Comparison of Stand Density Measures for Loblolly Pine Cubic-Foot Growth Prediction

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Abstract. Four expressions of stand density were tested in cubic-foot growth multiple regressions in even-aged, managed stands. Stahelin's percent of full stocking, total basal area, Reineke's stand density index, and initial merchantable cubic-foot volume were tested as expressions of density, in combination with age and site in regression analyses of cubic-foot growth. Essentially, the same amount of variation in annual cubic-foot growth was explained by each expression in combination with age, site, and certain of their significant interactions from data involving repeat measurements of stands throughout a wide range of initial age, site, and stand density. It is concluded that the choice among the variates expressing stand density is primarily at the discretion of the investigator. However, ease of computation and application and the advantages occurring from the use of a direct measure rather than an index suggest the use of basal area unless bioeconomic analyses are desired.

LOBLOLLY PINE is commonly grown in even-aged stands and the expression of age and site are more or less standard; age is expressed as years from seed, and site as site index computed from the logarithm of height and the reciprocal of age (Coile 1952). The form or expression of stand density has not been uniform, however, nor have various measures of stand density been compared in relation to growth.

This paper discusses the results of testing some common methods of expressing stand density in relation to cubic-foot volume growth of natural loblolly pine stands.

Expressions of Stand Density

Bickford *et al.* (1957) point out that stand density[1] is usually measured in terms of some combination of diameter, height, form, and number of trees. In characterizing stand density of southern pines in general, and more particularly loblolly pine, the usual combinations of these factors include number of trees in relation to diameter and the number of trees in relation to diameter, height, and form. The common expressions of loblolly pine stand density utilizing the combination of number of

[1]Stand density is used in this paper and by Bickford *et al.* (1957) according to the Society of American Foresters' definition in its *Forestry Terminology*, Revised Edition, 1950; i.e., density of stocking expressed in numbers of trees, basal area, volume, or other criteria, on a per-acre basis. Its use is interpreted in this paper to include those relative measures which can be readily expressed in quantitative terms. Stocking, on the other hand, includes only relative measures; e.g., well-stocked, overstocked, and partially stocked.

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trees and diameter include basal area, stand density index, and percent of full stocking (density percent). One of the common expressions using all four factors is initial merchantable cubic-foot volume.

Basal area is perhaps the most commonly used expression of stand density in loblolly pine growth and yield tables (Brender 1960, Schumacher and Coile 1960, Forest Service 1929). Its advantages and disadvantages for analytical mensurational analyses are similar to its advantages and disadvantages in field application. To its credit, it is simple, objective, and easy to use. With practice, most foresters can readily conjure a mental picture of a stand with a given basal area.

On the other hand, Bickford *et al.* (1950) point out that basal area gives the same weight to equal areas of biologically dead heartwood and functioning sapwood, to young trees and old trees, and to suppressed trees and dominant trees. They also raise the question of whether or not the square is the proper exponent of diameter when the cambial area as represented by the circumference is given by the first power.

The advantages of basal area as an expression of stand density are clearcut and proven; the disadvantages listed are principally conjecture. No good evidence is documented to show that the concern about dead heartwood and live sapwood is valid. Tree age should not be a factor when dealing with even-aged stands, especially when stand age and its interaction with stand density are provided for. The range of stand structure for a given basal area may be slight in managed stands if age and site are accounted for. And we have little evidence that cambial surface is of any value in expressing stand density.

Although other criticisms of basal area as an analytical tool can readily be raised, most of the other expressions of stand density utilizing a combination of number of trees and diameter share, to a lesser or greater degree, many of the disadvantages of basal area.

Reineke's stand density index (1933) has been used in southern pine yield studies (Langdon 1961). It utilizes a formula in which the logarithm of the number of trees in a fully stocked stand is equated to the logarithm of the diameter of the tree of average basal area:

$$\log N = -1.605 \log D + K$$

It has been stated that the advantages of stand density index include lesser correlation with age and possibly with site than with use of basal area as a measure of stand density (Spurr 1952). It is not clear, however, how this purported advantage can be justified in the presence of modern techniques of analyses. Its disadvantages include relative complexity of calculation and its lack of absolute value; i.e., it is an approximation of average stand relationships that does not necessarily apply to a given stand.

Stahelin's full stocking curve and percent of full stocking (1949) were derived from Chisman and Schumacher's tree-area ratio (1940). Wenger *et al.* (1958) used Stahelin's percent of full stocking as the expression of stand density in the first 5-year analysis of the same study in which the 5- to 10-year growth data are now being used as a vehicle for these comparisons, and Gruschow and Evans (1959) used it in their growth study in young slash pine stands. Stahelin converted Chisman and Schumacher's expression "Area required by a single tree = $.0000480 + .0000668d + .0000267d^2$" to "Basal area in

$$\text{sq. ft. per acre} = \left(\frac{d}{.086936 + .07007d}\right)^2\text{"}$$

where d = average dbh of trees 1.6 inch dbh and larger, and computed a series of curves for various percentages of full stocking. Percent of full stocking, like basal area, is simple, objective, and easy to use. An individual stand may deviate from the original sample used and the expression may not be valid for a particular stand.

Past stand history may have dictated abnormal growing space and dbh-crown diameter relations. Spurr (1952) also questions the second-degree parabolic relationship between growing space and dbh. In fact, he argues that from a theoretical standpoint the relationship between growing space and dbh is more likely to produce an S-shaped curve, as is the case when crown diameter is plotted against dbh. Finally, percent of full stocking is also an index.

Bickford and his fellow committeemen (1957) frown upon volume as a measure of stand density. Nevertheless, because it appeared to offer a sound wedge into bioeconomic analysis of growth data, it was included in the comparison of expressions of stand density.

Methods

The source of the data reported in this paper is the 5- to 10-year remeasurements of the managed portion of the Southeastern Forest Experiment Station loblolly pine stand density study, the physical details of which have been adequately described elsewhere (Nelson *et al.* 1961, Wenger *et al.* 1958). Briefly, in 1949-50, a total of 103 circular ¼-acre plots with ½-chain isolation strips were selected in 20- to 60-year-old stands with a range in site index from a little less than 60 to a little more than 100; densities ranged from 40 to 130 of Stahelin's percent of full stocking. Only pure loblolly pine stands, even-aged, uniformly spaced, and relatively insect- and disease-free were used.

Some plots were located in Georgia on the Hitchiti Experimental Forest, others were in South Carolina on the Santee and Westvaco Experimental Forests, and the remainder in Virginia on the Camp Experimental Forest.[2]

About one-half of the previously unthinned plots were thinned to specified residual densities at plot establishment. These plots were rethinned and the remaining received their first thinning at the end of the first 5-year growth period. The thinnings were from below, modified as necessary to permit uniform spacing of residual stems and the removal of undesirable trees in all crown classes. Hardwoods 4.6 inches dbh and larger were poisoned.

Merchantable cubic-foot growth was expressed in terms of net periodic annual increment. Using percent of full stocking as a measure of stand density, the form of the function fitted to the data was:

$$Y = b_0 + b_1\left(\frac{1}{A}\right) + b_2\,(S) + b_3\,(D) + b_4\,(D^2) + b_5\left(\frac{S}{A}\right) + b_6\left(\frac{D}{A}\right) + b_7\left(\frac{D^2}{A}\right) + b_8\,(SD) + b_9\,(SD^2) \qquad (1)$$

Where Y = merchantable periodic net annual growth of trees 4.6 inches dbh and larger; A = total age; S = site index; and D = stand density at the beginning of the growth period.

All possible combinations of the nine independent variables shown in Eq. 1 were screened for the most reasonable solution with significant variates. Reasonable solutions were those with a high coefficient of determination (R^2), a low standard error, and that also made biological sense when the curves of predicted growth were plotted over age, site, and stand density. Residuals were tested for randomness and further variates were screened as necessary to eliminate discernible trends in residuals when plotted in relation to age, site, stand density, and the difference between actual and estimated values. Equation development is shown in Table 1.

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[2] The Westvaco Experimental Forest is maintained by the West Virginia Pulp and Paper Company near Georgetown, S. C. The Camp Experimental Forest is maintained in Sussex County, Va., by the U. S. Forest Service in cooperation with Union Bag-Camp Paper Corporation.

TABLE 1. Regression equations for estimating annual cubic-foot growth per acre.

	Regression coefficients (b)				
Site index					−8.11990
(Site index)^2				+.00458	+ .05328
$\frac{10{,}000}{\text{Age}}$			+.27796	+.27025	+ .26005
(Percent of full stocking)^2		−.03653	−.03606	−.02821	− .03087
Site × percent of full stocking	+.03017	+.06595	+.06564	+.05434	+ .05824
Constant	− 1.1	− 38.8	− 113.4	− 117.6	+ 205.0
	Summary statistics				
r^2 or R^2	.515	.692	.761	.763	.781
Mean cubic-foot growth	130	130	130	130	130
SD of residuals	56	45	40	40	38

The resulting equation, using percent of full stocking as a measure of stand density, produced results with a sufficiently high coefficient of determination and low error of estimate that a similar model was used in testing the remaining expressions of stand density:

$$Y = b_0 + b_1\left(\frac{1}{A}\right) + b_2\,(S) + b_4(D^2) + b_8\,(SD) + b_{10}\,(S^2) \quad (2)$$

Using the other measures of stand density, the variates in Eq. 2 were tested for significance. Curves were plotted, residuals were tested, and a final expression selected as previously described.

Results and Discussion

The analyses led to the development of the following equations:

$$Y = 205.0 + .26005\left(\frac{10{,}000}{A}\right) - 8.11990(S) - .03087(D_1^2) + .05824(SD_1) + .05328(S^2) \quad (3)$$

Where D_1 = Stahelin's percent of full stocking.

$$Y = 148.85409 + .28630\left(\frac{10{,}000}{A}\right) - 6.79599(S) - .012(D_2^2) + .0358(SD_2) + .04451(S^2) \quad (4)$$

Where D_2 = total basal area per acre of all stems 0.6 inch dbh and larger.

$$Y = 208.51483 + .17002\left(\frac{10{,}000}{A}\right) - 7.95149(S) - .00147(D_3^2) + .01215(SD_3) + .05942(S^2) \quad (5)$$

Where D_3 = Reineke's stand density index.

$$Y = -134.70055 + .49694\left(\frac{10{,}000}{A}\right) - .00593\,\frac{(D_4^2)}{1{,}000} + .00081(SD_4) \quad (6)$$

Where D_4 = initial merchantable cubic-foot volume at the beginning of the growth period.

Table 2 indicates reduction sums of squares and the coefficients of determination for Eq. 3 to 6. Solutions of these equations are shown graphically in Figure 1.

The coefficients of determination obtained in these analyses are large enough to provide reasonable predicting equations (77 to 79 percent). This particular study, coupled with previous work, leads the authors to the conclusion that, within the age and site span of loblolly pine under study, little improvement can be expected through substitution or addition of other forms of the density variate than those tested in the predicting equations.

In independent growth analyses conducted on loblolly pine at the Hitchiti Experimental Forest (Brender 1960), basal

TABLE 2. Some components of variation for equations 3 to 6 showing reduction sums of squares.

Source	Eq. 3	Eq. 4	Eq. 5	Eq. 6
	Sums of squares			
$(\text{Density})^2$	135,046	175,300	146,083	196,725
Other terms	372,025	331,771	351,898	316,189
Error	142,187	142,187	151,277	136,344
Total	649,258	649,258	649,258	649,258
R^2	.781	.781	.767	.790

area per acre of trees 5.6 inches dbh and larger and Reineke's stand density index were used as measures of stand density. The results were similar to those obtained in the study reported in this paper in that neither expression showed an appreciable improvement of the predicting equation in comparison with the other.

The goodness of fit and shape of the curves from Eq. 3 to 6 are strikingly simi-

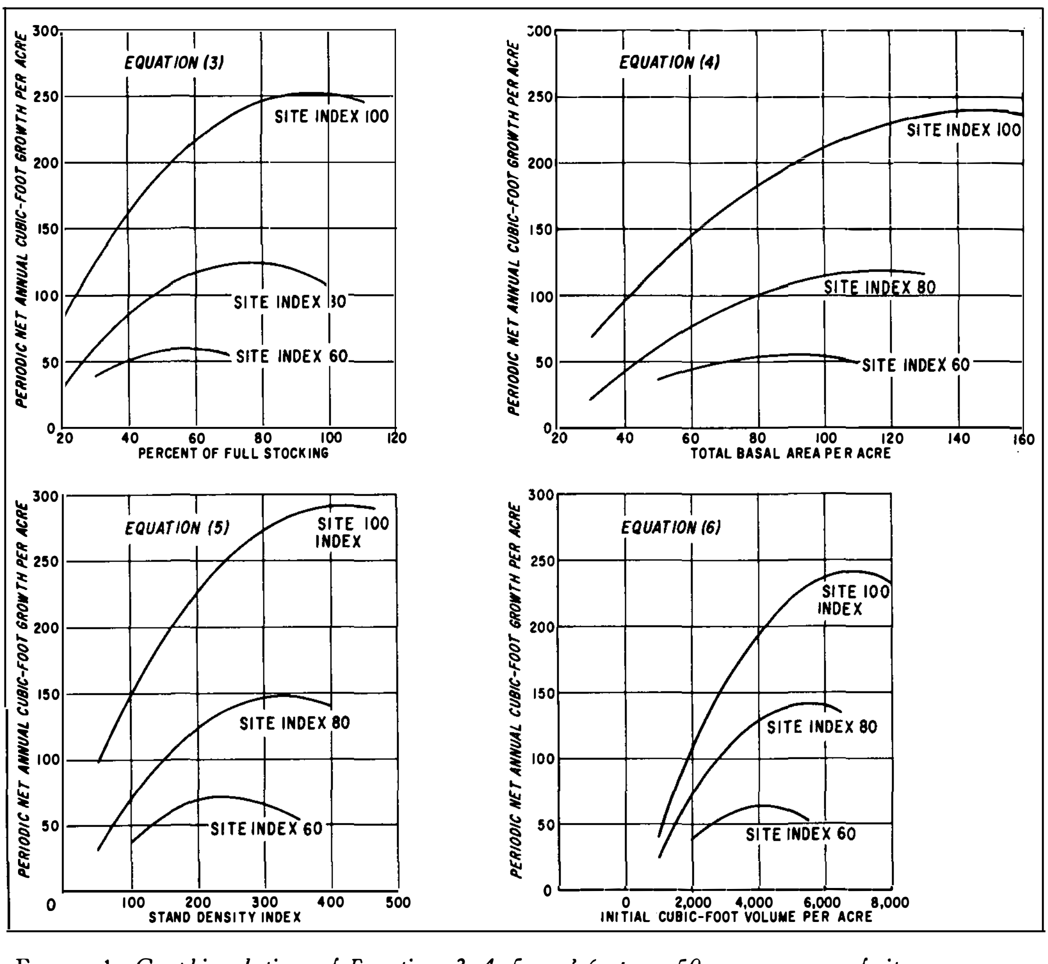

FIGURE 1. *Graphic solutions of Equations 3, 4, 5, and 6 at age 50 over a range of sites.*

lar (Fig. 1). There appears little to choose among expressions of the density variate in these respects. They all indicate site-stand density interactions and the effect of the squared term for stand density. The equations all show culmination of growth within the range of sites. They first show growth increasing rapidly with an increase in density but soon attain a relatively flat curve form that indicates a wide range in density with little change in growth.

Some differences among the expressions are quite evident. The effect of age is more pronounced in Eq. 6 where cubic-foot volume was used as a measure of stand density than in the other three equations. In Eq. 6, the terms site index and site index squared were not necessary. Height, of course, is included in the calculation of the stand density expression of initial cubic-foot volume and reduces the contribution of site index per se as a variate.

The correlation coefficients of the expressions of stand density used in this study with cubic-foot growth, and the relationships of growth to stand density expressed as number of trees and average dbh are as follows:

Expression of stand density	*Cubic-foot growth* (r^2)
Number of trees	.024
Average dbh	.063
(Percent of full stocking)^2	.208
(Basal area)^2	.270
(Stand density index)^2	.225
(Initial cubic-foot volume)^2	.303

The expressions using either the single factor of number of trees or average dbh as a variate account for too small a portion of the total variation to be seriously considered in multiple regression analyses. Expressions using the combination of number of trees and dbh (percent of full stocking, basal area, and stand density index) account for sizeable amounts of variation and are of similar magnitudes.

The simple correlation coefficients of expressions of stand density with growth, however, do not indicate the amount of explained variation attributable to the stand density variate. The true measure of the value of the density variate in these analyses should be the amount of variation explained by the density variable and its interactions after the effects of age, site, and their interactions have been accounted for. Insofar as age and site variates were essentially constant among Eq. 3 to 6 in terms of the variation accounted for, and the total variation accounted for by the entire equations is nearly equal, it is concluded that there is little to choose from among the variates in the estimation of cubic-foot growth.

The relationship of the variates to one another indicates why one might reasonably expect little difference among variates utilizing the same factors. The high degree of correlation among density percent, basal area, and stand density index are shown in Table 3. The correlations between initial cubic-foot volume and the other expressions of stand density are considerably lower and the differences are not as amenable to explanation.

The results of this study cannot be extrapolated at this time to other measures of stand increment such as board-foot or basal area growth. The high degree of correlation between the two-way combinations of density percent, basal area, and stand density index, plus a series of less complete comparisons indicate, however, that a rather free substitution among them could be made in board-foot and basal area growth analyses.

TABLE 3. Comparison of correlation coefficients (r^2) between various expressions of loblolly pine stand density.

	Basal area	SDI	Initial cubic-foot volume
Stahelin's percent of full stocking	.953	.929	.526
Basal area		.806	.719
SDI			.318

The use of basal area as an expression of stand density is recommended in future cubic-foot growth analyses of loblolly pine unless bioeconomic interpretations are desired. Point sampling techniques will give basal area stocking simply and rapidly. Also, no analytical advantages accrue in using more complex and difficult-to-obtain measures of stand density, and basal area provides a direct measure instead of an index. If stand density is expressed directly, growth surfaces can be developed that negate the use of normality concepts in loblolly pine.

With the statistical tools available today, and with species for which models have been developed that adequately describe growth surfaces, it seems highly desirable to definitively describe their growth responses in terms of measures of stand density such as basal area instead of using measures of stocking in relation to an ideal such as full stocking.

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