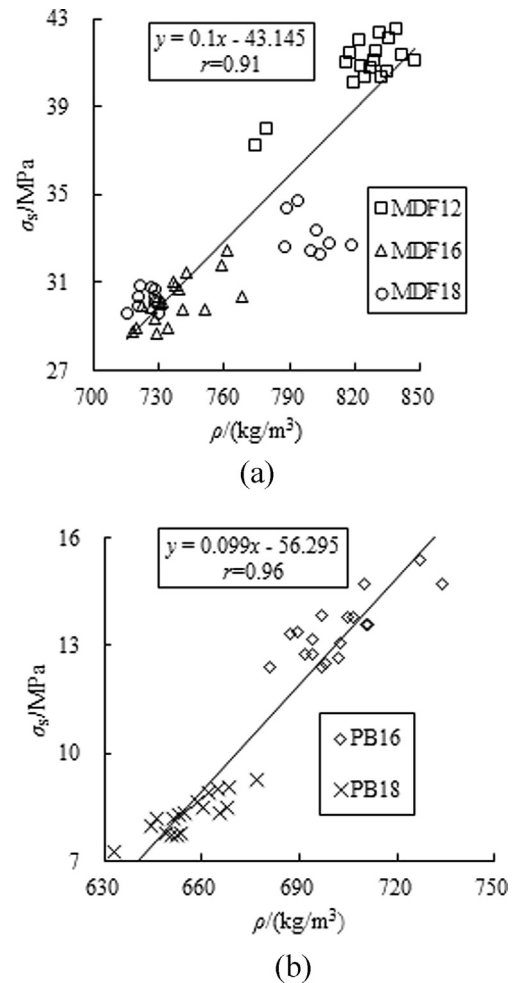


Fig. 7. Relationships between panel density and MOR of small samples. (a) MDF panels. (b) PB panels.

Table 4

Linear regression equations and related parameters between MOR and dynamic MOE for each thickness series of MDF and PB panels.

Panel	No. of panels	$y = cx + e$		Correlation coefficient (r)	F-statistic	Level of significance
		c	e			
MDF12	18	0.006	13.078	0.89	59.04	0.001
MDF16	18	0.004	16.573	0.81	31.01	0.001
MDF18	18	0.013	−16.266	0.89	62.47	0.001
PB16	18	0.007	−10.249	0.82	32.7	0.001
PB18	18	0.003	0.504	0.88	55.82	0.001

Table 5

Linear regression equations and related parameters between MOR and density for each thickness series of MDF and PB panels.

Panel	No. of panels	$y = jx + k$		Correlation coefficient (r)	F-statistic	Level of significance
		j	k			
MDF12	18	0.059	−8.010	0.83	36.35	0.001
MDF16	18	0.051	−7.187	0.67	13.24	0.01
MDF18	18	0.036	4.001	0.87	49.75	0.001
PB16	18	0.050	−21.900	0.78	25.17	0.001
PB18	18	0.045	−21.084	0.86	46.71	0.001

Table 6Binary linear regression equations and related parameters between MOR with dynamic MOE (x_1) and thickness (x_2) for both MDF and PB panels.

Panel	No. of panels	$y = c_1x_1 + e_1x_2 + k_1$			Correlation coefficient (r)	F-statistic	variable	Standard regression coefficient	Level of significance
		c_1	e_1	k_1					
MDF	27	0.007	−874.2	19.98	0.983	340.5	x_1	0.681	0.001
							x_2	−0.459	0.001
PB	18	0.005	624.6	−14.06	0.998	1550	x_1	1.249	0.001
							x_2	0.257	0.05

panel density and MOR. The cause for the relatively weak correlation between panel density and MOR for MDF16 and PB16 was unknown.

4.5. Multivariate regression models for MOR prediction

4.5.1. MOR versus dynamic MOE and panel thickness

Half of the test data for all the thicknesses of MDF and PB panels was chosen to develop multivariate regression models using both dynamic MOE (E_d) and panel thickness (h) as independent variables (x_1 and x_2) and MOR as dependent variable (y). The following linear regression model was chosen for the two-variate regression analysis:

$$y = c_1x_1 + e_1x_2 + k_1 \quad (13)$$

where c_1 , e_1 , and k_1 are regression coefficients.

The results of the two-variate regression analysis are summarized in Table 6. The two-variate regression analysis resulted in a significant improvement in MOR prediction for MDF panels. The correlation coefficient was increased from 0.91 for the model with E_d as a single variable to 0.983 for the model with both E_d and h as variables. For PB panels, the two-variate regression model resulted in a slight increase in MOR prediction, with the correlation coefficient increased from 0.96 with E_d as a single variable to 0.998 with both E_d and h as variables. The full-size PB panels used in this study only had two thickness groups, which certainly had a bias effect on the regression analysis.

The other half of the test data of all thicknesses groups was used for validation test of the two-variate regression models with the plots in Fig. 8. The predicted MOR showed a strong linear relationship with the MOR obtained from static bending test with determination coefficients of 0.980 for MDF and 0.961 for PB. The average difference found between predicted MOR and measured MOR was 2.37% for MDF and 4.61% for PB.

4.5.2. MOR versus natural frequency, density, and thickness

For the transverse free vibration of a full-size panel supported on two nodal-lines, the dynamic MOE of the panel is calculated based on the theoretical Eq. (10), which is a simplified formula with some assumptions and approximations as discussed early. From a practical stand point, empirical models can also be developed to predict MOR by using the natural frequency directly measured through vibration testing as a predictor, coupled with panel density and thickness. Therefore, we developed the following

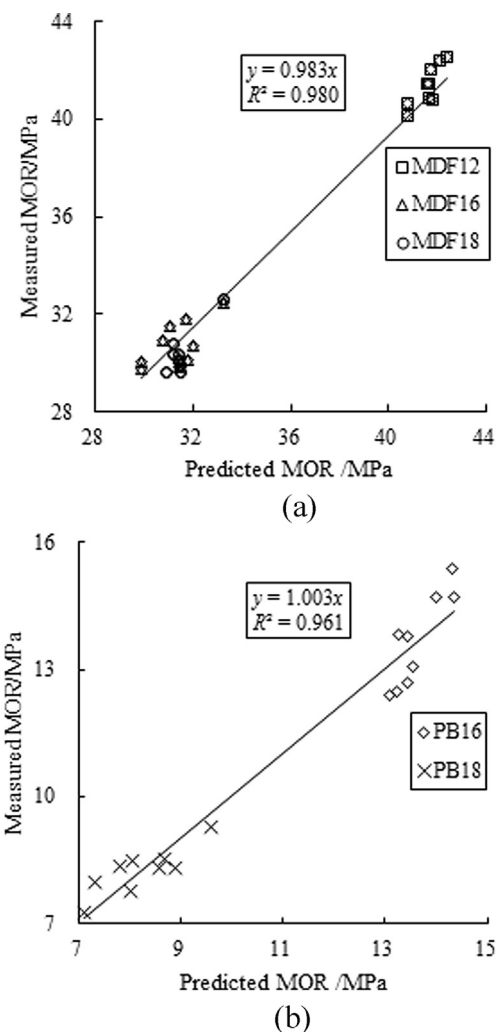


Fig. 8. Relationships between MOR obtained from static bending test and predicted MOR based on the two-variate regression models. (a) MDF panels. (b) PB panels.

three-variate regression models for predicting MOR of MDF and PB panels:

$$y = c_2x_1 + e_2x_2 + j_2x_3 + k_2 \quad (14)$$

Table 7Ternary linear regression equations and related parameters between MOR with natural frequency (x_1), density (x_2) and thickness (x_3) for both MDF and PB panels.

Panel	No. of panels	$y = c_2x_1 + e_2x_2 + j_2x_3 + k_2$				Correlation coefficient (r)	F-statistic	variable	Standard regression coefficient	Level of significance
		c_2	e_2	j_2	k_2					
MDF	27	6.034	0.053	−3196	6.309	0.988	319.4	x_1	1.15	0.001
								x_2	0.486	0.001
								x_3	−1.677	0.001
PB	18	3.628	0.031	−1050	−14.3	0.998	1043	x_1	0.333	0.001
								x_2	0.268	0.001
								x_3	−0.431	0.001

where c_2 , e_2 , j_2 and k_2 are regression coefficients, x_1 is natural frequency of the panel, x_2 is panel density, and x_3 is panel thickness.

The three-variate regression analysis resulted in strong linear relationships between three predicting variables and MOR of the panels as shown in Table 7. The three-variate regression models were found statistically significant at 0.001 level, with the correlation coefficient of 0.988 for MDF panels and 0.998 for PB panels. These empirical models were validated by the 2nd half of the test data using the plots in Fig. 9. The predicted MOR showed a strong linear relationship with the MOR obtained from static bending test with determination coefficients of 0.989 for MDF and 0.956 for PB.

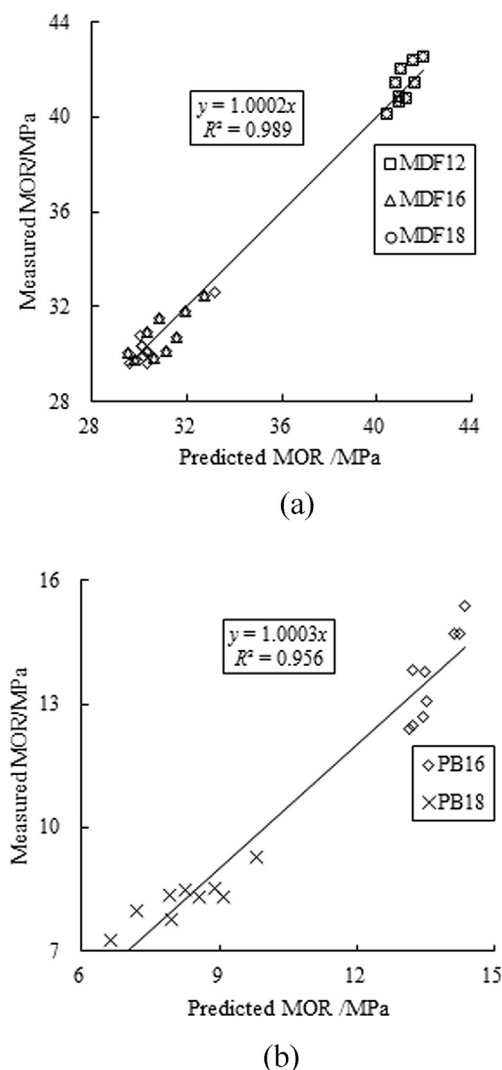


Fig. 9. Relationships between MOR obtained from static bending test and predicted MOR based on the three-variate regression models. (a) MDF panels. (b) PB panels.

The average difference between predicted MOR and measured MOR was 1.37% for MDF and 5.17% for PB.

It appears that the empirical models with natural frequency, density, and thickness as predicting variables are as effective as the two-variate regression models with dynamic MOE and thickness as predicting variables.

5. Conclusions

In this study, we introduced a transverse free vibration testing method for evaluating the stiffness and strength of full-size MDF and PB panels. Static bending tests were conducted on the small samples cut from the full-size panels to validate the results obtained from vibration testing. The relationships between dynamic MOE, natural frequency, density and thickness of the panels and the static bending properties were examined through statistical regression analysis. Based on the results and analysis, we conclude the following:

- (1) The transverse free vibration of a thin plate supported on two nodal-lines proved to be a valid testing approach for rapid determination of dynamic modulus of elasticity of full-size MDF and PB panels. Strong linear relationships were found between the dynamic MOE of full-size panels and static MOE of small samples, with the dynamic MOE approximately 8% higher than the static MOE.
- (2) Both dynamic MOE and panel density were found to have a positive linear relationship with the modulus of rupture (MOR). When the test data of all thickness groups was combined, the coefficient of correlation between dynamic MOE and MOR was 0.91 for MDF and 0.99 for PB.
- (3) Multivariate regression models were developed to predict the bending strength of full-size MDF and PB panels. The two-variate models with dynamic MOE and thickness as predicting variables and the three-variate models with natural frequency, density and thickness as variables were found equally effective in predicting the modulus of rupture of full-size MDF and PB panels.

The wood composite panels used in this study were limited to MDF and PB only with a relatively small sample size. Large scale test should be carried out in future to test the applicability of the transverse free vibration testing as a quality control tool to evaluate the bending stiffness and strength of full-size panels. Further research should expand the panel type to include oriented strand board and plywood and focus on assessing the panel stiffness in both major and minor axis and in-plane shear modulus of full-size WCPs.

Declaration of Competing Interest

None.

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