Estimation of Optimal Stocking Levels And Rotation Ages of Loblolly Pine

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Abstract. Models are developed for estimating optimal growing stock levels and rotation ages for loblolly pine. Under a profit maximization goal, they provide for varying stumpage prices, stand ages, site indexes, stocking levels, alternative rates of return, and regeneration costs for two single-product objectives. Quality differential as expressed in stumpage prices is recognized. A system is also provided for determining optimal stocking levels at the same marginal rate of return as that selected for determining optimal rotation age. Using a loblolly pine growth study in the Southeast for the biological data, solutions of stocking levels at various alternative rates of return indicate that stocking standards for profit maximization are much less than levels which optimize physical production. Rotation age solutions indicated that rotations lengthened as the alternative rate of return is reduced, as the site index increased and as a quality differential is introduced. In general, the optimal rotation age is less flexible with changes in the variables affecting it than optimal stocking levels.

In THE PRODUCTION of loblolly pine pulpwood and sawtimber two interrelated questions confront the forest manager. First, what stand density should be maintained under management and second, how long should the rotation be? Theoretically, it is necessary to maintain stand density at the optimal level throughout the rotation to attain the economically optimal rotation. Conversely, if the length of rotation is fixed, it exerts an influence on the optimal growing stock level.

This paper presents models for estimating optimal stocking and rotation ages for loblolly pine pulpwood and sawtimber objectives. Because of the product restraints and the particular assumptions introduced into the analyses for simplifying calculations, the stocking levels and rotations presented here should not be considered prescriptions. They are better viewed as boundary markers that indicate estimated optimal stocking and optimal rotation age, subject to the limitations of the restraints, assumptions, and growth functions, for growers and sellers of loblolly pine stumpage. The analytical models, however, can be used to determine prescriptions when solved with the specific product objectives and restraints, cost circumstances, price forecasts, and stand management alternatives of individual producers of loblolly pine stumpage.

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Objectives, Restraints, and Assumptions

It is assumed in this paper that the usual goal of the forest manager (and therefore our definition of "optimal") is to maximize net profit per annum, and since all solutions are in terms of 1 acre, also per acre. Our reason for taking pure profit maximization as the goal simply is one of convenience, since this goal may be readily quantified. The constraints commonly applied to profit maximization are omitted in this analysis because they vary a great deal from ownership to ownership. Costs and product prices likewise vary greatly from ownership to ownership. The costs and product prices that have been assumed in deriving stocking levels and rotation ages have been selected as typical for the Southeast.

The optimal growing stock levels and rotation ages determined in this analysis are based upon single product objectives, i. e., pulpwood or sawtimber. Analysis introducing joint production of these products or of other products was not undertaken. The available quantity and quality of data would not permit such analysis.

If a multiple product approach were used, one would expect pulpwood thinnings in the sawtimber solution to lengthen the rotation, raise the soil rent, and perhaps change the optimal stocking levels. Thinnings provide early revenue and thus lessen the financial burden of carrying costs on the timber capital and compounded regeneration cost until the time of the first sawtimber thinning.

The concept of the guiding or alternative rate of return is central to decision-making by the forest manager; it is an opportunity cost, a marginal unit cost, associated with sacrificed opportunities. It may be defined as the minimum rate, expressed in compound interest form, that the timber grower will accept on investments of equal risk and uncertainty. It is largely determined for each enterprise by the factors of (1) liquidity preference, (2) time preference, and (3) capital limitations (Chappelle 1959).

The optimal solution to any maximization problem may be found where the marginal unit cost is equal to the marginal unit revenue. This paper follows the general convention that both marginal unit cost and marginal unit revenue are expressed in terms of percentage of capital value. Here the marginal unit cost is the alternative rate of return, and the marginal unit revenue is the value growth rate of the timber.

Operationally, there are several levels in the woods where economic decisions must be made. These levels are (1) the individual tree, (2) the stand, and (3) the forest. Through the use of financial maturity guides, individual trees may be evaluated as investments, with recognition of the potential earnings in terms of the capital required and the risks and the uncertainty involved.

If the financial maturity model for individual trees could be applied without qualification, it would be a simple matter to decide how many trees should be removed, *i. e.*, to determine optimal stocking. The number of trees to remove in a thinning would be the sum of those trees earning at or below the alternative rate of return, with the added stipulation that an economic cut is attained. But in practice it is not possible to interpret such guides as being rules. The use of the financial maturity concept requires a large measure of judgment on the part of the user because, being an analysis based on individual trees, it neglects the influence of adjoining trees on the trees being evaluated, and the effects of release on residual trees.

Much of the difficulty relates to the inevitable clustering of trees found in natural stands, and to a lesser extent, in plantations. In view of the above, it is useful to develop guides for thinning based on the stand rather than the individual tree, and to use such guides to estimate optimal stocking. The financial maturity guides are then used to assist in marking trees within the stand in order to achieve such a stocking level. This is the scheme of management adopted in this paper.

Financial maturity guides are not presented; this paper deals only with the stand. The first level of functional analysis is concerned with optimizing the growing stock level of individual stands. For optimal rotation age determinations, it is assumed *a priori* that correct tree decisions have been made in thinnings, which are of optimal timing and intensity.

Again, to simplify solutions, flat rates for stumpage prices are assumed without regard for the many factors affecting each individual tract.

The Economic Models

Two models are used in the analysis that follows. Both models are applications of marginal analysis to timber production. The first model determines optimal growing stock levels. The second, the traditional soil rent formula, determines optimal rotation length.

Optimal growing stock model. Optimal stocking of a forest stand is indicated when the marginal unit revenue is equal to the marginal unit cost (Duerr and Bond 1952). That is, the problem is solved when

$$MC = MR \tag{1}$$

where

$$MC = \text{marginal cost}$$
$$MR = \text{marginal revenue}$$

In this model, the marginal cost is represented by the firm's alternative rate of return, and the marginal revenue by the value growth rate of the stand. Equation (1) may be further defined (Chappelle 1959).

$$p = \sqrt[n]{\frac{g + s}{s}} - 1 \tag{1'}$$

where

p = alternative rate of return, decimal
n = growth or investment period, years
g = marginal value growth for the period n, dollars
s = capital value of an additional unit of growing stock, dollars

Equation (1′) is simply an application of the familiar compound interest formula

$$(1 + p)^n = \frac{V_n}{V_o} \tag{1''}$$

where

V_n = value at end of investment period n, dollars
V_o = value at beginning of investment period n, dollars
p = rate of return, decimal
n = investment period, years

In order to solve this problem of optimal stocking, therefore, it is necessary to know for any stand the additional or marginal growth obtainable from an additional or marginal unit of stocking, and the net values associated with these marginal units. Development of such values requires data from stand density growth studies. One of the aims of these studies is generally to develop reliable stand growth predictions. It is well known that physical growth of the forest (the physical production function), either in total or in merchantable units, may be predicted if one has measures of site, age, and stand density. For example, the growth model for cubic-foot production may be of the form:

$$G_n = b_0 + b_1(A^{-1}) + b_2(SD) + b_3(D^2) \tag{2}$$

where

G_n = growth in merchantable units over period n
A = age, years
D = stand density in merchantable volume units
S = site index, feet in 50 years
n = growth period, years
b_0, b_1, b_2, b_3 = constant and partial regression coefficients

The next step is to estimate value growth, that is, growth in dollars. To do this it is necessary to determine the conversion surplus at each intensity of management. Conversion surplus is the difference between end product price and all the variable costs of production to the firm (Guttenberg and Duerr 1949). In this analysis the end product is assumed to be stumpage. In such a case, conversion surplus is the difference between stumpage price and the variable costs of growing and selling the timber, e.g., timber marking costs. It is here hypothesized that stumpage price is a function of stand age:

$$P = b_4 + b_5A \tag{3}$$

where

P = price per merchantable unit of volume, dollars
A = age of stand, years
b_4, b_5 = constant and partial regression coefficients

It should be noted that since the growth occurs over a period of n years, the price of the growth is not identical to the price of the growing stock. The price of the growing stock is given directly by equation (3). However, the price of the growth will be

$$P_G \quad b_4 + b_5(A + n) \tag{3'}$$

where

P_G = price per merchantable unit of volume growth, dollars
n = length of cutting cycle, years

Value growth of the stand may therefore be expressed by the following equation:

Authors' Correction

In our recent article, "Estimation of Optimal Stocking Levels and Rotation Ages of Loblolly Pine" (*Forest Science* 10:471–502), an error occurred in the development of the general optimal stocking models.

The value growth equations (4) and (13) were in error in that the increase in value over the investment period per unit volume of growing stock was not taken into consideration. The corrected value growth equation for the pulpwood objective is:

$$VG_n = P_G(b_0 + b_1A^{-1} + b_2SD + b_3D^2) + b_5n(D) - VC(D) \tag{4}$$

This correction results in the following equation:

$$(1 + p)^n \cong 1 + b_2S + b_3(2D + \Delta D) + \frac{nb_5[1 + (b_2S) + (2D + \Delta D)(b_3)]}{(b_4 + b_5A)} \tag{9}$$

It should be noted that the intermediate equations (4′), (5), (8′), (8″), (8‴), and (10) are also affected by this error. Equation (11), the final model for the pulpwood solutions, is not affected, however.

The error also is reflected in the sawtimber optimal stocking model. The corrected value growth equation is:

$$VG_n = P_G[c_0 + c_1(D) + c_2(SD) + c_3(SD^2) + c_4(A^{-1})] + c_6n(D) - VC(D) \tag{13}$$

The corrected optimal stocking model for sawtimber is as follows:

$$D \cong \frac{(c_5 + c_6A)(1 + p)^n - (c_5 + c_6A + c_6n)[1 + c_1 + c_2S + c_3S\Delta D]}{2\,c_3S(c_5 + c_6A + c_6n)} \tag{20}$$

The intermediate equations (16), (17), (18), (18′), (18″), and (19) are similarly affected. However equation (21), the optimal stocking model for sawtimber without a quality differential, is not affected by the correction.

Also, equation (31) is in error in one term. The corrected equation is:

$$D \cong \frac{\left(.024 + \frac{A}{5000}\right)(1 + p)^n - \left(.025 + \frac{A}{5000}\right) - .0001755625\ (S)}{[-\ .000000003375(S) - .000000000027\ (AS)]} + \frac{.006585025 - .0000014045\ (AS) + .0000526802\ (A)}{[-\ .000000003375(S) - .000000000027\ (AS)]} \tag{31}$$

These corrections do not affect the tabular values presented in the article as all solutions were generated from the corrected equations. Unfortunately, the equations in the text were not corrected prior to publication of the original article.

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$$VG_n = P_G(b_0 + b_1A^{-1} + b_2SD + b_3D^2) - VC(D) \tag{4}$$

where

VG_n = value growth of the stand over period n, dollars
VC = variable costs of growing and selling the timber, per merchantable un't of stocking, dollars

Substituting equation (3′) into equation (4)

$$VG_n = (b_4 + b_5A + b_5n)(b_0 + b_1A^{-1} + b_2SD + b_3D^2) - VC(D) \tag{4′}$$

and taking the density derivative of this equation yields the marginal va'ue growth equation

$$\Delta VG_n = \Delta D[b_2S(b_4 + b_5A) + b_2S(b_5n) + (2D + \Delta D)(b_3)(b_4 + b_5A) + (2D + \Delta D)(b_3b_5n) - VC] \tag{5}$$

where

ΔD = increment of density, merchantable volume units
ΔVG_n = marginal value growth of the stand over the period n, dollars, i.e., the additional value growth associated with ΔD

To convert this to a percent of capital value it is necessary to recall that

$$(1 + p)^n = \frac{V_n}{V_o} \tag{1″}$$

In using formula (1″) to determine optimal stocking, V_o must be taken as the value of ΔD, i.e., the value of an additional unit of stocking. Thus,

$$V_o = P\Delta D \tag{6}$$

then,

$$V_n = V_o + \Delta VG_n \tag{7}$$

Substituting into equation (1″) we find that

$$(1 + p)^n = \frac{P\Delta D + \Delta VG_n}{P\Delta D} \tag{8}$$

$$(1+p)^n = \frac{P\Delta D + \Delta D[b_2S(P) + b_2S(b_5n) + (2D+\Delta D)(b_3)(P) + (2D+\Delta D)(b_3b_5n) - VC]}{P\Delta D} \tag{8′}$$

$$(1+p)^n = 1 + b_2S + b_3(2D + \Delta D) + \frac{b_2S(b_5n) + (2D + \Delta D)(b_3b_5n) - VC}{P} \tag{8″}$$

$$(1+p)^n = 1 + b_2S + b_3(2D + \Delta D) + \frac{n[(b_2b_5S) + (2D + \Delta D)(b_3b_5)] - VC}{(b_4 + b_5A)} \tag{8‴}$$

If the manager is selling timber as stumpage and if the management costs, which are variable with respect to stand density, are assumed negligible, then conversion surplus is approximated by stumpage price. An alternative assumption would be that the stumpage price used in the analysis be assumed net of the variable costs of growing and selling the timber. In this study such a condition is assumed to hold. Therefore,

$$(1 + p)^n \cong 1 + b_2S + b_3(2D + \Delta D) + \frac{nb_5[(b_2S) + (2D + \Delta D)(b_3)]}{(b_4 + b_5A)} \tag{9}$$

Note that if b_4 is equal to some positive constant,say 1.0, and b_5 is equal to zero, then

$$(1 + p)^n \cong 1 + b_2S + b_3(2D + \Delta D) + \frac{n(0)[(b_2S) + (2D + \Delta D)(b_3)]}{(1 + 0)} \tag{10}$$

$$(1 + p)^n \cong 1 + b_2S + b_3(2D + \Delta D) \tag{10'}$$

Thus, stumpage is assumed to be positive and constant.

The optimal stocking level for the pulpwood objective is found by solving equation (10′) for D,

where

p = the alternative rate of return that is

$$D \cong \frac{(1 + p)^n - 1 - b_2S - b_3\Delta D}{2b_3} \tag{11}$$

The sawtimber stocking model hypothesized is similar to the above, but the form of the variables included differs slightly. A growth model of the following form is stipulated:

$$G_n = c_0 + c_1(D) + c_2(SD) + c_3(SD^2) + c_4(A^{-1}) \tag{12}$$

where

D = initial board foot volume, Int. 1/4-inch rule
c_0, c_1, c_2, c_3, c_4 = constant and partial regression coefficients
and value growth is expressed by

$$VG_n = P_G[c_0 + c_1(D) + c_2(SD) + c_3(SD^2) + c_4(A^{-1})] - VC(D) \tag{13}$$

The price functions, as before, are

$$P = c_5 + c_6A \tag{14}$$

$$P_G = c_5 + c_6(A + n) \tag{15}$$

Hence, by substituting equation (15) into equation (13) we find

$$VG_n = (c_5 + c_6A + c_6n)[c_0 + c_1(D) + c_2(SD) + c_3(SD^2) + c_4(A^{-1})] - VC(D) \quad (16)$$

As before, the density derivative of this equation yields the marginal value growth equation

$$\Delta VG_n = \Delta D[(c_5 + c_6A + c_6n)(c_1 + c_2S + c_3S(2D + \Delta D)) - VC] \quad (17)$$

To convert to a percentage of capital value it is again necessary to substitute into equation (8) yielding

$$(1 + p)^n = \frac{P\Delta D + \Delta D[(P + c_6n)(c_1 + c_2S + c_3S(2D + \Delta D)) - VC]}{P\Delta D} \quad (18)$$

$$(1 + p)^n = 1 + c_1 + c_2S + c_3S(2D + \Delta D) + \frac{c_6n[c_1 + c_2S + c_3S(2D + \Delta D)] - VC}{P} \quad (18')$$

$$(1 + p)^n = 1 + c_1 + c_2S + c_3S(2D + \Delta D) + \frac{c_6n[c_1 + c_2S + c_3S(2D + \Delta D)] - VC}{c_5 + c_6A} \quad (18'')$$

If the variable costs are again considered negligible, then

$$(1 + p)^n \cong 1 + c_1 + c_2S + c_3S(2D + \Delta D) + \frac{c_6n[c_1 + c_2S + c_3S(2D + \Delta D)]}{c_5 + c_6A} \quad (19)$$

Optimal stocking may then be determined by setting p equal to the alternative rate of return and solving equation (19) for D, yielding:

$$D \cong \frac{(c_5 + c_6A)[(1 + p)^n - (1 + c_1 + c_2S + c_3S\Delta D)] - c_6n(c_1 + c_2S + c_3S\Delta D)}{2c_3S(c_5 + c_6A + c_6n)} \quad (20)$$

If stumpage price is constant regardless of stand age (c_5 equal to 1.0, and c_6 equal to zero) then

$$D \cong \frac{[(1 + p)^n - (1 + c_1 + c_2S + c_3S\Delta D)]}{2c_3S} \quad (21)$$

Equations (20) and (21) are the mathematical models for optimal stocking in the case of the sawtimber objective.

These models constitute an application of marginal analysis to the problem of ascertaining optimal stocking levels in forest management. It is a problem of equating a marginal unit cost, the alternative rate of return, to the marginal unit revenue, the value growth rate of a marginal or additional unit (ΔD) of growing stock.

Optimal rotation model. For a given stand of timber the optimal rotation may be determined using the traditional soil rent approach. This method designates as an optimum that rotation length which yields a maximum capitalized value of net income from a perpetual stream of rotations. The soil rent approach provides solutions which fully take into account both the interest charges on invested capital and the influence of future rotations on optimal rotation length. The soil rent approach, as applied here, assumes that all the stands and values of future rotations will be identical to those of the present rotation. This is certainly a heroic assumption, but it should be remembered that inasmuch as future returns are discounted, the rotations beyond the first are of decreasing importance the more distantly they occur in the future. If a firm were to buy the land for no more than the soil rent value per acre and follow the stipulated program, and if the assumptions proved correct, the firm would make no less than the alternative rate of return on its investment before income taxes (Duerr 1960, p. 242).

The soil rent model can be expressed in the following form:

$$S_r = \frac{a}{(1 + p)^n - 1} \tag{22}$$

where

S_r = soil rent value, dollars
p = alternative rate of return, decimal
n = length of rotation, years

a, the net income at rotation age, is defined as follows:

$$a = Y_n + T_a(1 + p)^{n-a} + T_b(1 + p)^{n-b} + \cdots + T_z(1 + p)^{n-z} - C(1 + p)^n \tag{23}$$

where

Y_n = harvest yield, dollars
$T_a, T_b, \cdots, T_z$ = thinning yields at ages $a, b, \cdots z$, dollars
C = regeneration cost, dollars
n = length of rotation, years

Fixed costs are omitted from the mode' expressed in equation (22) and from the application made in this paper, because they do not affect the marginal values and hence have no bearing on the determination of optimal rotation age. The objective of this paper is to determine at what intensity the stumpage enterprise should be operated, not to answer the entirely different question of whether one should grow timber at all. The answer to the latter question requires calculation of net profit, and accordingly, would take account of fixed costs.

Biological Model

To provide biological data for the economic models previously discussed, a loblolly pine stand density study in natural stands, established by the Southeastern Forest

Experiment Station in 1949–50, was used.[1] The 5- to 10-year remeasurements of the managed or cut portion of this study provide the biological data used in this paper.

The physical details describing these plots are more adequately described elsewhere (Nelson *et al.* 1961, Nelson *et al.* 1963). Briefly, 153 circular 1/4-acre plots with 1/2-chain isolation strips were selected in 20- to 60-year-old stands ranging in site index from less than 60 to more than 100. Using Stahelin's percent of full stocking as a base (Stahelin 1949), stocking ranged from 40 to 130 percent. Only pure stands, even-aged and with uniformly spaced, insect-and disease-free loblolly pine were used.

Approximately two-thirds of the previously unthinned 153 plots were thinned, one-third at the time of plot establishment and another third at the end of the first 5-year growth period. At this time the plots thinned at the time of establishment were rethinned. Except for modifications as necessary to permit uniform spacing of residuals and to remove undesirable trees in all crown classes, the thinning was from below. Hardwoods 4.6 inches dbh and larger were poisoned. Growth of only the thinned plots, expressed in terms of net periodic annual increment,was used in this report.

The analyses led to the development of the following equations for merchantable periodic annual cubic-foot growth and board-foot growth, respectively:

$$G = -134.7005 + 0.49694\left(\frac{10{,}000}{A}\right) + 0.00081\,(SD) - 0.00593\,\frac{(D)^2}{1{,}000} \qquad (24)$$

$$R^2 = .790$$

where

D = initial merchantable cubic-foot volume of trees 4.6 inches dbh and larger per acre
G = merchantable periodic annual cubic-foot growth of trees 4.6 inches dbh and larger

$$G = -571.18 - 0.05268(D) + 0.001418(SD) - 0.0000000135(SD^2) + 2.80391\left(\frac{10{,}000}{\text{Age}}\right) \qquad (25)$$

$$R^2 = .750$$

where

D = initial board-foot volume, Int. 1/4-inch rule, of trees 9.6 inches dbh and larger per acre
G = periodic annual board-foot growth, Int. 1/4-inch rule

Periodic annual cubic-foot growth tables are presented in Table 1 as calculated

[1] Plots were located on the Hitchiti Experimental Forest in Georgia, on the Santee and Westvaco Experimental Forests in South Carolina, and on the Camp Experimental Forest in Virginia. The Westvaco Experimental Forest is maintained by the West Virginia Pulp and Paper Company; the Camp Experimental Forest is maintained by the U.S. Forest Service in cooperation with the Union Bag-Camp Paper Corporation.

TABLE 1. Relation of periodic annual cubic-foot growth to age, site, and initial cubic-foot volumes.

Age	Initial merchantable cubic-foot stocking per acre[1]															
	500	1000	1500	2000	2500	3000	3500	4000	4500	5000	5500	6000	6500	7000	7500	8000
								Site index 60								
20	137	156	173	187	198	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	
25	87	107	124	138	148	156	162	. .	. .	. .	. .	. .	. .	. .	. .	
30	54	73	90	104	115	123	128	130	129	. .	. .	. .	. .	. .	. .	
35	30	50	67	81	92	100	105	107	106	102	95	. .	. .	. .	. .	
40	12	32	49	63	74	82	87	89	88	84	77	68	. .	. .	. .	
45	−1	18	35	49	60	68	73	75	74	70	64	54	41	. .	. .	
50	−13	7	24	38	49	57	62	64	63	59	53	43	30	14	. .	
55	−22	−2	15	29	40	48	53	55	54	50	44	34	21	5	. .	
60	−29	−9	8	22	33	41	46	48	47	43	36	26	14	−2	. .	
								Site index 70								
20	141	165	185	203	218	230	. .	. .	. .	. .	. .	. .	. .	. .	. .	.
25	91	115	136	154	169	181	190	196	199	. .	. .	. .	. .	. .	. .	
30	58	82	102	120	135	148	157	163	166	166	163	. .	. .	. .	. .	
35	34	58	79	97	112	124	133	139	142	142	140	134	125	. .	. .	
40	16	40	61	79	94	106	115	121	125	125	122	116	108	96	. .	
45	3	26	47	65	80	92	102	108	111	111	108	102	94	82	. .	.
50	−8	15	36	54	69	81	90	97	100	100	97	91	83	71	. .	.
55	−17	6	27	45	60	72	81	88	91	91	88	82	74	62	. .	
60	−25	−1	20	38	53	65	74	80	83	84	81	75	66	55	. .	
								Site index 80								
20	145	173	198	220	239	255	268	278	. .	. .	. .	. .	. .	. .	. .	.
25	95	123	148	170	189	205	218	228	236	240	241	. .	. .	. .	. .	.
30	62	93	115	137	156	172	185	195	202	207	208	206	201	194	. .	
35	38	66	91	113	132	148	161	172	179	183	184	183	178	170	160	146
40	20	48	73	95	114	131	144	154	161	165	167	165	160	153	142	128
45	7	35	60	82	101	117	130	140	147	151	153	151	146	139	128	115
50	−4	24	49	71	90	106	119	129	136	140	142	140	135	128	117	104
55	−13	14	40	62	80	97	110	120	127	131	133	131	126	119	108	94
60	−21	7	32	54	73	89	102	113	120	124	125	124	119	111	101	87
								Site index 90								
20	149	181	210	236	259	279	296	310	322	330	. .	. .	. .	. .	. .	.
25	99	131	160	186	209	229	247	261	272	280	286	288	287	284	. .	.
30	66	98	127	153	176	196	213	228	239	247	252	255	254	251	244	234
35	42	74	103	129	152	173	190	204	215	224	229	231	231	227	220	211
40	24	57	86	112	135	155	172	186	198	206	211	213	213	209	203	193
45	11	43	72	98	121	141	158	172	184	192	197	200	199	195	189	179
50	0	32	61	87	110	130	147	161	173	181	186	189	188	184	178	168
55	−9	23	52	78	101	121	138	152	164	172	177	180	179	175	169	159
60	−17	15	44	70	93	114	131	145	156	165	170	172	172	168	161	152
								Site index 100								
20	153	189	222	252	279	303	325	343	358	371	380	386	. .	. .	. .	.
25	103	139	172	202	230	254	275	293	308	321	330	336	340	340	338	333
30	70	106	139	169	196	220	242	260	275	288	297	303	307	307	305	299
35	46	82	115	146	173	197	218	236	252	264	273	280	283	284	281	276
40	29	65	98	128	155	179	200	219	234	246	256	262	266	266	263	258
45	15	51	84	114	141	165	186	205	220	232	242	243	252	252	250	244
50	4	40	73	103	130	154	176	194	209	221	231	237	241	241	239	233
55	−5	31	64	94	121	145	166	185	200	212	222	228	232	232	230	224
60	−13	23	56	87	114	138	159	177	193	205	214	221	224	224	222	217

[1] At the beginning of each 5-year growth period.

from equation (24) and board-foot growth tables are shown in Table 2 as calculated from equation (25).

The data of Tables 1 and 2 are the basic growth and stocking information used to calculate optimal stocking levels and optimal rotation ages. The values in these tables approach ideal tree distribution and growing conditions for a given age, site, and stand density. As stated previously, the stand density study plots providing

TABLE 2. Periodic annual board-foot growth (Int. 1/4– rule) per acre of trees 9.6 inches dbh and larger to a 7-inch top diameter inside bark as related to age, initial board-foot volume, and site index.

Age	Initial board-foot stocking (MBF) per acre[1]																							
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	Site index 60																							
30	425	453	480	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..					
35	291	319	346	371	395	417	..	..	..	..	..	..	..	..	..	..	..	..						
40	191	219	246	271	295	317	337	355	..	..	..	..	..	..	..	..	..	..	..					
45	113	141	168	193	217	239	259	277	294	..	..	..	..	..	..	..	..	..	..					
50	51	79	106	131	155	177	197	215	232	248	262	..	..	..	..	..	..	..	..					
55	0	28	55	80	104	126	146	164	181	197	211	223	233	..	..	..	..	..	..					
60	..	..	19	38	62	84	104	122	139	155	169	181	191	200	..	..	..	..	..					
65	..	..	..	2	26	48	68	86	103	119	133	145	155	164	171	..	..	..	..					
70	..	..	..	..	..	17	37	55	72	88	102	114	124	133	140	146	..	..	..					
	Site index 70																							
30	453	497	535	573	609	643	676	706	735	761	786	..	..	..	..	..	..	..	..	..	..	..	..	..
35	319	363	401	439	475	509	542	572	601	627	652	675	696	716	733	..	..	..	..	..	..	..	..	..
40	219	263	301	339	375	409	442	472	501	527	552	575	596	616	633	648	662	673	..	..	..	..	..	..
45	141	185	223	261	297	331	364	394	423	449	474	497	518	538	555	570	584	595	605	613	619	622	625	..
50	79	123	161	199	235	269	302	332	361	387	412	435	456	476	493	508	522	533	543	551	557	560	563	563
55	28	72	110	148	184	218	251	281	310	336	361	384	405	425	442	457	471	482	492	500	506	509	512	512
60	..	30	68	106	142	176	209	239	268	294	319	342	363	383	400	415	429	440	450	458	464	467	470	470
65	..	..	32	70	106	140	173	203	232	258	283	306	327	347	364	379	393	404	414	422	428	431	434	434
70	..	..	1	39	75	109	142	172	201	227	252	275	296	316	333	348	362	373	383	391	397	400	403	403

TABLE 2. (ctd.)

Age	Initial board-foot stocking (MBF) per acre (Continued)																							
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	Site index 80																							
30	481	536	589	640	689	736	780	823	863	901	937	971	1,002	1,032	1,059	1,084	1,107	1,128	1,147	1,163	..	..	..	..
35	347	402	455	506	555	602	646	689	729	767	803	837	868	898	925	950	973	994	1,013	1,029	1,043	1,056	1,066	1,073
40	247	302	355	406	455	502	546	589	629	667	703	737	768	798	825	850	873	894	913	929	943	956	966	973
45	169	224	277	328	377	424	468	511	551	589	625	659	690	720	747	772	795	816	835	851	865	878	888	895
50	107	162	215	266	315	362	406	449	489	527	563	597	628	658	685	710	733	754	773	789	803	816	826	833
55	56	111	164	215	264	311	355	398	438	476	512	546	577	607	634	659	682	703	722	738	752	765	775	782
60	14	69	122	173	222	269	313	356	396	434	470	504	535	565	592	617	640	661	680	969	710	723	733	740
65	..	33	86	137	186	233	277	320	360	398	434	468	499	529	556	581	604	625	644	660	674	687	697	704
70	..	2	55	106	155	202	246	289	329	367	403	437	468	498	525	550	573	594	613	629	643	656	666	673
	Site index 90																							
30	508	559	644	708	769	829	885	940	991	1,041	1,088	1,132	1,174	1,214	1,251	1,286	1,319	1,349	1,376	1,401	1,424	1,444	1,462	1,478
35	374	425	510	574	635	695	751	806	857	907	954	998	1,040	1,080	1,117	1,152	1,185	1,215	1,242	1,267	1,290	1,310	1,328	1,344
40	274	325	410	474	535	595	651	706	757	807	854	898	940	980	1,017	1,052	1,085	1,115	1,142	1,167	1,190	1,210	1,228	1,244
45	196	247	332	396	457	517	573	628	679	729	776	820	862	902	939	974	1,007	1,037	1,064	1,089	1,112	1,132	1,150	1,166
50	134	185	270	334	395	455	511	566	617	667	714	758	800	840	877	912	945	975	1,002	1,027	1,050	1,070	1,088	1,104
55	83	134	219	283	344	404	460	515	566	616	663	707	749	789	826	861	894	924	951	976	999	1,019	1,037	1,053
60	41	92	177	241	302	362	418	473	524	574	621	665	707	747	784	819	852	882	909	934	957	977	995	1,011
65	5	56	141	205	266	326	382	437	488	538	585	629	671	711	748	783	816	846	873	898	921	941	959	975
70	..	25	110	174	235	295	351	406	457	507	554	598	640	680	717	752	785	815	842	867	890	910	928	944
	Site index 100																							
30	536	619	698	775	850	921	990	1,056	1,120	1,180	1,238	1,294	1,347	1,396	1,444	1,488	1,530	1,569	1,606	1,640	1,671	1,699	1,725	1,748
35	402	485	564	641	716	787	856	922	986	1,046	1,104	1,160	1,213	1,262	1,310	1,354	1,396	1,435	1,472	1,506	1,537	1,565	1,591	1,614
40	302	385	464	541	616	687	756	822	886	946	1,004	1,060	1,113	1,162	1,210	1,254	1,296	1,335	1,372	1,406	1,437	1,465	1,491	1,514
45	224	307	386	463	538	609	678	744	808	868	926	982	1,035	1,084	1,132	1,176	1,218	1,257	1,294	1,328	1,359	1,387	1,413	1,436
50	162	245	324	401	476	547	616	682	746	806	864	920	973	1,022	1,070	1,114	1,156	1,195	1,232	1,266	1,297	1,325	1,351	1,374
55	111	194	273	350	425	496	565	631	695	755	813	869	922	971	1,019	1,063	1,105	1,144	1,181	1,215	1,246	1,274	1,300	1,323
60	69	152	231	308	383	454	523	589	653	713	771	827	880	929	977	1,021	1,063	1,102	1,139	1,173	1,204	1,232	1,258	1,281
65	33	116	195	272	347	418	487	553	617	677	735	791	844	893	941	985	1,027	1,066	1,103	1.137	1,168	1,196	1,222	1,245
70	2	85	164	241	316	387	456	522	586	646	704	760	813	862	910	954	996	1,035	1,072	1,106	1,137	1,165	1,191	1,214

[1] At the beginning of each 5-year growth period.

TABLE 3. Optimal stocking for various alternative rates of return in merchantable cubic feet per acre.

Alternative rate of return	Site index				
	60	70	80	90	100
	merchantable cubic feet per acre				
3	1,162	1,845	2,528	3,211	3,894
4	194	877	1,560	2,243	2,926
5	----	----	555	1,238	1,921
6	----	----	----	193	876
7	----	----	----	----	----

the original raw growth data were limited to one-quarter-acre in size; trees were uniformly spaced and free of insects and disease; hardwood competition was completely absent.

Optimal Levels of Growing Stock

Pulpwood production. Optimal stocking levels for pulpwood production on sites 60 to 100 are presented in Table 3 for five alternative rates of return. For the indicated stocking levels, marginal value growth rates are equated to the corresponding alternative rates of return.

Marginal value growth rates were calculated by first substituting the coefficients and constant from equation (24), into equation (11) and taking $\Delta D = 500$ merchantable cubic feet, deriving equation (26):

$$D \cong \frac{0.985175 + 0.00405(S) - (1 + p)^5}{0.0000593} \tag{26}$$

where

D = stocking at the beginning of the growth period in merchantable cubic feet

Then substitution of values of S and p into equation (26) provides a direct solution to the optimal stocking level for each combination of site index and guiding rate of return.

In this analysis optimal stocking does not vary with stand age except when a quality differential is assumed for sawtimber, in which case stumpage price is assumed to be a function of stand age. This means that marginal growth is taken to be constant for a given stocking increase regardless of stand age. This rather surprising effect occurs because, within the range of data considered in this study, age enters the physical growth model only as a main effect. For optimal stocking to vary with age, it would be necessary that an age-density interaction be a significant variable in the physical growth model. The age-density interaction was tested in this study and found not to approach statistical significance once significant terms were removed from the predicting equation (most potent variable regression analysis was used in this study). The age-density interaction might be significant if

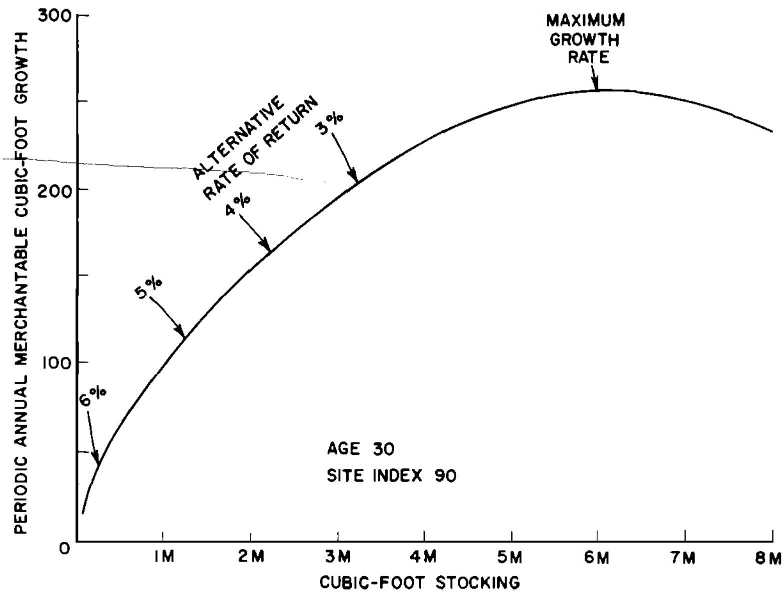

FIGURE 1. *Loblolly pine stocking for various alternative rates of return for pulpwood production.*

the range of the age variable were extended beyond that of the study. All that can be said at this time is that the age-density interaction requires further testing in growth studies.

Since equation (11) was derived on the assumption that the unit value of growing stock is constant and positive for all ages and sizes of timber, the same assumption applies to solutions of optimal stocking based on equation (26). In other words, the optimal stocking levels given in Table 3 have a built-in assumption that the merchantable cubic-foot growth during any 5-year period commands the same price in the market as the growing stock upon which it accrues.

TABLE 4. Comparison of maximum cubic-foot growth rate and optimal growth at various alternative rates of return at age 30.

Site index	Maximum growth rate	Optimal growth rates: Alternative rates of return 3%	4%	5%
	Periodic annual merchantable cubic-foot growth			
60	130	79	41	--
70	166	116	77	--
80	208	157	118	65
90	255	204	165	113
100	307	256	218	165

TABLE 5. Optimal stocking at age 30[1] *for pulpwood production for alternative rates of return with stocking expressed in terms of basal area per acre.*

Alternative rate of return	Site index				
	60	70	80	90	100
			Square feet		
3	75	84	92	100	108
4	47	55	63	71	79
5	--	--	34	42	50
6	--	--	--	11	19
7	--	--	--	--	--

[1] For an age other than 30 years, the following correction factors can be applied:

Age (years)	*Basal Area Correction (square feet per acre)*
20	+31
25	+12
35	—9
40	—16

Table 4 compares the growth rates per acre at the several optimal stocking levels with the maximum growth rate for each site index at age 30. It is apparent that optimal cubic-foot production for the indicated rates of return is much less than the maximum growth rate at age 30 using site index 90 as an example (Fig. 1). The low growth rates on poor sites at the alternative rates of return shown in Table 4 indicate that the better sites deserve the attention and effort of the landowner.

Table 5 presents optimal stocking levels for pulpwood production in terms of basal area. These were derived from equation (27), which is based on the correlation between initial basal area and initial merchantable cubic-foot volume from the same stand density plots used to provide the basic growth data for this study.

TABLE 6. Optimal stockings in board feet per acre for various alternative rates of return without a quality differential.

Alternative rate of return	Site index				
	60	70	80	90	100
			Board feet per acre		
3	----	7,290	12,880	17,230	20,710
4	----	1,220	7,570	12,510	16,460
5	----	----.	2,050	7,600	12,040
6	----	----	----	2,500	7,450
7	----	----	----	----	2,690

$$Y = 50.7 + .18847\left(\frac{10{,}000}{\text{Age}}\right) - 1.20216(\text{Site}) + .02940\ (\text{Initial Cubic-Ft. Volume})$$

$$R^2 = .90 \qquad (27)$$

TABLE 7. Comparison of maximum board-foot growth[1] rate and optimal growth at various alternative rates of return at age 50.

Site index	Maximum growth rate	Optimal growth rates: Alternative rates of return 3%	4%	5%
60	314	--	--	--
70	564	279	47	--
80	844	593	389	109
90	1,145	922	741	492
100	1,460	1,257	1,093	869

[1] Perodic annual merchantable board-foot growth.

TABLE 8. Optimal stockings in board feet per acre for various alternative rates of return with a quality differential.

Alternative rate of return	Age in years: 30	35	40	45	50	55	60	65	70
					Site index 60				
3	4,450	4,310	4,170	4,050	3,920	3,810	3,700	3,600	3,510
4	......	......	......	......	......	......	......	......	......
					Site index 70				
3	11,250	11,120	11,010	10,900	10,800	10,700	10,610	10,520	10,440
4	5,370	5,240	5,120	5,010	4,900	4,800	4,700	4,610	4,520
5	......	......	......	......	......	......	......	......	......
					Site index 80				
3	16,340	16,240	16,130	16,040	15,950	15,860	15,780	15,710	15,630
4	11,200	11,090	10,980	10,880	10,790	10,700	10,610	10,530	10,460
5	5,860	5,740	5,630	5,520	5,420	5,330	5,240	5,160	5,080
6	310	180	70	......	......	......	......	......	......
7	......	......	......	......	......	......	......	......	......
					Site index 90				
3	20,310	20,210	20,120	20,040	19,960	19,880	19,810	19,740	19,680
4	15,740	15,640	15,540	15,450	15,370	15,290	15,210	15,140	15,080
5	10,990	10,880	10,780	10,690	10,600	10,520	10,440	10,360	10,290
6	6,050	5,940	5,840	5,740	5,650	5,560	5,480	5,400	5,330
7	930	820	710	600	510	410	330	250	170
					Site index 100				
3	23,480	23,390	23,310	23,230	23,160	23,090	23,030	22,970	22,910
4	19,370	19,280	19,190	19,110	19,030	18,960	18,890	18,830	18,770
5	15,090	15,000	14,910	14,820	14,740	14,670	14,600	14,530	14,470
6	10,650	10,550	10,460	10,370	10,290	10,210	10,130	10,060	10,000
7	6,040	5,940	5,840	5,740	5,660	5,570	5,500	5,420	5,350

where

Y = basal area per acre of all stems 0.6 inches dbh and larger

Since values of the dependent variable D from equation (26) are used for the independent variable "Initial Cubic-Foot Volume" in equation (27), the basal area estimates of optimal stocking are somewhat less accurate than the cubic-foot volume estimates in Table 3. Although somewhat less accurate, they do provide another measure of the relative influence of site and the alternative rate of return on the optimal level of stocking.

Sawtimber production. Optimal stocking levels for sawtimber are presented in Table 6. These were determined from equation (28) in a manner analogous to the optimal stocking level determinations for pulpwood production.

$$D \cong \frac{.73660 + .0070225(S) - (1 + p)^5}{.000000135(S)} \tag{28}$$

where

D = stocking at the beginning of the growth period in board feet, Int. 1/4-inch rule

Equation (28) was derived by substituting the coefficients and constant from equation (25) into equation (21) and taking ΔD = 1,000 board feet. The Table 6

TABLE 9. Loblolly pine: yields of well-stocked stands used for initial stocking.

Site index	*Pulpwood yield at age 20*[1] *Merch. cu ft*
60	1,170
70	1,557
80	1,971
90	2,484
100	3,042
	Sawtimber yield at age 30 *Bd ft (Int. 1/4" rule)*[2]
60	2,100
70	6,000
80	10,100
90	16,600
100	24,700

[1] Adapted from Table 1, pp. 6–7. F. X. Schumacher and T. S. Coile. Growth and yields of natural stands of the southern pines. T. S. Coile, Inc., Durham, N.C., 115 pp.

[2] Yields in Schumacher and Coile appear in terms of the Scribner log rule. The yields were converted to International 1/4" rule by multiplying by 1.14, the conversion factor recommended by Cruikshank and Anderson 1955. Pine sawtimber stumpage prices in South Carolina, 1948–1954. U. S. Forest Service, Southeastern Forest Expt. Sta., Station Paper 57, 14 pp.

estimates are based on the assumption of a constant, positive unit value per board foot for all ages and sizes of timber.

In Table 7, growth rates per acre for the various optimal stocking levels are compared with the maximum growth rates for each site index at age 50. They are notably less than the potential maximum growth per acre. On site index class 60, there is no practicable stocking level even at the low guiding rate of 3 percent.

Table 8 presents optimal stocking levels for sawtimber production based on the assumption of a price differential for quality of \$1 more per MBF of growing stock per 5-year period beyond age 30. Stumpage value at age 30 was assumed to be \$30 per MBF (Int. 1/4-inch rule). Optimal stocking levels were determined from equation (31), which was derived by substituting the coefficients and constants from equation (25) and equations (29) and (30) into equation (20). The price functions fitting the assumptions concerning the price differential follow the form of equations (14) and (15) respectively:

$$P = .024 + .0002(A) \tag{29}$$

$$P_G = .024 + .0002(A + 5) \tag{30}$$

$$D \cong \frac{(.024 + \frac{A}{5{,}000})(1+p)^n - (.024 + \frac{A}{5{,}000}) - .0001755625(S) + .006585025 - .0000014045(AS) + .0000526802(A)}{[-.000000003375(S) - .000000000027(AS)]} \tag{31}$$

where

D = stocking at the beginning of the growth period in board feet, Int. 1/4-inch rule.

Age becomes a factor in estimating optimal stocking when a quality differential related to age is introduced. The influence of the quality differential is to increase the optimal stocking level (see Table 6 for comparison). The influence is relatively greater on the lower site index classes and at the higher alternative rates of return.

Optimal Rotation Ages

Optimal rotation ages were calculated by use of the soil rent formula for the alternative rates of return of 3 percent to 7 percent, two levels of regeneration cost, \$12 and \$28 per acre, site index classes 60 to 100, and two single-product objectives, pulpwood and sawtimber (with and without a quality premium).

In the analyses presented here, the following additional assumptions were made:

1. A uniform price would be paid for all tracts regardless of costs. Stumpage prices were \$0.06667 per merchantable cubic foot for pulpwood (equivalent to \$6 per 90-cubic-foot cord) and \$30 per MBF (Int. 1/4-inch rule) for the sawtimber objective without a quality premium.

2. For the sawtimber objective, a quality differential equal to \$1 per MBF per 5-year period beyond age 30 for both growing stock and thinnings, the base price on which this premium was added being \$30 per MBF.

3. No price change occurs over the period of time being considered, i.e., that present prices will hold for the future.

TABLE 10. Soil rent values (excluding annual costs)—pulpwood rotation with a regeneration cost of $12 per acre.

Age	3%	4%	5%	6%	7%
			Site index 60		
20	$69.88	$43.41	------	------	------
25	97.87	60.69	------	------	------
30	*104.59*	*64.47*	------	------	------
35	103.19	63.32	------	------	------
40	98.69	60.42	------	------	------
45	93.16	57.05	------	------	------
50	87.51	53.75	------	------	------
55	82.15	50.71	------	------	------
60	77.23	47.99	------	------	------
			Site index 70		
20	$101.89	$65.08	------	------	------
25	129.13	80.81	------	------	------
30	*135.68*	*83.59*	------	------	------
35	134.17	81.76	------	------	------
40	129.57	78.37	------	------	------
45	123.98	74.63	------	------	------
50	118.28	71.05	------	------	------
55	112.87	67.81	------	------	------
60	107.92	64.96	------	------	------
			Site index 80		
20	$136.13	$88.25	$60.22	------	------
25	163.75	103.35	69.63	------	------
30	*171.04*	*105.72*	*70.62*	------	------
35	170.05	103.61	68.84	------	------
40	165.84	100.01	66.28	------	------
45	160.53	96.13	63.70	------	------
50	155.05	92.43	61.35	------	------
55	149.83	89.11	59.33	------	------
60	145.02	86.19	57.63	------	------
			Site index 90		
20	$178.56	$116.96	$80.91	$57.60	------
25	207.15	131.82	89.22	63.28	------
30	*215.50*	*134.03*	*89.51*	*63.33*	------
35	215.25	131.81	87.26	61.71	------
40	211.58	128.13	84.37	59.75	------
45	206.68	124.19	81.55	57.91	------
50	201.52	120.45	79.03	56.34	------
55	196.54	117.09	76.88	55.04	------
60	191.94	114.15	75.08	54.00	------

Alternative rates of return (spanning 3%–7% columns)

(Table 10 ctd. on next page)

TABLE 10. (*ctd.*)

Age	3%	4%	5%	6%	7%
			Site index 100		
20	$224.71	$148.20	$103.41	$74.45	------
25	255.50	163.58	*111.18*	*79.05*	------
30	265.65	*166.14*	111.13	78.42	------
35	*266.65*	164.15	108.65	76.35	------
40	263.90	160.65	105.61	74.09	------
45	259.70	156.83	102.67	72.04	------
50	255.08	153.19	100.06	70.31	------
55	250.52	149.90	97.85	68.91	------
60	246.26	147.02	96.00	67.79	------

TABLE 11. Soil rent values (excluding annual costs)—pulpwood rotation with a regeneration cost of $28 per acre.

		Alternative rates of return			
Age	3%	4%	5%	6%	7%
			Site index 60		
20	$34.03	$13.98	------	------	------
25	67.24	35.09	------	------	------
30	77.38	41.34	------	------	------
35	*78.37*	*41.89*	------	------	------
40	75.62	40.21	------	------	------
45	71.40	37.75	------	------	------
50	66.78	35.13	------	------	------
55	62.23	------	------	------	------
60	57.96	------	------	------	------
			Site index 70		
20	$66.04	$35.64	------	------	------
25	98.50	55.21	------	------	------
30	108.47	*60.46*	------	------	------
35	*109.35*	60.33	------	------	------
40	106.50	58.16	------	------	------
45	102.22	55.33	------	------	------
50	97.55	52.43	------	------	------
55	92.95	49.72	------	------	------
60	88.65	47.28	------	------	------

(Table 11 ctd. on next page)

TABLE 11. (ctd.)

Age	3%	4%	5%	6%	7%
			Site index 80		
20	$100.28	$58.82	$34.55	------	------
25	133.12	77.75	46.92	------	------
30	143.83	*82.59*	*49.80*	------	------
35	*145.23*	82.18	49.29	------	------
40	142.77	79.80	47.63	------	------
45	138.78	76.82	45.69	------	------
50	134.32	73.81	43.82	------	------
55	129.91	71.01	42.16	------	------
60	125.75	68.51	40.73	------	------
			Site index 90		
20	$142.71	$87.53	$55.23	$34.35	------
25	176.52	106.21	66.51	42.42	------
30	188.29	*110.90*	*68.69*	*43.95*	------
35	*190.43*	110.38	67.71	43.31	------
40	188.51	107.92	65.72	42.02	------
45	184.93	104.89	63.54	40.66	------
50	180.79	101.83	61.50	39.42	------
55	176.62	99.00	59.70	38.37	------
60	172.67	96.47	58.17	------	------
			Site index 100		
20	$188.86	$118.76	$77.74	$51.20	------
25	224.87	137.98	88.48	58.19	------
30	238.44	*143.01*	*90.32*	*59.04*	------
35	*241.83*	142.72	89.11	57.96	------
40	240.83	140.44	86.96	56.36	------
45	237.94	137.53	84.66	54.78	------
50	234.35	134.57	82.53	53.39	------
55	230.60	131.81	80.67	52.23	------
60	226.99	129.34	79.10	51.29	------

4. The concept of minimum cut was ignored in this analysis because it tends to vary greatly from operation to operation.

5. A 5-year cutting cycle is assumed.

Since the stand density study produced only growth equations, it was necessary to introduce supplementary data for yields of pulpwood at age 20 and for yields of sawtimber at age 30. The yield table for natural stands of loblolly pine presented by Schumacher and Coile (1960) provides such a starting point. The yields shown in Table 9 were adapted from Table 1 in that publication.

The growth equations (24) and (25) were applied to the initial yields (stocking) to obtain estimates of stocking 5 years hence. Thinnings were assumed to be applied whenever actual stocking exceeded optimal stocking, as indicated by the optimal stocking equations (26), (28), and (31). In every case stands were thinned at 5-year

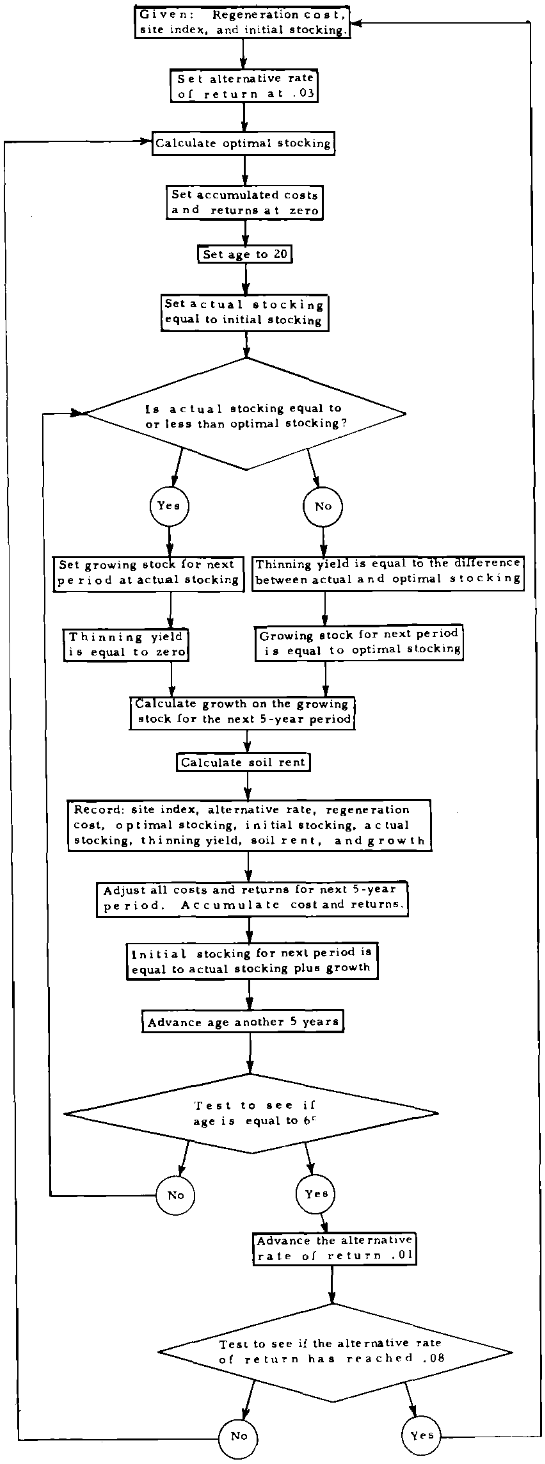

FIGURE 2. *Generalized flow chart used to develop optimal rotation ages for pulpwood objective.*

intervals, to their appropirate optimal stocking level, if a thinning was indicated at all.

The method of analysis used is illustrated compactly in a generalized flow chart (Fig. 2). This chart shows, step by step, the method used to develop optimal rotation ages for the pulpwood objective. The computer flow charts for the three specific analyses (pulpwood objective, sawtimber objective without quality premium, and sawtimber objective with quality premium) can be obtained from the authors upon request. The analyses were completed using an IBM 7070 electronic digital computer located at Duke University; the FORTRAN computer programs for these analyses can also be obtained from the authors.

The soil rent values obtained for the pulpwood objective are found in Table 10 for a regeneration cost of $12 and Table 11 for $28. The higher regeneration cost is intended to represent what may be expected if moderate site preparation measures are necessary; the lower figure applies where they are not. The individual may adjust these soil rent values to reflect his own situation by adding back into the soil

TABLE 12. Capitalized regeneration cost components of soil rent values.[1]

	Alternative rates of return				
Age	3%	4%	5%	6%	7%
		Regeneration Cost = $12 per acre			
20	$26.89	$22.08	$19.26	$17.44	$16.18
25	22.97	19.20	17.03	15.65	14.71
30	20.41	17.35	15.61	14.53	13.81
35	18.62	16.07	14.66	13.80	13.24
40	17.30	15.16	13.99	13.29	12.86
45	16.31	14.48	13.50	12.94	12.60
50	15.55	13.97	13.15	12.69	12.42
55	14.94	13.57	12.88	12.51	12.30
60	14.45	13.26	12.68	12.38	12.21
65	14.06	13.02	12.53	12.28	12.15
70	13.73	12.82	12.41	12.21	12.11
		Regeneration Cost = $28 per acre			
Age	3%	4%	5%	6%	7%
20	$62.73	$51.51	$44.93	$40.69	$37.76
25	53.60	44.81	39.73	36.51	34.32
30	47.62	40.48	36.43	33.90	32.23
35	43.44	37.50	34.20	32.19	30.89
40	40.38	35.37	32.64	31.02	30.00
45	38.07	33.78	31.51	30.19	29.40
50	36.27	32.59	30.67	29.61	28.98
55	34.86	31.66	30.05	29.18	28.69
60	33.72	30.94	29.58	28.88	28.49
65	32.80	30.37	29.23	28.65	28.35
70	32.05	29.92	28.95	28.48	28.25

[1] Capitalized regeneration cost component $= \dfrac{C(1.0 + p)^n}{(1.0 + p)^n - 1.}$

rent value the regeneration cost assumed in the analysis (either $12 or $28 capitalized) and then substracting the capitalized value of his own regeneration cost. Table 12, which lists capitalized regeneration cost components of soil rent value by age of stand and alternative rate of return, is included to facilitate this adjustment. To remove the effect of regeneration cost from the soil rent values presented in Tables 10 and 11, it is merely necessary to add the appropriate figure from Table 12. Then, in order to account for his actual regeneration costs, the grower

TABLE 13. Soil rent values (excluding annual costs)—sawtimber rotation without a quality differential and regeneration cost of $12 per acre.

Age	Alternative rates of return				
	3%	4%	5%	6%	7%
			Site index 60		
30	------	------	------	------	------
35	------	------	------	------	------
40	------	------	------	------	------
45	----	------	------	------	------
50	------	------	------	------	------
55	------	------	------	------	------
60	------	------	------	------	------
65	----	------	------	------	------
70	------	------	------	------	------
			Site index 70		
30	$105.71	$62.89	------	------	------
35	130.98	76.90	------	------	------
40	140.79	81.69	------	------	------
45	*142.81*	*82.14*	------	------	------
50	140.92	80.56	------	------	------
55	137.14	78.12	------	------	------
60	132.54	75.42	------	------	------
65	127.72	72.75	------	------	------
70	122.99	70.25	------	------	------
			Site index 80		
30	$191.89	$117.72	$75.60	------	------
35	220.13	131.13	83.27	------	------
40	232.68	135.50	*85.34*	------	------
45	*236.78*	*135.62*	84.95	------	------
50	236.49	133.81	83.55	------	------
55	233.98	131.19	81.82	------	------
60	230.40	128.34	80.08	------	------
65	226.41	125.55	78.47	------	------
70	222.35	122.97	77.05	------	------

(Table 13 ctd. on next page)

TABLE 13. (ctd.)

Age	3%	4%	5%	6%	7%
			Site index 90		
30	$328.52	$204.64	$134.30	$90.46	------
35	361.18	218.55	140.80	94.27	------
40	376.31	223.27	*142.04*	*94.73*	------
45	382.34	*223.65*	141.05	93.87	------
50	*383.57*	222.04	139.21	92.58	------
55	382.24	219.57	137.15	91.26	------
60	379.62	216.84	135.16	90.06	------
65	376.40	214.15	133.37	89.03	------
70	372.97	211.64	131.31	88.18	------
			Site index 100		
30	$498.77	$312.96	$207.45	$141.69	$98.25
35	535.22	328.20	213.88	*144.58*	*100.00*
40	553.04	333.88	*215.07*	144.40	99.74
45	561.10	*334.98*	214.05	143.10	98.80
50	*563.89*	333.90	212.18	141.50	97.75
55	563.81	331.84	210.10	139.95	96.79
60	562.19	329.44	208.10	138.58	95.97
65	559.77	327.00	206.29	137.43	95.32
70	557.01	324.69	204.72	136.49	94.82

should subtract his own capitalized regeneration cost component, which may be calculated using the following formula:

$$\text{Capitalized regeneration cost} = \frac{C(1.0 + p)^n}{(1.0 + p)^n - 1}$$

where

C = regeneration cost, dollars
p = alternative rate of return, decimal
n = rotation age, years

For example, if a grower found that his regeneration cost averaged $40 per acre, he would compute the capitalized regeneration cost at age 50 with an alternative rate of return of 4 percent as follows:

$$\text{Capitalized regeneration cost} = \frac{40(1.04)^{50}}{(1.04)^{50} - 1} = \$46.55$$

Looking then to Table 12, one may find a comparable figure to work with that is already used in the analysis. Taking that part of the table applicable to $28.00 per acre, the capitalized regeneration cost already subtracted from the solutions presented here for the age-alternative rate combination is $32.59. One may then

TABLE 14. Soil rent values (excluding annual costs)—sawtimber rotation without a quality premium and regeneration cost of $28 per acre.

Age	Alternative rates of return 3%	4%	5%	6%	7%
			Site index 60		
30	------	------	------	------	------
35	------	------	------	------	------
40	------	------	------	------	------
45	------	------	------	------	------
50	------	------	------	------	------
55	------	------	------	------	------
60	------	------	------	------	------
65	------	------	------	------	------
70	------	------	------	------	------
			Site index 70		
30	$78.50	$39.78	------	------	------
35	106.16	55.47	------	------	------
40	117.72	61.48	------	------	------
45	*121.06*	*62.83*	------	------	------
50	120.19	61.94	------	------	------
55	117.22	60.03	------	------	------
60	113.27	57.74	------	------	------
65	108.98	55.39	------	------	------
70	104.67	53.15	------	------	------
			Site index 80		
30	$164.68	$94.58	$54.78	------	------
35	195.31	109.70	63.73	------	------
40	209.61	115.29	66.69	------	------
45	215.02	*116.32*	*66.95*	------	------
50	*215.76*	115.19	66.02	------	------
55	214.06	113.10	64.64	------	------
60	211.13	110.66	63.17	------	------
65	207.66	108.20	61.77	------	------
70	204.04	105.87	60.51	------	------
			Site index 90		
30	$301.30	$181.51	$113.48	$71.08	------
35	336.36	197.12	121.25	75.88	------
40	353.24	203.06	*123.39*	*77.00*	------
45	360.59	*204.35*	123.05	76.61	------
50	*362.84*	203.42	121.68	75.66	------
55	362.32	201.48	119.98	74.58	------
60	360.35	199.16	118.26	73.56	------
65	357.65	196.79	116.67	72.66	------
70	354.66	194.54	115.26	71.90	------

(Table 14 ctd. on next page)

TABLE 14 (*ctd.*)

Age	3%	4%	5%	6%	7%
			Site index 100		
30	$471.56	$289.82	$186.64	$122.31	$79.83
35	510.40	306.77	194.33	126.19	82.35
40	529.96	313.67	*196.42*	*126.68*	*82.59*
45	539.35	*315.67*	196.04	125.85	82.00
50	543.17	315.28	194.65	124.58	81.19
55	*543.89*	313.75	192.93	123.28	80.39
60	542.92	311.76	191.19	122.08	79.69
65	541.03	309.64	189.59	121.06	79.12
70	538.70	307.60	188.18	120.22	78.68

TABLE 15. Soil rent values (excluding annual costs)—sawtimber rotation with a quality differential and regeneration cost of $12 per acre.

		Alternative rates of return			
Age	3%	4%	5%	6%	7%
			Site index 60		
30	$23.73	------	------	------	------
35	53.84	------	------	------	------
40	67.62	------	------	------	------
45	72.49	------	------	------	------
50	*72.51*	------	------	------	------
55	69.91	------	------	------	------
60	65.92	------	------	------	------
65	61.27	------	------	------	------
70	56.39	------	------	------	------
			Site index 70		
30	$105.71	$62.89	------	------	------
35	135.96	79.09	------	------	------
40	151.24	85.56	------	------	------
45	157.69	*87.16*	------	------	------
50	*159.03*	86.31	------	------	------
55	157.54	84.25	------	------	------
60	154.51	81.67	------	------	------
65	150.68	78.94	------	------	------
70	146.52	76.26	------	------	------

(Table 15 ctd. on next page)

TABLE 15 (*ctd.*)

Age	3%	4%	5%	6%	7%
			Site index 80		
30	$191.89	$117.72	$75.60	$49.35	------
35	228.09	135.82	84.94	55.29	------
40	249.28	143.85	88.25	*57.24*	------
45	260.42	146.68	*88.71*	------	------
50	265.60	*146.81*	87.85	------	------
55	*267.31*	145.56	86.42	------	------
60	266.98	143.64	84.81	------	------
65	265.47	141.46	83.23	------	------
70	263.30	139.25	81.77	------	------
			Site index 90		
30	$328.52	$204.64	$134.30	$90.46	$61.50
35	373.84	225.55	144.05	95.51	64.80
40	400.63	235.68	147.72	*96.86*	*65.59*
45	416.02	240.20	*148.48*	96.60	65.35
50	424.78	*241.73*	147.87	95.69	64.74
55	429.54	241.64	146.66	94.58	64.05
60	431.86	240.70	145.24	93.48	63.41
65	*432.66*	239.35	143.82	92.49	62.85
70	432.52	237.84	142.49	91.63	62.39
			Site index 100		
30	$498.77	$312.96	$207.45	$141.69	$98.25
35	552.34	337.08	218.49	146.83	100.91
40	584.20	349.76	223.16	*148.28*	*101.27*
45	603.84	356.35	224.74	148.12	100.74
50	616.21	359.60	*224.79*	147.30	99.94
55	624.12	360.96	224.13	146.28	99.12
60	629.18	*361.24*	223.16	145.26	98.38
65	632.40	360.94	222.11	144.32	97.76
70	*634.42*	360.32	221.09	143.52	97.26

select the set of soil rent values he wishes to alter, e. g., the pulpwood soil rent values (Table 11) for site index 90. The calculation would be

$$\text{Soil rent value} = \$101.83 + \$32.59 - \$46.55 = \$87.87$$

Therefore, the soil rent value for this particular combination of owner and circumstances would be $87.87.

The soil rent values applicable to sawtimber without a quality premium are shown in Table 13 for a regeneration cost of $12 and in Table 14 for a regeneration cost of $28. Tables 15 and 16 include the soil rent values for the sawtimber objective with the assumed quality premium.

TABLE 16. Soil rent values (excluding annual costs)—sawtimber rotation with a quality differential and regeneration cost of $28 per acre.

	Alternative rates of return				
Age	3%	4%	5%	6%	7%
			Site index 60		
30	------	------	------	------	------
35	$29.02	------	------	------	------
40	44.55	------	------	------	------
45	50.74	------	------	------	------
50	*51.79*	------	------	------	------
55	49.99	------	------	------	------
60	46.65	------	------	------	------
65	42.53	------	------	------	------
70	38.08	------	------	------	------
			Site index 70		
30	$78.50	$39.76	------	------	------
35	111.14	57.66	------	------	------
40	128.17	65.35	------	------	------
45	135.94	*67.86*	------	------	------
50	*138.31*	67.69	------	------	------
55	137.62	66.16	------	------	------
60	135.24	63.99	------	------	------
65	131.94	61.58	------	------	------
70	128.21	59.16	------	------	------
			Site index 80		
30	$164.68	$94.58	$54.78	$29.97	------
35	203.27	114.39	65.40	36.89	------
40	226.21	123.64	69.60	*39.52*	------
45	238.67	127.38	*70.71*	------	------
50	244.87	*128.19*	70.32	------	------
55	247.39	127.46	69.24	------	------
60	*247.71*	125.96	67.91	------	------
65	246.72	124.10	66.53	------	------
70	244.98	122.15	65.23	------	------
			Site index 90		
30	$301.30	$181.51	$113.49	$71.08	$43.08
35	349.02	204.12	124.51	77.12	47.15
40	377.56	215.47	129.07	79.13	48.44
45	394.27	220.90	*130.48*	*79.34*	*48.55*
50	404.05	223.11	130.34	78.77	48.18
55	409.63	*223.54*	129.49	77.90	47.66
60	412.59	223.02	128.34	76.98	47.13
65	413.92	221.99	127.12	76.12	46.65
70	*414.21*	220.74	125.95	75.36	46.25

(Table 16 ctd. on next page)

TABLE 16 (ctd.)

Age	3%	4%	5%	6%	7%
			Site index 100		
30	$471.56	$289.83	$186.64	$122.31	$79.83
35	527.52	315.65	198.95	128.44	83.25
40	561.13	329.55	204.52	130.55	*84.12*
45	582.08	337.05	206.74	*130.86*	83.94
50	595.49	340.98	*207.27*	130.38	83.38
55	604.20	342.86	206.95	129.60	82.72
60	609.91	343.56	206.25	128.76	82.10
65	613.66	*343.58*	205.41	127.95	81.56
70	*616.10*	343.23	204.54	127.24	81.11

TABLE 17. Optimal rotation length (years) with regeneration cost of $12 per acre.

Alternative rate of return (pct.)	Site index (feet at 50 years)				
	60	70	80	90	100
			Pulpwood		
3	30	30	30	30	35
4	30	30	30	30	30
5	--	--	30	30	25
6	--	--	--	30	25
7	--	--	--	--	--
			Sawtimber (without quality differential)		
3	--	45	45	50	50
4	--	45	45	45	45
5	--	--	40	40	40
6	--	--	--	40	35
7	--	--	--	--	35
			Sawtimber (with quality differential)		
3	50	50	55	65	70[1]
4	--	45	50	50	60
5	--	--	45	45	50
6	--	--	--	40	40
7	--	--	--	40	40

[1] Soil rent had not maximized at 70 years. The limits of the biological data did not allow further extrapolation.

It should be observed in the soil rent value tables that the soil rent value at any given age on a given site decreases as the alternative rate of return increases. This effect is ascribable, of course, to increased interest charges on invested capital (growing stock and regeneration costs), which reflect the attractiveness of the

TABLE 18. Optimal rotation length (years) with regeneration cost of $28 per acre.

Alternative rate of return (pct.)	*Site index (feet at 50 years)*				
	60	70	80	90	100
			Pulpwood		
3	35	35	35	35	35
4	35	30	30	30	30
5	--	--	30	30	30
6	--	--	--	30	30
7	--	--	--	--	--
			Sawtimber (without quality differential)		
3	--	45	50	50	55
4	--	45	45	45	45
5	--	--	45	40	40
6	--	--	--	40	40
7	--	--	--	--	40
			Sawtimber (with quality differential)		
3	50	50	60	70[1]	70[1]
4	--	45	50	55	65
5	--	--	45	45	50
6	--	--	--	45	45
7	--	--	--	45	40

[1] Soil rent had not maximized at 70 years. The limits of the biological data did not allow further extrapolation.

timber grower's forgone opportunities outside the forest. Soil rent value increases with site quality because more timber is produced. It should also be noted that soil rent value decreases with increased regeneration costs, for a given site, age and alternative rate of return. This reduction is equivalent to the capitalized difference between the regeneration costs.

In the soil rent value tables, the maximum soil rent values are shown in italics for each combination of site index and alternative rate of return. As pointed out previously, the age at which the highest soil rent value occurs is the optimal rotation length, given the assumptions of this study. Table 17 contains the optimal rotations given a regeneration cost of $12 per acre, and Table 18 applies to a regeneration cost of $28 per acre. Optimal rotations lengthen with increased site quality because value growth increases. Rotations shorten as the alternative rate of return increases because the marginal unit revenue, the value growth rate of the stand, is decreasing with age.

Optimal rotations with thinnings are ordinarily longer than optimal rotations without thinnings, partially because the volume of timber capital on which interest is charged is reduced. To be sure, the influence of future rotations on optimal length of rotations would tend to have the opposite effect, i.e., to shorten rotation length. It has generally been found, however, that the effects of the former prevail (Gaffney 1957).

Rotations lengthen as regeneration cost increases because it then becomes more profitable to defer reproduction cuts. The effect of the sawtimber quality premium is to increase the soil rent value and increase the rotation length. The increasing quality premium with age counteracts a decreasing volume growth rate with age.

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