

Site Index for Loblolly Pine in the Atlantic Coastal Plain of the Carolinas and Virginia

by

Kenneth B. Trousdell, Principal Silviculturist
Charleston, South Carolina

Donald E. Beck, Principal Silviculturist
Bent Creek Experimental Forest, Asheville, North Carolina

and

F. Thomas Lloyd, Mathematical Statistician
Research Triangle Park, North Carolina

Site index, the height of the dominant stand at a selected index age, is a useful indicator of relative yield potential for even-aged stands. It is the most direct and probably the best tool now available for evaluating productivity.

Height measurements for determination of site index were made at 5-year intervals over a period of 15 years on a series of permanent growth plots of loblolly pine (*Pinus taeda* L.) in the middle Coastal Plain of Virginia and the lower Coastal Plain of South Carolina. The measured height-growth trend on these plots did not conform to several existing site curves for natural stands (USDA Forest Service 1929; Coile 1952; Schumacher and Coile 1960). In general, the measured rate of growth was more rapid than the rate depicted by the site curves. Consequently, when the existing site curves were applied to the measured values, the stands appeared to be increasing in site index as they increased in age.

Many possible weaknesses of the conventional methods of preparing harmonic site index curves could lead to disagreement between site curves and measured growth trends, as has been thoroughly discussed by Spurr (1952, 1956). Recent studies have shown that these weaknesses may contribute to major errors in estimating site index (Curtis 1966; Beck 1971). It is possible to avoid many of the pitfalls by using data developed from stem analysis or periodic remeasurement of stands on permanent plots. Curves constructed from such data would show true growth trends for given sites.

This paper presents site index curves for natural stands of loblolly pine in the Atlantic Coastal Plain of Virginia, North Carolina, and South Carolina.¹ The curves were developed from stem analysis and allow for varying curve shapes on sites of differing quality.

¹The North Carolina and Virginia portions of the study were conducted in cooperation with Union Camp Corp., Franklin, Virginia.

FIELD PROCEDURES

Twenty-two 2-tree plots were located in stands 50 years old or older. The trees ranged from 70 to 120 feet in height at age 50. The criteria set for choosing and accepting a sample tree were that the tree (a) was a dominant or codominant in the stand, (b) showed no evidence in the increment core of prior suppression, (c) did not show evidence of heart rot along the bole, and (d) showed no evidence of past damage to the crown or leader. If no physical evidence to disqualify it was found, the tree was felled.

After felling, total height was measured and the tree was cut into 6-foot bolts if it was in site index class 60 to 70, 8-foot bolts if it was in class 80 to 90, or 10-foot bolts if it was in class 100 to 120. Increment cores were taken at the 1-foot height to determine total age and at each cut to determine at what ages the tree reached intermediate heights. Exact height of the tree at these intermediate ages was determined by locating the nearest primary whorl² below the increment core and measuring height to this point. In the lower part of the bole where branch knots were overgrown, the bolt was split to expose all branch whorls at the pith. This field procedure furnished 12 to 16 height/age measurements per tree.

LABORATORY PROCEDURES

In the laboratory, all increment cores were sliced and the annual rings were counted. Two independent age counts were made, and differences were settled by close inspection and recounting. Total tree age was the age determined at the 1-foot height plus 1 year. The ages corresponding to the primary whorls below each increment core were calculated as total tree age minus the age count of that particular core.

ANALYSIS AND RESULTS

Site index curves for index ages of 25 and 50 years were developed from the data on stem analysis by (a) fitting for each tree a mathematical curve to the height/age data by means of a least squares estimation technique, (b) determining if and how the pattern of growth varied among trees from plots of varying site index, and (c) providing for change in curve shape in the model, if needed.

When loblolly pine grows under favorable environmental conditions and in the absence of extreme competition from its neighbors, one expects the rate of height growth to increase in early years and decrease thereafter, approaching zero as the tree grows old. The sigmoid growth model described by Richards (1959) provided an adequate description of the growth of the trees in this study when it was fit to the data with a nonlinear least squares technique (Middleton 1969). This extremely flexible growth

²Observations by the senior author indicate that loblolly pine annually produces about three growth flushes, at least in its first 20 to 25 years. The initial spring flush, with the primary whorl at its base, is the longest, and the last flush in the growing season is the shortest. In the tops of the mature pines on the study plots, only one flush of height growth occurred.

model has been used to describe the height growth of a number of species in recent years (Brickell 1966, 1968; Lundgren and Dolid 1970; Beck 1971). The form of the equation used in this study is

$$H = A \left[1 - e^{-Bt} \right]^{\frac{1}{1-m}} \quad (1)$$

where H equals the height of trees at age t and A, B, and m are parameters to be estimated.

Fitting equation 1 to the height/age data for each tree individually resulted in 44 sets of estimates ($\hat{A}_i$, $\hat{B}_i$, $\hat{m}_i$, $i=1, 2, \dots, 44$) of the three growth function parameters. Because the heights of the two trees measured on a plot were not generally observed at the two index ages of 25 and 50 years, site index (S) was taken to be the predicted height at index age from an estimated form of equation 1 with estimates of the parameters obtained by using both trees on a plot. The success of our approach to introducing site index into the model depended on the existence of a relationship between at least one of the growth function parameters and site index. We looked only at those functions

$$A = f_1(S) \quad (2)$$

$$B = f_2(S) \quad (3)$$

$$m = f_3(S) \quad (4)$$

that were linear in the parameters. We then used nonlinear least squares estimation on the model

$$H = f_1(S) \left[1 - e^{-f_2(S)t} \right]^{\frac{1}{1-f_3(S)}} \quad (5)$$

to obtain estimates of the parameters in the equations f_1 , f_2 , and f_3 . Although equation 1 provided a good fit with the height/age data on individual trees, four of the 44 trees appeared to have parameter/site index relationships different from the other 40 trees, especially for parameter A. The following tabulation of R^2 values for the equations

$$f_1(S) = b_1 + b_2 S \quad (6)$$

$$f_2(S) = b_3 + b_4 S + b_5 S^2 \quad (7)$$

$$f_3(S) = b_6 + b_7 S + b_8 S^2 \quad (8)$$

shows how they were changed when the four problem trees were deleted:

<u>Parameters</u>	<u>Index age (Years)</u>	<u>Fit for data set 1* (R²)</u>	<u>Fit for data set 2 ** (R²)</u>
A	25	0.01	0.35
	50	.02	.49
B	25	.55	.49
	50	.46	.37
m	25	.25	.13
	50	.27	.13

*Data set 1 includes all observations.

**Data set 2 does not contain the four problem trees.

Because of (a) the increased R^2 for f_1 , (b) the small difference in curve shape between the problem trees and the others, and (c) the relatively small number of problem trees (9 percent), we decided to estimate the parameters of equation 5 by using the reduced data set of 40 trees. The resulting parameter estimates for both index ages are

<u>Parameter</u>	<u>Estimate for index age 50</u>	<u>Estimate for index age 25</u>
b_1	6.31415×10	8.30075×10
b_2	6.35080×10^{-1}	6.57918×10^{-1}
b_3	6.43041×10^{-3}	1.57700×10^{-2}
b_4	1.24189×10^{-4}	-4.72909×10^{-5}
b_5	1.62545×10^{-6}	5.31718×10^{-6}
b_6	1.72741×10^{-1}	3.53269×10^{-1}
b_7	-2.91877×10^{-3}	-9.36396×10^{-3}
b_8	3.10915×10^{-5}	1.05419×10^{-4}

Tables 1 and 2 present the absolute values for the deviations of predicted from observed site index when all trees were included, that is, the problem trees were used in the construction of these tables even though they were not used in the estimation of parameters in equation 5. Although not presented in this paper, similar tables were calculated from equation 5 in which the parameters were estimated by using all 44 trees. The predictions were much more aberrant when all the trees were used in estimating the parameters, with some differences between observed and

Table 1.--Deviations of predicted from observed site index at index age 50 years

Stand age (years)	Deviation in feet at age 50				Observations of height/age
	±0.0 to 2.5	±2.6 to 5.0	±5.1 to 7.5	±7.6 to 10.0	
	----- <u>Percent of the deviations</u> -----				<u>No.</u>
10	32	23	32	13	22
20	41	50	9	--	22
30	73	27	--	--	22
40	100	--	--	--	22
50	100	--	--	--	22
60	95	5	--	--	22
Average or total	74	17	7	2	132

Table 2.--Deviations of predicted from observed site index at index age 25 years

Stand age (years)	Deviation in feet at age 25			Observations of height/age
	±0.0 to 2.5	±2.6 to 5.0	±5.1 to 7.5	
	----- <u>Percent of the deviations</u> -----			<u>No.</u>
10	64	23	13	22
20	100	--	--	22
30	100	--	--	22
40	100	--	--	22
50	73	27	--	22
60	55	41	4	22
Average or total	82	15	3	132

predicted site index being greater than 20 feet. The heights for the predictions in tables 1 and 2 were obtained from the average height growth curves for each of the 22 plots at the ages given in the tables. Notice that, for the predictions made from the curves for index age 50, 91 percent of the predicted values were within 5 feet of the observed site index; and, for the predictions made from the curves for index age 25, 97 percent were within 5 feet of the observed site index. As one would expect, tables 1 and 2 show that the further the age of a stand is from the index age at the time the prediction is made, the more aberrant the site index predictions will be. It follows that a set of site index curves with index age near the average age of the stands to be predicted should generate the most accurate predictions. For example, if rotation age for a forest property is 50 years or less, curves for index age 25 are preferred. If, however, the rotation is longer, curves for index age 50 might be preferable.

Equation 5, along with the estimates of the b_i ($i=1, 2, \dots, 8$) substituted in it, were used to calculate curves for selected site indices, as shown in figures 1 and 2. As noted by Lundgren and Dolid (1970), the height-age function used does not specify that the height at age 50 must be exactly equal to site index, but only that it be proportional to it. However, the error in predicted height at age 50 is usually small, and the curves in figures 1 and 2 have been adjusted to pass through the indicated site index at age 50 and 25.

In order to determine the deviations of site index predicted by our curves from observed site index, the stands were grouped into 7-foot site classes for index age 25 and 10-foot site classes for index age 50, and the deviations were plotted over the age at which the predictions were made. Where three or more plots were available in a site index class, there did not appear to be any trend in the deviations over the age at prediction.

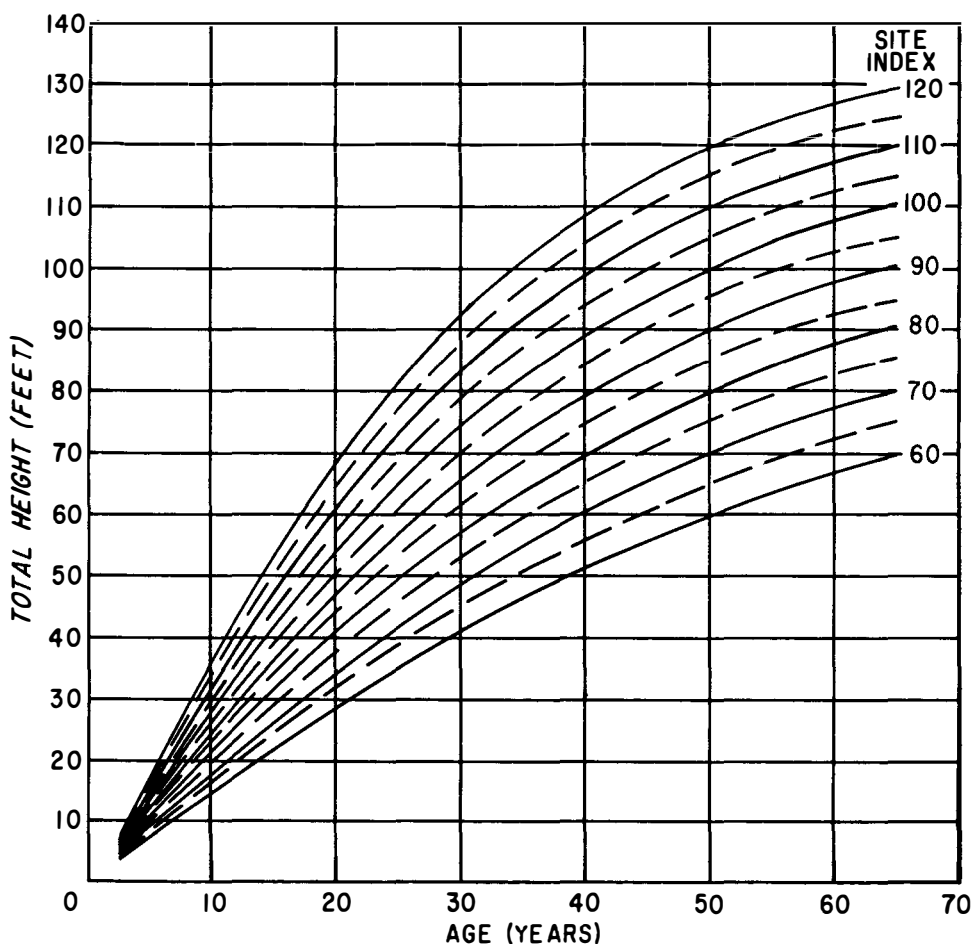


Figure 1.--Site index curves for loblolly pine at index age 50 years in the Coastal Plain of Virginia, North Carolina, and South Carolina. (These curves are based on stem analysis of 40 dominant trees in the middle and lower Coastal Plain.)

Notice that, because equation 5 cannot be solved for S in terms of H and t , it is necessary to use an iterative technique to calculate site index ($\hat{S}_0$) from stand age and sample mean height (t_0, H_0) of a particular plot. For further explanation of this procedure, see the Appendix. For practical purposes, site index may be estimated by using either figure 1 or figure 2.

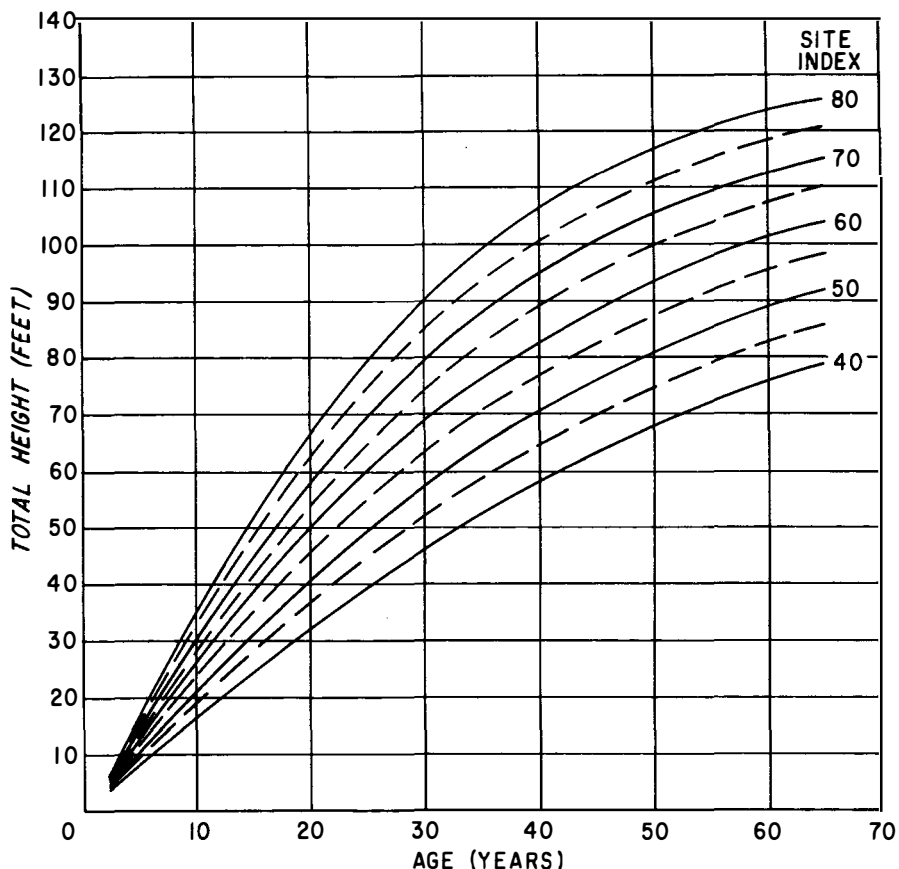


Figure 2.--Site index curves for loblolly pine at index age 25 years in the Coastal Plain of Virginia, North Carolina, and South Carolina. (These curves are based on stem analysis of 40 dominant trees in the middle and lower Coastal Plain.)

TEST AND COMPARISON OF SITE INDEX CURVES

Seventy-four permanent growth plots in another study provided independent data for testing our new site index curves with those of Schumacher and Coile (1960). We chose the Schumacher-Coile curves for purposes of comparison because they are widely used in estimating growth and yield of loblolly pine in the Southeastern Coastal Plain. When the growth plots were established in 1949, they ranged in age from 16 to 50 years and in site index (age 50) from 63 to 119 feet. Measurements of these 74 plots at 5-year intervals for a 20-year period furnished 364 separate estimates of age and height.

In our test of each curve, we wished to determine if site index estimates changed with age and if this relationship varied by broad site classes (that is, low, medium, and high sites). The grouping of the plots was as follows: low sites included site indices 63 to 89 (age 50); medium site included indices 90 to 104; and high sites included indices 105 to 119. The regression model used for these tests was

$$Y = a + bx + \epsilon$$

where Y equals site index and x equals age when site index was estimated. Separate regressions were computed for our new curves on the basis of low (Y_1), medium (Y_2), and high (Y_3) site groupings and for the Schumacher-Coile curves also on the basis of low (Y_4), medium (Y_5), and high (Y_6) site groupings.

The regressions of x and Y_1 , Y_2 , and Y_3 for our new site index curves were nonsignificant:

$$Y_1 = 83.3793 - 0.0303 x ; R^2 = 0.3 \text{ percent}$$

$$Y_2 = 96.3655 + 0.0827 x ; R^2 = 3.9 \text{ percent}$$

$$Y_3 = 107.7456 + 0.0235 x ; R^2 = 0.4 \text{ percent}$$

However, the regressions of x and Y_4 , Y_5 , and Y_6 for the Schumacher-Coile curves were all highly significant:

$$Y_4 = 60.7662 + 0.4098 x ; R^2 = 36.2 \text{ percent}$$

$$Y_5 = 77.5144 + 0.4335 x ; R^2 = 53.3 \text{ percent}$$

$$Y_6 = 88.8352 + 0.3814 x ; R^2 = 44.6 \text{ percent}$$

The slopes of the regressions for the Schumacher-Coile curves were not significantly different.

These tests and comparisons of the two site index curves show that site index estimates of remeasured plots do not change with increasing ages when our new curves are used but that the estimates do change when the Schumacher-Coile curves are used. The changing estimates generated by the latter curves indicate that the curves are biased. We therefore conclude that our new curves give unbiased estimates of site index that are consistently better than those generated by the Schumacher-Coile curves.

LITERATURE CITED

- Beck, Donald E.
1971. Polymorphic site index curves for white pine in the Southern Appalachians. Southeast. For. Exp. Stn., USDA For. Serv. Res. Pap. SE-80, 8 pp.
- Brickell, J. E.
1966. Site index curves for Engelmann spruce in the northern and central Rocky Mountains. Intermt. For. & Range Exp. Stn., USDA For. Serv. Res. Note INT-42, 8 pp.
1968. A method for constructing site index curves from measurements of tree age and height--its application to inland Douglas-fir. Intermt. For. & Range Exp. Stn., USDA For. Serv. Res. Pap. INT-47, 23 pp.
- Coile, T. S.
1952. Soil productivity for southern pines. Part 1. Shortleaf and loblolly pines. For. Farmer 11(7): 10-11, 13.
- Curtis, Robert O.
1966. A comparison of site curves for Douglas-fir. Pac. Northwest For. & Range Exp. Stn., USDA For. Serv. Res. Note PNW-37, 8 pp.
- Lundgren, Allen L., and Dolid, William L.
1970. Biological growth functions describe published site index curves for Lake States timber species. North Cent. For. Exp. Stn., USDA For. Serv. Res. Pap. NC-36, 8 pp.
- Middleton, J. A.
1969. Least-square estimation of non-linear parameters. NLIN [Nonlinear], IBM Share Program 360-13.2.003.
- Richards, F. J.
1959. A flexible growth function for empirical use. J. Exp. Bot. 10: 290-300.
- Schumacher, F. X., and Coile, T. S.
1960. Growth and yields of natural stands of the southern pines. 115 pp. Durham, N.C.: T. S. Coile, Inc.
- Spurr, Stephen H.
1952. Forest inventory. 476 pp. New York: The Ronald Press Co.
1956. Soils in relation to site-index curves. Soc. Am. For. Proc. 1955: 80-85.
- USDA Forest Service
1929. Volume, yield, and stand tables for second-growth southern pines. USDA For. Serv. Misc. Publ. 50, 202 pp.

APPENDIX

The least squares fit of equation 5 is

$$H = (\hat{b}_1 + \hat{b}_2 S) \left[1 - e^{-t(\hat{b}_3 + \hat{b}_4 S + \hat{b}_5 S^2)} \right] \frac{1}{1 - \hat{b}_6 - \hat{b}_7 S - \hat{b}_8 S^2} \quad (1A)$$

where the $\hat{b}_i$ ($i=1,2,\dots,8$) are the nonlinear least squares estimates of the model parameters. We want to obtain the value of site index (S^*) for particular values of height (H^*) and age (t^*) observed in some stand. However, equation 1A can not be solved for S in terms of H and t , and, therefore, it is necessary to find by iterative, numerical methods an approximation to S^* .

Equation 1A satisfies the mathematical properties necessary for the following procedure to work. Consider the equation in site index (S)

$$g(S) = 0 = H^* - \hat{f}_1(S) \left[1 - e^{-t^* \hat{f}_2(S)} \right] \frac{1}{1 - \hat{f}_3(S)} \quad (2A)$$

where H^* and t^* are numbers, that is, observed values of height and age from some stand. We want to find a value of site index (S_i) that is "close" to the value of site index (S^*) which satisfies equation 2A above, that is

$$g(S^*) = 0.$$

In other words, S^* is the predicted value of site index that would be obtained if equation 1A could be solved for S in terms of H and t . Start with an initial guess of site index, say S_0 . Compute

$$D_0 = \frac{g(S_0) - g(S_0 + h)}{h} \quad (3A)$$

where h is some small number, say $h = 0.001$. The quantity D_0 is an approximation to the value of the derivative function of g evaluated at $S = S_0$. One could find the derivative function, but this process is rather complicated. Then compute

$$\delta_0 = - \frac{g(S_0)}{D_0} \quad (4A)$$

and test whether

$$|\delta_0| \leq \Delta \quad (5A)$$

where Δ equals some small, positive number, such as $\Delta = 0.00005$. If 5A is not true, then set

$$S_1 = S_0 + \delta_0$$

and compute

$$\delta_1 = - \frac{g(S_1)}{D_1}$$

where

$$D_1 = \frac{g(S_1) - g(S_1 + h)}{h}$$

Now test if

$$|\delta_1| \leq \Delta . \quad (6A)$$

Continue computing δ_i 's until the statement

$$|\delta_i| \leq \Delta \quad (7A)$$

is true. When 7A is true for some small Δ , then δ_i is a good measure of how close S_i is to the unknown S^* .