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Volume Tables and Point-Sampling Factors for SHORTLEAF PINES in Plantations on Abandoned Fields in Tennessee, Alabama, and Georgia Highlands

Glendon W. Smalley
David R. Bower

SOUTHERN FOREST EXPERIMENT STATION
T. C. Nelson, Director

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Acknowledgments

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LIST OF TABLES

Volume Tables and Point-Sampling Factors for Shortleaf Pines in Plantations on Abandoned Fields in Tennessee, Alabama, and Georgia Highlands

Glendon W. Smalley[1] and David R. Bower[2]

The tables and equations published here provide ways to estimate total and merchantable cubic-foot volumes, both inside and outside bark, of shortleaf pines (*Pinus echinata* Mill.) planted on abandoned fields in the Ridge and Valley, Cumberland Plateau, Eastern Highland Rim, and Western Highland Rim regions of Tennessee, Alabama, and Georgia (fig. 1). There already are about 60,000 acres of shortleaf plantations cn such sites, and more are likely to be established. The tables and equations should prove useful in growth and yield studies, forest inventories, timber sales, and management plans.

Basic data were obtained by felling and measuring a total of 305 trees at points throughout the four physiographic regions. Tree ages ranged from 10 to 35 years.

Similar tables are available for loblolly pine *(4)*.[3]

Study Methods

All sample trees were cut off 3 inches above ground on the uphill side. Total height (H) was measured to the nearest foot from ground level on the uphill side to the tip. Forked or other deformed trees were avoided. Additional measurements included diameter outside bark and inside bark at stump height, at breast height (D), and at maximum intervals of 5 feet to tops of 4.0, 3.0, and 2.0 inches o.b. For the tables, the minimum tree considered in each utilization class was 5.0 inches o.b. at breast height. Thus, the tables are based on

[1] Soil Scientist, stationed at the Silviculture Laboratory, maintained at Sewanee. Tennessee, by the Southern Forest Experiment Station in cooperation with the University of the South.

[2] Statistician, Southern Forest Experiment Station, New Orleans, Louisiana.

[3] Italic numbers in parentheses refer to Literature Cited, p. 5.

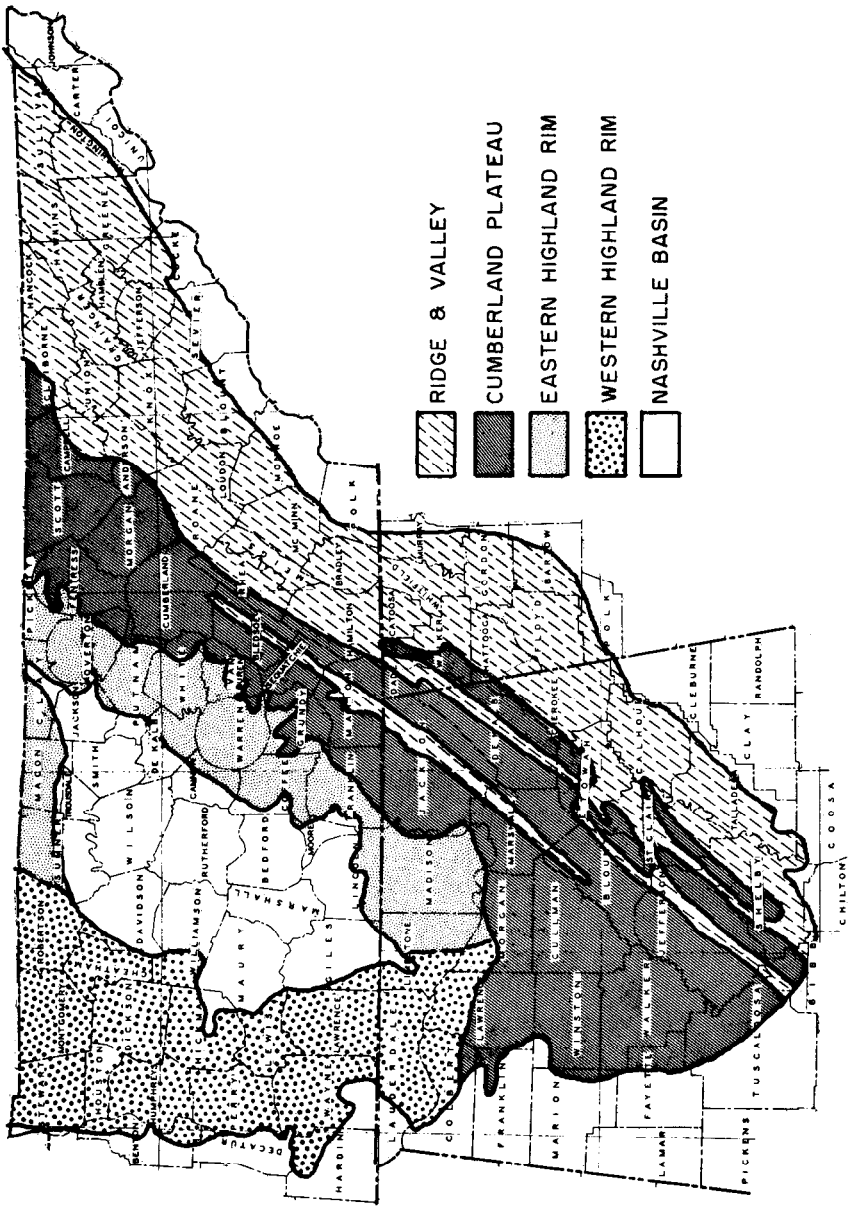

Figure 1.—*Physiographic regions of middle Tennessee, north Alabama, and northwest Georgia.*

data from 168 trees; number of stems measured in each diameter-height category is indicated in table 1. Separate equations were fitted, however, for a larger sample that included 137 stems less than 5.0 inches d.b.h.

Cubic volume of each tree was computed **by Smalian's formula.** Volumes in the 3-inch stumps were excluded.

Experience with the data on loblolly pine (4) indicated that a regression of volume on D^2H would provide an adequate model. For various threshold diameters, three statistical weights were tested: unity, the inverse of D^2H, and the inverse of $(D^2H)^2$. By Furnival's index of fit *(2),* there was little difference between the inverse of D^2H and the inverse of $(D^2H)^2$. The latter was chosen, as it minimized problems of intersection. The volume tables were therefore derived from weighted linear regressions of the form:

$$V(D^2H)^{-1} = b_1 + b_0(D^2H)^{-1}$$

where

V = gross volume in the appropriate units

D = diameter at breast height, outside bark, in inches

H = total height in feet

b_0, b_1 = regression constants

Fitting the equation by ordinary least squares is the same as weighting the squared residuals of $V = b_0 + b_1D^2H$ by $1/(D^2H)^2$.

A covariance analysis of regressions for total cubic-foot volume outside bark showed no differences, at the 0.01 level of significance, between physiographic regions in either slope or intercept values. The data were therefore combined for computing the values in tables 1, 3, 5, 7, 9, 11, 13, and 15. The equation, the geometric mean of D^2H, **and the** standard error of estimate are shown below each table.

Tables 2, 4, 6, 8, 10, 12, 14, and 16 give point-sampling factors for each combination of tree diameter and height appearing in the corresponding volume table. Volumes per square foot of basal area were derived by dividing each volume equation by $0.005454D^2$, a formula for basal area.

In all tables, midpoints of the total height classes are multiples of 5 feet, e.g., 30 = 28 through 32 feet. The midpoint of each diameter class is given, e.g., 7 = 6.5 through 7.4 inches. The boxed portion of each volume table indicates the extent of basic data.

Using the Tables and Equations

The volumes and point-sampling factors in the tables should prove applicable throughout the area indicated in figure 1. For developing the equations on which the tables were based, tree diameters were measured, to tenths of an inch and heights were measured to the nearest foot. The same limits of accuracy are recommended in application of the tables or equations.

A point-sampling cruise may be made by measuring diameters and heights of trees counted through a prism or relascope and applying the volume-conversion factors from the even-numbered tables. Volume per acre is computed as follows:

1. Multiply the number of counted trees in each diameter-height class by the point-sampling factor for the class.
2. Sum the products of step one.
3. Multiply the total of step two by the basal area factor of the prism or other angle gage used.
4. Divide the product of step three by the number of points sampled in the plantation.

If a sampling error for the overall average volume per acre is desired, individual point estimates of volume per acre must be calculated, i.e., the tally of each point must be kept separate. Instructions can be found in Beers and Miller *(1)* or Hunt, Baker, and Biskamp (3).

When maximum precision and accuracy are desired, volumes should be computed from the equations rather than read from the tables. For a conventional cruise of trees larger than 5.0 inches d.b.h., volumes for any combinations of diameters and heights can be obtained by summation of the appropriate equation. Thus, outside-bark volumes of entire stems would be obtained by summing the equation for table 1 over *n* sample trees:

$$\Sigma V_c(\text{o.b.}) = 0.0027379\ \Sigma D^2H - 0.0326\ (n)$$

For a point-sampling cruise the corresponding equation (table 2) would be:

$$\Sigma V_p(\text{o.b.}) = BAF(0.5020(\ CH) + 5.9771(\Sigma D^{-2}))$$

Individual heights and reciprocals of squared diameters are accumulated over all trees for each sample point.

If the inventory includes trees smaller than 5.0 inches d.b.h., the following equations may be used for total tree volume:

$$V_c(\text{o.b.}) = 0.03532 + 0.002777120D^2H$$

$$V_c(\text{i.b.}) = 0.00885 + 0.001997667D^2H$$

These two equations are not to be compared with those having a minimum-diameter threshold of 5.0 inches. Including smaller trees modifies the slope, so that these equations may intersect with the 5.0-inch equations within the range of sampled data.

The equations used to develop volumes for different top-utilization levels (and 5.0-inch threshold diameters) do not intersect one another within the range of diameters and heights given in the tables.

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Table l.-Outside-bark volume of entire stem [1]

D.b.h., o.b. (inches)	Total height in feet												Basis: trees
	20	25	30	35	40	45	50	55	60	65	70	75	
	Cubic feet												*No.*
5	1.34	1.68	2.02	2.36	2.71	3.05	3.39	3.73	4.07				32
6		2.43	2.92	3.42	3.91	4.40	4.90	5.39	5.88	6.37			45
7		3.32	3.99	4.66	5.33	6.00	6.68	7.35	8.02	8.69	9.36		36
8			5.22	6.10	6.98	7.85	8.73	9.60	10.48	11.36	12.23		29
9				7.73	8.84	9.95	11.06	12.16	13.27	14.38	15.49		19
10						12.29	13.66	15.03	16.39	17.76	19.13	20.50	
11							16.53	18.19	19.84	21.50	22.16	24.81	
12								21.65	23.62	25.59	22.57	29.54	
Basis: No. of trees	1	3	13	19	29	27	39	26	8	2	1	0	168

[1]Derived from V = 0.0027379(D^2H) — 0.0326. Geometric mean of D^2H = 2,055; standard error of estimate = 0.0002544 cu. ft./D^2H.

Table 2.-Outside-bark volume of *entire stem, per* square *foot of basal area* [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	9.8	12.3	14.8	17.3	19.8	22.4	24.9	27.4	29.9			
6		12.4	14.9	17.4	19.9	22.4	24.9	27.4	30.0	32.5		
7		12.4	14.9	17.4	20.0	22.5	25.0	27.5	30.0	32.5	35.0	
8			15.0	17.5	20.0	22.5	25.0	27.5	30.0	32.5	35.0	
9				17.5	20.0	22.5	25.0	27.5	30.0	32.6	35.1	
10						22.5	25.0	27.5	30.1	32.6	35.1	37.6
11							25.0	27.6	30.1	32.6	35.1	37.6
12								27.6	30.1	32.6	35.1	37.6

[1]Derived from V/B.A. = 0.5020(H) — 5.9771/D^2.

Table 3.-*Inside-bark volume of entire stem* [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	0.78	1.06	1.33	1.60	1.87	2.14	2.42	2.69	2.96			
6		1.65	2.05	2.44	2.83	3.22	3.61	4.00	4.40	4.79		
7		2.36	2.89	3.43	3.96	4.49	5.03	5.56	6.09	6.63	7.16	
8			3.87	4.57	5.27	5.96	6.66	7.36	8.05	8.75	9.45	
9				5.87	6.75	7.63	8.51	9.39	10.27	11.16	12.04	
10						9.49	10.58	11.67	12.75	13.84	14.93	16.02
11							12.86	14.18	15.50	16.81	18.13	19.45
12								16.93	18.50	20.07	21.64	23.20

[1] Derived from $V = 0.0021767(D^2H) - 0.3052$. Geometric mean of D^2H = 2,055; standard error of estimate = 0.0002178 cu. ft./D^2H.

Table 4.—*Inside-bark volume of entire stem, per square foot of basal area* [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	5.7	7.7	9.7	11.7	13.7	15.7	17.7	19.7	21.7			
6		8.4	10.4	12.4	14.4	16.4	18.4	20.4	22.4	24.4		
7		8.8	10.8	12.8	14.8	16.8	18.8	20.8	22.8	24.8	26.8	
8			11.1	13.1	15.1	17.1	19.1	21.1	23.1	25.1	27.1	
9				13.3	15.3	17.3	19.3	21.3	23.3	25.2	27.2	
10						17.4	19.4	21.4	23.4	25.4	27.4	29.4
11							19.5	21.5	23.5	25.5	27.5	29.5
12								21.6	23.6	25.6	27.5	29.5

[1] Derived from V/B. A. = 0.3991 (H) — 55.9574/D^2.

Table 5.-Outside-bark volume to a 2.0-inch top [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	1.28	1.62	1.96	2.31	2.65	2.99	3.33	3.68	4.02			
6		2.37	2.87	3.36	3.85	4.35	4.84	5.33	5.83	6.32		
7		3.26	3.94	4.61	5.28	5.95	6.62	7.29	7.96	8.63	9.31	
8			5.17	6.05	6.92	7.80	8.68	9.55	10.43	11.31	12.18	
9				**7.68**	**8.79**	**9.89**	11.00	12.11	13.22	14.33	15.44	
10						12.24	13.61	**14.98**	16.35	17.72	19.09	20.46
11							16.48	18.14	19.80	**21.46**	**23.11**	24.77
12								21.61	23.58	**25.55**	**27.52**	29.50

[1] **Derived from V = 0.0027396 (D^2H) — 0.09130. Geometric mean of D^2H = 2,055; standard error of estimate = 0.0002572 cu. ft./D^2H.**

Table 6.-Outside-bark volume to a 2.0-inch top, per square foot of basal area [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	**9.4**	11.9	14.4	16.9	19.4	**21.9**	**24.4**	**27.0**	29.5			
6		12.1	14.6	17.1	19.6	22.1	24.6	27.2	29.7	32.2		
7		12.2	14.7	17.2	19.8	22.3	24.8	27.3	29.8	32.3	34.8	
8			14.8	17.3	19.8	22.3	24.9	27.4	29.9	32.4	34.9	
9				17.4	19.9	22.4	24.9	27.4	29.9	32.4	35.0	
10						**22.4**	25.0	27.5	30.0	32.5	35.0	37.5
11							25.0	27.5	30.0	32.5	**35.0**	37.5
12								27.5	30.0	32.5	35.0	37.6

[1] **Derived from V/B. A. = 0.5023 (H) — 16.7395/D^2.**

Table 7.—*Inside-bark* volume *to a 2.0-inch* top [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	0.74	1.01	1.29	1.56	1.83	2.10	2.37	2.65	2.92			
6		1.61	2.00	2.40	2.79	3.18	3.57	3.96	4.36	4.75		
7		2.32	2.85	3.39	3.92	4.45	4.99	5.52	6.06	6.59	7.12	
8			3.83	4.53	5.23	5.93	6.62	7.32	8.02	8.71	9.41	
9				5.83	6.71	7.59	8.47	9.36	10.24	11.12	12.00	
10						9.45	10.54	11.63	12.72	13.81	14.90	15.99
11							12.83	14.15	15.47	16.78	18.10	19.42
12								16.90	18.47	20.04	21.61	23.18

[1] Derived from V = 0.0021782(D²H) — 0.3482. Geometric mean of D²H = 2,055; standard error of estimate = 0.0002186 cu. ft./D²H.

Table 8.-Znside-bark volume to a *2.0-inch top, per square foot of basal area* [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	5.4	7.4	9.4	11.4	13.4	15.4	17.4	19.4	21.4			
6		8.2	10.2	12.2	14.2	16.2	18.2	20.2	22.2	24.2		
7		7	10.7	12.7	14.7	16.7	18.7	20.7	22.7	24.7	26.7	
8			11.0	13.0	15.0	17.0	19.0	21.0	23.0	25.0	27.0	
9				13.2	15.2	17.2	19.2	21.2	23.2	25.2	27.2	
10						17.3	19.3	21.3	23.3	25.3	27.3	29.3
11							19.4	21.4	23.4	25.4	27.4	29.4
12								21.5	23.5	25.5	27.5	29.5

[1] Derived from V/B. A. = 0.3994(H) — 63.8413/D².

Table 9.-*Outside-bark volume to a 3.0-inch top* [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	1.07	1.42	1.76	2.11	2.45	2.80	3.14	3.49	3.83			
6	2.18	2.67	3.17	3.67	4.16	4.66	5.16	5.65	6.15			
7	3.07	3.75	4.42	5.10	5.78	6.45	7.13	7.81	8.48	9.16		
8		4.99	5.87	6.76	7.64	8.52	9.41	10.29	11.17	12.06		
9			7.52	8.63	9.75	10.87	11.99	13.10	14.22	15.34		
10					12.11	13.49	14.86	16.23	17.63	19.01	20.39	
11									21.40	23.07	24.74	
12							21.55	23.54	25.52	27.51	29.50	

[1] Derived from $V = 0.0027599(D^2H) - 0.3085$. Geometric mean of $D^2H = 2{,}055$; standard error of estimate = 0.0002765 cu. ft./D^2H.

Table 10.-Outside-bark volume to *a 3.0-inch top, per square foot of basal area'*

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	7.9	10.4	12.9	15.4	18.0	20.5	23.0	25.6	28.1			
6	11.1	13.6	16.1	18.7	21.2	23.7	26.3	28.8	31.3			
7	11.5	14.0	16.6	19.1	21.6	24.1	26.7	29.2	31.7	34.3		
8	14.3	16.8	19.4	21.9	24.4	26.9	29.5	32.0	34.5			
9		17.0	19.5	22.1	24.6	27.1	29.7	32.2	34.7			
10				22.2	24.7	27.3	29.8	32.3	34.9	37.4		
11					24.8	27.4	29.9	32.4	35.0	37.5		
12						27.4	30.0	32.5	35.0	37.6		

[1] Derived from V/B. A. $= 0.5060(H) - 56.5624/D^2$.

Table ll.-Inside-bark volume to *a 3.0-inch top* [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	0.59	0.86	1.14	1.41	1.68	1.96	2.23	2.51	2.78			
6		1.47	1.86	2.25	2.65	3.04	3.44	3.83	4.23	4.62		
7		2.18	2.71	3.25	3.79	4.33	4.86	5.40	5.94	6.47	7.01	
8			3.70	4.40	5.10	5.80	6.51	7.21	7.91	8.61	9.31	
9				5.71	6.59	7.48	8.37	9.26	10.14	11.03	11.92	
10						9.35	10.45	11.55	12.64	13.74	14.83	15.93
11							12.75	14.08	15.40	16.73	18.05	19.38
12								16.85	18.43	20.00	21.58	23.16

[1] Derived from $V = 0.0021914(D^2H) - 0.5070$. Geometric mean of $D^2H = 2{,}055$; standard error of estimate = 0.0002279 cu. ft./D^2H.

Table 12.—*Inside-bark* volume *to a 3.0-inch top, per square foot of basal area* [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	4.3	6.3	8.3	10.3	12.4	14.4	16.4	18.4	12.4	14.4	16.4	18.4	20.4			
6		7.5	9.5	11.5	13.5	15.5	17.5	19.5	21.5	23.5		
7		8.1	10.2	12.2	14.2	16.2	18.2	20.2	22.2	24.2	26.2	
8			10.6	12.6	14.6	16.6	18.6	20.6	22.7	24.7	26.7	
9				12.9	14.9	16.9	18.9	21.0	23.0	25.0	27.0	
10					17.2	19.2	21.2	23.2	25.2	27.2	29.2
11						19.3	21.3	23.3	25.3	27.4	29.4
12							21.5	23.5	25.5	27.5	29.5

[1] Derived from V/B. A. = $0.4018(H) - 92.9567/D^2$.

Table 13.-Outside-bark volume to a *4.0-inch* top [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	0.62	0.96	1.31	1.66	2.00	2.35	2.70	3.05	3.39			
6		1.73	2.23	2.73	3.23	3.73	4.23	4.73	5.23	5.72		
7		2.63	3.31	3.99	4.67	5.35	6.03	6.71	7.39	8.07	a.75	
a			4.56	5.45	6.34	7.22	8.11	9.00	9.89	10.78	11.67	
9				7.10	8.22	9.35	10.47	11.60	12.72	13.85	14.97	
						11.72				17.28	18.66	20.05
10							16.03	14.30	19.89	21.06	22.74	24.42
12								21.22	23.22	25.22	27.21	29.21

[1] Derived from V = 0.0027765 (D^2H) — 0.7722. Geometric mean of D^2H = 2,055; standard error of estimate = 0.0003220 cu. ft./D^2H.

Table 14.-Outside-bark volume to *a 4.0-inch top, per square foot of* basal *area* [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	4.5	7.1	9.6	12.2	14.7	17.2	19.8	22.3	24.9			
6		8.8	11.3	13.9	16.4	19.0	21.5	24.1	26.6	29.2		
7		9.8	12.4	14.9	17.5	20.0	22.6	25.1	27.7	30.2	32.7	
a			13.1	15.6	la.2	20.7	23.2	25.8	28.3	30.9	33.4	
9				16.1	18.6	21.2	23.7	26.3	28.8	31.3	33.9	
10						21.5	24.0	26.6	29.1	31.7	34.2	36.8
11							24.3	26.8	29.4	31.9	34.5	37.0
12								27.0	29.6	32.1	34.7	37.2

[1] Derived from V/B. A. = 0.5091 (H) — 141.5803/D^2.

Table 15.—*Inside-bark* volume to a *4.0-inch top* [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	0.25	0.53	0.80	1.08	1.35	1.63	1.90	2.17	2.45			
6		1.13	1.53	1.92	2.32	2.71	3.11	3.50	3.90	4.29		
7		1.84	2.38	2.92	3.46	4.00	4.54	5.07	5.61	6.15	6.69	
8			3.37	4.07	4.78	5.48	6.18	6.89	7.59	8.29	8.99	
9				5.38	6.27	7.16	8.05	8.94	9.83	10.72	11.61	
10						9.04	10.14	11.24	12.33	13.43	14.53	15.63
11							12.44	13.77	15.10	16.43	17.76	19.09
12								16.55	18.13	19.71	21.30	22.88

[1] Derived from $V = 0.0021966(D^2H) - 0.8459$. Geometric mean of $D^2H = 2{,}055$; standard error of estimate $= 0.0002521$ cu. ft./D^2H.

Table 16.—*Inside-oark* volume to a *4.0-inch* top, per square foot of basal area [1]

D.b.h., o.b. (inches)	Total height in feet											
	20	25	30	35	40	45	50	55	60	65	70	75
	Cubic feet											
5	1.9	3.9	5.99	7.9	.	11.9	913.9	15.9	) 18.0			
6		5.8	7.8	9.8	11.8	13.8	15.8	17.8	19.9	21.9		
7		6.9	8.9	10.9	12.9	15.0	17.0	19.0	21.0	23.0	25.0	
8			9.7	11.7	13.7	15.7	17.7	19.7	21.7	23.8	25.8	
9				12.2	14.2	16.2	18.2	20.2	22.2	24.3	26.3	
10						16.6	18.6	20.6	22.6	24.6	26.6	28.7
11							18.9	20.9	22.9	24.9	26.9	28.9
12								21.1	23.1	25.1	27.1	29.1

[1] Derived from V/B. A. $= 0.4027(H) - 155.0929/D^2$.