DIFFERENCES IN FIXED-RADIUS PLOT SAMPLE SIZE ESTIMATES WHEN USING THE INHERENT VARIABILITY AMONG DIAMETER CLASSES IN STAND TABLES RATHER THAN AVERAGE VALUES

Curtis L. VanderSchaaf

Abstract—As part of many forest inventories, sample sizes needed to meet some desired level of precision are estimated. In many timber inventories, based on estimates of the inherent variability (e.g., coefficient of variation, CV), sample sizes are estimated for average total merchantable volume or for the most valuable product class such as average sawlog volume. However, often foresters are equally interested in producing accurate and precise stand tables. Stand tables contain number of trees by diameter and/or height classes while stock tables contain amount of volume by diameter and/or height classes. The most important component of most timber prospectuses in the Southeastern United States is actually stand and/or stock tables, not average values. This study attempted to quantify differences in sample size estimates of fixed-radius plots when using variability among diameter classes in stand tables as opposed to the variability in average values in the sample size equation. Several methods were proposed to quantify the CV among diameter classes within stand tables. Fixed-radius plots of varying sizes from many inventories conducted across the Southeastern United States and a few from other parts of the country were examined. In most cases, sample sizes needed for stand tables were at least twice as large when compared to estimating average trees per acre. However, when using Method Four at an 80 percent probability, sample size estimates (n = 27) were similar to sample sizes for average volume per acre at 95 percent probability (n = 29).

INTRODUCTION

Forest inventories are used to estimate the resources that exist on a site and aim to balance both accuracy and precision while minimizing time and cost. As part of many inventories, sample sizes needed to meet some desired level of precision are estimated. In many timber inventories, sample sizes are estimated for variables such as average total merchantable volume or for the most valuable product class such as average sawlog volume, etc. (e.g., Avery and Burkhart 2002: 37-39, 255; Shiver and Borders 1996: 22, 106-108).

However, foresters are often equally interested in producing accurate and precise stand and stock tables. Stand tables contain average number of trees by diameter and/or height classes while stock tables contain the amount of volume by diameter and/or height classes. The most important component of most timber prospectuses and even management plans in the Southeastern United States are stand and stock tables rather than average or total values. These stand tables and stock tables allow loggers, procurement foresters, timber buyers, managers, etc., to see what product classes exist and how many trees are in each product class. Other examples in forestry where stand tables are important are forest nursery inventories, where managers are more interested in number of seedlings by diameter class rather than total number of seedlings.

This study examined several ways to determine sample sizes using stand tables. It then examines the differences in sample size estimates for stand tables compared to the more traditional means of using some average value per acre for fixed-radius plots. Although most data are from pine plantations in the Southeastern United States, some studies are of hardwoods in the Southeastern United States and some studies are of species in other regions of the United States.

METHODS

Quantifying Variability within Stand Tables

Sample size is often estimated using equation (1):

$$n = \left[\frac{z\ CV}{\%E}\right]^2 \tag{1}$$

where

n = sample size needed to meet some desired level of precision, or a given bound on the error of estimation with a specified degree of confidence (Shiver and Borders 1996: 106),

Author information: Curtis L. VanderSchaaf, Assistant Professor, Louisiana Tech University, Ruston, LA 71272.

Citation for proceedings: Bragg, Don C.; Koerth, Nancy E.; Holley, A. Gordon, eds. 2020. Proceedings of the 20th Biennial Southern Silvicultural Research Conference. e–Gen. Tech. Rep. SRS–253. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station. 338 p.

z = z-value needed for a particular percent confidence (probability),
CV = coefficient of variation, and
%E = allowable error expressed as a percent.

Occasionally, with fixed-area plots, the finite population correction sample size equation is used. For this study equation (1) was used.

Allowable error of 10 percent is common (e.g., State of Florida Timber Cruise/Timber Appraisal Standards, https://floridadep.gov/lands/bureau-appraisal/documents/timber-cruise-timber-appraisal), although some have used values such as 15 percent (Coble and Grogan 2007). Agencies may allow probability levels as low as 68 percent when conducting inventories for management plans (State of Florida). However, timber appraisals will likely require probability levels no lower than 80 percent.

All proposed methods to quantify uncertainty (e.g., CV in equation (1)) are based on using separate plot stand table estimates. There are generally three acceptable ways to estimate CV within a stand/site prior to conducting the operational cruise. First, conduct a pilot/preliminary study where 2 to 15 plots are measured throughout the stand. These plots are then used to estimate the CV. Second, use CVs from inventories of similar stands. Three, use CVs from previous inventories of the stand/site. This study will use the first approach, estimating sample sizes using a pilot/preliminary study.

It is assumed the CV estimated on the pilot/preliminary studies is our best estimate of the true value, and hence it is assumed this estimate equals the true value. Thus, the *z*-value is used within equation (1) and not a *t*-value. Some foresters use the *t*-value based on the pilot/preliminary sample size.

Method One—The first proposed method to quantify variability (CV) is to calculate sample size within each observed diameter class across all plots within a site and then take an average of the sample size estimates across all diameter classes. Hence, first the CV is calculated across all plots for a particular diameter class, sample size is then estimated by diameter class, and then the final sample size needed to estimate a stand table for a particular site is the average sample size across all diameter classes.

Method Two—Method Two is the same as Method One, except the sample sizes within diameter classes are weighted such that diameter classes with a greater number of trees have a greater impact on the sample size estimate. This will likely reduce the needed sample size to meet a desired level of precision while also ensuring that, similar to the concept of variable-radius sampling, you are not spending a substantial amount of time establishing many plots because of a rare diameter class being found in one plot.

For example, five pilot plots may have been established in a chip-and-saw stand and in all five plots diameters range from 8 to 11 inches, with the exception of one plot that had a 5-inch tree. Because that one small tree is going to substantially increase the sample size estimate for the stand, weighting by the number of trees within diameter classes will reduce the impact of that individual.

Method Three—The third proposed method to quantify uncertainty is to calculate the CV by diameter class among all plots within a site, obtain an average CV across all diameter classes, and then use this average CV in equation (1) for a particular site.

Method Four—In a similar fashion to Method Two, the fourth method is the same as Method Three, except the CVs within diameter classes are weighted such that diameter classes with a greater number of trees have a greater impact on the sample size estimate.

Method Five—First, the standard deviation among all plots for a particular diameter class is calculated, followed by the CV of the diameter class standard deviations within a site. This CV is then placed into equation (1).

Method Six—First, the CV among all plots for a particular diameter class is calculated, and then the CV of the CVs is calculated and inserted into equation (1).

Method Seven—this is the traditional means of determining sample size, using some average value per acre.

Data Sources

A variety of data sources were examined based on availability of the data. Table 1 provides a list of studies that were used during this analysis. DBHs were placed into 1-inch diameter classes. As examples, the 5-inch class was defined as all d.b.h.s ranging from 4.6 inches to 5.5 inches, and the 9-inch class was defined as all d.b.h.s ranging from 8.6 inches to 9.5 inches. Table 2 contains average stand-level summary statistics by measurement age for each study described in table 1.

Table 1—Study descriptions

Study	Location	Species	Treatments	Ages	Site	Operational/ Research	Plot shape and size	*n*	Reference
1	Monticello, AR	LP	Thinning (30, 50, 70, and 90 sq ft/acre) and pruning (25, 40, and 50%) study. Plus unthinned/unpruned	12,15,16,19, 24,27	OF	O, R	S, 0.1	3, 4	General et al. (2013)
2	Crossett, AR	LP	Thinning study	10,13,16,19,22, 25,28,31,34,36, 40,46,53,57	OF	O, R	S, 0.25	3	Baldwin et al. (1998)
3	East Texas	LP	Operational plantings	Varies	CO	O	S, 0.25	2	Coble (2009)
		SP					S, 0.25		
4	Winston Co., MS	LP	Planting density trial (1,742, 778, and 484 seedlings/acre)	8,10,12,14,15, 17,19,20,23, 25, 28,30	OF	R	R, 0.5	4	VanderSchaaf (2010)
5	Tattnall Co., GA & Hampton Co., SC	LP	Seedling size and management intensity study	3,4,8,10	CO	R	R, 0.11	3	VanderSchaaf and South (2003)
6	Macon Co., AL	LP	Seedling size and planting density study	3	CO	R	-	4	South et al. (2005)
7	Haile, LA	LP	Thinning trial (70 sq ft to 105 sq ft/acre). Plus unthinned	25	CO	O	-	5	Carroll Walls/C. McDavid Hughes
8	Saucier, MS	LP	Management intensity study	8,9,25,39	CO	R	S, 0.52	4	Schmidtling (1987)
		SP							
		LL							
9	SE AR (6 sites)	LP	Operational plantings. Unthinned/Thinned	11,12,18,19, 24,25	CO	O	C, 0.10 or 0.25	2 or 3	Bragg et al. (2004)
10	Colt, AR	LP	Operational planting	8	OF	O	S, 0.16	2	-
12	Homerville, GA	SP	Site preparation study	7	CO	O	-	5	Shrock (1994)
13	Quitman, LA	LP	Operational planting	Mature	CO	O	C, 0.1	14	-
14	Monticello, AR	SG	Fertilization study	5,6,7,8,9,13, 14,15,21,24, 25,26,27	OF	R	S, 0.045	6	Guo et al. (1998)
15	Potlatch, ID	DF	Multi-nutrient fertilization study	14,15	CO	O	S	2	VanderSchaaf (2008)
		PP	Multi-nutrient fertilization study	14,15	CO	O	S	2	
16	Farmington, IA	EWP	Operational planting	Mature	-	O	C, 0.016	2	-
17	Arbo, MN	WS	Operational planting	Mature	-	O	C, 0.016	2	-
18	Grand Rapids, MN	RP	Operational planting	Mature	-	O	C, 0.03	4	-

Species: LP is loblolly pine (*Pinus taeda* L.), SP is slash pine (*Pinus elliottii* Engelm.), LL is longleaf pine (*Pinus palustris* P. Mill.), SG is sweetgum (*Liquidambar styraciflua* L.), DF is Douglas-fir (*Pseudotsuga menziesii* var. *glauca*), PP is ponderosa pine (*Pinus ponderosa* Lawson & C. Lawson), EWP is eastern white pine (*Pinus strobus* L.), WS is white spruce (*Picea glauca* (Moench) Voss), RP is red pine (*Pinus resinosa* Aiton).

Site: OF is old-field, CO is cutover.

Operational/Research: O is operational, R is research.

Plot shape and size: R is rectangular, S is square, C is circular; area is in acres.

n is number of replications.

Note: Study 11 was dropped.

Table 2—Stand-level summary statistics of the studies

Study	Age	Species	TPA	Dq	BAA	Vol/acre
1	12	LP	334	6.6	80	1400
	15		271	8.0	95	2125
	16		155	8.8	66	1559
	19		112	10.5	68	1916
	24		109	12.2	89	2921
	27		84	13.2	79	2633
2	10	LP	1083	3.5	71	1597
	13		718	5.2	87	1959
	16		337	7.4	72	1627
	19		251	9.3	83	2332
	22		166	10.8	83	2337
	25		126	12.3	88	2471
	28		104	13.6	91	2573
	31		78	15.0	84	2378
	34		63	16.2	80	2261
	36		63	16.8	87	2437
	40		63	18.1	101	2834
	46		63	19.1	112	3160
	53		56	20.2	111	3140
	57		56	20.7	117	3303
3	Many	LP	445	5.9	89	2492
4	8	LP	893	4.4	85	1499
	10		861	5.4	119	2088
	12		813	6.1	147	2577
	14		772	6.7	167	2931
	15		747	7.0	175	3937
	17		687	7.4	187	4432
	19		641	7.8	195	5489
	20		614	7.9	195	5481
	23		513	8.5	193	5434
	25		478	8.9	197	5555
	28		411	9.7	205	5763
	30		364	10.1	196	5505
5	3	LP	508	0.9	2	14
	4		555	1.2	4	28
	8		539	5.5	89	861
	10		550	6.0	108	1897
6	3	LP	436	1.6	6	46
7	25	LP	161	13.4	159	4465
8	8	LP	258	5.0	35	619
		SP	227	3.5	15	272
		LL	262	4.7	31	545
	9	LP	242	5.4	38	664
		SP	212	3.9	17	309
		LL	243	5.0	33	583
	25	LP	132	7.7	42	1192
		SP	154	7.6	48	1349
		LL	151	8.0	53	1480
	39	LP	99	9.5	49	1370
		SP	140	9.6	70	1982
		LL	120	9.9	64	1811
9	11,12	LP	621	5.4	100	1764
	18,19		230	8.3	86	2423
	24,25		95	10.3	55	1547
10	8	LP	395	6.3	86	834
12	7	SP	483	3.8	39	688
13	Mature	LP	54	17.6	92	2984
14	5	SG	406	2.0	9	63
	6		407	2.3	12	95
	7		390	2.8	16	142
	8		381	3.5	25	257
	9		366	3.4	23	278
	13		366	4.6	42	132
	14		362	5.0	49	345
	15		365	5.4	57	525
	21		366	6.3	79	850
	24		360	6.9	93	1264
	25		359	7.0	95	1401
	26		366	7.1	100	1618
	27		362	7.2	102	1719
15	14,15	DF	539	2.61	20	-
	14,15	PP	379	4.67	45	-
16	Mature	EWP	246	14.3	276	-
17	Mature	WS	339	11.1	227	-
18	Mature	RP	234	11.4	167	-

Species: LP is loblolly pine, SP is slash pine, LL is longleaf pine, SG is sweetgum, DF is Douglas-fir, PP is ponderosa pine, EWP is eastern white pine, WS is white spruce, RP is red pine.

TPA is trees per acre, BAA is basal area per acre (feet^2), Dq is quadratic mean diameter (inches), and Vol/acre is cubic foot volume per acre.

RESULTS AND DISCUSSION

Table 3 and figure 1 clearly show that the d.b.h. class-weighted sample size estimates were reasonable (Methods Two and Four). However, sample size estimates using the non-d.b.h. class-weighted methods were restrictively high (Methods One, Three, Five, and Six). On average, to obtain a stand table at a probability level of 90 percent when using a 10 percent error, Method Four requires around five times as many plots as compared to just estimating the total number of trees per acre (Method Seven using trees per acre). Figure 1 shows the general trend in sample size estimate across the methods as compared to simply estimating average number of trees per acre (Method Seven using trees per acre) when using a probability level of 90 percent.

However, when using an 80 percent probability level sample sizes were reduced essentially consistently around 40 percent. For the 80 percent probability level, only three times as many plots were required for Method Four relative to just estimating the average number of trees per acre (Method Seven using trees per acre) when using a 90 percent probability level (e.g., 27 plots for Method Four at 80 percent versus 8 plots at 90 percent for Method Seven using trees per acre).

When using 95 percent probability, the required sample sizes obviously greatly increase. As compared to the 90 percent probability level, they consistently increase around 41 percent. Foresters may use 95 percent probability when estimating sample sizes for average trees per acre. This will require on average a sample size of 12 (e.g., 12 plots for Method Seven using trees per acre at 95 percent).

If a forester is willing to accept 80 percent probability levels when estimating sample sizes for stand tables, then sample sizes are only about twice that of current commonly used methods when using Method Four (e.g., 27 plots for Method Four at 80 percent versus 12 plots for Method Seven using trees per acre at 95 percent. If a forester is willing to accept 80 percent probability levels when estimating sample sizes for stand tables, then sample sizes (when using Method Four, n = 27) are on average almost identical to using the currently used method of calculating sample sizes using average tree volume at a 95 percent probability level (Method Seven using volume, n = 29).

Table 3—Mean, minimum, maximum, and standard deviation (SD) of sample size estimates at 80 percent, 90 percent, and 95 percent probability levels, by method

	80 percent confidence				90 percent confidence				95 percent confidence			
Method	Mean	Minimum	Maximum	SD	Mean	Minimum	Maximum	SD	Mean	Minimum	Maximum	SD
	trees per acre											
One	101	0.0	1366	75.7	166	0.0	2250	124.6	235	0.0	3194	176.9
Two	38	0.0	629	48.2	63	0.0	1035	79.4	89	0.1	1470	112.7
Three	66	0.0	1175	64.1	108	0.0	1935	105.5	153	0.0	2747	149.7
Four	27	0.0	483	45.0	45	0.0	796	74.1	64	0.1	1130	105.1
Five	135	0.0	603	80.1	222	0.0	993	131.9	315	0.0	1410	187.3
Six	142	0.0	764	88.2	233	0.0	1257	145.2	331	0.0	1785	206.2
Seven	5	0.0	146	9.8	8	0.0	241	16.2	12	0.0	341	23.0

Sample size is 1907.

A 10 percent error was used (%E in equation (1))

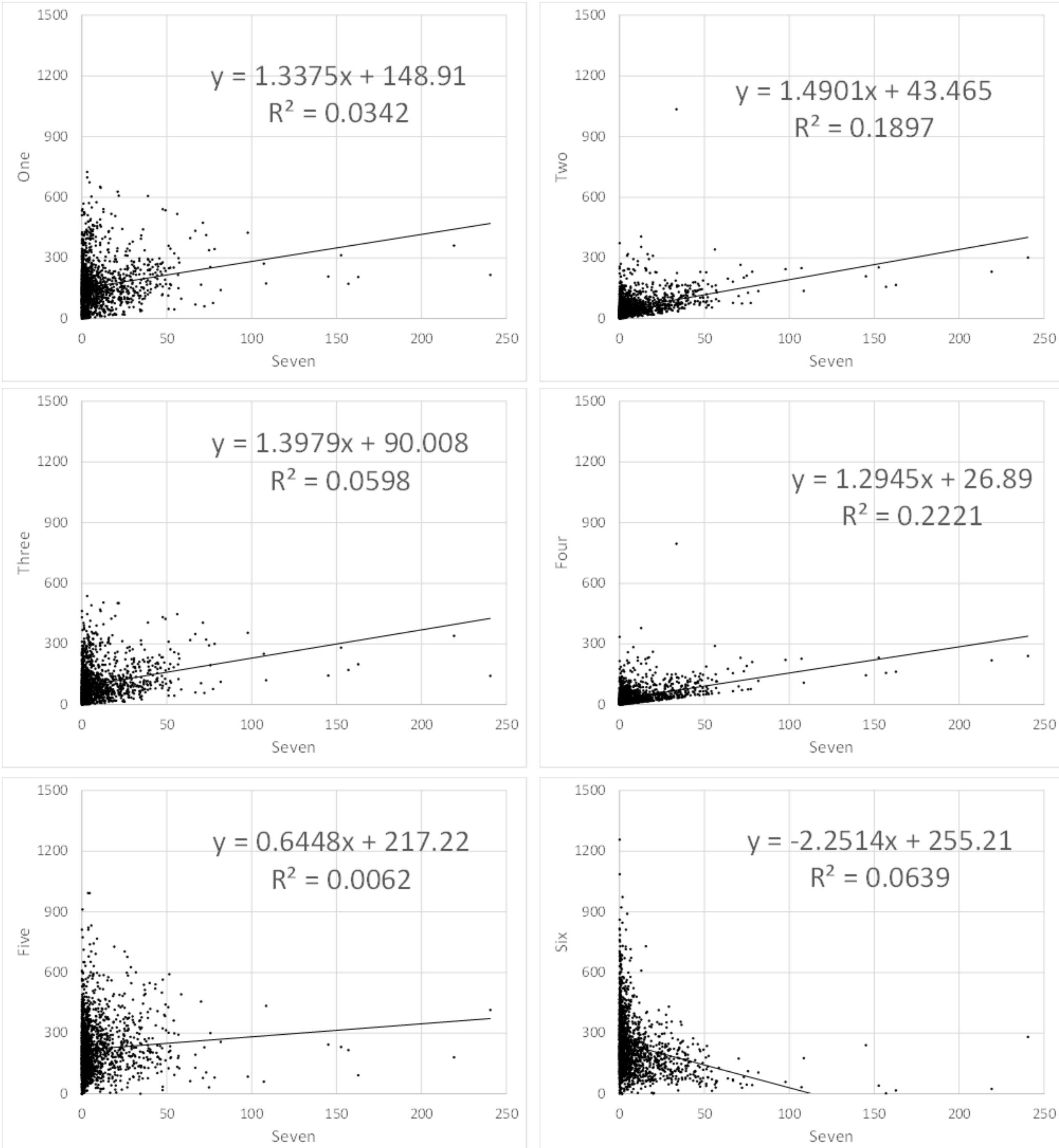

Figure 1—General trend in sample size estimates across the methods as compared to simply estimating average number of trees per acre (Method Seven using trees per acre) when using a probability level of 90 percent (n = 1907).

LITERATURE CITED

Avery, T.E.; Burkhart, H.E. 2002. Forest measurements. New York, NY: McGraw Hill. 456 p.

Baldwin, V.C., Jr.; Leduc, D.J.; Ferguson [and others]. 1998. The not-so-sudden results of the sudden saw log study—growth and yield through age 45. In: Waldrop, T.A., ed. Proceedings of the ninth biennial southern silvicultural research conference. Gen. Tech. Rep. SRS-20. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station: 574-578.

Bragg, D.C.; Shelton, M.G.; Heitzman, E. 2004. Relative impacts of ice storms on loblolly pine plantations in central Arkansas. In: Connor, K.F., ed. Proceedings of the 12th biennial southern silvicultural research conference. Gen. Tech. Rep. SRS-71. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station: 132-137.

Coble, D.W. 2009. A new whole-stand model for unmanaged loblolly and slash pine plantations in east Texas. Southern Journal of Applied Forestry. 33: 69–76.

Coble, D.W.; Grogan, J. 2007. Comparison of systematic line-point and double sampling designs for pine and hardwood forests in the Western Gulf. Southern Journal of Applied Forestry. 31: 199–206.

General, D.R.; VanderSchaaf, C.L.; Zeide, B. 2013. The longest active thinned and pruned loblolly pine permanent plots: the last measurement. In: Guldin, J.M., ed. Proceedings of the 15th biennial southern silvicultural research conference. e-Gen. Tech. Rep. SRS-175. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station: 479-486.

Guo, Y; Lockhart, B.R; Ku, T.T. 1998. Effect of nitrogen and phosphorus fertilization on growth in a sweetgum plantation in southeastern Arkansas. Southern Journal of Applied Forestry. 22: 163-168.

Schmidtling, R.C. 1987. Relative performance of longleaf compared to loblolly and slash pines under different levels of intensive culture. In: Phillips, D.R., comp. Proceedings of the fourth biennial southern silvicultural research conference. Gen. Tech. Rep. SE-42. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station: 395-400.

Shiver, B.D.; Borders, B.E. 1996. Sampling techniques for forest resource inventory. New York, NY: John Wiley and Sons, Inc. 356 p.

Shrock, P.D. 1994. Integration of seedling quality with silvicultural practices to improve seedling establishment and growth. M.S. thesis. Auburn, AL: Auburn University. 91 p.

South, D.B.; VanderSchaaf, C.L.; Britt, J.R. 2005. Reforestation costs can be decreased by lowering initial stocking and outplanting morphologically improved seedlings. Native Plants Journal. 6: 76–82.

VanderSchaaf, C.L. 2008. Estimating understory vegetation response to multi-nutrient fertilization in Douglas-fir and ponderosa pine stands. Journal of Forest Research. 13: 43–51.

VanderSchaaf, C.L. 2010. Estimating individual stand size-density trajectories and a maximum size-density relationship species boundary line slope. Forest Science. 56: 327–335.

VanderSchaaf, C.L.; South, D.B. 2003. Effect of planting depth on growth of open-rooted *Pinus elliottii* and *Pinus taeda* seedlings in the United States. Southern African Forestry Journal. 198: 63–73.