



Measure twice, cut once: Optimal inventory and harvest under volume uncertainty and stochastic price dynamics[☆]



Matthew R. Sloggy^a, David M. Kling^b, Andrew J. Plantinga^{c,*}

^a Research Economist, USDA Forest Service Pacific Southwest Research Station, USA

^b Department of Applied Economics, Oregon State University, USA

^c Bren School of Environmental Science & Management, University of California, Santa Barbara, USA

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ABSTRACT

Natural resources are often subject to state uncertainty: resource abundance is not known with certainty, but can be measured. Measurements are typically imperfect and costly to obtain. The decision of whether to invest in resource measurement may be influenced by other state variables, for example a resource commodity price. We introduce a mixed-observability model of optimal forest management featuring a partially-observable forest resource and perfectly-observable stochastic price. The decision maker optimizes the expected net present value of forest returns by choosing when to measure current forest volume (conduct an inventory), harvest and replant, or delay action. Parameter values are obtained from numerous forestry data sources. Optimal investment in inventory reduces the cost of uncertainty about timber volume and increases the predictability of returns. Moreover, price stochasticity interacts with inventory decisions to produce asymmetric effects of high and low prices on inventory timing. We also produce the first graphical Faustmann rule analogues for jointly-optimal inventory and harvest.

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1. Introduction

Many studies addressing resource management under uncertainty and learning focus on parameter uncertainty (e.g. [Springborn and Sanchirico, 2013](#)). This orientation of the literature has left the problem of making choices when key state variables are either unobserved or partially observed relatively overlooked. Along with parameter uncertainty, resource managers also face state uncertainty, which differs from parameter uncertainty both conceptually and in terms of how it is operationalized in optimal resource management models. While parameters of resource systems can often be approximated as static (if unknown) quantities, most state variables are inherently stochastic and dynamic. Crucially, because the dynamics of these state variables are stochastic, in the absence of new information (obtained either through direct measurement or indirect signals) a resource manager's uncertainty about the current value of a state variable will typically grow from one decision period to the next

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* Corresponding author.

E-mail address: plantinga@bren.ucsb.edu (A.J. Plantinga).

(MacLachlan et al., 2017).¹

In practice, investment in information about the state of a system plays a central role in natural resource management, and previous studies demonstrate how the lack of such information can adversely affect management (e.g., Holopainen et al., 2010a). Despite the fact that natural resource managers routinely invest in information on the abundance of resources (e.g., fishery stock assessments, threatened species abundance), nearly all models of natural resource management ignore state uncertainty. Recent advances in optimization techniques have enabled a number of studies to overcome the high computational expense of modeling state uncertainty in resource management (Fackler and Haight, 2014; MacLachlan et al., 2017; Kling et al., 2017; Memarzadeh et al., 2019); however, these studies are limited to cases where all of the dynamic state variables are uncertain. There are many problems in natural resource economics that involve mixed observability, where some state variables are partially observable and others are perfectly observable (Fackler and Pacifici, 2014). Stocks of fish, trees, and minerals are generally not freely or perfectly observable to the resource manager, although costly investment in information can improve estimates of stock sizes. On the other hand, current input and output prices can readily be observed by the manager, even if future prices are stochastic. Dynamic optimization models that accommodate mixed observability have the potential to yield important new insights into natural resource management because they can uncover whether and how observable state variables influence optimal management and learning about resource stocks subject to state uncertainty. Mixed-observability models are also the theoretically appropriate framework for characterizing the value of information about uncertain state variables in systems where perfectly-observable states and uncertain states are likely to have an economically-significant link in decision making.

The aim of this paper is to address the problem of optimal measurement and harvest of a renewable resource under conditions of mixed observability. We examine the specific case of timber stand management, where the current per-unit price of timber is observable to the forest manager, while a costly investment in an inventory² is required to obtain an estimate of current timber volume (Scott and Gove, 2002; Pukkala and Kellomäki, 2012). Despite timber inventories being a common activity for forest managers (Scott and Gove, 2002), the economic rationale for these investments remains largely unexplored. To construct our model, we employ a framework known as a continuous-state Mixed Observability Markov Decision Process (MOMDP). Examples of MOMDPs addressing natural resource management have so far been limited to highly stylized applications where the resource state variables are discretized into a small number of categories (e.g., Chadès et al., 2012; Fackler and Haight, 2014). A continuous-state MOMDP approach allows for a more realistic description of resource dynamics, especially those of timber volume. An obstacle to applying MOMDPs in economics is the high computational cost of solving them. In order to preserve the continuous-state dynamics of timber volume, we extend an approximate solution technique from Zhou et al. (2010) that has been previously applied to simpler problems where all state variables are measured with error (MacLachlan et al., 2017; Kling et al., 2017). To the best of our knowledge, our model is the first continuous-state MOMDP in the natural resource economics literature.

We parameterize our model using volume, price, and cost data for timber stands in the southern United States and evaluate scenarios with and without inventories and under alternative informational assumptions. The empirical grounding of our optimization model is uncommon in the literature on state uncertainty.³ We find that the option to conduct inventories raises the expected net present value of the timber stand such that mean returns nearly reach their value in the counterfactual where timber volumes are perfectly observable. A key insight from our results is that stochastic price dynamics have an important influence on inventory decisions. Inventories are most likely to be chosen (specifically, the range of expected timber volumes at which inventories take place is greatest) at prices near the mean of the stationary price process. This is partly because the cost of delaying harvest in order to conduct an inventory is low because only small changes in price are expected. In contrast, when prices are high, the cost of delaying harvest is also high because prices are expected to fall, and so inventories are less common. Finally, at low prices, the manager is better off waiting to see if prices rise before committing to either a harvest or an inventory. This asymmetry in inventory decisions is magnified by the degree of uncertainty over the timber volume estimate. When there is a high degree of confidence in the estimate, inventories are only conducted at prices near the mean. However, as confidence decreases, it becomes optimal to conduct inventories at ever higher prices. With low confidence, an inventory may indicate a large timber volume that can be harvested before prices fall further. To our knowledge, this is the first characterization of the relationship between stochastic price dynamics and inventory in the literature.

Our analysis yields results of particular significance for forest economics. To our knowledge, we introduce the first economic theory of forest inventory. While the science and practice of forest measurement is central to contemporary forestry, investment in information has so far not figured into predictive models or normative economic prescriptions for fundamental activities like harvest planning. Using the solution to our model, we simulate the stochastic dynamics of timber stand manage-

¹ The multi-disciplinary literature on learning in optimal resource management is often grouped under the banner of “adaptive management” (Walters, 1986). The majority of papers in this literature restrict their attention to stochasticity and parameter uncertainty, and so this adaptive management is sometimes understood to be synonymous with parameter uncertainty. This has changed recently as state uncertainty has begun to receive more attention, and so a current understanding of adaptive management should be inclusive of models that account for state uncertainty (e.g., Chadès et al., 2017). However, because of this informal understanding and in order to emphasize our contribution to the economics of state uncertainty, we do not identify our analysis with the adaptive management literature.

² In order to keep the description of our model consistent with common terminology in forestry, we use the label “inventory” to describe an investment in a noisy measurement of current timber volume. A common label for this type of activity used in other economic research on state uncertainty is “monitor”. One method of inventory is called a “timber cruise”. See Section 2.2 for further details.

³ Notable recent examples of studies that strive to tie model parameter values closely to data include MacLachlan et al. (2017) and Memarzadeh et al. (2019).

ment. This allows us to study the timing of inventory investment alongside harvest choices. Notably, we find that inventories do not typically occur in young stands, even at very low levels of certainty. The reason is that resolving uncertainty is more valuable later in the stand's development, when it is more likely to be harvested. While our model is more complex than the classical Faustmann bioeconomic model, we are able to construct intuitive graphical analogues to standard visual depictions of the Faustmann model-based harvest timing. For the first time, we are able to generate bioeconomically-based "harvest and inventory schedules" that numerically characterize Faustmann-like rules for planning harvest *and* inventory jointly.

The remainder of this paper is structured as follows. Sections 2 reviews the relevant literature on models of state uncertainty and inventories for forest management. Our MOMDP model of a timber stand is presented in section 3, along with discussion of the solution technique and model parameterization. Results are presented in section 4 and conclusions are provided in the final section. Additional material on our analysis, including parameter values used and a detailed description of the solution method, may be found in the [Supplementary Information \(SI\)](#).

2. Literature review

2.1. State uncertainty in models of natural resource management

Studies that address state uncertainty in optimal resource management differ from a related literature on learning that focuses on structural determinants of system dynamics ([LaRiviere et al., 2018](#)). One variant is parameter uncertainty, where the target of learning is a static (but unknown) parameter in the objective function or state equation (e.g., the growth rate of a harvested stock) ([Walters, 1986](#)). As [MacLachlan et al. \(2017\)](#) note, while uncertainty regarding a static parameter may often be resolved given a sufficient number of opportunities to learn, in general state uncertainty cannot be resolved completely. A measurement of a stochastic-dynamic state variable in a particular period is generally less informative in a future period, because in the intervening time the state has undergone stochastic change. In practical terms, modeling state uncertainty requires different solution procedures, and insights from studies that focus on static targets for learning may not be transferable to applications where state variables are not perfectly observed.

There are a small number of early studies recognizing the significance of state uncertainty in resource economics (e.g. [Dixon and Howitt, 1980](#); [Clark and Kirkwood, 1986](#)). As computing power has improved, new approaches have been developed that have allowed researchers to overcome past constraints in modeling state uncertainty. A common thread among these contributions is their formalization of the management problem as Partially Observable Markov Decision Processes (POMDP) ([Kaelbling et al., 1998](#)). A majority of problems in the literature are addressed with discrete-state models, an approach which is most fruitful for contexts in which the state variable can be broken into a handful of meaningful categories (e.g., [White, 2005](#); [Fackler and Haight, 2014](#)).

Despite their difficulty to solve, discrete-state POMDPs still offer a less computationally demanding approximation technique compared to continuous-state POMDPs ([Kling et al., 2017](#)). However, continuous-state POMDPs can provide more realistic representation of state dynamics. This is especially true for models of forest management, as a model of timber harvest based on a small set of pre-specified volume categories would have limited usefulness. Only a few recent studies utilize continuous-state POMDPs to model natural resource management. [MacLachlan et al. \(2017\)](#) examine the treatment and control of bovine tuberculosis outbreaks, finding substantial efficiency losses in adaptive management models that ignore learning. [Kling et al. \(2017\)](#) investigate invasive species management and demonstrate that investments in monitoring can make control more efficient in the presence of state uncertainty. [Memarzadeh and Boettiger \(2018a\)](#) demonstrate that state uncertainty can lead to costly mismanagement of ecosystems, and that POMDP-based decision rules outperform those that do not consider state uncertainty. [Memarzadeh and Boettiger \(2018b\)](#) go on to apply POMDP methods to fisheries management, finding that neither optimal control approaches nor observed management adequately account for uncertainty in fish stocks, and that POMDP-based management outperforms all others. Although these studies demonstrate how state uncertainty can be incorporated into natural resource models to uncover important insights, neither study includes perfectly-observable stochastic-dynamic state variables that may shape investment in information about imperfectly observed variables.

By explicitly modeling stochastic price dynamics, we offer an economic application of continuous-state MOMDP methodology. As with POMDP applications, MOMDPs in the natural resource literature have typically been confined to discrete-state problems because of the curse of dimensionality. [Chadès et al. \(2012\)](#) outline a methodology for solving MOMDPs as discrete hidden model MDPs (hmMDPs). [Fackler and Pacifici \(2014\)](#) develop related discrete-state methods capable of incorporating several observed and unobserved variables simultaneously. Our model extends this work by including continuous-state dynamics, integrating empirically derived transition functions, and by making monitoring a control variable. One contribution of our model is illustrating how recent advances in addressing state uncertainty in continuous-state natural resource models can now be leveraged to analyze more general problems where some important stochastic-dynamic state variables are perfectly observed (or very accurately observed at negligible cost), while others are not.

2.2. Inventory in forest management

Investment in forest inventories is commonplace (Barlow and Levendis, 2015),⁴ but has largely been excluded from economic models of forest management. There are a wide variety of inventory options available to a forest manager. These methods range from collecting data on every tree in a forest or sampling the forest instead. Because a complete accounting of each tree is very costly, forest managers tend to survey their land instead to obtain an accurate yet imperfect signal of available timber volume, as well as other forest characteristics. These surveys are frequently carried out on the ground in what is called a timber cruise (Scott and Gove, 2002), but information can also be obtained from aerial observation (Naesset, 1997), as well as satellite imagery (Haapanen et al., 2000). Each of these techniques has an associated cost and expected level of error.

Even though addressing state uncertainty through inventory investments is common practice in forestry, previous research on forest management under uncertainty focuses on the influence of stochasticity from sources including price volatility (e.g. Thomson, 1992), wildfire risk (Reed, 1984), climate change (Guo and Costello, 2013), and stochastic biomass growth (Reed and Clarke, 1990). Morck et al. (1989) evaluates the case where both prices and volume evolve stochastically. Willassen (1998) generalizes the Faustmann formula to account for stochastic forest growth, modeled as a Brownian motion process. Others have modeled forest management under stochastic growth using Markov models (e.g. Buongiorno and Zhou, 2015; Buongiorno and Zhou, 2017) and have shown that stochasticity in forest growth influences planting and harvest decisions. Studies in this field do not address state uncertainty, with the notable early exception of Dixon and Howitt (1980) who investigates timber removals in the Stanislaus National Forest using a stochastic optimal control technique, finding that the specification of the objective function plays an important role in harvest levels as well as the cost of uncertainty in management.

Although there are few economic studies of forest inventories, there is a large forest science literature on the topic (Scott and Gove, 2002; Zobrist et al., 2012). The focus of these studies is often on the development of statistical methods for better inventory design (e.g. Korhonen and Kangas, 1997). Several studies in this area attempt to quantify the management implications of poor information on timber volume. Eid et al. (2000) find that suboptimal management decisions stemming from inaccurate measurements could potentially result in losses in the net present value of the forest. Holopainen et al. (2010b) find that inventory error was the largest contributor to errors in predicted levels of timber. Holopainen et al. (2010a) calculates the loss in a forest's value resulting from inventory error and results in Kangas et al. (2015) further demonstrate the role of information quality in meeting specific forest management objectives. Waggoner et al. (2009) discusses how even seemingly small errors in regional forest measurements can have enormous impacts on carbon stock estimates. The preceding studies find that lack of information on resource abundance can reduce the value of timber stands, but do not solve the problem of how to cost-effectively collect such information. We explore this solution in the next section.

3. Methods

We introduce a model of optimal harvest timing for a forest resource that includes both uncertainty about the volume of timber and stochastic prices. The forest manager's objective is to maximize the expected net present value (NPV) of timber rotations: the sequence of intervals beginning with the planting of the stand and ending with the harvest and sale of timber. A second control variable, which we label inventory, provides a measurement (with error) of the current volume of the timber stand.⁵ Each period, the forest manager chooses whether to harvest and replant the timber stand, to conduct an inventory, or do neither.⁶ The decision to conduct an inventory is costly. The manager observes the current timber price (but not future values) and maintains a belief state regarding timber volume, which takes the form of a probability density over volume levels.

3.1. Timber volume growth model

We begin by presenting the transition equation for timber volume:

$$X_{t+1} = f(X_t)Z(X_t, u_t) \quad (1)$$

In Equation (1), X_t is the timber volume in period t , $f(X_t)$ is a deterministic component of growth, and $Z(X_t, u_t)$ is a stochastic factor that is a function of current volume and an iid standard normal random shock, u_t . The deterministic component, $f(X_t)$, has

⁴ In addition to private investment in forest inventories, the U.S. Forest Service's Forest Inventory and Analysis (FIA) program received \$80 million of funding in 2015 (Vogt and White, 2017).

⁵ In the literature on state uncertainty, a control variable that generates an observation of a state variable is often called "monitoring." We use the label inventory instead because it is more common in the forest science literature. We do so at the risk of some confusion because inventory is sometimes also used as a synonym for the current level of the state variable (e.g., inventory of a durable good). In what follows, inventory is a control variable, not the timber volume state variable.

⁶ In practice, forest management involves other activities such as thinning, the removal of biomass to promote the growth of remaining trees (Schultz et al., 1997; Brazee and Bulte, 2000; Tahvonen, 2016). An area for future research is to include thinning as a control variable in the model, a possibility considered in the concluding section.

a standard Beverton-Holt form:

$$f(X_t) = X_t \left(\frac{r}{1 + \left[\frac{r-1}{K} \right] X_t} \right) \quad (2)$$

Here r is a growth parameter and K a parameter governing density-dependence of the deterministic component of growth.⁷

The stochastic factor $Z(X_t, u_t)$ is specified so that, conditional on current volume, it takes the form of a lognormal shock with volume-dependent moments. To simplify notation, we express $Z(X_t, u_t)$ using two functions, $\varphi_1(X_t)$ and $\varphi_2(X_t)$:

$$Z(X_t, u_t) = \exp(\varphi_1(X_t)u_t + \varphi_2(X_t)) \quad (3)$$

$$\varphi_1(X_t) = \frac{\sigma_{g1}}{X_t + 1} \quad (4)$$

$$\varphi_2(X_t) = -\frac{\sigma_{g2}^2}{2(X_t + 1)^2} \quad (5)$$

Here σ_{g1} and σ_{g2} are positive parameters. The form taken by the stochastic factor ensures that stand volume remains positive. We assume that $\sigma_{g1}^2 > \sigma_{g2}^2$, which is confirmed by a fit of Equation (1) parameters to data as described in the SI. Conditional on a specific value of X_t , denoted X_c , the mean and variance of Z_t are given by:

$$E[Z(X_t, u_t)|X_t = X_c] = \exp\left(\frac{\varphi_1(X_c)^2}{2} + \varphi_2(X_c)\right) = \exp\left(\frac{\sigma_{g1}^2 - \sigma_{g2}^2}{2(X_c + 1)^2}\right) \quad (6)$$

$$\text{Var}[Z(X_t, u_t)|X_t = X_c] = \exp\left(\frac{2\sigma_{g1}^2 - \sigma_{g2}^2}{(X_c + 1)^2}\right) - \exp\left(\frac{\sigma_{g1}^2 - \sigma_{g2}^2}{(X_c + 1)^2}\right) \quad (7)$$

It can be shown that when $\sigma_{g1}^2 > \sigma_{g2}^2$, the conditional mean and variance of the stochastic growth factor both decrease as the conditioning value X_c increases. At the same time, the stochastic factor has a multiplicative effect on the deterministic component of growth. This means that a particular value of the stochastic factor will have a larger absolute effect on growth when the current stand volume is larger, all being else equal.⁸

There are many stochastic influences that can affect tree growth, including weather, pests, tree disease, and fire. Our choice for the form of $Z(X_t, u_t)$ is based on patterns observed in timber growth data (which we describe in detail in the SI).⁹ In the literature on stochastic biological resource management using discrete time, it is common to specify a multiplicative growth shock with mean and variance that are independent of resource abundance (e.g., Reed (1979)). In our application, this assumption leads to implausibly large changes in timber volume at higher volume levels. We follow the general modeling approach for stochastic biological resource growth advocated by Sims et al. (2017) (who were inspired by Melbourne and Hastings (2008)), which we believe produces more realistic growth dynamics. We compare our growth model to the classical approach involving a volume-independent multiplicative growth shock in Section S3 of the SI.¹⁰

3.2. The forest Manager's problem

The forest manager is a price-taker who decides, at the start of each period, whether to harvest and replant the stand. Resource rent (net revenue) from harvesting is given by:

$$\pi(X_t, P_t) = P_t X_t - C_H \quad (8)$$

⁷ In particular, note that for conventional growth parameter values ($r > 1$), if $Z(X_t, u_t)$ were restricted to equal 1, volume would increase over time and reach $\bar{X}$ for any initial volume $\bar{X} \in (0, K)$.

⁸ One feature of the growth model is that, due to the role of the stochastic growth factor, the parameters in $f(X_t)$ do not correspond perfectly to, respectively, the intrinsic growth parameter r and density dependence parameter K (i.e., the long-run average abundance of the stand). In weighing these properties, it is important to remember that stand volume in practice rarely falls significantly below the replanting volume $\bar{X}$. Letting $f(\cdot)$ and $\bar{Z}(\cdot)$ denote the corresponding functions evaluated at the fitted parameter vector $[\hat{r}, \hat{K}, \hat{\sigma}_{g1}, \hat{\sigma}_{g2}, \hat{\bar{X}}]$, for the calibrated model, we find that $\hat{r} \approx 1.18$ and $\hat{\bar{X}}^{-1} f(\hat{\bar{X}}) E_{u_t}[\bar{Z}(X_t, u_t)|X_t = \hat{\bar{X}}] \approx 1.44$. Also, $\hat{K} \approx 2,519.4$ cubic ft. per acre, while the expected long-run volume of an unharvested stand is approximately 2593.8 cubic ft. per acre.

⁹ An alternative presentation of the growth model suggested by a reviewer is provided in the SI (Section S1).

¹⁰ In particular, we describe how fitting Equation (1) to observed data produces greater growth variability in relative terms when the stand is young compared to when it is more mature. The reasonable behavior of the fitted model over the range of realized dynamics, in particular the absence of implausibly high variation in growth when the stand is mature, leads us to conclude that it is a suitable specification for our study. Also in the SI, we compare the properties of our preferred growth model specification (including the associated fit to growth data and full model solution) with several alternatives that differ with respect to the structure of the stochastic growth factor (Equation (3)).

where C_H is a parameter measuring the fixed cost of preparing and replanting the stand, and P_t is the per-unit timber price, which we model as a first-order autoregressive process¹¹:

$$\ln(P_{t+1}) = \beta_0 + \beta_1 \ln(P_t) + \epsilon_{t+1} \quad (9)$$

where ϵ_{t+1} is a mean-zero normally-distributed error term. At the time the harvest decision is made, the manager is assumed to observe P_t , but not future values of the price.

Timber volume is not perfectly observed, but the forest manager can obtain information about the volume by conducting an inventory. An inventory is done at the end of a period and yields a measurement of the volume, Y_t , at the start of the next period. The measurement Y_t is related to the true volume X_t by:

$$Y_t = \omega_t X_t \quad (10)$$

where ω_t is an iid random shock with nonnegative support and expected value equal to one. The constant variance of ω_t implies that the expected error of an inventory is constant in proportion to the stand volume, which accords with our treatment of inventories as a binary decision. The forest manager incurs a cost of C_I for each inventory.

At the start of a period, the forest manager forms a belief about current timber volume, $B_t(X_t)$, using her knowledge of: the timber volume growth function and the relationship in Equation (10); the distributions of the random shocks u_t and ω_t ; the past history of harvest and inventory decisions; and any inventory measurements received since the last harvest. In the period immediately after the stand is harvested and replanted, the belief state is initialized at $B_0(X_t)$. The initial belief state is such that it is an accurate estimate of the true underlying replanting volume $\bar{X}$ ($E_{B_0}[X_t] = \bar{X}$) and is characterized by high confidence (low coefficient of variation (cv)).

If an inventory is conducted in period $t - 1$, the manager updates her belief about the current timber volume X_t using the estimate Y_t and Bayes' rule:

$$B_t(X_t) = \frac{p(Y_t|X_t) \int p(X_t|X_{t-1})B_{t-1}(X_{t-1})dX_{t-1}}{p(Y_t|B_{t-1}(X_{t-1}))} = \psi_I(X_t, Y_t, B_{t-1}(X_{t-1})) \quad (11)$$

where $p(\cdot|\cdot)$ is a conditional probability. As in Zhou et al. (2010), we specify $\psi_I(\cdot)$ as the Bayesian update function where the subscript I indicates the availability of a new inventory. The denominator of Equation (11) is the probability of obtaining the estimate Y_t given prior beliefs about timber volume, $B_{t-1}(X_{t-1})$. It is expanded below as:

$$p(Y_t|B_{t-1}(X_{t-1})) = \iint p(Y_t|X_t)p(X_t|X_{t-1})B_{t-1}(X_{t-1})dX_t dX_{t-1} \quad (12)$$

If the forest manager does not conduct an inventory or harvest the stand in period $t - 1$, the belief state is used as the basis for prediction:

$$B_t(X_t) = \psi_N(X_t, B_{t-1}(X_{t-1})) = \int p(X_t|X_{t-1})B_{t-1}(X_{t-1})dX_{t-1} \quad (13)$$

where the N indexing the update function $\psi_N(\cdot)$ indicates that no inventory or harvest was done.

Our MOMDP can be represented as a dynamic optimization problem over the product of the belief state space and the price state space (Chadès et al., 2012). The “belief $\times$ price” Bellman equation is presented below. To simplify the notation, we drop time subscripts and adopt the convention of denoting next period variables with a superscript “+”:

$$\begin{aligned} V(B(X), P) = \max_{H, I} \int & \left\{ H \times [\pi(X, P) + \delta \int V(B_0(X^+), P^+)p(P^+|P)dP^+] \right. \\ & + I \times \left[-C_I + \delta \iiint V(\psi_I(X^+, Y^+, B(X)), P^+) \times p(Y^+|X^+)p(X^+|X)p(P^+|P)dY^+ dX^+ dP^+ \right] \\ & \left. + (1 - H)(1 - I) \times \left[\delta \iiint V(\psi_N(X^+, B(X)), P^+) \times p(X^+|X)p(P^+|P)dX^+ dP^+ \right] \right\} B(X)dX \\ \text{s.t. } & H \in \{0, 1\}, \quad I \in \{0, 1 - H\} \end{aligned} \quad (14)$$

In Equation (14), H and I are indicators for the forest manager's choice of Harvest and Inventory, respectively, and δ is a discount factor. Given the current belief state and observed price, if the forest manager chooses to harvest ($H = 1$), she expects to receive net revenue estimated using the current belief state plus a discounted continuation value. The continuation value in this case depends on both the initial belief about the newly planted stand ($B_0(X^+)$) and expectations regarding the next-period price. If the manager chooses to conduct an inventory rather than harvest ($I = 1$), she spends C_I and the discounted expected continuation value accounts for the expected effect of the inventory measurement (Y^+) on next-period beliefs about volume. Finally, if neither

¹¹ In our model the per-unit price of the resource is the “stumpage” price, the price a harvester pays for the right to harvest a unit of timber on a parcel of land. The stumpage price is implicitly adjusted for any variable costs of harvesting.

harvest nor inventory is chosen ($H = 0, I = 0$), the discounted continuation value reflects expectations regarding both the next-period price and the belief state that results from permitting the stand to grow without being measured.¹²

The structure of the Bellman equation highlights how the forest manager weighs an inventory investment. Conducting an inventory involves foregoing harvest and paying an up-front cost in order to update beliefs about stand volume using an inventory measurement. The value of inventory must also be compared with the continuation value associated with doing nothing. In our application, the forest manager generally expects to have greater confidence in her volume estimate after receiving an inventory measurement. An inventory will be selected if, given the current belief state and price, the expected NPV of the forest stand associated with a belief update that uses an inventory measurement exceeds that of the alternatives (harvesting or taking no action).

The Bellman equation can in principle be solved for the optimal value function $V^*(B(X), P)$ and a stationary policy function that provides the optimal control choice given the current belief state and price. There is no closed form solution for the Bellman equation in this problem, so we approximate the solution numerically as described in the following section.

3.3. Solution method

In this section, we describe key concepts behind our solution method. Our aim in what follows is to provide intuition; additional technical information is provided in the SI. The high dimensionality of MOMDPs and POMDPs make them computationally difficult to solve (Papadimitriou and Tsitsiklis, 1987).¹³ We extend a technique developed by Zhou et al. (2010) that approximates the solution to a continuous-state POMDP by forming and solving a problem with greatly reduced dimension. The belief state in this method is approximated by a continuous parameterized density with support over possible values of the uncertain states. The parameters of approximating density become state variables in the resulting dynamic programming problem.

Following Kling et al. (2017), we use the lognormal density to approximate beliefs about timber volume. The lognormal density has a number of desirable properties for this application. These include a non-negative support, and the ability to summarize beliefs using two variables: mean and coefficient of variation (cv). A key challenge is that posterior densities defined by Equations (11) and (13) are not lognormal and must be simulated. As discussed in Zhou et al. (2010), an approach to fitting a lognormal density to the (simulated) posterior belief state involves finding the density function parameters that minimize the Kullback-Leibler (KL) divergence from the posterior. Because the lognormal is a member of the exponential family, the solution to this minimization problem has a computationally convenient closed-form: match the values of the lognormal sufficient statistics to those of the simulated posterior.

Even with the dimension-reducing properties of this solution method, numerically solving the dynamic programming problem efficiently and at a high resolution is a non-trivial problem. This is because it includes three continuous state variables (two for belief about timber volume, and one for the price). We first discretize a range of values for the belief state parameters, and use these values to form a mesh along with discrete price values. Each node on the mesh represents a particular combination of the belief mean, belief cv, and current price.

The next step is an application of the expected value approach to forming the Bellman equation suggested by Fackler (2018). To our knowledge, we are the first to exploit this technique to solve a continuous-state MOMDP for an economic application. Briefly, a direct approach would entail computing and storing the entire set of state transition probabilities for points included in the mesh. This would require a great deal of time and large amounts of memory for our application. Fortunately, since the price and approximate belief state variables are independent conditional on a control choice, we are able to calculate and store the associated transition probabilities separately: one transition matrix for belief mean and cv, and a separate transition matrix for the price state variable. This technique provides a means of forming the Bellman equation while realizing substantial computational time and memory savings. We also adopt a number of practices from Kling et al. (2017), including Halton draws for posterior belief state simulation and bilinear interpolation to construct the belief state transition matrix.¹⁴ We use modified policy iteration to solve for the policy and value functions. Additional details are provided in Section S6 of the SI.

¹² We assume that the forest manager must either harvest, conduct an inventory, or do nothing in each time step of the model. This means that an inventory and harvest may not occur in the same decision period. We have two reasons for this modeling choice. First, a delay between inventory and harvest is realistic given that an inventory takes time to plan, implement, and analyze. Second, a meaningful representation of both controls applied within the same time step would require a so-called multi-stage dynamic program, where each time step includes at least two decision periods. Without a multi-stage structure, an inventory coinciding with harvest within a single decision period would have no value since a pre-commitment to harvest will have already been made. Extending our model to accommodate multiple decision stages within each time step would be a further methodological advance relative to the continuous-state MOMDP we study in this paper Fackler and Haight (2014). We save this extension for future research.

¹³ Discrete-state POMDPs may be analyzed as dynamic optimization problems requiring a probability state variable for all but one category of each discrete state variable (Kaelbling et al., 1998). As Porta et al. (2006, p. 2333) note, when an uncertain state variables is continuous rather than discrete, the infinite number of possible values the state may take on would require the same number of probability states to represent. The difficulty only increases in a mixed-observability problem, since the observable states each require an additional dimension. This property of decision problems involving state uncertainty has oriented the technical literature towards developing approximate numerical solution methods (e.g., Kurniawati et al. (2008)).

¹⁴ Bilinear interpolation is used to construct the belief state transition matrix.

3.4. Model parameterization

To parameterize our model, we use a combination of reported values as well as relationships estimated from data on prices and forest growth. We use our model to investigate the management of loblolly pine stands in the southern United States. Loblolly pine is one of the most commercially important tree species in the U.S. and tends to occur in pure stands (Gaby, 1985). We use data from the Forest Inventory and Analysis database (FIADB)¹⁵ to parameterize the timber growth model. The FIADB is the best source of publicly available data on forest growth from the United States; however, it provides limited temporal variation. For instance, the most any plot has been measured in our sample is twice. By restricting our analysis to a single site class, we are able to minimize variation that is due to cross-sectional differences in site productivity, focusing instead on temporal variation in factors such as precipitation, temperature, and inherent randomness of biological growth. We select the most common site class for private forests in the FIADB. Data on real stumpage prices are taken from Howard and Jones (2016) for the period 1965–2013 and used to estimate the autoregressive price function in Equation (9). The parameter estimates indicate a stationary price process. Inventory, harvest preparation, and replanting costs are taken from Dooley and Barlow (2013) and data from Reynolds (2013) is used to characterize a typical timber inventory in the region and to derive the expected error of an inventory. We use a discount factor equal to 0.972, which is an estimate derived by Provencher (1995) for Southern Pine timber management. All parameters used in the numerical analysis that follows are reported in the SI.

4. Results

Our main results correspond to the model presented above that includes state uncertainty, price stochasticity, and forest inventories. We refer to this model as the “with inventory” (WI) model and its solution as the WI policy. We solve two additional cases of our model for the purpose of comparison. The first case is one in which the forest manager cannot invest in inventory but still confronts state uncertainty and price stochasticity. We label this the “no inventory” (NI) model. The second case is a stochastic Faustmann model (e.g. Willassen, 1998) that does not incorporate state uncertainty (stand volume in each period is freely and perfectly observable), but still maintains stochastic price dynamics. The solution to this model, referred to as the “perfect observability” (PO) policy, is a useful benchmark for the WI and NI policies because it is well understood (e.g., Brazee and Mendelsohn, 1988; Plantinga, 1998) and because the WI and NI policies are natural extensions of the PO policy. Wherever applicable, each alternative uses the same solution procedure and parameter set used to solve the WI model.

4.1. Optimal harvest and inventory decisions

The solution to the forest manager’s problem is a policy function that maps current values of the state variables (price and timber volume mean and cv) to optimal actions (harvest, inventory, and delay). We illustrate the WI policy in Fig. 1 as a plot of price against volume mean, holding the volume cv fixed at a low (Panel A) and high level (Panel B).¹⁶ For convenience, in what follows we refer to the mean and cv of the belief state as expected volume and confidence, respectively.

The PO policy is also represented in Fig. 1 by a dashed line. Since in the PO model the stand is perfectly observable in each period, the PO policy is a function of the current (actual) timber volume and the current per-unit price. For a given volume, the PO policy defines a price threshold above which harvest is optimal. For low volumes it is optimal to delay harvest and take advantage of high rates of timber growth, unless the current price is sufficiently high. Because price is stationary, it is expected to revert to the long-term mean ($P = 1.86$ from estimates in Table A2) and so harvesting at low volumes is done, in part, to take advantage of a high current price. At higher timber volumes, growth rates are low and so it becomes optimal to harvest at lower prices.

For a given level of confidence, the WI policy is defined by price-expected volume combinations for which it is optimal to conduct an inventory, harvest and replant, or do nothing. When confidence is high (Panel A), inventories are only optimal for current prices near the mean. To understand this result, recall that inventories are done at the end of a period in which the decision to not harvest is also made. Thus, when the current price is close to the mean, foregoing harvest to conduct an inventory has a lower opportunity cost since the price is not expected to change by much. An inventory in this case gives the manager better information to optimally time the harvest. As the price rises, the opportunity cost of delaying the harvest also rises because prices are expected to decline in subsequent periods. For high enough prices, it is better to harvest right away before prices drop. This is because at higher prices the value of an inventory is smaller (relative to doing nothing) as the result of expected future price declines. When prices are low, it becomes optimal to delay both harvests and inventories at high volume

¹⁵ Forest Inventory and Analysis Database. (2018). St. Paul, MN: U.S. Department of Agriculture, Forest Service, Northern Research Station. Available at: <https://apps.fs.usda.gov/fia/datamart/> (last accessed February 27, 2020).

¹⁶ In Fig. 1, the upper bound on expected volume shown in the policy function exceeds the long-run average volume of an unharvested stand. This choice is useful for numerical purposes, and also allows us to characterize behavior in rare cases where positive productivity shocks lead to very large expected stand volumes. The cv values chosen to show “iso-confidence” transects of the policy function are drawn from the distribution of realized cv values obtained from simulations of forest management under the WI policy, which we introduce below in section 4.2. The cv values are 0.13 and 0.52, respectively, and were selected to illustrate differences in the WI policy function as cv increases. The average cv at which the forest manager conducts an inventory is 0.56. The cv immediately after a harvest is always 0.033, by assumption.

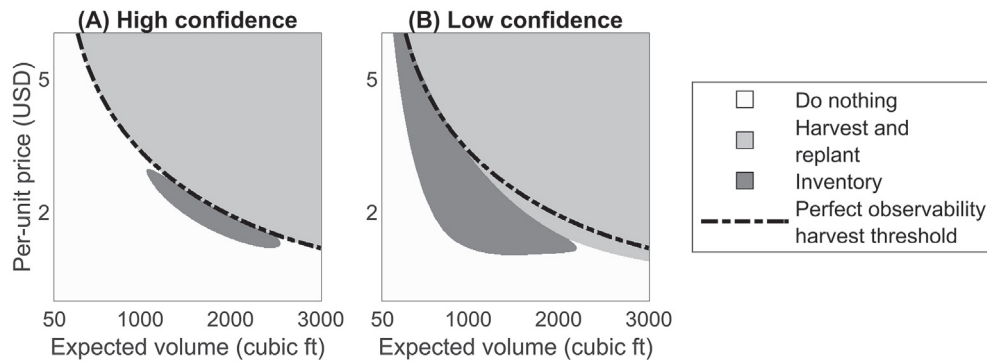


Fig. 1. Policy function for the with inventory (WI) case, holding confidence constant at a high level (A) and a low level (B). The perfect observability (PO) harvest threshold is overlaid for comparison. Note: The belief state cv values chosen for the high and low confidence levels are 0.13 and 0.52, respectively.

estimates because the price is expected to rise from its current value.

The inventory region expands when confidence in expected volume falls (belief cv increases) (Panel B). Inventories are more valuable when confidence is low because they are likely to have larger effects on the manager's belief about timber volume. The largest expansion in the inventory region between Panel A and Panel B occurs at prices near the mean because, as before, delaying the harvest to conduct an inventory has a lower opportunity cost. However, with low confidence, inventories are optimal even at high prices. Even though prices are expected to decline, an inventory may reveal a large timber volume that can make it optimal to harvest in the near term while prices are still relatively high. In contrast, inventories do not become optimal at low prices. Because prices are expected to rise, the value of information from future inventories is expected to increase and the manager should delay inventories to take advantage of this gain. Further insights into the role of stochastic prices in influencing the timing of inventories is obtained by considering the case of constant prices (see [SI Section S4](#)).

When confidence is high, the harvest region of the WI policy is identical to that for the PO policy ([Fig. 1](#), Panel A). However, with low confidence, there are some price-expected volume combinations at which harvest is optimal under the WI policy when delay is chosen under the PO policy (Panel B). This is a surprising feature of the policy function, in that it implies that the manager is *more likely* to make an irreversible harvesting decision under volume uncertainty compared to when volume is perfectly observable. The explanation is that for a given expected volume, a decrease in confidence (an increase in the cv) raises the likelihood of very high timber volumes at which harvesting is optimal. Changes in the probabilities assigned to low timber volumes have no effect since harvesting is never warranted in these cases. In effect, lower confidence makes the manager believe that harvesting is more likely to be the best action.

In the next section, we study simulations of stand management under the WI policy. Since each rotation is initialized from the same initial belief state $B_0(X_t)$ characterized by low expected volume (equal to $\bar{X}$) and high confidence, it is rare for applications of the WI policy produce state dynamics that reach the high expected volume/low price/high cv zone of the state space where the harvest region substantially diverges from the PO harvest threshold. However, the behavior of the harvest and inventory zones when cv is high can be interpreted as providing insight into the incentives facing a manager who acquires a forest stand and has poor information about current timber volume.

4.2. Harvest and inventory dynamics

We explore the dynamics of harvest and inventory investment to gain further insight into the model. For each of three versions of the forest manager's problem—management with inventory (WI), no inventory (NI), and perfect observability (PO)—we simulate 20,000 dynamic realizations of timber stand management over 200 years in order to compare management choices and performance across policies. Like other models that consider state uncertainty, there are two sets of initial conditions required to simulate dynamics: the true initial conditions of the bioeconomic system (price and stand volume) and the forest manager's initial belief about stand volume. All dynamic realizations considered in this section begin with a newly-replanted stand. The forest manager's beliefs under the WI and NI policies are initialized at $B_0(X_t)$, and return to this value immediately following a decision to harvest and replant. We use realization-specific sequences of exogenous shocks to produce output from the growth, price, and inventory measurement equations. This means that each realization under the WI policy can be matched with another realization in either the PO and NI policies that feature the same price path and same exogenous growth shocks. Differences among these realizations of stand management are therefore a consequence of different choices made by the forest manager. For the WI and NI policies, we also record the true dynamics of timber volume in addition to the forest manager's belief state. In the [SI](#), we include a supplementary analysis that finds changing the initial conditions has a modest effect on outcomes evaluated over the long-run.

[Fig. 2](#) provides an example of counterfactual dynamic realizations of stand management under the WI and NI policies. To simplify the illustration, instead of plotting prices and the dynamics of belief regarding volume separately, we show the dynam-

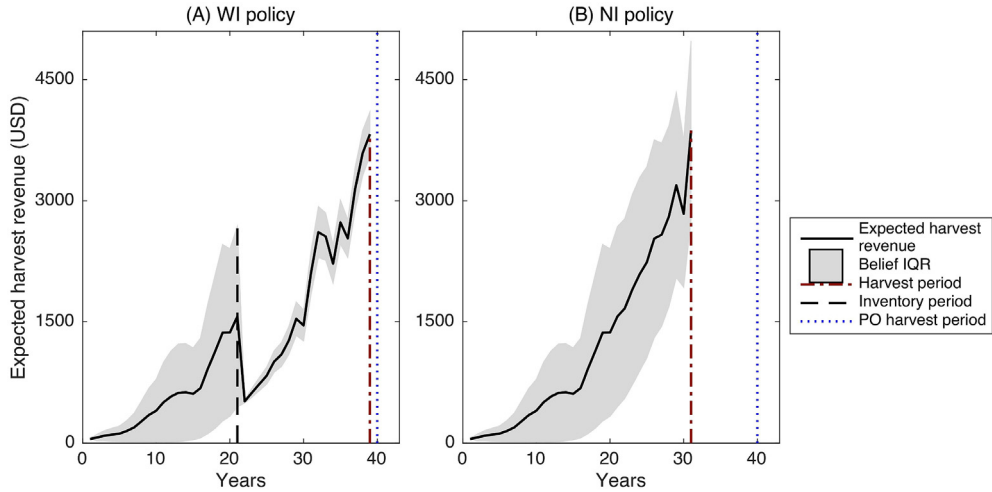


Fig. 2. Example of belief dynamics, harvest, and inventory decisions under different policies. (A) Expected harvest revenue dynamics under the with inventory (WI) policy. (B) Expected harvest revenue dynamics under the no inventory (NI) policy. Each panel shows a single realization of stand management, matched to counterfactual dynamics in the other panel. The forest manager's confidence is represented by the interquartile range (IQR) of the belief state. The timing of harvest under the counterfactual perfect observability (PO) policy is overlaid for comparison.

ics of belief over harvest revenue: the belief state over volume scaled by the current price.¹⁷ Both panels plot expected harvest revenue, and the forest manager's confidence is represented by plotting the interquartile range (IQR) of the belief state scaled by the current price. Because differences in harvest timing among the policies accumulate over time, we plot the first rotation only. The timing of harvest under both policies is indicated, as is the timing of inventory selected by the WI policy in this example. The timing of harvest under the matched PO counterfactual is overlaid for comparison.

This example highlights a few noteworthy features of the dynamics of the forest manager's problem. First, investment in inventory increases confidence (narrows the belief IQR in this illustration) (Fig. 2, Panel A).¹⁸ The increase in confidence persists over several periods but can degrade over time without new information from another inventory or a decision to harvest. Although confidence in the current estimate can be low (the belief IQR can be wide) early in the rotation, the forest manager does not typically invest in an inventory because information on stand volume has little value. This is because the volume estimate is too far below levels that may trigger harvest (depending on the current price). In other words, uncertainty in itself is not worth reducing with an inventory unless the present value of the expected benefit of improved harvest timing is less than the up-front inventory cost. Another feature of the dynamics is the stochasticity induced by the price state variable. While the variability of stand growth declines as the stand matures (Equation (7)), price dynamics can induce swings in expected harvest revenue, with implications for harvest timing. In particular, without the price fluctuations, the evolution of the belief mean for expected revenue would be smooth prior to and after an inventory.¹⁹ Lastly, relative to the NI policy, the timing of WI harvest tends to be nearer to that of the PO counterfactual in the first rotation.

Table 1 summarizes selected features of the dynamics.²⁰ The WI policy tracks features of the PO policy closely. For example, the true volume at harvest under the WI policy is nearly the same as under the PO policy, and expected volume at harvest on average deviates from true volume by 0.15%. The average rotation length (number of years between harvest and replanting) chosen by the NI policy is similar to the alternatives, however results in terms of realized harvest are more variable. The standard deviation of the harvested volume within realizations is approximately 76% higher than under the WI policy. Unsurprisingly, without the ability to invest in inventories the volume expected by the forest manager at harvest is on average 7.85% off of true volume in absolute terms. Although the WI policy comes closest to mimicking properties of the PO counterfactual, the similar results from the NI policy reflects the sophistication of the Bayesian decision rule under the NI policy.

¹⁷ For a particular policy $k \in \{WI, NI\}$ we plot $R_t^k = P_t E_{b_t^k}(X_t)$, where the expectation is taken over the policy-specific belief state b_t^k . The interquartile range of belief about current expected harvest revenue in period t is composed of an upper bound, $IQR_t^k = [P_t \exp(\mu_t^k + \sigma_t^k \Phi^{-1}(0.25))]$, and a lower bound, $IQR_t^k = [P_t \exp(\mu_t^k - \sigma_t^k \Phi^{-1}(0.25))]$, where (μ_t^k, σ_t^k) are the density parameters characterizing b_t^k and $\Phi^{-1}(\cdot)$ is the inverse CDF of the standard normal distribution.

¹⁸ Inventories always increase precision of beliefs in expectation, but for a given realization it is possible for uncertainty to increase following an inventory if the new observation is far from what would otherwise have been next period's projected point estimate.

¹⁹ It is worth noting also that for any given realization of stand growth under the WI policy, belief regarding stand volume will evolve the same way from $B_0(X_t)$ up until the initial inventory.

²⁰ For example, we compute the average number of harvests for each realization and then compute the standard deviation of these averages to obtain the standard deviation of the average within realizations.

Table 1

Summary of dynamic harvest and inventory behavior over a 200 year time horizon.

Feature	Policy ^a		
	With Inventory (WI)	No Inventory (NI)	Perfect Observability (PO)
	Mean Value [Standard Deviation]		
Harvest rotation length (years)	32.92 [7.14]	33.03 [6.51]	32.89 [7.14]
Number of harvests	5.56 [0.62]	5.54 [0.58]	5.57 [0.62]
Per-unit price at harvest ^b	2.23 [0.46]	2.27 [0.47]	2.23 [0.46]
True volume at harvest (cubic ft. per acre)	1905.30 [351.23]	1851.20 [511.11]	1905.70 [347.63]
Percent deviation between expected volume and true volume ^c	0.15% [3.89%]	7.85% [79.62%]	–
Timing of first inventory (years following planting) ^d	20.03 [3.40]	–	–
Number of inventories per rotation	1.09 [0.12]	–	–

Notes:

^a For each policy, we simulate 20,000 dynamic realizations over 200 years starting from a newly replanted stand. Each set of policy-specific realizations is generated using a common set of exogenous shocks.^b The average per-unit price is \$1.86.^c We calculate this as the average of $(X_{t,i}^{k,true} - E(X_{t,i}^k))/E(X_{t,i}^k)$ for each policy k taking the average first within realizations over time t and then across realizations i .^d Time is measured starting from either the initial planting or the harvest and replanting of the stand. At least one inventory is conducted prior to harvest in 100% of rotations under the WI policy. On average, at least two inventories occur prior to harvest in 8.6% of rotations under the WI policy.

4.3. The value and predictability of forest management

In this section, we study the performance of alternative policies. We use the same set of dynamic realizations employed above. We also add a fourth counterfactual: a fixed inventory (FI) policy that modifies the NI policy by assuming that the forest manager invests in inventory at regular intervals. This alternative is meant to represent common rules-of-thumb for how long to wait before measuring a stand. We assume under the FI policy that the forest manager invests in an inventory every seven years, which is consistent with recommendations from the forestry extension literature (e.g., [Northwest Natural Resource Group and Stewardship Forestry, 2014](#)).

We compare the average performance of the WI, NI, and FI policies relative to the PO counterfactual in [Table 2](#). In terms of mean net present value (NPV), the WI policy comes closest on average to matching the value obtained under perfect observability. The average loss of NPV under the WI policy is far less than that of the NI policy. The tendency of the FI policy to over-invest in inventories, and do so at sub-optimal times, results in it having the worst performance when compared to the PO counterfactual. The gap is smaller than might be expected, however, because the frequent inventories under the FI policy provide the forest manager with the most accurate beliefs on average out of the three state uncertainty policies.

To provide further insight into the performance of the policy alternatives, [Table 2](#) also reports average realized harvest rent (Equation (8)). The loss associated with the NI policy comes entirely from lost harvest rent. In contrast, the WI and FI policies both come close to matching the PO policy. Comparing the average discounted harvest rent obtained from the WI policy relative to the NI policy, instead of the PO baseline used up to now, provides an estimate of the average (gross) value of information (VOI) generated by inventories.²¹ On average, the use of inventory increases the NPV of harvest revenue by 4.52%, or \$85.53 per acre in gross terms.

When comparing results for rent generation reported in [Table 2](#), it is important to recall the properties of the policy alternatives. While the WI policy is the first-best solution to the forest manager's problem, the NI policy is itself a sophisticated second-best Bayesian response to volume uncertainty and price dynamics. The FI policy is a different type of second-best policy, where the constraint involves mis-timed investment in inventory rather than removing the control. As such, the NI and FI policies present credible competition for the WI policy. Lastly, we note that the timber volume growth model employed here does

²¹ As discussed by [Kling et al. \(2017\)](#) and others, state uncertainty typically presents a different problem than parameter uncertainty in the following sense. In parameter uncertainty problems, it is typically assumed that obtaining and using information in order to learn about the parameter is costless. As a result, VOI calculations using results for models with and without learning involve no cost of learning, aside from different actions that may be induced by incentives for experimentation. In contrast, problems involving state uncertainty often include the option to invest in measurement of the state directly, as is the case with the inventory control in the forest manager's problem. To make our VOI calculation more comparable to similar calculations in the adaptive management literature, we follow [Kling et al. \(2017\)](#) and emphasize a gross calculation. The corresponding net VOI calculation, where the cost of inventories is netted out, produces a 3.75% average difference between the WI and NI policies, or \$64.56 per acre.

Table 2

Value and predictability of forest management policy alternatives over a 200 year time horizon.

Feature	Policy		
	With Inventory (WI)	No Inventory (NI)	Fixed Inventory (FI)
NPV relative to the perfect observability (PO) policy (% difference) ^a	−0.96%	−4.58%	−3.63%
	[5.94%] ^b	[16.28%]	[6.40%]
Difference in NPV compared to PO policy (\$/acre)	−22.15	−86.71	−92.68
	[153.98]	[375.57]	[150.14]
Harvest rent relative to PO policy (% difference) ^c	−0.21%	−4.58%	−0.20%
	[5.87%]	[16.28%]	[5.89%]
Difference in harvest rent compared to PO policy (\$/acre)	−1.18	−86.71	−1.26
	[154.10]	[375.57]	[149.91]
NPV coefficient of variation (cv) relative to PO policy (% difference) ^d	0.63%	10.64%	3.33%
Difference in NPV standard deviation compared to PO policy (\$/acre)	0.82	43.38	0.26

Notes:

^a The mean NPV of forest management under the PO policy taken, calculated over 200 years, is \$2914.30 per acre.^b Bracketed terms report standard deviations of the corresponding calculation taken over matched counterfactual realizations.^c Harvest rent is $p(\tau_i^k)X(\tau_i^k) - C$ where τ_i^k is the timing of harvest i for policy-specific dynamic realization k .^d The PO policy NPV standard deviation is \$589.64 per acre.

not include various shocks that may affect harvestable volume, like insect infestations (Huang et al., 2010). Taken together, our conjecture is that these modeling choices act to mute the value of the WI policy relative to the alternatives. We return to this issue in our concluding discussion.

Another performance measure we consider is the predictability of returns from forest management. Relative to the PO policy, the coefficient of variation of realized harvest NPV under the WI policy is 0.63% greater, as compared to 10.64% greater for the NI policy (Table 2). As the result of frequent inventories, the FI policy generates less variable rents than the NI policy (3.33% difference in NPV coefficient of variation relative to the PO policy); however, the returns to the FI policy are still less predictable than those under the WI policy because of sub-optimal timing of inventories.

4.4. Generating schedules for inventory and harvest

The Faustmann rule is a simple and descriptively powerful result from natural resource economics that indicates the optimal timing of harvests (Amacher et al., 2009). Although our model generalizes the classical Faustmann model, it is possible for us to generate inventory schedules, much like how the Faustmann rule can be used in practice to produce harvest schedules. To generate inventory schedules, one could summarize only the average behavior taken over all dynamic realizations. This is essentially what is done in Table 1 above. However, we can take advantage of the WI policy function to generate rules-of-thumb for inventory and harvest. Drawing from our 20,000 simulations, we extract the realizations meeting criteria for price—price paths below the 40th percentile (“low prices”) or above the 60th percentile (“high prices”)—and volume—volume paths from below the 40th percentile (“slow growth”) or above the 60th percentile (“fast growth”). This yields four sets of realizations. For each set, we calculate the average time of the first inventory, the second inventory if applicable, and harvest. The timing of inventories and harvests are indicated on a graph of the average *actual* volume.²² We label the result a “harvest and inventory schedule”. While the harvest and inventory schedules correspond to a single rotation, they reflect incentives inherent in the long-run multiple rotation problem.

The harvest and inventory schedules generated from this procedure are shown in Fig. 3. They are heuristic reinterpretations of the policy function, and can be used similarly to growth tables, which recommend harvest timing following Faustmann logic and are commonly used by foresters. (Baldwin and Feduccia, 1987). The patterns across the cases are intuitive. For example, a shorter rotation length is recommended when prices trend high (e.g., compare Panels A and B of Fig. 3). Earlier investment in inventory is also a response to high prices. When prices are high, the first inventory occurs in about year 18 (Panels A and C), compared to year 22 when prices are low (Panels B and D).²³ Following the first inventory, the forest manager expecting to be on a slow growth (or low productivity) path delays harvest and a second inventory becomes more common (Panels C and D). The average timing of the second inventory responds to price trends similarly to the timing of the first inventory, occurring sooner when prices are high (compare Panels C and D of Fig. 3).

The basic Faustmann model assumes stand volume is always perfectly observable, and that volume evolves deterministically. In practice, current price is perfectly observable (or approximately so) but stand volume must be measured, which raises a question: how might a forest manager use harvest and inventory schedules similar to the cases included in the figure given that true volume is not known with certainty? The role of inventory provides some insight. In practice, the result of an inventory may change the harvest and inventory schedule the forest manager uses going forward. For example, a forest manager may expect

²² It is important to remember that under the WI policy timber volume is imperfectly observed by the manager. Thus, the average volume depicted in the graph is based on realizations and is not the same as the manager's expectation of the volume.

²³ After the stand is planted, the belief state for volume evolves identically for all stands and so only prices affect the timing of the first inventory.

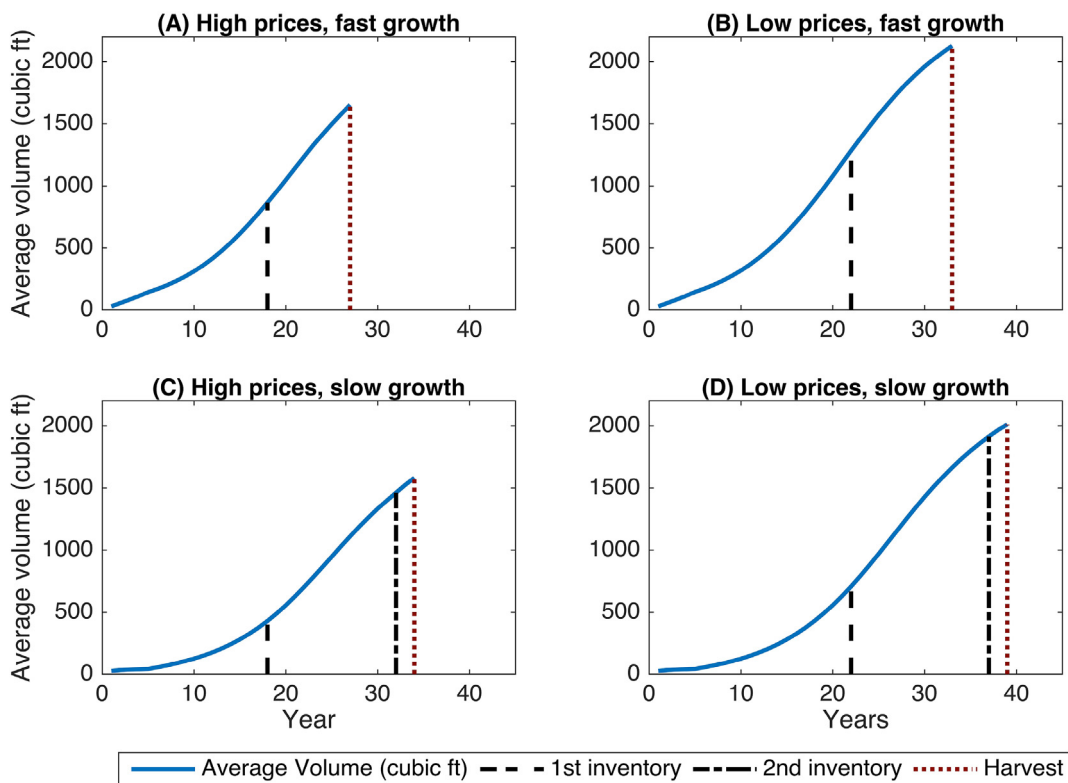


Fig. 3. Harvest and inventory schedules at low and high price levels, varying the average volume growth history. Note: Price levels were selected from percentiles of the realized price distribution, where the low price corresponds to prices below the 40th percentile and high price to prices above the 60th percentile. The slow and fast growth paths shown here are the average realized volumes from the bottom and top 40 percent of realizations, respectively.

a slow growth path and observe lower prices (Panel D). In response to a higher-than-expected inventory measurement, a shift to a faster growth (higher productivity) schedule may help the manager determine whether harvest is overdue (Panel B). A low inventory measurement, in contrast, would recommend delaying harvest and potentially a second inventory.

5. Conclusion

Natural resources differ from other forms of capital in the economy, such as built capital, in that they are often not perfectly observable to economic agents. Instead, management decisions must be based on imperfect information about dynamic states. Increasingly, theory is being introduced to account for state uncertainty and its implications, in the process developing explanations for why natural resource managers invest in information about resource status. So far, however, the emerging literature on state uncertainty within natural resource economics has mostly overlooked how observable states, like current commodity prices, influence decision making with respect to partially-observable states.

To study the link between observable and partially-observable state variables in natural resource management, we introduce a mixed-observability model of timber stand management under stochastic price dynamics. To the best of our knowledge, our analysis introduces the first continuous-state, mixed-observability Markov decision process (MOMDP) model in natural resource economics. The flexibility and intuitive Bayesian structure of the decision problem make MOMDPs a useful platform for studying decision making in natural resource systems where states are characterized by varying degrees of observability. Moreover, we demonstrate that it is possible to approximately solve the MOMDP problem while maintaining continuous-state representations of the observable and partially observable variables, rather than ad-hoc discrete-state simplifications where states are forced to jump among a small number of pre-determined bins. We expect that our methodology can be adapted to study a wide variety of economically-significant natural resource management problems that feature mixed observability. For example, in public health surveillance local economic indicators like the employment rate may influence where monitoring resources can efficiently be directed to track and treat disease outbreaks (Fenichel et al., 2011).

Our study stands out from the stylized approach commonly taken in the literature on state uncertainty in resource management through its use of multiple data sources to derive parameter estimates for biological and economic processes. This empirical grounding allows us to generate results that we regard as generalizable to other systems, in addition to findings that have particular relevance to forest economics. Although parameter values are drawn from disparate data sources, our model

reproduces observed patterns of inventory and harvest timing for loblolly pine stands.

Our numerical analysis demonstrates the extent to which the optimal management of an uncertain state variable (timber volume) is influenced by a key observable state variable (timber commodity price). For a given expected volume and level of confidence, a low or high current price relative to the average can disincentivize inventory investment, but for different reasons. High prices can tip the decision from inventory to harvest in order to avoid price declines, all else being equal. The expectation of future price increases given low current prices may delay inventory investment as well as harvest. This result indicates that it is not always economical to resolve uncertainty. Indeed, we find that inventories are rarely conducted for young stands, despite the fact that timber volumes are highly variable.

We contribute to forest economics by introducing the first bioeconomic theory of optimal joint forest inventory and harvest. This paper's forestry application re-starts a line of inquiry into state uncertainty in forest management that has early origins, but has since seen only a few contributions (Dixon and Howitt, 1980). Our simulation results reveal how inventory investment improves the timing of harvest, driving down losses in harvest revenue, but at the cost of expenditures on measurement. For the silvicultural system examined, we find that the manager can use inventories to nearly reach the stand value achieved under perfect observability.

There are several avenues for building on this study in future research. In order to keep our model tractable, we make several simplifying assumptions that can be revisited. We limit the control space by modeling inventory as a binary control, while in practice the forest manager may select from a broad menu of options with different costs and error properties. The model could be expanded to include multiple inventory methods. A multi-stage version of our model, where multiple decision stages may occur within a single time step, could be constructed that would allow for the option of harvesting just after an inventory measurement is received (without pre-committing to harvest) (Fackler and Haight, 2014). Additionally, our single-state biomass model of timber volume overlooks age structure and multi-species composition, two important components of forest management (Tahvonen, 2004; Tahvonen et al., 2018). Incorporating non-market ecosystem services to this problem also represents a viable contribution. Adding any of these features to our model will increase the dimensionality of the model, potentially making it more difficult to solve. It may be possible to handle larger models involving state uncertainty by embedding our belief state approximation scheme within an approximate dynamic programming solver (Powell, 2010).

State uncertainty features in important areas of forest policy, and we expect that methods introduced here can be further developed to inform policy design. First, in addition to methodological refinements, the same analysis could be replicated for other silvicultural systems (e.g., Douglas fir management in North America). Two additional topics that stand out are non-timber forest ecosystem services and forest-based carbon offset markets. State uncertainty is potential challenge for managing ecosystem services, for example habitat utilization within a forest. There is an opportunity to extend this research to develop a generalized Hartman rule for forest management and resource measurement (including measurement of the ecosystem service) (Hartman, 1976). A component of forest-based carbon offsets is measurement requirements for forest owners participating in the program, which may be cost-prohibitive to the forest owner. Methods related to those developed in this paper may be useful for investigating the efficiency of measurement ("verification") requirements in terms of adoption and overall program performance. One path forward would entail generalizing the MOMDP framework to a multiple agent model, in which offset buyers and sellers behave strategically, and then using the resulting framework to evaluate measurement requirements from a mechanism design perspective.

Appendix: Supplementary Information Supplementary data to this article can be found online at <https://doi.org/10.1016/j.jeem.2020.102357>.

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