Diameter Distribution And Growth of Loblolly Pine

BY
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Abstract. The relationships between cubic-foot growth and diameter distribution, and residuals from cubic-foot growth equations and diameter distribution were tested on 103 plots of even-aged managed loblolly pine. A series of diameter distribution characterizations using mathematical expressions of the entire distribution curve, mathematical expressions of individual attributes of the diameter distribution curve, and biological characterizations of it were computed. The gamma distribution and cumulative frequency distribution by dbh classes of the Pearl-Reed population growth curve were computed. Average dbh, number of trees per acre, and the range of tree diameters were ascertained. The second, third, and fourth moments, as well as the standard deviations, the coefficients of skewness, and the coefficients of excess were calculated. Biological characterizations of the right-hand tail of the diameter distribution included ratios of basal area in various percentages of the tail in number of trees to total basal area as well as an expression involving number of trees. All of the 15 expressions of diameter distribution were rejected as useful expressions for characterizing merchantable cubic-foot growth. The probability of markedly reducing unexplained variation in merchantable cubic-foot growth in managed, even-aged loblolly pine with characterizations of diameter distribution is small.

GROWTH of pure loblolly pine (*Pinus taeda* L.) stands has been quantitatively related to age, site, and stand density, but the effects of genetic variation, clustering, and diameter distribution upon growth have yet to be quantified.

This paper reports on an attempt to relate expressions of diameter distribution in even-aged managed stands to cubic-foot growth itself and to growth residuals about the fitted relationship between cubic-foot growth and age, site, stocking, and various interactions of these variables.

Growth Relationships

More effort has probably been expended upon growth studies of loblolly pine than upon any single species in the United States. Only recently, however, has a reasonable portion of differences in growth rate been explained. Brender (1960) explained variation in merchantable cubic-foot growth; Clutter,[1] using a growth model obtained through the differentiation of a previously accepted yield model, accounted for variation about the mean in total cubic-foot growth; and Nelson *et al.* (1961) explained variation in merchantable cubic-foot growth with the expression quoted later in this paper. All three of these studies utilized the independent variates of age, site, some measure of stand density or stocking, and the various interactions.

There are numerous hypotheses about

[1]Clutter, Jerome L. The development of compatible analytical models for growth and yield of loblolly pine. Doctorial dissertation, School of Forestry, Duke University, Durham, N. C. 62 pp. 1961.

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the nature of the unexplained variation in growth of loblolly pine. No doubt some of the variation can be attributed to measurement error, although this can and has been minimized for a relatively fast-growing species by careful measurements with time enough between them to permit discernible changes in the measured characteristics. Imperfections in the indexes used to describe site quality and stocking could also have contributed to the unexplained variation.

Genetic variation, clustering, and diameter distribution—the subject of this study—are three other possible sources that could explain growth variation. Diameter distribution has been used extensively as a characterization of the forest stand. The "méthode du contrôle" determines periodic gross increment of the forest and characterizes growth and volume by diameter class groupings (Meyer *et al.* 1952); stand table projection methods such as Smithers (1949) described utilize growth by diameter classes. Meyer (1930), in his excellent work on diameter distribution, has characterized distribution series in even-aged forest stands.

The utility of crown classes in correlations with growth is discussed by Spurr (1952) and there unquestionably is a relationship between crown class and diameter class in even-aged, pure loblolly pine.

With this background, the hypothesis was developed that trees of different diameter classes or crown classes do not make equal contributions to cubic-foot growth. A considerable variation in diameter class distribution among areas was known to exist from previous unpublished work at the Southeastern Forest Experiment Station. It was further reasoned that a portion of the growing stock as represented by the measure of stand density was contributing less than its proportionate share to the total growth and that some of the growing stock perhaps was moribund.

To explore the idea that a portion of the total number of trees exerts an influence upon growth differing from its proportionate stocking, or that an expression of diameter distribution was significantly related to

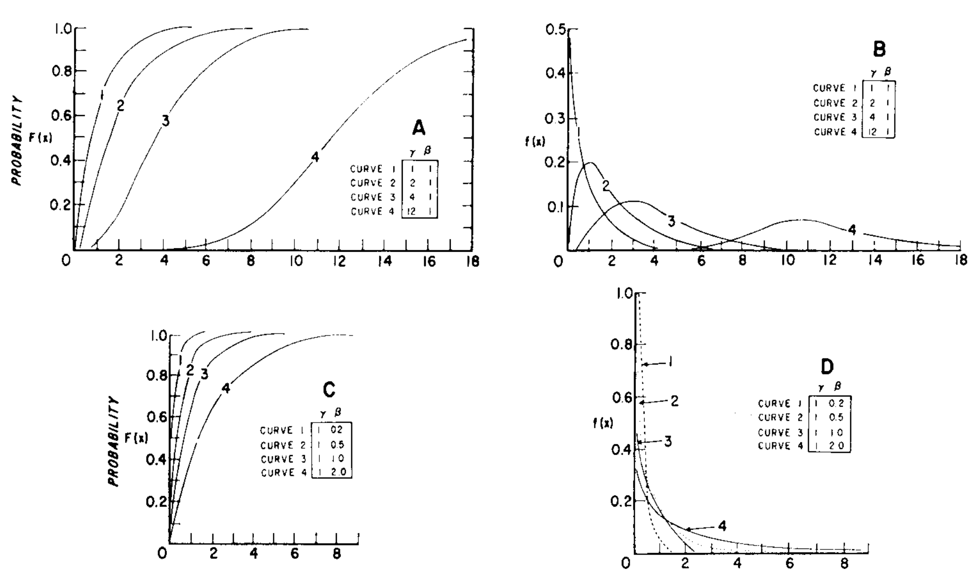

FIGURE 1. *The gamma distribution* (A) *cumulative form and* (B) *non-cumulative form with B held constant and various values of* γ. (C) *cumulative form and* (D) *non-cumulative form with* γ *held constant and various values of B. Adapted from Friedman and Janes (1957).*

growth, a series of characterizations of diameter distribution was first developed.

Characterization of Diameter Distribution

It was reasoned that two approaches could be used to find the portion of the stand having a disproportionate effect upon growth: (1) characterization of the entire distribution curve, or (2) characterization of particular attributes of the curve. The first approach is mathematical; the second can be mathematical or biological.

A literature search and consultations with specialists led to the selection of the gamma distribution and cumulative frequency distributions using the Pearl-Reed population growth curves as characterizations of the entire diameter distribution curve.

Examination of the gamma distribution indicated that it might characterize curve forms of diameter distribution encountered throughout the range of study plots. Barger and Thom (1949) suggested this model to approximate rainfall distributions, and Friedman and Janes (1957) successfully used the gamma distribution to smooth irregularities in periodic rainfall data. Thom (1954) developed approximate solutions of the maximum likelihood equation for estimation of the parameters γ and B of the gamma distribution. Figure 1 shows the gamma distribution in cumulative and non-cumulative form for a series of values of γ and B. For the diameter distribution data, the estimated parameters were obtained as:

$$\gamma = \frac{1 + \sqrt{1 + 4A/3}}{4A} \quad (1)$$

$$B = \bar{x}/\gamma \quad (2)$$

where

$$A = \log \bar{x} - \frac{1}{n} \Sigma \log x \quad (3)$$

$$\bar{x} = \Sigma\, x/n \quad (4)$$

and

x = tree diameter

Osborne and Schumacher (1935) successfully constructed red gum stand tables by application of the Pearl-Reed population growth curve to cumulative frequency distribution by dbh classes.

A logarithmic solution of the Pearl-Reed curve can be obtained by least squares fitting of the data in each plot to the form of equation:

$$\log \left\{ \frac{100 - y}{y} \right\} = a + b_1 x + b_2 x^2 + b_3 x^3 \quad (5)$$

where

y = cumulative frequency to the upper limit of any diameter class

x = corresponding upper limit itself

Instead of characterizing the entire diameter distribution curve per se, a number of its significant attributes provide descriptive information concerning the nature of the distributions. Meyer (1930) effectively used such characterizations with shortleaf pine in earlier work. Some of the more common attributes include average dbh, number of trees per acre, and range of tree diameters. The second, third, and fourth moments themselves (μ_2, μ_3, μ_4), as well as the standard deviation, the coefficient of skewness or asymmetry [μ_3/(standard deviation)^3], and the coefficient of excess $\frac{[\mu_4/(\mu_2)^2 - 3]}{24}$ are other common measurable attributes of diameter distributions.

Attempts were also made biologically to characterize attributes of diameter distribution curves. It appeared logical that, in managed stands of loblolly pine where thinnings had been made from below, the effects of diameter distribution upon growth should be a function of the right-hand tail of the distribution. The left-hand tail of the distribution will be truncated or approach a truncated condition at higher diameters at each subsequent thinning. Therefore, the ratios of the largest 10, 20, and 30 percent of the number of trees in terms of basal area to total basal area were computed. Another ratio

$$\frac{\text{Mean dbh of upper 10\% in No. of trees} - \text{mean dbh of all trees}}{\text{Mean dbh of all trees}}$$

was also tested.

Methods

Diameter distributions from 103 plots of loblolly pine were characterized. These plots, installed during 1948-50 and more adequately described elsewhere (Nelson *et al.* 1961), consisted of circular ¼-acre plots (with ½-chain isolation strips) having a rectangular distribution in age, site, and stand density. Only pure stands, even-aged, with uniformly spaced, insect- and disease-free loblolly pine between the ages of 20 and 60 years were used. Site indexes ranged from approximately 60 to 100 and densities from 40 to 130 percent of full stocking. Plots were located in the Georgia Piedmont, in the low country of South Carolina, and in Tidewater Virginia.[2]

One-half of the plots were cut to specified residual stockings immediately following plot establishment. Thinning was from below, modified as necessary to obtain uniform spacing of residual stems and to remove undesirable trees in all crown classes. The thinned plots were rethinned at the end of the first 5-year growth period if necessary, and the remaining plots were thinned at that time. Analyses of measurements following the fifth and tenth growing seasons indicated that merchantable net periodic annual cubic-foot growth was related to age, site, and stocking as follows (Nelson *et al.* 1961):

$$\text{Cubic-foot growth} = 205.0 + .26005 \left(\frac{10{,}000}{\text{age}}\right) - .03087\ (\text{Density percent})^2 + .05824\ (\text{Site} \times \text{Density percent}) - 8.11990(\text{Site}) + .05328\ (\text{Site})^2 \qquad (6)$$

where

Cubic-foot growth = periodic 5-year net annual merchantable growth in cubic feet per acre of trees 4.6 inches dbh and larger

age = age of stand in years at the beginning of the 5-year period

site = site index of stand

density = percent of theoretical full stocking after thinning at the beginning of the 5-year period (Stahelin 1949).

The standard error of estimate for Eq. 6 is ± 38 cubic feet at the mean.

To avoid confounding initial and residual stocking, plots to be cut in each initial stocking class were randomly assigned to residual stocking classes insofar as possible. Thus, the diameter distributions at the beginning of the growth period were essentially random and represented most distributions one would expect to find in thinned, pure, even-aged loblolly pine stands in the Southeast.

There was a surprisingly wide range of distributions represented on the study plots. Some of the plots tended to be skewed to the left or right (Fig. 2); both directions of excess were represented (Fig. 3). Some plots approached a truly normal distribution (Fig. 4), and a limited number of plots tended to be multi-modal in distribution (Fig. 5).

Plots within the same age, site index, and stocking classes had distributions that varied in a number of characteristics. Figure 6 illustrates the diameter distribution of such a pair of plots in young stands and Figure 7 in two older stands.

The various characterizations of diam-

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[2]Plots were located on the Westvaco Experimental Forest of the West Virginia Pulp and Paper Company near Georgetown, S. C., and on the Camp Experimental Forest maintained in Surry County, Virginia, by the U. S. Forest Service in cooperation with the Union Bag-Camp Paper Corporation. Plots were also located on the Hitchiti Experimental Forest near Macon, Georgia, and the Santee Experimental Forest near Charleston, S. C.

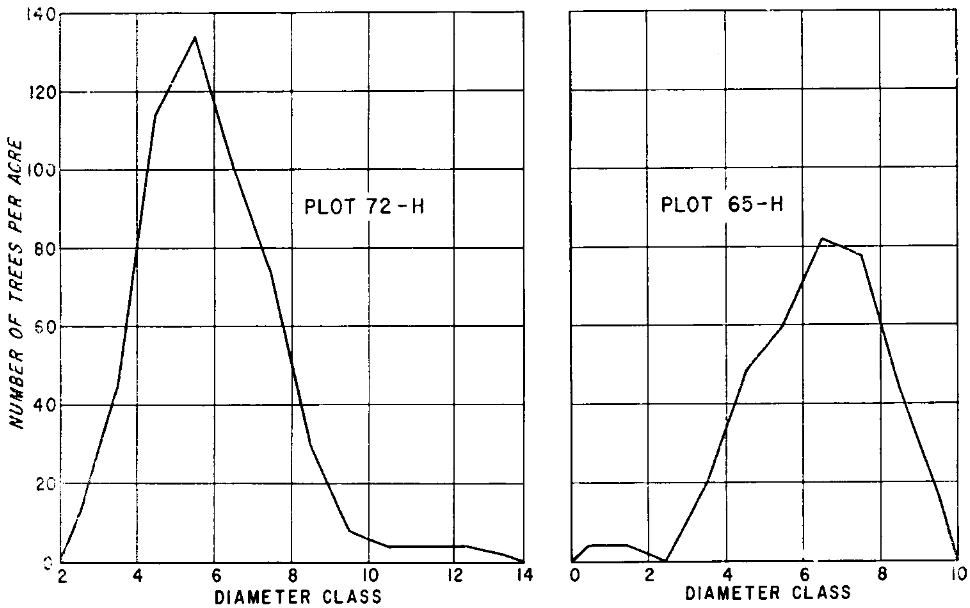

FIGURE 2. *Examples of plots with skewed diameter distributions.*

eter distribution were then related to merchantable cubic-foot growth and to the residuals (observed-expected) from Eq. 6.

Results and Discussion

Diameter distribution as characterized by gamma distribution parameters was not significantly related to cubic-foot growth[3] or growth residuals following removal of significant age, site, and stocking relationships (Table 1).

[3]Cubic-foot growth refers to periodic 5-year net annual merchantable growth in cubic feet per acre of trees 4.6 inches dbh and larger to a 4-inch top diameter outside bark.

Solution of the Pearl-Reed population growth curves using cumulative frequency distributions by dbh classes provided excellent fits to the data. For given plots, from 93.9 to 100 percent of the variation in tree diameters was accounted for by the solutions. The resulting variates, however, were not significantly related to cubic-foot growth or residuals from Eq. 6. Analyses of variance are shown in Tables 2 and 3.

The ease of solution of Pearl-Reed population growth curves plus the excellent fits of the distributions make it a more desirable tool for characterizing diameter distributions than the gamma distribution. There

TABLE 1. Relationship of gamma distribution parameters to cubic-foot growth and to residuals from Equation 6 which accounted for variation in growth attributable to age, site, and stocking.

Regression values	Periodic annual cubic-foot growth		Residuals, Equation 6	
	γ	B	γ	B
b, (slope)	+.1958	−6.5613	+.0199	−2.5882
Percent of sample variation explained	<1.0	<1.0	<1.0	<1.0
t, (b/S_b)	.646 n.s.	.709 n.s.	.140 n.s.	.598 n.s.

n.s. Not significant.

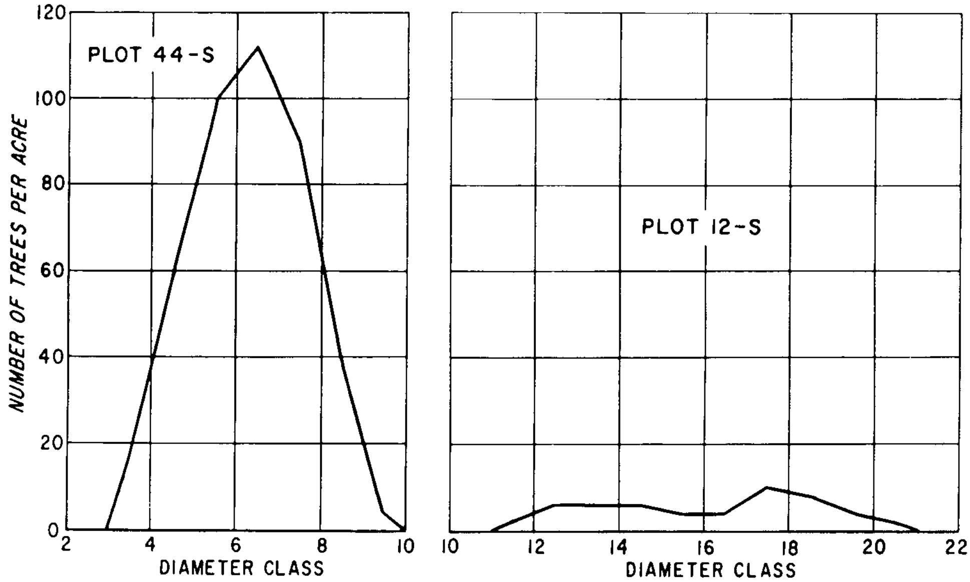

FIGURE 3. *Examples of plots with excess.*

are, of course, a number of other distributions which could be used to characterize loblolly pine stands; investigators desiring such characterizations should check the Charlier Type A and B distributions described by Meyer (1930) and the Fisher-Tippett Type II described by Thom (1954).

The only significant relationship of cubic-foot growth itself with any of the independent variates was found with average dbh (Table 4). Average dbh also partially expresses stand density or stocking (Bickford *et al.* 1957) but it accounted for only 6.3 percent of the variation about the mean as compared to 20.8 percent accounted for by percent of full stocking as a measure of stocking. The significance of this variate was lost after the effects of age, site, and stocking were removed (Table 5).

Number of trees per acre and the range of diameters within a plot were not significantly related to cubic-foot growth nor was number of trees per acre related to the residuals from Eq. 6.

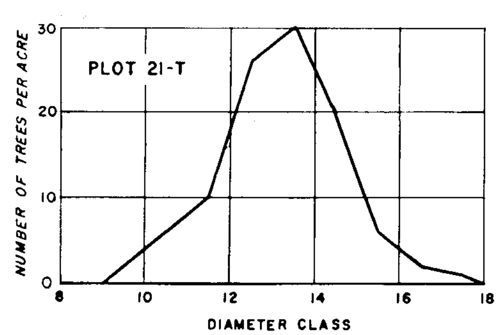

FIGURE 4. *Some plots had near-normal diameter distribution.*

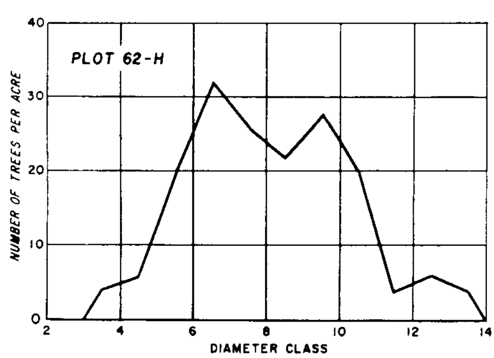

FIGURE 5. *A few plots had multi-modal diameter distributions.*

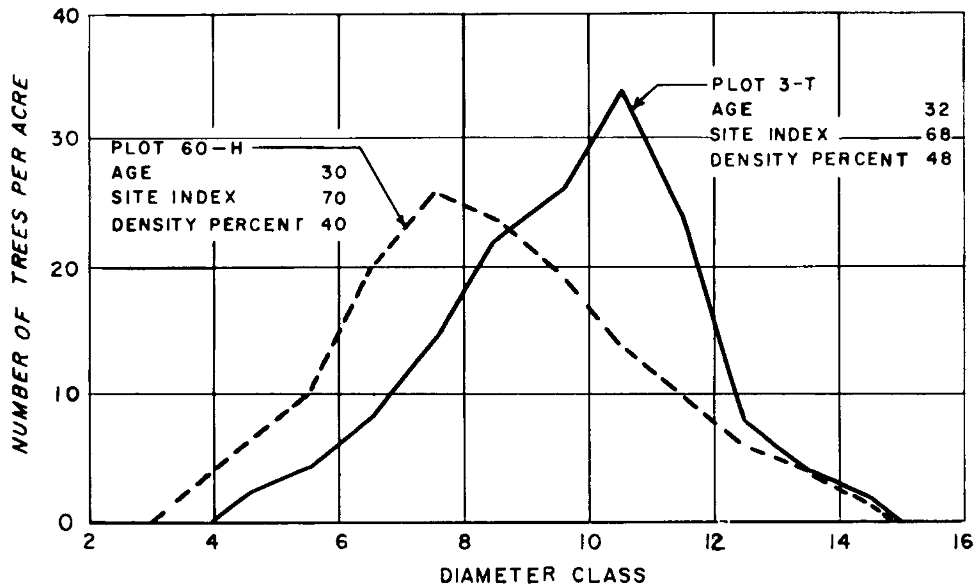

FIGURE 6. *Variation in diameter distribution between two plots of younger ages which were within the same age, site index, and stocking classes.*

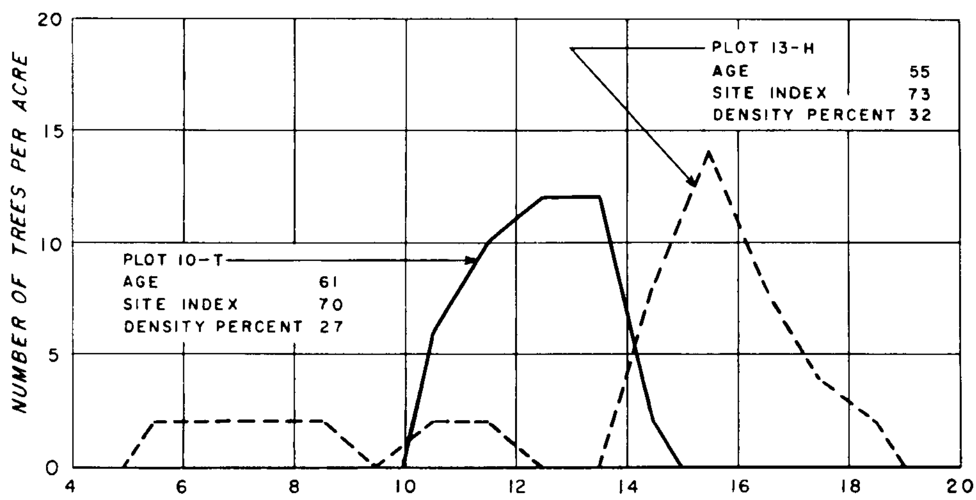

FIGURE 7. *Variation in diameter distribution between two plots in older stands which were within the same age, site index, and stocking classes.*

Range of diameters, however, was related to the residuals from Eq. 6, indicating significance at the 5 percent level. Although mathematically significant, the range of diameters within plots was rejected as a useful expression. It was reasoned that in screening, one out of 15 variates approaches a chance significance at the 5 percent level. There was not a very wide range in this variate; the mean range was 9.7 diameter classes with a standard deviation of 3.2 diameter classes, and the actual

TABLE 2. Analysis of variance. Relationship of independent variates, Equation 5, to cubic-foot growth.

Source	DF	S.S.	M.S.
Constant	1	3,116	3,116 n.s.
b_2	1	15,949	15,949 n.s.
b_3	1	16,716	16,716 n.s.
b_1	1	4	4 n.s.
Error	98	613,945	6,265
Total	102	649,730	

n.s. Not significant.

TABLE 3. Analysis of variance. Relationship of independent variates, Equation 5 to residuals from Equation 6 which accounted for variation in growth attributable to age, site, and stocking.

Source	DF	S.S.	M.S.
b_3	1	2,599	2,599 n.s.
Constant	1	445	445 n.s.
b_1	1	583	583 n.s.
b_2	1	395	395 n.s.
Error	93	137,933	1,483
Total	97	141,955	

n.s. Not significant.

TABLE 4. Relationships of common attributes of diameter distribution to cubic-foot growth.

Regression values	Ave. dbh	No. trees per acre	Range of diameters
b, (slope)	+6.1939	+.1208	+1.7767
Percent of sample variation explained	6.3	2.4	<1.0
t, (b/S_b)	2.6481**	.4986 n.s.	.6287 n.s.

**Significant at 1% level.
n.s. Not significant.

TABLE 5. Relationships of common attributes of diameter distribution to residuals from Equation 6 which accounted for variation in growth attributable to age, site, and stocking.

Regression values	Ave. dbh	No. trees per acre	Range of diameters
b, (slope)	+.4798	+.0612	−2.8745
Percent of sample variation explained	<1.0	<1.0	4.7
t, (b/S_b)	.4187n.s.	.5405n.s.	−2.2248*

*Significant at 5% level.
n.s. Not significant.

increase in the coefficient of determination of Eq. 6 occasioned by the addition of this variate was only in the magnitude of 1.0 percent.

None of the measures of moments or terms calculated from moments showed a significant relationship with cubic-foot growth or residuals from the growth equation (Tables 6 and 7). There was lack of significance in spite of a considerable range within the expression of these attributes. Nor was success achieved in using biological expressions of diameter distribution in relationships with growth and residuals from Eq. 6 (Tables 8 and 9). Thus we were unable to demonstrate disproportionate growth rates attributable to the right-hand tail of the diameter distribution curve.

It is possible that an analysis of the interaction between some characterization of diameter distribution and stocking would be worthwhile in reducing variation of residuals from Eq. 6. At low stocking, it is probable that diameter distribution would have little effect in a well spaced stand. That is to say, under these conditions, all components of the stand individually and collectively might be approaching their maximum production for a specific site and age. Exploration of this interaction was not attempted in the present study.

It is also possible that a high threshold of diameter would increase the effect and sensitivity of diameter distribution upon growth; e.g., use of board-foot growth instead of cubic-foot growth. The threshold in this study was low enough so that not many stems were below it.

The lack of consistent relationship of diameter distribution to cubic-foot growth equations offers no proof that such relationships do not exist in managed, even-aged stands of loblolly pine. The lack of a rectangular distribution in plot selection for the diameter distribution variate and an intentional rectangular distribution for age, site, and stand density could have reduced the sensitivity of the diameter distribution variate.

The 15 characterizations of diameter

TABLE 6. Relationships of moments and expressions of moments to cubic-foot growth.

Regression values	μ_2	μ_3	μ_4	Standard deviation	Skewness $\mu_3/(S.D.)^3$	Excess $\frac{(\mu_4/(\mu_2)^2-3)}{24}$
b, (slope)	—.1534	+.1897	—.0083	—.0083	+4.1326	+16.3439
Percent of sample variation explained	<1.0	<1.0	<1.0	<1.0	<1.0	<1.0
t, (b/S_b)	.0880 n.s.	.4208 n.s.	.2971 n.s.	.2546 n.s.	.3231 n.s.	.1190 n.s.

n.s. Not significant.

TABLE 7. Relationships of moments and expressions of moments to residuals from Equation 6 which accounted for variation in growthattributable to age, site, and stocking.

Regression values	μ_2	μ_3	μ_4	Standard deviation	Skewness $\mu_3/(S.D.)^3$	Excess $\frac{(\mu_4/(\mu_2)^2-3)}{24}$
b, (slope)	—.6171	+.0048	—.0091	—4.9867	—.3880	—57.0423
Percent of sample variation explained	<1.0	<1.0	<1.0	<1.0	<1.0	<1.0
t, (b/S_b)	.7589 n.s.	.0227 n.s.	.6994 n.s.	1.0227 n.s.	.0649 n.s.	.8918 n.s.

n.s. Not significant.

TABLE 8. Relationships of biological characterizations of diameter distribution to cubic-foot growth.

Regression values	Basal area top percent of trees / Total basal area			Mean dbh of upper 10% in number of trees—mean dbh / Mean dbh
	10%	20%	30%	
b, (slope)	—406.57	—235.40	—200.15	—76.92
Percent of sample variation explained	2.2	1.5	1.5	1.6
t, (b/S_b)	1.4952 n.s.	1.2565 n.s.	1.3346 n.s.	1.3019 n.s.

n.s. Not significant.

TABLE 9. Relationships of biological characteristics of diameter distribution to residuals from Equation 6 which accounted for variation attributable to age, site, and stocking.

Regression values	Basal area top percent of trees / Total basal area			Mean dbh of upper 10% in number of trees—mean dbh / Mean dbh
	10%	20%	30%	
b, (slope)	—211.83	—98.43	—78.19	—27.38
Percent of sample variation explained	2.7	1.2	1.0	<1.0
t, (b/S_b)	1.6705 n.s.	1.1219 n.s.	1.0335 n.s.	.9875 n.s.

n.s. Not significant.

distribution used in this study, however, were valid and logical expressions of diameter distribution with at least recognized examples of each of the various groups of statistics used in frequency distribution work. None showed a consistent relationship to merchantable cubic-foot growth, either singularly or after the effects of age, site, and stocking had been removed. It is logical, therefore, to conclude that the probability of other expressions of diameter distribution showing consistent and significant relationship with these dependent variates and markedly reducing unexplained variation in managed, even-aged loblolly pine is indeed small.

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