



Analysis of Continuous Beams with Joint Slip

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Abstract

A computer analysis with user guidelines to analyze partially continuous multi-span beams is presented. Partial continuity is due to rotational slip which occurs at spliced joints at the supports of continuous beams such as floor joists. Beam properties, loads, and joint slip are input; internal forces, reactions, and deflections are output.

Introduction

Floor joist deflection and maximum design moment are decreased by utilizing continuous joists over two or more spans compared to simply supported joists. Splices are used to attain the lengths required for continuous members. The splices, whether nailed, bolted, glued, or truss plated, if located at an interior support, will have rotational slip occur when subjected to bending moment. Thus the beam continuity is disrupted with the joist acting somewhere between a simply supported and a fully continuous beam.

The finite element method of analysis, using discrete elements referred to as matrix structural analysis, is the state-of-the-art method used to analyze continuous structures. The usual procedure in matrix structural analysis to account for partial fixity is to model a short or fictitious member with a low stiffness value. This

allows the input of some percentage of complete fixity but has the disadvantage of not being able to input the specific amount of joint slip since its relationship to the fictitious member stiffness is not known.

This study presents a computer analysis method and input user guidelines to determine internal forces, reactions and deflections of continuous beams with rotational slip at supports. Although developed specifically for two-span floor joist analysis and design, the method and computer program are applicable to any continuous beam structure.

Joist design is based on satisfying both stiffness and strength requirements. The stiffness criterion most commonly used is to limit maximum joist deflection in a floor system to span/360 when subjected to static live load of 40 pounds per square foot. The strength criterion limits bending stress to an allowable value based on species and grade of lumber. Design tables (3)³ are available for floor joist design for either simple or two-span continuous beams. The continuous beam tables assume full moment capacity over the entire span (i.e., no splices or other loss of continuity).

The National Association of Homebuilders (NAHB) (2) and American Plywood Association (APA) (1) investigated two-span continuous beams with splices near the inflection points. The splices were designed to transmit shear force since moments near the inflection points are assumed small. Splice slip was not considered.

¹ In cooperation with U.S. Dep. of Housing and Urban Development.

² Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

³ Italicized numbers in parenthesis refer to literature cited at the end of this report.

Splice slip is due to the behavior of the mechanical fastening system. Nails (and truss plates) slip due to bending of the nail (tooth) and crushing of the wood. Many studies have established nonlinear load-slip behavior for nailed joints in single shear. When bolts are used they are installed in oversized holes; thus a certain amount of movement occurs before loads are transmitted. Slip measurements related to design load levels are required for various splice configurations.

Splice slip affects both joist stiffness and strength. Slippage of a splice at an interior support will result in increased deflection throughout the span and in a decreased moment capacity at the support.

Matrix analysis (4) of continuous beams assumes supports either fixed or pinned. Intermediate end conditions are generally modeled by addition of a fictitious member with low stiffness or a rotational spring. Matrix analysis assumes each joint having discrete displacements corresponding to degrees of freedom; thus no discontinuity in displacement can occur. Loads at or between joints and support settlements can be included in the analysis.

Theory

The philosophy of the analysis is illustrated in figure 1. Deflections due to loads are found by matrix method assuming no joint slip. An experimentally determined slip, θ_s , is input and allocated, θ_r , θ_l , to the adjacent right and left spans. Deflections caused by adjacent slips are calculated and superimposed on the deflections due to loads. Member end actions and support reactions are calculated from the member deflections.

The member stiffness matrix assumes the member being subjected to lateral loads which induce bending moments and shear forces. The effects of shear are neglected; thus two degrees of freedom at each node,

vertical and rotational, are necessary. The degrees of freedom are numbered sequentially from left to right; thus the vertical translation of the left end is 1, the rotation 2, and those of the right end are 3 and 4, respectively, as shown in figure 2. Also shown is the member stiffness matrix with forces corresponding to unit displacements for each degree of freedom. The positive sign convention for the member forces and displacements are as shown, with vertical translation upward and rotation counterclockwise being positive.

The beam is modeled with nodes at points of support, changes in cross-section properties, and at any other point(s) where shear forces and bending moments, and/or vertical and rotational displacements are to be computed. Figure 3 shows node locations for an example two-span beam. It includes the support locations plus an arbitrary interior location, a distance x from the left support, where forces and displacements are to be calculated. Again, there are two degrees of freedom at each node; they are numbered sequentially starting at the left end of the beam. Member numbers (circled) are also sequential from the left end. Positive node forces and displacements are, as shown, upward and counterclockwise. The member stiffness matrices are superimposed, as shown symbolically, to form the structure stiffness matrix, $[S]$.

The structure stiffness matrix is rearranged and partitioned related to the unknown displacements, D_u , and the known boundary displacements, D_s , representing support conditions:

$$[S] = \begin{bmatrix} S_{dd} & S_{ds} \\ S_{sd} & S_{ss} \end{bmatrix}$$

where the subscript s refers to support degrees of freedom (with known displacements of zero or support settlement value) and d refers to degrees of freedom with unknown displacements.

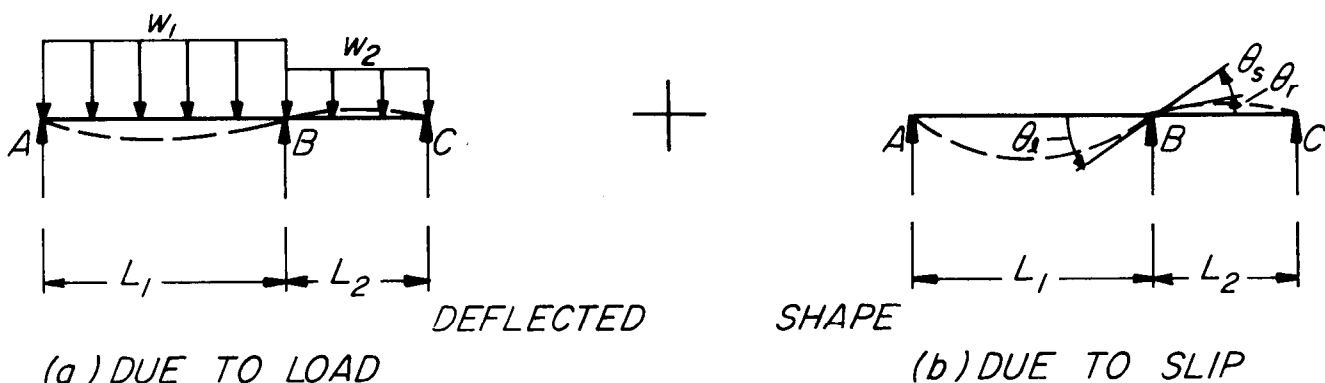
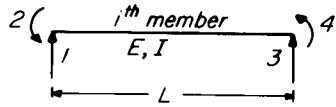


Figure 1.—Superposition of load and slip deflected shapes.

(M 149 237)



(a) Member

$$[S_m]_i = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

(b) Member Stiffness Matrix

Figure 2.—Member degrees of freedom and stiffness matrix.

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Node forces and/or support settlements (if any) are input with sign convention in figure 3b. Fixed end shears and moments corresponding to member loads are input with sign convention in figure 2a.

The unknown joint displacements, $\{D_d\}$, due to load, $\{A\}$, are found by:

$$\begin{aligned} \{D_d\} &= [S_{dd}]^{-1} \{A\} \\ \{A\} &= \{A_j\} - \{A_{fem}\} - [S_{ds}] \{D_s\} \end{aligned}$$

where $\{A\}$ is the general load matrix consisting of the specified joint loads, $\{A_j\}$, fixed end reactions due to between-the-joint loads, $\{A_{fem}\}$, and the forces, $[S_{ds}] \{D_s\}$, due to support settlement, $\{D_s\}$.

The known joint slip is allocated to adjacent spans based on compatibility and equilibrium as indicated in figure 4 in which moments and rotations are shown in the positive direction. The effects of spans other than the adjacent spans are neglected. The sign convention for the slip rotation is that the slip angle is measured from the tangent to the elastic curve in the right span to the tangent of the elastic curve in the left span with counterclockwise rotation being positive.

From figure 4a, the compatibility equation is:

$$\theta_s = \theta_l - \theta_r \quad (1)$$

where θ_s is the slip rotation, and θ_l, θ_r are the rotational allocation of the slip to the left and right adjacent spans, respectively.

From figure 4b, the moment equilibrium equation at the interior support is:

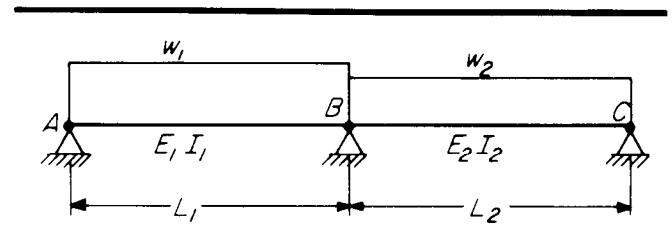
$$M_l = -M_r \quad (2)$$

where M_l, M_r are the moments corresponding to θ_l and θ_r . From standard beam theory, the relationship between moment and rotation is:

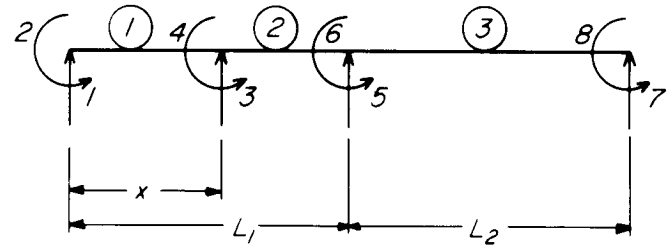
$$M_l = \frac{3E_l I_l \theta_l}{L_l} \quad (3a)$$

$$M_r = \frac{3E_r I_r \theta_r}{L_r} \quad (3b)$$

where E is the modulus of elasticity, I the moment of inertia, and L the length of the left and right spans (subscripts l, r), respectively. The relationship is applicable to members having a rotation at one end of the member.



(a) REAL BEAM



(b) MODELED BEAM

$$[S] = \begin{bmatrix} S_{m1} & & \\ & S_{m2} & \\ & & S_{m3} \end{bmatrix}$$

(c) STRUCTURE STIFFNESS MATRIX

Figure 3.—Structure degrees of freedom and stiffness matrix.

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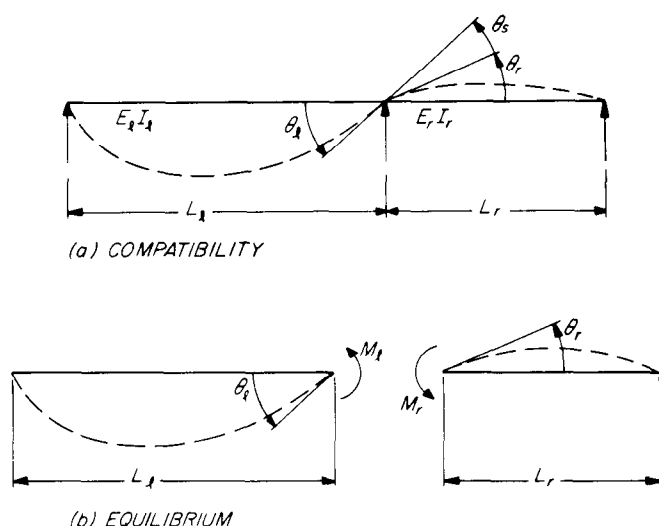


Figure 4.—Joint slip compatibility-equilibrium relations.

(M 149 240)

Substituting equations (3) into (2) yields a relationship between θ_l and θ_r , and substituting this into equation (1) results in the following allocation of slip to adjacent spans (assuming $E_r = E_l$):

$$\theta_l = \theta_s \left[\frac{L_l I_r}{L_l I_r + L_r I_l} \right] \quad (4a)$$

$$\theta_r = -\theta_s \left[\frac{L_r I_l}{L_l I_r + L_r I_l} \right] \quad (4b)$$

To determine displacements due to a known slip, the continuous beam is separated into two structures at the support where the slip occurs. The slip allocations, θ_l and θ_r , are applied as support displacements to each side of the separated structure. Deformations are then determined by matrix analysis.

The member end displacements due to slip are superimposed on the displacements due to loads. Member end displacements multiplied by the member stiffness matrix summed with the fixed end moments due to load result in member end forces. Adjacent member end forces combined with applied joint forces determine support reactions.

Procedure

The Fortran program using this theory is given in appendix A. The program is intended to be as complete as possible so that it can be easily modified for future research; it is not a production tool and no effort has been made to make it as efficient as possible. The units used must be compatible; the example in this paper uses input lengths in inches, forces in pounds, moments in inch-pounds, slip in radians, modulus of elasticity in pounds per square inch, and moment of inertia in inches⁴. Output is in the same units.

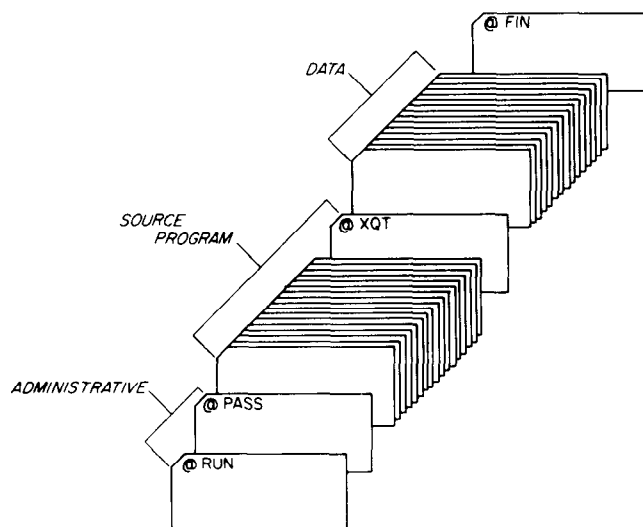


Figure 5.—Schematic representation of the makeup of computer deck.

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The program is limited to one slip per member, and no slip at either the first or last support. Slips at adjacent joints are permissible provided each is indexed to a separate adjacent member. The program is arbitrarily limited to a maximum of 10 elements which is considered adequate for most 2- or 3-span floor joist systems (extra nodes may be included between supports). For larger problems, this limitation can be removed by modification of the dimension statement.

The degrees of freedom are numbered sequentially from the leftmost support with the vertical translation first and the rotation second. Sign conventions have been previously defined and are shown in the positive direction in figures 2, 3, and 4.

The Fortran program is stored on both tape and punched cards on the FPL/MACC system. Input data required are described in table 1. The command sequence for the FPL/MACC system to access the tape is given in table 2; that for the punched deck is shown in figure 5.

The following example illustrates both input data required and output generated.

Example

The two-span beam of figure 3a is assumed to have a splice at B which slips + 0.00432 radian when loaded. Other values for this example are:

$$\begin{aligned} W_1 &= 50 \text{ lb/ft} = 4.16 \text{ lb/in.} & I_1 &= I_2 = 20.8 \text{ in.}^4 \\ L_1 &= 12 \text{ ft} - 0 \text{ in.} = 144 \text{ in.} & W_2 &= 10 \text{ lb/ft} = 0.833 \text{ lb/in.} \\ E_1 &= E_2 = 1,700,000 \text{ lb/in.}^2 & L_2 &= 9 \text{ ft} - 0 \text{ in.} = 108 \text{ in.} \end{aligned}$$

Shear and moment diagrams, and deformed shape for this partially continuous beam example are required.

Table 1.-Data cards

Number of cards required ¹	Information required ²	Input data in columns numbered	Fortran format
1	a. NM = Total number of members b. NS = Total number of supports corresponding to degree of freedom numbers (i.e., NS = 2 for shear and moment at fixed support) c. NF = Total number of joint slips d. NA = Total number of degrees of freedom corresponding to joint loads (i.e., NA = 2 for joint with applied vertical load and moment)	1 through 3* 4 through 6* 7 through 9* 10 through 12*	I3 I3 I3 I3
NS	For each card (support): a. Structure degree of freedom number b. Support settlement (in. or radians)	1 through 3* 4 through 13	I3 F10.6
NM	For each card (member): a. Member number b. Modulus of elasticity (lb/in. ²) c. Moment of inertia (in. ⁴) d. Length (in.) e. Left end fixed end shear due to member loads (lb) f. Left end fixed end moment (in.-lb) g. Right end fixed end shear (lb) h. Right end fixed end moment (in.-lb)	1 through 3* 4 through 13 14 through 21 22 through 28 29 through 37 38 through 46 47 through 55 56 through 64	I3 F10.0 F8.2 F7.2 F9.2 F9.2 F9.2 F9.2
NA	For each card (joint load): a. Structure degree of freedom number corresponding to joint load b. Joint load (lb or in.-lb)	1 through 3* 4 through 13	I3 F10.2
NF	a. Member number (may be either left or right span adjacent to slip) b. Member degree of freedom number (either 2 or 4 corresponding to member selected in "a") c. Structure degree of freedom number d. Slip (radians)	1 through 3* 4 through 6* 7 through 9* 10 through 19	I3 I3 I3 F10.6

¹ Cards must be sequenced in this order.² Zero values may be entered by blanks.

* Values must be right-adjusted.

Table 2.-Tape command sequence

```

@RUN...
@PASS...
@CAT HUD*CONTINUEBEAM.
@ASG, AX HUD*CONTINUEBEAM.
@ASG, TH DIMEN*LUMBER.,U9H, 7639
@TGET DIMEN*LUMBER., HUD*CONTINUEBEAM.
@XQT HUD*CONTINUEBEAM.SLIP
Data as per table 1
@FIN

```

The beam is modeled with joints at points of support and at other arbitrary locations deemed necessary to define the shear and moment diagrams and the deformed shape. For illustrative purposes, only one arbitrary location 5 feet (60 in.) from the left support ($x = 60$ in.) is selected. Thus the beam is modeled with structure degrees of freedom and members numbered sequentially from the left as in figure 3b.

The input data corresponding to table 1 are:

Card 1

NM = 3
 NS = 3 (corresponds to structure degrees of freedom 1, 5, and 7)
 NF = 1 (corresponds to structure degree of freedom 6)
 NA = 0 (no joint loads, only member loads)

Cards 2, 3, and 4, respectively

Structure degree of freedom = 1, 5, and 7, respectively
 Support settlement = 0 (all cards)

Cards 5, 6, and 7, respectively

Member number = 1, 2, and 3, respectively
 Modulus of elasticity = $1,700,000 \text{ lb/in.}^2$ (all cards)
 Moment of inertia = 20.8 in.^4 (all cards)
 Length = 60, 84, and 108 in., respectively
 Left end fixed end shear for member 1 =

$$+ \frac{W_1 X}{2} = \frac{50 \times 5}{2} = + 125.0 \text{ lb}$$

Left end fixed end moment for member 1 =

$$+ \frac{W_1 X^2}{12} = \frac{50 \times 5^2 \times 12}{12} = + 1250.0 \text{ in.-lb}$$

Fixed end reactions for right end and for other members similarly found.

(Note, sign convention as per figure 2; thus right end fixed end moment is negative.)

No NA cards (NA = 0)

Card 8 (NF = 1)

Member number of left adjacent span = 2 (alternately, member 3, the right adjacent span could be selected)
 Member degree of freedom (figure 2a) = 4 (alternately, member degree of freedom = 2 could be selected to correspond to member 3)
 Structure degree of freedom (figure 3b) = 6 Slip = + 0.00432 radian

The output forces and displacements are given in appendix B to illustrate the output format (which is referenced to the structure degree of freedom numbering). Joint displacements are not given directly since the slip creates a discontinuity; member end

displacements to the left and right of the joint are given.

The results are plotted as case II in figure 6. Results for simply supported and fully continuous beams, found by standard structural analysis, are given as cases I and III for comparison. As expected, the behavior of the partially continuous beam is bounded by the simple and fully continuous cases. The displacement at $x = 5$ feet is 0.6318 and 0.4126 inch for the simply supported, and fully continuous beams, respectively. The partially continuous beam results in a deflection of 0.4738 inch. Use of a joint at B reduces the simply supported deflection by about 25 percent. The negative moment over the center support is reduced from the fully continuous value of 558 to 406 foot-pounds for the partially continuous beam; however, the corresponding positive moment at $x = 5$ feet is increased from 643 to 706 foot-pounds. This is still about 20 percent less than the simply supported positive moment. Figure 7 details the discontinuity of the deformed shape at support B.

Program Alteration

The program is arbitrarily limited to 10 elements (NM = 10); the corresponding number of joint degrees of freedom is 22 ($2NM + 2$). Dimensioned arrays have values of 10, 22, or 4 corresponding to number of elements, number of joint degrees of freedom or number of degrees of freedom per element. To increase the number of elements, change the dimension statements in the first six cards of the source program as follows:

- Dimensions of value 10 are increased to the new number of elements, NM.
- Dimensions of value 22 are increased to a value = $2NM + 2$.
- Dimensions of value 4 are unchanged.
- Dimensions of the array called SCRACH are increased to $3 \times (2NM + 2)$.

——— CASE I = SIMPLY SUPPORTED BEAMS
 - - - - - CASE II = PARTIALLY CONTINUOUS BEAM
 ——— CASE III = CONTINUOUS BEAM

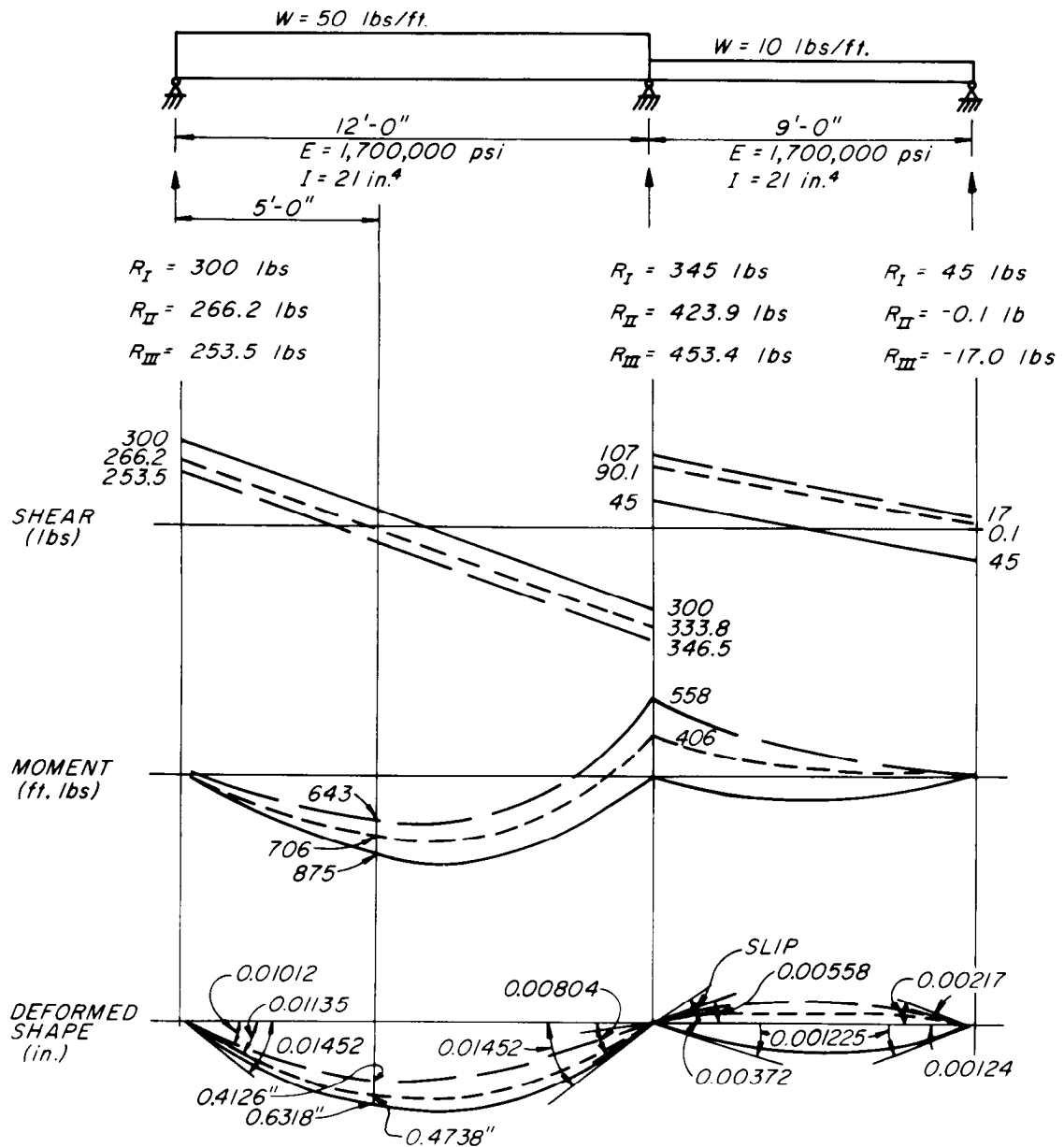


Figure 6.—Example-continuous beam with splice slip at support.

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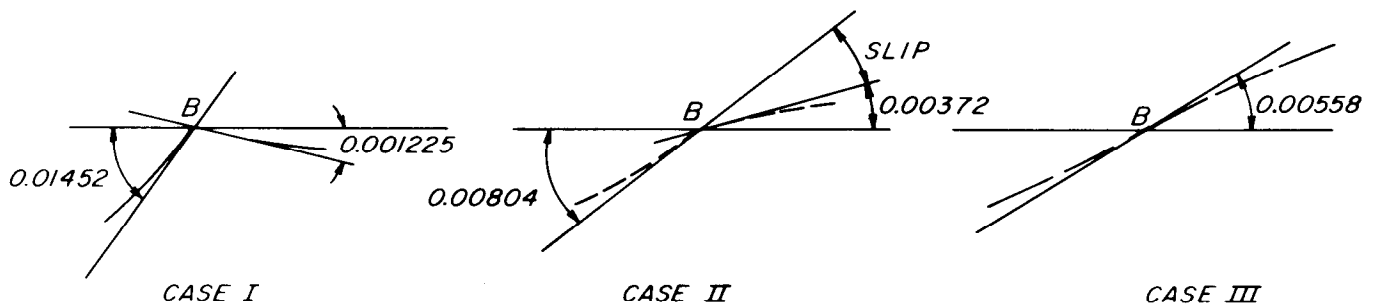


Figure 7.—Example-splice slip detail at support.

(M 149 316)

Appendix A

Computer Source Program

```
1 C      MATRIX ANALYSIS OF CONTINUOUS BEAMS
2 C      MAXIMUM NUMBER OF SPANS = 10
3      DIMENSION JNS(22),DS(22),JNSS(22),INS(22),AJ(22),SR(22),DDD(22), T
4      1(22),B(10),V(10),W(10),X(10),SM(10,4,4),E(10),XI(10),XL(10),AM(10,
5      24),AMF(10,4),COORD(22),XLS(10),S(22,22),A(22),JND(22),RR(22,22),R(2
6      32,22),SDD(20,20),SSD(22,22),SDS(22,22),SSS(22,22),AS(22),AD(22),C(
7      422),Y(22),SDDI(20,20),SCRACH(60,60),DD(22),D(22),SLIP(22),SLIP1(22
8      5),SLIP2(22),DL(10,4),DR(10,4),AMM(10,4),AJJ(22)
9 C
10 C      INPUT DATA
11      READ(5,1)NM,NS,NF,NA
12      DO 5 I=1,NS
13      READ(5,2)JNS(I),DS(I)
14      5 JNSS(I)=JNS(I)
15      1 FORMAT(4I3)
16      2 FORMAT(I3,F10.6)
17 C
18 C      INITIALIZE CONDITIONS
19      NIF=0.
20      NSSJJ=0.
21      NMM=NM
22      NSS=NS
23      DO 10 I=1,22
24      INS(I)=0.
25      AJJ(I)=0.
26      AJ(I)=0.
27      SR(I)=0.
28      10 DDD(I)=0.
29 C
30 C      MEMBER STIFFNESS MATRIX
31      DO 15 J=1,NM
32      READ(5,3)M,E(M),XI(M),XL(M), (AM(M,I),I=1,4)
33      3 FORMAT(I3,F10.0,F8.2,F7.2,4F9.2)
34      B(M)=6.*E(M)*XI(M)/XL(M)**2.
35      V(M)=2.*B(M)/XL(M)
36      W(M)=4.*E(M)*XI(M)/XL(M)
37      X(M)=W(M)/2.
38      SM(M,1,1)=V(M)
39      SM(M,2,2)=W(M)
40      SM(M,3,3)=V(M)
41      SM(M,4,4)=W(M)
42      SM(M,1,3)=-V(M)
43      SM(M,3,1)=-V(M)
44      SM(M,2,4)=X(M)
45      SM(M,4,2)=X(M)
46      SM(M,1,2)=B(M)
47      SM(M,1,4)=B(M)
48      SM(M,2,1)=B(M)
49      SM(M,4,1)=B(M)
50      SM(M,3,2)=-B(M)
51      SM(M,3,4)=-B(M)
52      SM(M,2,3)=-B(M)
53      15 SM(M,4,3)=-B(M)
54      DO 16 J=1,NMM
55      DO 16 L=1,4
56      16 AMF(J,L)=AM(J,L)
57 C
58 C      STRUCTURE SPAN LENGTHS
59      NJ=2*NM+2
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60      NOJ=NJ
61      COOR(1)=0.
62      DO 20 I=3,NJ,2
63      K=(I-1)/2
64      20 COOR(I)=COOR(I-2)+XL(K)
65      NSL=NS-1
66      I=1
67      DO 35 J=3,NJ,2
68      DO 30 K=2,NS
69      IF(J.EQ.JNS(K))GO TO 25
70      GO TO 30
71      25 JJ=JNS(K-1)
72      XLS(I)=COOR(J)-COOR(JJ)
73      I=I+1
74      GO TO 35
75      30 CONTINUE
76      35 CONTINUE
77 C
78 C      STRUCTURE JOINT LOADS
79      IF(NA)50,50,40
80      40 DO 45 I=1,NA
81      READ(5,4)NAM,AJ(NAM)
82      4 FORMAT(I3,F10.2)
83      AJJ(NAM)=AJ(NAM)
84      45 CONTINUE
85      50 CONTINUE
86 C
87      999 CONTINUE
88 C
89 C      STRUCTURE STIFFNESS AND LOAD MATRIX
90      DO 55 I=1,22
91      A(I)=0.
92      DO 55 J=1,22
93      55 S(I,J)=0.
94      DO 75 I=1,NM
95      M=I
96      M1=2*M-1
97      M2=2*M
98      M3=2*M+1
99      M4=2*M+2
100     M=M+NSSJJ/2
101     S(M1,M3)=SM(M,1,3)
102     S(M1,M4)=SM(M,1,4)
103     S(M2,M3)=SM(M,2,3)
104     S(M2,M4)=SM(M,2,4)
105     S(M3,M1)=SM(M,3,1)
106     S(M3,M2)=SM(M,3,2)
107     S(M4,M1)=SM(M,4,1)
108     S(M4,M2)=SM(M,4,2)
109     S(M1,M1)=S(M1,M1)+SM(M,1,1)
110     S(M1,M2)=S(M1,M2)+SM(M,1,2)
111     S(M2,M1)=S(M2,M1)+SM(M,2,1)
112     S(M2,M2)=S(M2,M2)+SM(M,2,2)
113     IF(I-NM)60,65,70
114     60 S(M3,M3)=S(M3,M3)+SM(M,3,3)
115     S(M3,M4)=S(M3,M4)+SM(M,3,4)
116     S(M4,M3)=S(M4,M3)+SM(M,4,3)
117     S(M4,M4)=S(M4,M4)+SM(M,4,4)
118     GO TO 70
119     65 S(M3,M3)=SM(M,3,3)
120     S(M3,M4)=SM(M,3,4)
121     S(M4,M3)=SM(M,4,3)

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122      S(M4,M4)=SM(M,4,4)
123      70 CONTINUE
124      A(M1)=A(M1)-AM(M,1)+AJ(M1)
125      A(M2)=A(M2)-AM(M,2)+AJ(M2)
126      A(M3)=A(M3)-AM(M,3)+AJ(M3)
127      A(M4)=A(M4)-AM(M,4)+AJ(M4)
128      75 CONTINUE
129 C
130 C      REARRANGE STIFFNESS MATRIX FOR JOINT RESTRAINT
131 C      DETERMINE JND
132      L=1
133      DO 95 I=1,NJ
134      DO 90 K=1,NS
135      IF (1-JNS(K))80,95,80
136      80 CONTINUE
137      IF (K-NS)90,85,90
138      85 JND(L)=I
139      L=L+1
140      90 CONTINUE
141      95 CONTINUE
142 C      MOVE VERTICAL COLUMNS
143      ND=NJ-NS
144      DO 100 I=1,NJ
145      DO 100 J=1,ND
146      KK=JND(J)
147      100 RR(I,J)=S(I,KK)
148      DO 105 I=1,NJ
149      DO 105 J=1,NS
150      KK=JNS(J)
151      105 RR(I,J+ND)=S(I,KK)
152 C      MOVE HORIZONTAL ROWS
153      DO 110 J=1,NJ
154      DO 110 I=1,ND
155      KK=JND(I)
156      110 R(I,J)=RR(KK,J)
157      DO 115 J=1,NJ
158      DO 115 I=1,NS
159      KK=JNS(I)
160      115 R(I+ND,J)=RR(KK,J)
161 C      TO SUBDIVIDE MATRIX
162      DO 120 I=1,ND
163      DO 120 J=1,ND
164      120 SDD(I,J)=R(I,J)
165      DO 125 I=1,NS
166      DO 125 J=1,ND
167      125 SSD(I,J)=R(I+ND,J)
168      DO 130 I=1,ND
169      DO 130 J=1,NS
170      130 SDS(I,J)=R(I,J+ND)
171      DO 135 I=1,NS
172      DO 135 J=1,NS
173      135 SSS(I,J)=R(I+ND,J+ND)
174 C      REARRANGE LOAD MATRIX
175      DO 140 I=1,ND
176      K=JND(I)
177      140 AD(I)=A(K)
178      DO 145 I=1,NS
179      K=JNS(I)
180      145 AS(I)=A(K)
181 C
182 C      TO FIND JOINT DISPLACEMENTS
183      DO 150 I=1,ND

```

```

184      C(I)=0.
185      DO 150 K=1,NS
186 150  C(I)=C(I)+SDS(I,K)*DS(K)
187      DO 155 I=1,ND
188 155  Y(I)=AD(I)-C(I)
189      CALL MTINV2(SDD,SDDI,ND,20,20,'GEN',0,$610,SCRACH)
190      DO 160 I=1,ND
191      DD(I)=0.
192      DO 160 K=1,ND
193 160  DD(I)=DD(I)+SDDI(I,K)*Y(K)
194      DO 162 I=1,ND
195      J=JND(I)+NSSJJ
196      DDD(J)=DD(I)+DDD(J)
197 162  D(J)=DD(I)
198      DO 165 I=1,NS
199      K=JNS(I)+NSSJJ
200      DDD(K)=DS(I)+DDD(K)
201 165  D(K)=DS(I)
202 C
203 C      IF SLIP AT SUPPORT OCCURS
204      IF(NF)340,340,170
205 170  CONTINUE
206      DO 175 I=1,NMM
207      DO 175 J=1,4
208      DO 175 K=1,22
209      AM(I,J)=0.
210      AJ(K)=0.
211 175  DS(K)=0.
212      IF(NIF-2*NIF)180,340,340
213 180  NIF=NIF+1
214      KNIF=NIF/NF
215      GO TO (190,275),KNIF
216 190  CONTINUE
217      READ(5,6)M,KKK,NFM,SLIP(NFM)
218 6    FORMAT(3I3,F10.6)
219      INS(M)=NFM
220      NSJ=2*((NFM+1)/2)-1
221      L=0
222      DO 195 I=1,NSS
223      IF(JNSS(I).EQ.NSJ) GO TO 200
224      IF(JNSS(I).EQ.NFM) GO TO 200
225 195  L=L+1
226 200  GO TO(210,205,210,205),KKK
227 205  SLIP1(NFM)=SLIP(NFM)*XLS(L)*XI(M+1)/(XI(M+1)*XLS(L)+XI(M)*XLS(L+1)
228 1)
229      SLIP2(NFM)=SLIP1(NFM)-SLIP(NFM)
230 210  CONTINUE
231 C
232 C      FIND NUMBER OF JOINTS AND MEMBERS IN DIVIDED STRUCTURE
233      NLJ=2*((NFM+1)/2)
234      NLM=(NLJ-2)/2
235      NRM=NMM-NLM
236      NRJ=2*NRM+2
237 C      DETERMINE NUMBER OF SUPPORTS IN LEFT SPANS
238      L=0
239      DO 220 I=1,NLJ
240      DO 220 J=1,NSS
241      IF(I-JNSS(J))220,215,220
242 215  L=L+1
243 220  CONTINUE
244      DO 225 I=1,NSS
245      IF(JNSS(I).EQ.NSJ)GO TO 230
246      IF(JNSS(I).EQ.NFM)GO TO 230

```

```

247 225 CONTINUE
248 GO TO 235
249 230 L=L-1
250 235 CONTINUE
251 GO TO (250,245,250,245),KKK
252 245 NLS=L+2
253 JNS(NLS-1)=2*((NFM-1)/2)+1
254 JNS(NLS)=NFM
255 DS(NLS)=SLIP1(NFM)
256 250 CONTINUE
257 NM=NL1
258 NJ=NLJ
259 NS=NLS
260 GO TO 999
261 275 CONTINUE
262 C DETERMINE NUMBER OF SUPPORTS IN RIGHT SPANS
263 KNFM=2*((NFM-1)/2)+1
264 NSSJJ=NSJ-1
265 L=0
266 DO 285 I=KNFM,NOJ
267 DO 285 J=1,NSS
268 IF(I-JNSS(J))285,280,285
269 280 I=L+1
270 285 CONTINUE
271 DO 290 I=1,NSS
272 IF(JNSS(I).EQ.NSJ)GO TO 295
273 IF(JNSS(I).EQ.NFM)GO TO 295
274 290 CONTINUE
275 GO TO 300
276 295 L=L-1
277 300 GO TO (325,310,325,310),KKK
278 310 NRS=L+2
279 JNS(2)=NFM-NSSJJ
280 JNS(1)=NFM-1-NSSJJ
281 DS(2)=SLIP2(NFM)
282 LM=2
283 DO 320 I=1,NSS
284 IF(JNSS(I).GT.NFM)GO TO 315
285 GO TO 320
286 315 LM=LM+1
287 JNS(LM)=JNSS(I)-NSSJJ
288 320 CONTINUE
289 325 CONTINUE
290 NM=NRM
291 NJ=NRJ
292 NS=NRS
293 GO TO 999
294 340 CONTINUE
295 C
296 C TO SUPERIMPOSE DEFLECTIONS AND FIND MEMBER END FORCES
297 DO 345 M=1,NMM
298 M1=2*M-1
299 M2=2*M
300 M3=2*M+1
301 M4=2*M+2
302 DL(M,1)=DDD(M1)
303 DL(M,2)=DDD(M2)
304 DR(M,3)=DDD(M3)
305 345 DR(M,4)=DDD(M4)
306 DO 395 M=1,NMM
307 IF(NF)350,395,350
308 350 K=INS(M)
309 M1=2*M-1

```

```

310      M2=2*M
311      M3=2*M+1
312      M4=2*M+2
313      IF(K.GE.M1.AND.K.LE.M4)GO TO 355
314      GO TO 395
315 355 KK=K-2*M+2
316      GO TO (395,360,395,370),KK
317 360 DL(M,2)=DL(M,2)-SLIP1(K)
318      IF(M-1)365,395,365
319 365 DR(M-1,4)=DR(M-1,4)-SLIP2(K)
320      GO TO 395
321 370 DR(M,4)=DR(M,4)-SLIP2(K)
322      IF(M-NMM)375,395,375
323 375 DL(M+1,2)=DL(M+1,2)-SLIP1(K)
324 395 CONTINUE
325      DO 400 M=1,NMM
326      DO 400 J=1,4
327 400 AMM(M,J)=SM(M,J,1)*DL(M,1)+SM(M,J,2)*DL(M,2)+SM(M,J,3)*DR(M,3)+SM(
328 1M,J,4)*DR(M,4)+AMF(M,J)
329 C
330 C      TO DETERMINE SUPPORT REACTIONS
331      T(1)=AMM(1,1)-AJJ(1)
332      T(2)=AMM(1,2)-AJJ(2)
333      IF(NMM.EQ.1)GO TO 410
334      NMML=NMM-1
335      NMMT=2*NMM
336      DO 405 M=1,NMML
337      DO 405 I=3,NMMT,2
338      T(I)=AMM(M,3)+AMM(M+1,1)-AJJ(I)
339 405 T(I+1)=AMM(M,4)+AMM(M+1,2)-AJJ(I+1)
340 410 T(2*NMM+1)=AMM(NMM,3)-AJJ(2*NMM+1)
341      T(2*NMM+2)=AMM(NMM,4)-AJJ(2*NMM+2)
342      DO 415 I=1,NSS
343      K=JNSS(I)
344 415 SR(I)=T(K)
345 C
346 C      PRINT OUTPUT
347      WRITE(6,500)
348 500 FORMAT('1',32X,'MATRIX ANALYSIS OF CONTINUOUS BEAM',////)
349      WRITE(6,505)
350 505 FORMAT(1X,'MEMBER',5X,'LENGTH',9X,'E',9X,'I',4X,'LEFT SHEAR',3X,'L
351 1EFT MOMENT',3X,'RIGHT SHEAR',3X,'RIGHT MOMENT',//)
352      DO 515 M=1,NMM
353      WRITE(6,510)M,XL(M),E(M),XI(M),(AMM(M,J),J=1,4)
354 510 FORMAT(1X,I3,8X,F5.0,6X,F9.0,1X,F7.1,4X,F7.1,5X,F8.1,8X,F7.1,6X,F8
355 1.1,/)
356 515 CONTINUE
357      WRITE(6,520)
358 520 FORMAT(/,1X,'DEGREE OF FREEDOM',21X,'VERTICAL DISPLACEMENT',21X,'R
359 1OTATION',/)
360      WRITE(6,525)
361 525 FORMAT(42X,'LEFT',6X,'RIGHT',22X,'LEFT',6X,'RIGHT')
362      WRITE(6,530)
363 530 FORMAT(43X,'OF',9X,'OF',24X,'OF',9X,'OF')
364      WRITE(6,535)
365 535 FORMAT(41X,'JOINT',6X,'JOINT',21X,'JOINT',6X,'JOINT',//)
366      I=1
367      WRITE(6,540)I,DL(1,1)
368 540 FORMAT(7X,I3,38X,F10.5)
369      NJJ=NMM-1
370      DO 550 M=1,NJJ
371      I=I+2
372      WRITE(6,545)I,DR(M,3),DL(M+1,1)

```

```

373 545 FORMAT(7X,I3,26X,F10.5,2X,F10.5)
374 550 CONTINUE
375     I=I+2
376     WRITE(6,555) I,DR(NMM,3)
377 555 FORMAT(7X,I3,26X,F10.5)
378     I=2
379     WRITE(6,560) I,DL(1,2)
380 560 FORMAT(7X,I3,75X,F10.5)
381     DO 570 M=1,NJJ
382     I=I+2
383     WRITE(6,565) I,DR(M,4),DL(M+1,2)
384 565 FORMAT(7X,I3,63X,F10.5,2X,F10.5)
385 570 CONTINUE
386     I=I+2
387     WRITE(6,575) I,DR(NMM,4)
388 575 FORMAT(7X,I3,63X,F10.5)
389     WRITE(6,580)
390 580 FORMAT(//,1X,'DEGREE OF FREEDOM',7X,'REACTION',7X,'MOMENT',//)
391     DO 605 I=1,NSS
392     K=JNSS(I)
393     KK=(K+1)/2
394     KJ=K/2
395     IF(KJ-KK)585,595,585
396 585 CONTINUE
397     WRITE(6,590)K,SR(I)
398 590 FORMAT(7X,I3,13X,F9.2)
399     GO TO 605
400 595 CONTINUE
401     WRITE(6,600)K,SR(I)
402 600 FORMAT(7X,I3,27X,F9.2)
403 605 CONTINUE
404 610 CONTINUE
405     STOP
406     END

```

*** STATEMENT NUMBERS ***

1	11	*15	
2	13	*16	
3	32	*33	
4	81	*82	
5	12	*14	
6	217	*218	
10	23	*28	
15	31	*53	
16	54	55	*56
20	62	*64	
25	69	*71	
30	68	70	*75
35	67	74	*76
40	79	*80	
45	80	*84	
50	79	*85	
55	90	92	*93
60	113	*114	
65	113	*119	
70	113	118	*123
75	94	*128	
80	135	*136	
85	137	*138	
90	134	137	*140

95	133	135	*141						
100	144	145	*147						
105	148	149	*151						
110	153	154	*156						
115	157	158	*160						
120	162	163	*164						
125	165	166	*167						
130	168	169	*170						
135	171	172	*173						
140	175	*177							
145	178	*180							
150	183	185	*186						
155	187	*188							
160	190	192	*193						
162	194	*197							
165	198	*201							
170	204	*205							
175	206	207	208	*211					
180	212	*213							
190	215	*216							
195	222	*225							
200	223	224	*226						
205	226	*227							
210	226	*230							
215	241	*242							
220	239	240	241	*243					
225	244	*247							
230	245	246	*249						
235	248	*250							
245	251	*252							
250	251	*256							
275	215	*261							
280	268	*269							
285	266	267	268	*270					
290	271	*274							
295	272	273	*276						
300	275	*277							
310	277	*278							
315	284	*286							
320	283	285	*288						
325	277	*289							
340	204	212	*294						
345	297	*305							
350	307	*308							
355	313	*315							
360	316	*317							
365	318	*319							
370	316	*321							
375	322	*323							
395	306	307	314	316	318	320	322	*324	
400	325	326	*327						
405	336	337	*339						
410	333	*340							
415	342	*344							
500	347	*348							
505	349	*350							
510	353	*354							
515	352	*356							
520	357	*358							
525	360	*361							
530	362	*363							
535	364	*365							
540	367	*368							

545	372	*373	
550	370	*374	
555	376	*377	
560	379	*380	
565	383	*384	
570	381	*385	
575	387	*388	
580	389	*390	
585	395	*396	
590	397	*398	
595	395	*400	
600	401	*402	
605	391	399	*403
610	189	*404	
999	*87	260	293

*** VARIABLES ***

A	3	*91	*124	*125	*126	*127	177	180				
AD	3	*177	188									
AJ	3	*26	*81	83	124	125	126	127	*210			
AJJ	3	*25	*83	331	332	338	339	340	341			
AI	3	*32	56	124	125	126	127	*209				
AMF	3	*56	327									
AMM	3	*327	331	332	338	339	340	341	353			
AS	3	*180										
B	3	*34	35	46	47	48	49	50	51	52	53	
C	3	*184	*186	188								
COOR	3	*61	*64	72								
D	3	*197	*201									
DD	3	*191	*193	196	197							
DDD	3	*29	*196	*200	302	303	304	305				
DL	3	*302	*303	*317	*323	327	367	372	379	383		
DR	3	*304	*305	*319	*321	327	372	376	383	387		
DS	3	*13	186	200	201	*211	*255	*281				
E	3	*32	34	36	353							
I	*12	*13	14	*23	24	25	26	27	28	*32	*62	63
	64	*66	72	*73	*80	*90	91	93	*94	95	113	*133
	135	138	*144	147	*148	151	*154	155	156	*158	159	160
	*162	164	*165	167	*168	170	*171	173	*175	176	177	*178
	179	180	*183	184	186	*187	188	*190	191	193	*194	195
	196	197	*198	199	200	201	*206	209	*222	223	224	*239
	241	*244	245	246	*266	268	*271	272	273	*283	284	287
	*337	338	339	*342	343	344	*366	367	*371	372	*375	376
	*378	379	*382	383	*386	387	*391	392	397	401		
INS	3	*24	*219	308								
J	*31	*54	56	*67	69	72	*92	93	*145	146	147	*149
	150	151	*153	156	*157	160	*163	164	*166	167	*169	170
	*172	173	*195	196	197	*207	209	*240	241	*267	268	*326
	327	353										
JJ	*71	72										
JND	3	*138	146	155	176	195						
JNS	3	*13	14	69	71	135	150	159	179	199	*253	*254
	*279	*280	*287									
JNSS	3	*14	223	224	241	245	246	268	272	273	284	287
	343	392										
K	*63	64	*68	69	71	*134	135	137	*176	177	*179	180
	*185	186	*192	193	*199	200	201	*208	210	211	*308	313
	315	317	319	321	323	*343	344	*392	393	394	397	401
KJ	*394	395										

KK	*146	147	*150	151	*155	156	*159	160	*315	316	*393	395
KKK	*217	226	251	277								
KNFM	*263	266										
KNIF	*214	215										
I	*55	56	*132	138	*139	*221	*225	227	*238	*242	*249	252
	*265	*269	*276	278								
LM	*282	*286	287									
M	*32	34	35	36	37	38	39	40	41	42	43	44
	45	46	47	48	49	50	51	52	53	*95	96	97
	98	99	*100	101	102	103	104	105	106	107	108	109
	110	111	112	114	115	116	117	119	120	121	122	124
	125	126	127	*217	219	227	*297	298	299	300	301	302
	303	304	305	*306	308	309	310	311	312	315	317	318
	319	321	322	323	*325	327	*336	338	339	*352	353	*370
	372	*381	383									
M1	*96	101	102	105	107	109	110	111	124	*298	302	*309
	313											
M2	*97	103	104	106	108	110	111	112	125	*299	303	*310
M3	*98	101	103	105	106	114	115	116	119	120	121	126
	*300	304	*311									
M4	*99	102	104	107	108	115	116	117	120	121	122	127
	*301	305	*312	313								
MTINV2	189											
NA	*11	79	80									
NAM	*81	83										
ND	*143	145	151	154	160	162	163	166	167	168	170	173
	175	183	187	189	190	192	194					
NF	*11	204	212	214	307							
NFM	*217	219	220	224	227	229	233	246	253	254	255	263
	273	279	280	281	284							
NIF	*19	212	*213	214								
NJ	*59	60	62	67	133	143	144	148	153	157	*258	*291
NJJ	*369	370	381									
NLJ	*233	234	239	258								
NLM	*234	235	257									
NLS	*252	253	254	255	259							
NM	*11	21	31	59	94	113	*257	*290				
NMM	*21	54	206	235	297	306	322	325	333	334	335	340
	341	352	369	376	387							
NMML	*334	336										
NMMT	*335	337										
NOJ	*60	266										
NRJ	*236	291										
HRM	*235	236	290									
NRS	*278	292										
NS	*11	12	22	65	68	134	137	143	149	158	165	169
	171	172	178	185	198	*259	*292					
NSJ	*220	223	245	264	272							
NSL	*65											
NSS	*22	222	240	244	267	271	283	342	391			
NSSJJ	*20	100	195	199	*264	279	280	287				
R	3	*156	*160	164	167	170	173					
RR	3	*147	*151	156	160							
S	3	*93	*101	*102	*103	*104	*105	*106	*107	*108	*109	*110
	*111	*112	*114	*115	*116	*117	*119	*120	*121	*122	147	151
SCRACH	3	189										
SDD	3	*164	189									
SDDI	3	189	193									
SDS	3	*170	186									
SLIP	3	*217	227	229								
SLIP1	3	*227	229	255	317	323						
SLIP2	3	*229	281	319	321							

SM	3	*38	*39	*40	*41	*42	*43	*44	*45	*46	*47	*48
		*49	*50	*51	*52	*53	101	102	103	104	105	107
		108	109	110	111	112	114	115	116	117	119	121
		122	327									
SR	3	*27	*344	397	401							
SSD	3	*167										
SSS	3	*173										
T	3	*331	*332	*338	*339	*340	*341	344				
V	3	*35	38	40	42	43						
W	3	*36	37	39	41							
X	3	*37	44	45								
XI	3	*32	34	36	227	353						
XL	3	*32	34	35	36	64	353					
XLS	3	*72	227									
Y	3	*188	193									

Appendix B

Example Output

MATRIX ANALYSIS OF CONTINUOUS BEAM							
MEMBER	LENGTH	E	I	LEFT SHEAR	LEFT MOMENT	RIGHT SHEAR	RIGHT MOMENT
1	60.	1700000.	20.8	266.2	- .0	-16.2	8469.3
2	84.	1700000.	20.8	16.2	-8469.3	333.8	-4873.6
3	108.	1700000.	20.8	90.1	4873.6	- .1	.0

DEGREE OF FREEDOM	VERTICAL DISPLACEMENT		ROTATION	
	LEFT OF JOINT	RIGHT OF JOINT	LEFT OF JOINT	RIGHT OF JOINT
1		.00000		
3	-.47382	-.47382		
5	.00000	.00000		
7	.00000			
2				-.01135
4			-.00205	-.00205
6			.00804	.00372
8			-.00124	

DEGREE OF FREEDOM	REACTION	MOMENT
1	266.16	
5	423.97	
7	-.13	

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C02-11-2

U.S. Forest Products Laboratory

Analysis of Continuous Beams with Joint Slip, by
Lawrence Soltis, Madison, Wis., FPL 1981.
20 p. (USDA For. Serv. Res. Note FPL 0244).

A computer analysis method and input user guideline
to determine internal forces, reactions and deflections
of continuous beams with rotational slip at supports is
presented. The method and computer program are applicable
to any continuous beam structure, although developed
specifically for two-span floor joist analysis and design.

Keywords: Joist design, rotational slip, two-span
floor joist, deflections, continuous beams.

2.0-20-7/81