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Height-Diameter Relationships at Age 15 in Loblolly Pine Seed Sources

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ABSTRACT. Data from nine locations not thinned by age 15 of the loblolly phase of the Southwide Pine Seed Source Study were analyzed to determine the effect of seed source on the shape and level of the height-diameter curves. Seed source affected the shape of the height-diameter curve at only one of the nine locations. Six of the nine locations showed a seed source effect on the level of the height-diameter regression. Seed source differences in height-diameter relationships can be integrated into growth and yield models by modifying the level of the height-diameter curve to reflect dominant height without altering the basic shape of the curve for a given site. Suggestions for further study are given. FOREST SCI. 32:812–818.

ADDITIONAL KEY WORDS. *Pinus taeda,* stand dynamics, growth and yield.

FOR THE PAST 10 TO 15 YEARS, there has been a steady increase in the proportion of genetically improved loblolly pine (*Pinus taeda* L.) seedlings planted, and it is expected that this general trend will continue.[1] As more acres are planted with genetically improved stock and the resulting stands mature, it becomes increasingly important to understand and model the development of these stands so that accurate yield forecasts and sound management decisions can be made. In modeling growth, we seek to mathematically interpret the biological relationships that underlie and drive stand development. Certain basic relationships are key components of stand development: (1) the development of dominant height through time, or the height-age relationship; (2) the development of height and diameter distributions through time; (3) the relationship of the mean height by diameter class to diameter across the range of diameters at a given age, or the height-diameter relationship; and (4) the mortality-time relationship. Quantifying the similarities and differences in the shapes and levels of these relationships for varied genetic material will lead to an understanding of the development of stands of improved stock and provide requisite information for modeling the growth of these stands.

Height-diameter relationships are used in many growth and yield models to predict the mean height for a given diameter or diameter class. The diameter and predicted height are used in stand volume and value calculations. In this note, the portion of the loblolly phase of the Southwide Pine Seed Source Study (Wells and Wakeley 1966) unthinned to age 15 is analyzed to assess the effect of seed source on the height-diameter relationship. This analysis provides a framework for building and testing hypotheses concerning genetic effects on the height-diameter relationship. Implications of the results for modifying existing growth and yield models to reflect seed source differences are discussed.

METHODS

The Southwide Pine Seed Source Study was established in 1952–53 to determine the degree that genetic variation in four southern pine species is associated with geographic variation in climate and physiography. Seed from 15 geographic areas was obtained and grouped

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TABLE 1. Analysis of variance results for the estimated coefficients b_i (eq. (2)) *and* a_i (eq. (3)) *for the fitted height-diameter equations at age 15 for nine locations of the Southwide Pine Seed Source Study.*

Location	Series	County or Parish	State	Results of ANOVA of b_i (slope) F_s[a]	MSE	Results of ANOVA of intercept (a_i) given a common slope, b F_s	MSE
E MD (1)	1	Worcester	Maryland	1.22	0.28939	29.43*	0.00064
SW GA (1)	1	Dooly	Georgia	4.43*	.06252	5.26*	.00147
S MS (1)	1	Pearl River	Mississippi	2.29	.04829	1.45	.01382
SE LA (1)	1	Washington	Louisiana	1.67	.24836	8.84*	.00041
W SC (2)	2	Newberry	South Carolina	1.41	.13029	1.21	.00423
C AL (2)	2	Coosa	Alabama	.76	1.0459	1.69	.00714
S MS (2)	2	Pearl River	Mississippi	1.06	.04884	3.80*	.00196
N MS (2)	2	Winston	Mississippi	1.08	5.8794	5.68*	.00494
SE LA (2)	2	Washington	Louisiana	1.17	.37347	13.53*	.00078

[a] F_s = MSS/MSE.

* An asterisk implies significance at the 0.05 level.

into two series. Three sources were common to each series and six sources were unique to each series. At each of 19 locations, the seedlings were planted in a randomized complete block design with four blocks. All locations were not planted with both series. Each seed source plot contained 121 trees on a 6 ft × 6 ft spacing. The inner 49 trees on each plot were measured at 1, 3, 5, 10, 15, 20, and 25 years after planting, although some measurements were made at 16, 22, and 27 years instead of 15, 20, and 25. Further details are given in Wells and Wakeley (1966).

Most of the plantations were thinned sometime after the age 10 measurement. Thinning confounds the effects of seed source on the height-diameter relationship. Plantations not thinned as of the age 15 measurement were used under the assumption that the height-diameter relationship would be well developed by age 15. This restriction provided 9 locations (plantations) for the analysis. Due to loss, two locations had two rather than four blocks available for the analysis, making a total of 32 block × location combinations. The locations used are given in Table 1.

The plotted curve of height vs. diameter for a given stand shows the expected mean height for a given diameter. Variation in this curve reflects variation in the underlying joint distribution of height and diameter. Variation attributable to seed source represents genetic differences in stand development, while variation attributable to blocks or locations represents environmental differences in stand development.

The analysis reported here consisted of the following: (1) fitting an appropriate model to the height-diameter pairs for each block × seed source combination (plot); (2) testing the hypothesis that the height-diameter curves do not differ by seed source using regression and analysis of variance techniques; and (3) examining the implications of the results of these tests.

The function chosen to model the height-diameter relationship is

$$\log(H) = a + bD^{-1} \tag{1}$$

where

H = total tree height for a given diameter
D = diameter breast height (Dbh)
a, b = estimated parameters
log = base e logarithm.

Equation (1) is commonly used to model height-diameter relationships (Curtis 1967) and has been shown to be useful for loblolly pine (Burk and Burkhart 1984). The slope parameter in equation (1), b, is the shape parameter of the fitted relationship when transformed to

TABLE 2. Form of analysis of variance for block and seed source effects on the parameters of the height-diameter relationship.

Source of variation	Degrees of freedom	Mean square	F-ratio
Blocks	$b - 1$		
Seed source	$s - 1$	MSS	F_s = MSS/MSE
Error	$(b - 1)(s - 1)$	MSE	

the height-diameter scale. The intercept parameter in equation (1), a, is related to the maximum height and governs the level of the fitted curve.

Equation (1) was fitted to the data from each plot. Values of r^2 were consistently larger than 0.88. Examination of residual plots showed no trends or evidence to indicate that equation (1) was inappropriate.

Equation (1) was expanded to the following to facilitate hypothesis testing by regression techniques:

$$\log(H) = a_1X_1 + a_2X_2 + \ldots + a_kX_k + b_1Z_1 + b_2Z_2 + \ldots + b_kZ_k \tag{2}$$

where: $X_i = \begin{cases} 1 \text{ if seed source } i, \\ 0 \text{ otherwise} \end{cases}$

$Z_i = \begin{cases} D^{-1} \text{ if seed source } i, \\ 0 \text{ otherwise} \end{cases}$

$k =$ number of seed sources in a given block at a given location.

Equation (2) was fitted to the data in each of the 32 block × location combinations separately and produced one (a, b) pair for each plot (block × seed source combination). Preliminary analysis indicated that there were significant seed source effects on the height-diameter relationship.

The four questions of primary interest in this study were:

1. Are there significant seed source effects on the shapes of the height-diameter relationships for the different seed sources on a given site?
2. Is there a common shape of the height-diameter relationships among the different seed sources on a given site?
3. If essential equality among the slope parameters, b, at a given location is assumed, are there significant seed source effects on the intercept or level parameter, a_i? and
4. If essential equality among the slope parameters, b, at a given location is assumed, do the slope parameters differ by location?

TABLE 3. r^2 *as a measure of association between the level parameter,* a_i, *of the height-diameter equation, and the plot dominant height for nine locations of the Southwide Pine Seed Source Study.*

Location	Series	Parameter estimates		r^2
		Intercept	Slope	
E MD	1	2.98	0.02416	0.80
SW GA	1	3.02	.020522	.88
S MS	1	2.95	.022282	.96
SE LA	1	3.71	.0107899	.64
W SC	2	3.01	.022475	.89
C AL	2	3.36	.0136071	.79
S MS	2	3.09	.019168	.87
N MS	2	3.10	.020214	.89
SE LA	2	3.45	.01496	.79

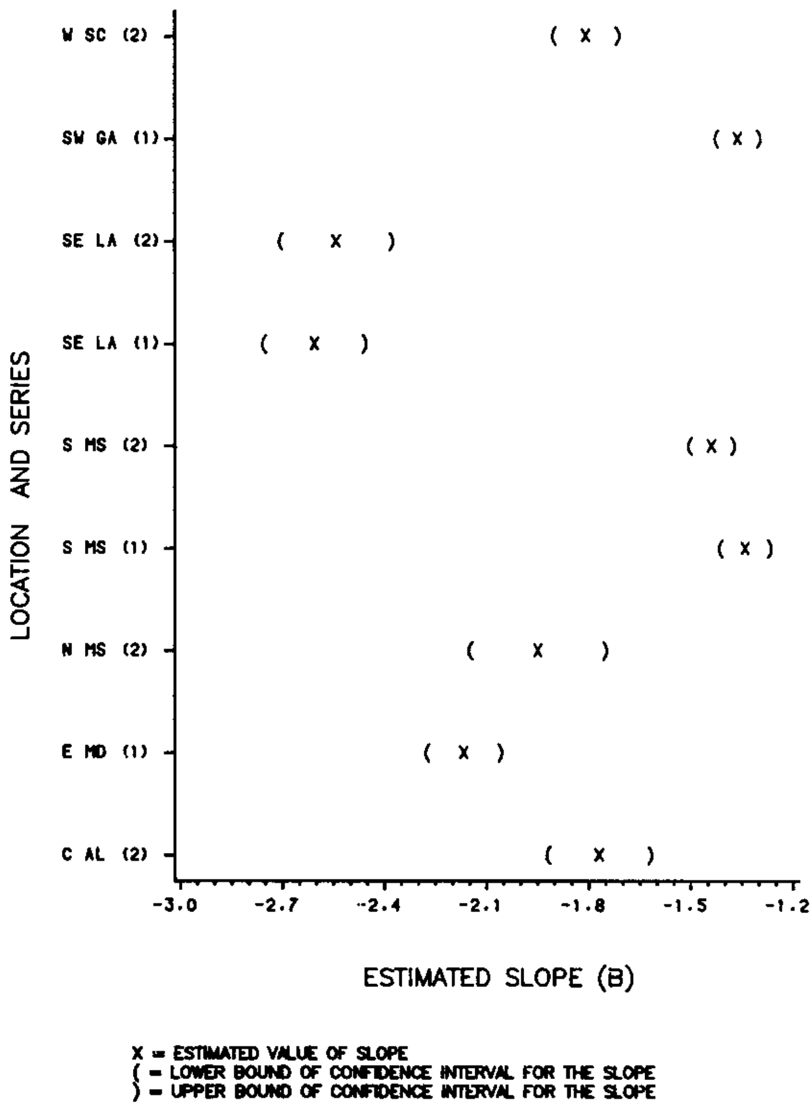

FIGURE 1. Family of confidence intervals for estimated slopes, b, for nine locations of the Southwide Pine Seed Source Study. Family confidence coefficient = 0.90.

The first question was addressed by performing an analysis of variance for each location following the form given in Table 2 on the b_i's estimated from Equation (2). For each location, the F-ratio, $\frac{\text{MSS}}{\text{MSE}}$, was compared to the tabulated $F_{0.05}$ with $(s - 1)$ and $(b - 1)(s - 1)$ degrees of freedom. The results are presented in Table 1.

The second question can be stated symbolically as

$$H_0\colon b_1 = b_2 = \ldots = b_k.$$

To test this hypothesis, the following equation was fitted to the data in each of the 32 block × location combinations:

$$\log(H) = a_1X_1 + a_2X_2 + \ldots + a_kX_k + bD^{-1} \tag{3}$$

where X_i is as defined above and b is a common slope. For each block × location combination, the F-ratio, $\left(\frac{SSE_3 - SSE_2}{SSE_2}\right)\left(\frac{n_2}{n_3 - n_2}\right)$, where SSE_i = error sum of squares and n_i = degrees of freedom for error, respectively, for equation (i), was compared to the tabulated $F_{0.05}$ with $n_3 - n_2$ and n_2 degrees of freedom. In only 3 of the 32 cases was the null hypothesis rejected.[2]

Question (3) was addressed by performing an analysis of variance for each location following the form given in Table 2 on the a_i's estimated from Equation (3). F-ratios, $\frac{MSS}{MSE}$, were compared to the tabulated $F_{0.05}$ with $(s - 1)$ and $(b - 1)(s - 1)$ degrees of freedom. The results are presented in Table 1.

Simple linear regressions were fitted to the a_i-dominant height pairs for each location to empirically assess their implicit association. The results are given in Table 3.

Question (4) was approached by calculating confidence intervals for the slope parameters estimated from equation (3) for the nine locations by the Bonferroni method. The lower bound on the family confidence coefficient is 0.90. The resulting family of confidence intervals is shown in Figure 1.

Results and Discussion

From Table 1, there are no significant seed source effects on the estimated slope parameters, b_i, for 8 of the 9 locations. Using a test described by Fisher (1946) for combining independent tests of significance, the probability of this aggregate of test results occurring by chance is 0.03. For these data, this implies that seed source does not affect the shape of the height-diameter relationship.

In 30 of the 32 block × location combinations, the test procedure failed to reject the hypothesis of equality of slopes among the seed sources at a given location. The probability of this aggregate of test results occurring by chance is less than 0.0001. For these data, this implies that equation (3) is an acceptable representation of the height-diameter relationship for the seed sources in a given block. Moreover, this result implies that a common shape for the height-diameter relationship may be assumed for the various seed sources in a given block.

The analysis of variance performed on the level parameter, a_i, estimated using equation (3) showed significant seed source effects on the a_i for 6 of the 9 locations (see Table 1). The probability of this aggregate of test results occurring by chance is less than 0.0001. This implies that the level of the height-diameter relationship is affected by seed source on a given site. The level parameter is the logarithm of the asymptotic height in the height-diameter relationship. That is, the antilog of a_i in the height-diameter relationship for a given age can be considered to be the maximum height for that given age. A strong relationship between the a_i and the dominant height of the i^{th} plot at that given age is implied by the model form. Results in Table 3 show that the r^2-values for a simple linear regression of the a_i on plot dominant height at age 15 for each location are 0.79 or greater for 8 of the 9 locations. Using the r^2 simply as a measure of association and noting the slopes of the regressions given in Table 3, these results show that there is a strong positive relationship between the level parameter and the dominant height at age 15 of the associated seed source on that site. That is, the greater the dominant height, the larger the intercept term and therefore, the higher the level of the height-diameter equation. While this is an expected result due to the model form (eq. (3)), the empirical result aids in understanding the association of a_i and height.

Figure 1 presents the Bonferroni confidence intervals for the nine slopes estimated using equation (3). The family confidence coefficient is 0.90, meaning that the probability is at least 0.90 that all 9 intervals are correct. Given this, it is a straightforward procedure to determine whether or not the slopes are the same across locations from the graph. Overlap of all intervals in Figure 1 would suggest accepting the hypothesis that the slopes are the

[2] A table of the calculated F-ratios may be obtained from the author. The table was not included in this note in the interest of space.

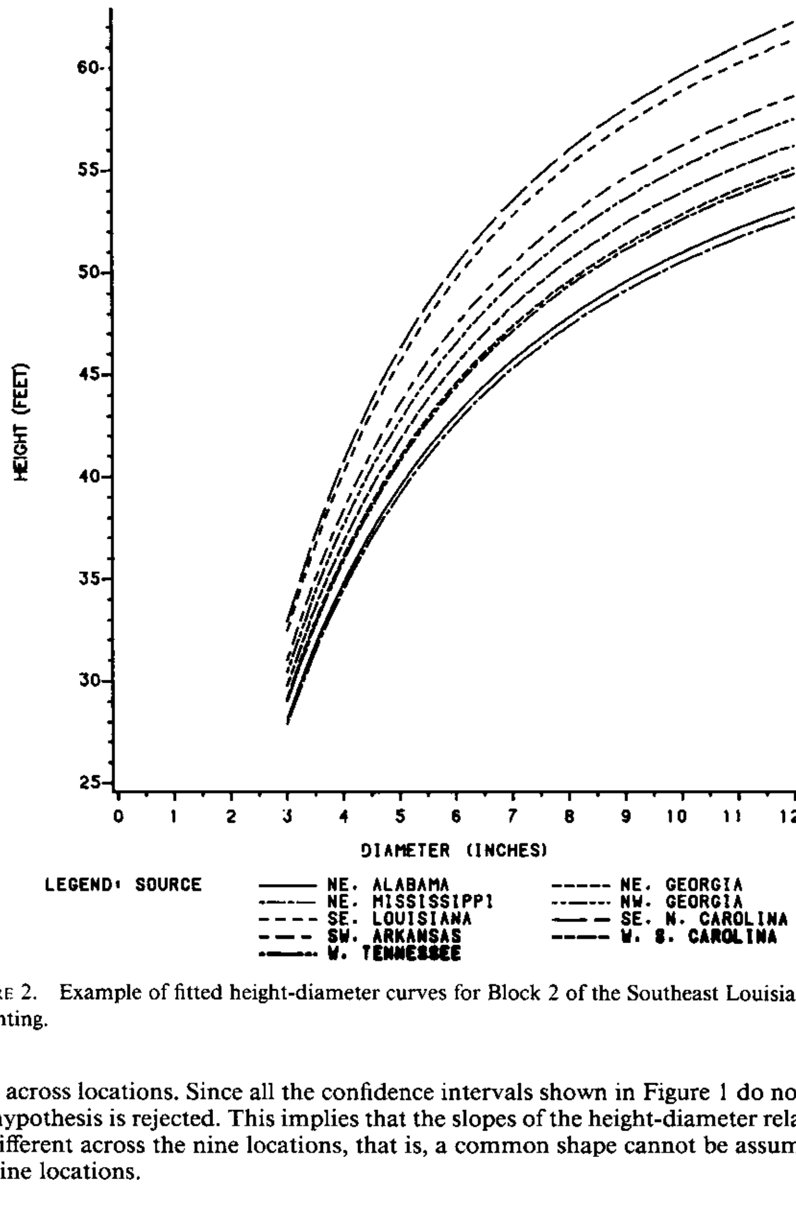

FIGURE 2. Example of fitted height-diameter curves for Block 2 of the Southeast Louisiana Series 2 planting.

same across locations. Since all the confidence intervals shown in Figure 1 do not overlap, this hypothesis is rejected. This implies that the slopes of the height-diameter relationships are different across the nine locations, that is, a common shape cannot be assumed across the nine locations.

CONCLUSION

The slopes of the fitted height-diameter curves at age 15 were the same for all seed sources in any given block at 8 of the 9 locations of the Southwide Pine Seed Source Study used in this analysis. The levels of the height-diameter relationship were different for the seed sources in any given block at 6 of the 9 locations. The general result implied is that there is a common curve shape for all the seed sources planted at a given site, but that there is displacement of the curves relative to each other at a given site. The level of a given curve is strongly related to the dominant height at age 15 and, from work reported by Wells (1983), to site index (base age 25). An example of a set of fitted curves for one site is given in Figure 2 showing a common shape and differing levels among the curves. In this instance the actual range of diameters was 2.2 inches to 11.0 inches and the actual range of heights was 16 feet to 65 feet.

These results indicate that seed source differences in height-diameter relationships can be integrated into any height-diameter equation of form similar to equation (1) by modifying the intercept (level) parameter to be a function of the dominant height of the desired seed source while not altering the shape of the height-diameter curve. Seed source differences in dominant height may be obtained by choice of level of the desired site index curve for a given site (Nance and Wells 1981, Wells 1983). The modified height-diameter equation can then be substituted into any growth and yield model employing a height-diameter equation of the appropriate form. For growth and yield modeling in general, these results indicate that the shape of this basic relationship is not altered by seed source differences, and that seed source differences can be represented as changes in the level of the height-diameter relationship for any given set of growing conditions.

While the results of this study apply only to seed sources, the framework presented here should be useful for investigating the dynamics of stands of open pollinated, half-sib, or full-sib family origin. It is suggested that testing the hypotheses considered here will provide important and useful information about these stands.

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