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Enhancing the precision of broad-scale forestland removals estimates with small area estimation techniques

John W. Coulston[1], P. Corey Green[2,*], Philip J. Radtke[2], Stephen P. Prisley[3], Evan B. Brooks[2], Valerie A. Thomas[2], Randolph H. Wynne[2] and Harold E. Burkhart[2]

[1]USDA Forest Service, Southern Research Station, 1710 Research Center Drive, Blacksburg, VA 24060 USA
[2]Department of Forest Resources and Environmental Conservation, Virginia Polytechnic Institute and State University, 310 W. Campus Drive, Blacksburg, VA 24061 USA
[3]National Council for Air and Stream Improvement, 541 Washington Ave SW, Roanoke, VA 24016 USA

*Corresponding author: E-mail: pcgreen7@vt.edu

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National Forest Inventories (NFI) are designed to produce unbiased estimates of forest parameters at a variety of scales. These parameters include means and totals of current forest area and volume, as well as components of change such as means and totals of growth and harvest removals. Over the last several decades, there has been a steadily increasing demand for estimates for smaller geographic areas and/or for finer temporal resolutions. However, the current sampling intensities of many NFI and the reliance on design-based estimators often leads to inadequate precision of estimates at these scales. This research focuses on improving the precision of forest removal estimates both in terms of spatial and temporal resolution through the use of small area estimation techniques (SAE). In this application, a Landsat-derived tree cover loss product and the information from mill surveys were used as auxiliary data for area-level SAE. Results from the southeastern US suggest improvements in precision can be realized when using NFI data to make estimates at relatively fine spatial and temporal scales. Specifically, the estimated precision of removal volume estimates by species group and size class was improved when SAE methods were employed over post-stratified, design-based estimates alone. The findings of this research have broad implications for NFI analysts or users interested in providing estimates with increased precision at finer scales than those generally supported by post-stratified estimators.

Introduction

National Forest Inventories (NFI) are designed with sample sizes sufficient to estimate forest parameters with designated precision at strategic scales. For example, the United States Department of Agriculture (USDA) Forest Inventory and Analysis (FIA) programme strives to provide annual estimates of forest removals with 5 per cent sampling error per 28.3 million m^3 of annual removals in the eastern US (USDA Forest Service 1970). Similar precision guidelines are also specified for forest area, forest volume and forest growth. However, many of today's tactical and strategic planning questions require estimates at finer spatial and temporal precision than are achievable given the current FIA sampling intensity (one field inventory plot per 2403 ha of land area, with remeasurement every ∼5–10 years (10 years in the west)) and post-stratified estimators (Bechtold and Patterson 2005). For example, a majority of forest inventory shifts in the southeastern US occur because of timber harvesting, and increased spatial and temporal precision of forest removal estimates are often needed to understand current wood supplies at relevant scales. Thus, the goal of this research is to increase the precision of removal estimates by employing small area estimation (SAE) techniques.

The term 'small area' refers to any domain of interest where post-stratified estimates cannot be made with a sufficient level of precision (Rao and Molina 2015). In this sense, SAE techniques may be applicable not only in situations where the geographic or temporal extent of a domain of interest (*d*) is small and hence has a small sample size but also in situations where increased estimate precision is required over any *d*. The terms 'sufficient' and 'small' are intended to be application-specific. Small area models borrow strength from areas outside the domain of interest and from auxiliary information, to increase the precision of parameter estimates (Lehtonen *et al*. 2003). There are two types of SAE models: unit-level and area-level. Unit-level models refer to estimators constructed at the level of sample units (e.g. field inventory plots in NFI), where auxiliary information is tied directly to sample units (Rao and Molina 2015). Area-level models refer to estimators constructed at the level of areas of interest (AOI), where an AOI is represented by multiple sample units (inventory plots) within a geographic or other domain. SAE applications in forestry have recently gained attention, but

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many applications to-date have focused on unit-level models. For example, Breidenbach and Astrup (2012) examined the use of photogrammetric canopy heights to improve the precision of mean canopy height estimates using a unit-level approach, while Mauro *et al*. (2017) used LiDAR-based auxiliary information, and Georndt *et al*. (2013) used 16 Landsat variables, land cover, tree cover and elevation as auxiliary information. McRoberts (2012) used unit-level approaches to improve the precision of volume per acre estimates using Landsat Thematic Mapper imagery as auxiliary data.

In contrast, Goerndt *et al*. (2011), Mauro *et al*. (2017) and Magnussen *et al*. (2017) provided examples of area-level applications. Georndt *et al*. (2011) and Mauro *et al*. (2017) used SAE area-level approaches to improve the precision of density, volume, basal area, quadratic mean diameter and average height estimates at the stand and management unit-levels, respectively. Magnussen *et al*. (2017) examined area-level SAE approaches to estimate volume based on auxiliary canopy height information derived from airborne laser scanning and aerial photography at various scales ranging from management units to municipalities. Even though most of the research towards SAE applications in forestry has focused on unit-level applications, Magnussen *et al*. (2017) identified several situations when area-level models may be more suitable than unit-level models when applied to forest inventories. One reason was that auxiliary data may not be available at the plot-level (unit-level). Another reason, in the case of field plots as sample units, is that low precision of global positioning system plot locations may degrade correlations between observed variables and auxiliary information (Green *et al*. 2019). Gopalakrishnan *et al*. (2015) noted some potential issues with registration error between FIA sample plots and auxiliary data. Additionally, the confidentiality of plot locations due to privacy concerns may make unit-level models impractical in some cases. Area-level approaches may also be more appropriate when an attribute of interest involves rare or infrequent occurrences, with forest harvesting as an example. At the unit-level, when an attribute of interest (Y) is rarely observed to be nonzero, observations become zero-inflated, which can be problematic for common model types (e.g. linear mixed models). One commonality among forest inventory applications of SAE techniques to-date is their focus on increasing the precision of static estimates at a single point in time. Little research has been aimed at understanding the utility of adopting SAE to increase precision in estimates of components of change (e.g. growth, removals, mortality).

Forests of the southeastern US produce >15 per cent of global wood products, with 89 per cent of removals occurring on privately owned forests, while simultaneously experiencing a range of other disturbances (Coulston *et al*. 2015). Because of the importance of southeastern US forests in a regional to global timber supply context, forest removal estimates of sufficient precision, both spatially and temporally, are needed to support strategic and tactical decisions by forest managers, policymakers and forest industry firms. In some situations, the precision targets noted earlier (5 per cent sampling error per 28.3 million m^3 harvested) are unobtainable or inadequate for highly relevant questions.

The FIA rotating panel design limits the temporal precision of change estimates. For example, under a five-panel system, the full set of panels is measured completely only after 5 years. Considering that components of change require remeasured panels, removal estimates in a five-panel design are compiled by constructing average annual estimates over a 10 year period. Many important forecasting efforts, such as those by Wear and Greis (2013), USDA Forest Service (2012) and Abt *et al*. (2000), are informed by removal estimates, but average annual removal estimates over a 10 year period are unlikely to provide the temporal resolution needed for informed decision making. As such, there is a need to increase the temporal precision of removal estimates using techniques like SAE to better inform decision-makers.

The overall goal of this research is to provide demonstrated improvement over post-stratified estimates in both the spatial and temporal domains of forest harvest removals estimates for the southeastern US. To achieve this goal, we evaluate the performance of area-level Fay-Herriot (FH) models (Fay and Herriot, 1979) including spatial models (Petrucci and Salvati 2006), to increase the precision of total removals estimates, removals by species group, and removals by species group and merchantability class. We focus on the following questions:

1. To what degree can the precision of the FIA survey unit scale (aggregates of counties ~2–4 million ha in area) and county-scale (~ 6700–410 000 ha) removal estimates be improved?
2. How does incorporating spatial correlation among domain means influence the results?
3. What are the magnitudes and directions of changes in estimates?
4. Given that the precision of survey unit post-stratified estimates based on full sets of panels are generally considered adequate, can we achieve similar precision at finer spatial scales (county estimates) and finer temporal scales (single-panel estimates)?

Methods

Study area and FIA data

The study area for this research was a seven-state region in the southeastern US (Figure 1). Each of the states in the study area was further divided into 3–7 survey units (our first spatial scale of interest). The survey units generally follow physiographic region boundaries (e.g. Coastal Plain, Piedmont, mountains) in the southeast (Coulston *et al*. 2014, Burrill *et al*. 2018). Counties (the second spatial scale of study) are political and administrative divisions within states. Because survey units followed county boundaries, in this case, each county also belonged to exactly one survey unit. There were 35 individual survey units and 687 counties. The study area contained 35 447 permanent plots (forest and non-forest). The average survey unit contained roughly 1000 permanent plots and the average county contained roughly 50 permanent plots across all land uses.

The FIA programme uses a rotating panel design with fixed sample locations. The design is assumed to produce an equal probability sample (McRoberts *et al*. 2006) for each panel and across panels. The number of panels (P) is either $P = 5$ or $P = 7$ in the southeastern US. The remeasurement period between successive plot visits is $\approx P$ years. Typically, the FIA programme uses P panels to construct point-in-time estimates. Change estimates are based on the remeasurement of P panels requiring

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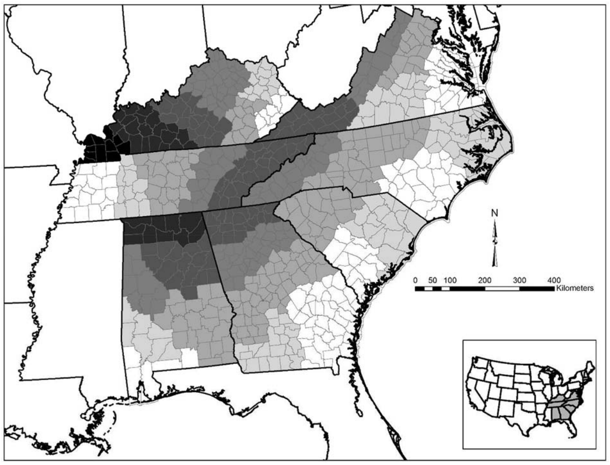

Figure 1 Survey unit and county spatial domains in the study area—seven states in the southeastern US: state boundaries are denoted by thick solid lines; county boundaries within states are denoted by narrow dashed lines, and survey units are distinguished by degrees of shading within each state.

2*P* years to obtain a full set of remeasurements. This practice is referred to as the temporally indifferent approach by Bechtold and Patterson (2005).

When constructing estimates of removals, the removal is assumed to occur at the midpoint of the remeasurement period (calculated on a plot basis). See Table 1 for a five-panel example where Panel 1 was measured in 2001 and 2006, and removals from this panel of measurements were assumed to occur ∼ 2004. Likewise, Panel 2 was measured in 2002 and 2007 with an assumed removal year of ∼2005. When panels are combined to a set of $P = 5$ panels, there are five different assumed removal years. For example, consider Panels 1 through 5 at the top half of Table 1. The assumed removal years for these single panels are ∼ 2004–2008. The first measurement (time 1) contributing to the change estimate occurred in 2001, and the last measurement (time 2) contributing to the estimate occurred in 2010. The midpoint of this full set of panel remeasurements was 2006. Survey unit and county full panel estimates of removals were constructed for 2007 and 2009. We constructed survey unit and county single panel estimates for 2007, 2009, 2011 and 2013. On average, there were 184 permanent plots used for single-panel survey unit estimates and nine permanent plots used for single-panel county estimates.

In this research, the parameters of interest were total removals (m^3 ha^{-1} yr^{-1}), hardwood removals (m^3 ha^{-1} yr^{-1}), softwood removals (m^3 ha^{-1} yr^{-1}), hardwood removals of pulpwood-sized trees 12.7 cm—31.75 cm dbh, (m^3 ha^{-1} yr^{-1}), hardwood removals of sawtimber-sized trees >31.75 cm dbh, (m^3 ha^{-1} yr^{-1}), softwood removal of pulpwood-sized trees 12.7 cm—21.59 cm (m^3 ha^{-1} yr^{-1}) and softwood removals of sawtimber-sized trees >21.59 cm dbh (m^3 ha^{-1} yr^{-1}). Publicly available data were downloaded from the FIA programme (https://apps.fs.usda.gov/fia/datamart/datamart.html) and used to construct *PS* estimates of the parameters of interest for both single panels and full panels for both survey units and counties in the study area (Figure 1). For all analyses, we considered the midpoint of the measurements as the year for which the removal estimate corresponded with and all removal estimates were put on a per-annum basis as suggested by Bechtold and Patterson (2005).

Auxiliary data

We used timber products output (TPO) and tree cover loss (TCL) data as auxiliary data for SAE. The TPO programme is a constituent programme of the FIA programme (Coulston *et al*. 2018) where roundwood-receiving facilities are canvassed every 2 years in the southern US (Bentley and Johnson, 2011). Each of these facilities receives a questionnaire and reports the amount of roundwood volume by timber products (e.g. sawlogs, pulpwood, poles), species group and source location (roundwood county of origin). Roundwood volumes by timber product and species across all facilities are aggregated by the county of origin to quantify county-level removals for products. We summarized county-level TPO data into total (TPO_{total}), hardwood (TPO_{hw}) and softwood (TPO_{sw}) removals based on TPO species group. Species group removals were also aggregated into product classes for hardwood pulpwood (TPO_{hwpulp}), hardwood solidwood ($TPO_{hwsolid}$), softwood pulpwood (TPO_{swpulp}) and softwood solidwood ($TPO_{swsolid}$). County-level TPO summaries were used for county-level FH and SFH models. The county-level TPO summaries were aggregated to the survey unit for survey unit-level FH and SFH models. Table 1 illustrates how TPO summaries were temporally aligned with FIA removal estimates.

TCL data were downloaded from Global Forest Watch (http://data.globalforestwatch.org/). These data were developed based on methodology from Hansen *et al*. (2013). The TCL data are Landsat-based, where each 30 m by 30 m pixel determined to have tree cover in 2000 is then tracked over time. If a disturbance causes greater than 50 per cent TCL in a pixel, then the year of the TCL is recorded. The total number of pixels with TCL was calculated for each county and survey unit for each year. The total number of such pixels was then divided by the total number of pixels for each county and survey unit by year, yielding proportion TCL by county, survey unit and year (Table 1).

Estimators

To conduct our analysis, we compare the estimated precision of the FIA post-stratified estimator (PS) for several removal parameters to the estimated precision of two area-level SAE models: the area-level FH model and the area-level spatial Fay Herriot. Each of these is described below.

Post-stratified estimator

The FIA programme uses a post-stratified estimator to construct estimates of means and totals and the variance of these parameter estimates. The post-stratification is typically performed using land cover maps derived from satellite imagery (Patterson *et al*. 2012). Based on Bechtold and Patterson (2005), the estimated

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Table 1 FIA and auxiliary data: example of FIA measurement years, assumed FIA removal years, TCL year and TPO year

	Measurement Year		Removal Year		Auxiliary Data Year	
Panel	Time 1	Time 2	Single panel	Full set of panels	TCL	TPO
1	2001	2006	2004		2004	
2	2002	2007	2005		2005	2005
3	2003	2008	2006	2006	2006	
4	2004	2009	***2007***	***2007***	***2007***	***2007***
5	2005	2010	2008	2008	2008	
1	2006	2011	***2009***	***2009***	***2009***	***2009***
2	2007	2012	2010	2010	2010	
3	2008	2013	***2011***	2011	***2011***	***2011***
4	2009	2014	2012		2012	
5	2010	2015	***2013***		***2013***	***2013***

Gray shading identifies how single panels are grouped for full set of panel estimates. The bold-italicized years denote the FIA data and auxiliary data used in this analysis.

population mean is.

$$\overline{Y} = \sum_{h}^{H} W_h \overline{Y_h} \tag{1}$$

where,

$$\overline{Y_h} = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{ih} \tag{2}$$

and W_h is the proportion of the population in stratum h, n_h is the number of plots in stratum h and y_{ih} is the ith plot-level observation (per unit area basis) in stratum h. The total is $\hat{Y} = E\overline{Y}$ where E is the total areal extent of the population. The estimated variance of $\overline{Y_h}$ is,

$$v\left(\overline{Y_h}\right) = \frac{\sum_{i=1}^{n_h} y_{ih}^2 - n_h \overline{Y}_h^2}{n_h\,(n_h - 1)} \tag{3}$$

and the estimated variance of $\overline{Y}$ is,

$$v\left(\overline{Y}\right) = \frac{1}{n}\left[\sum_{h}^{H} W_h n_h v\left(\overline{Y_h}\right) + \sum_{h}^{H} (1 - W_h)\frac{n_h}{n} v\left(\overline{Y_h}\right)\right] \tag{4}$$

The estimated variance of the total is,

$$v\left(\hat{Y}\right) = E^2 v\left(\overline{Y}\right). \tag{5}$$

For the purpose of using the FH model, $\overline{Y_d}^{PS} = \overline{Y}$ and $v(\overline{Y_d})^{PS} = v(\overline{Y})$ for a spatially defined domain (d). Equations (1) and (5) are population estimators but are also directly applicable to any domain (d) provided that E is known for the domain of interest (i.e. E_d). Our focus is on county-level and survey unit domains and hence E_d is known. For the purpose of using the FH models, $\overline{Y_d}^{PS} = \overline{Y}$ (equations (1)) and $v(\overline{Y_d})^{PS} = v(\overline{Y})$ for each individually estimated d.

Area-level FH model

The area-level FH model (Fay and Herriot, 1979) is motivated by a two-stage model based on $d = 1, \ldots, D$ mutually exclusive and exhaustive domains. As described by Molina and Marhuenda (2015), in the first stage.

$$\overline{Y_d}^{PS} = Y_d + e_d \tag{6}$$

where $e_d \sim^{ind} N(0, v(\overline{Y_d})^{PS})$, and Y_d is the true, but unobserved, value of the parameter. Model (6) is the sampling model because Y_d is unobserved but estimated by $\overline{Y_d}^{PS}$ which is assumed to be unbiased. In the second stage, Y_d is linearly related to m–1 auxiliary variables and an intercept ($\mathbf{x}_d$):

$$Y_d = \mathbf{x}_d^T \boldsymbol{\beta} + u_d \tag{7}$$

where $u_d \sim^{ind} N(0, A)$ (with A being the variance of the random effect) and $\boldsymbol{\beta}$ is the vector of regression coefficients conditioned on the spatial and temporal frame of survey domain, post-stratified estimates in $\mathbf{x}_d$. Models (6) and (7) are then combined:

$$\overline{Y_d}^{PS} = \mathbf{x}_d^T \boldsymbol{\beta} + u_d + e_d \tag{8}$$

where u_d is assumed to be independent of e_d.

The empirical best linear unbiased prediction (EBLUP) of (8) is.

$$\overline{Y_d}^{EBLUP} = \hat{\gamma}_d \overline{Y_d}^{PS} + (1 - \hat{\gamma}_d)\ \mathbf{x}_d^T \hat{\boldsymbol{\beta}} \tag{9}$$

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where $\hat{\boldsymbol{\beta}}$ is a set of estimated regression coefficients and the shrinkage $\hat{\gamma}_d$ is defined as.

$$\hat{\gamma}_d = \hat{A}/\left(\hat{A} + v\left(\overline{Y_d}\right)^{PS}\right) \tag{10}$$

When $v(\overline{Y_d})^{PS}$ is small compared to the variance of the random effect ($\hat{A}$) the EBLUP tends towards the *PS* estimate and when the *PS* variance is large compared to the estimated variance of the random effect $\hat{A}$ the EBLUP tends towards the synthetic estimate $\mathbf{x}_d^T\hat{\boldsymbol{\beta}}$.Rao and Molina (2015) describe the mean square error (MSE) of (9) as a combination of three components when $\hat{A}$ is estimated using restricted maximum likelihood (REML):

$$\mathrm{MSE}\left(\overline{Y_d}^{\mathrm{EBLUP}}\right) = g_{d1}\left(\hat{A}\right) + g_{d2}\left(\hat{A}\right) + 2g_{d3}\left(\hat{A}\right). \tag{11}$$

In the *R* sae package (Molina and Marhuenda 2015), the components are calculated:

$$g_{d1}\left(\hat{A}\right) = \hat{\gamma}_d\, v\left(\overline{Y_d}\right)^{PS} \tag{12}$$

$$g_{d2}\left(\hat{A}\right) = \left(1 - \hat{\gamma}_d\right)^2 \mathbf{x}_d^T\left(\left(\frac{\hat{\boldsymbol{\gamma}}}{\hat{A}}\mathbf{X}\right)^{\mathrm{T}}\mathbf{X}\right)^{-1}\mathbf{x}_d \tag{13}$$

$$g_{d3}\left(\hat{A}\right) = \left(1 - \hat{\gamma}_d\right)^2 \frac{2}{\sum\left(\frac{\hat{\gamma}}{\hat{A}}\right)^2}\left(\hat{A} + v\left(\overline{Y_d}\right)^{PS}\right)^{-1} \tag{14}$$

where $\mathbf{X}$ is a D x m matrix of $\mathbf{x}_d^T$ for each domain and $\hat{\boldsymbol{\gamma}}$ is a D x 1 vector of $\hat{\gamma}_d$ for each domain.

Area-level spatial FH model

Petrucci and Salvati (2006) combined the FH (8) with a simultaneously autoregressive model that incorporates spatial dependence (SFH) where $\mathbf{u}$ in the FH model is assumed to follow a first-order simultaneous autoregressive process (equation (15)).

$$\mathbf{u} = \rho_1\mathbf{W}\mathbf{u} + \boldsymbol{\varepsilon}, \qquad \varepsilon \sim \mathrm{N}\left(\mathbf{0}_{\mathrm{D}}, \sigma_1^2\mathbf{I}_{\mathrm{D}}\right) \tag{15}$$

where $\mathbf{0}_{\mathrm{D}}$ is a column vector of zeros, and $\mathbf{I}$ is the identity matrix.

The model requires a D x D matrix of spatial weights ($\mathbf{W}$) that in this case denotes the weights arising from adjacent domains. For example, if a specific domain had four adjacent domains, then each of those four adjacent domains would have a weight of 0.25 (i.e $\mathbf{W}$ is row-standardized). In the approach of Petrucci and Salvati (2006), both the variance of the random effects (A) and the spatial correlation coefficient (ρ) are estimated. In the *R* sae package (Molina and Marhuenda 2015), the EBLUP is:

$$\overline{\mathbf{Y}}^{\mathrm{EBLUP}} = \mathbf{X}\hat{\boldsymbol{\beta}} + \mathbf{G}\mathbf{V}^{-1}(\overline{\mathbf{Y}}^{\mathrm{PS}} - \mathbf{X}\hat{\boldsymbol{\beta}} \tag{16}$$

)where $\overline{\mathbf{Y}}^{\mathrm{EBLUP}}$ is a vector of d=1 … D estimates where,$\mathbf{G} = \hat{A}[(\mathbf{I} - \hat{\rho}\mathbf{W})(\mathbf{I} - \hat{\rho}\mathbf{W}^{\mathrm{T}})]^{-1}$, $\mathbf{I}$ = D x D identity matrix and $\mathbf{V} = \mathrm{diag}(\mathrm{v}(\overline{\mathbf{Y}})^{\mathrm{PS}}) + \mathbf{G}$.

In equation (16), ($\overline{\mathbf{Y}}^{\mathrm{PS}} - \mathbf{X}\hat{\boldsymbol{\beta}}$) is the vector of residuals, $\mathbf{G}$ is the model error covariance matrix, and $\mathbf{V}$ is the combined *PS* and model error covariance matrix. $\mathbf{G}\mathbf{V}^{-1}$ is analogous to the shrinkage factor in equation (9), except that for each domain the shrinkage is a linear combination that takes into account the relative error spatial covariance among domains and all residuals for each domain.

Molina and Marhuenda (2015) implemented the analytical formula derived by Singh *et al*. (2005) for approximating the MSE of the SFH EBLUP (15). The MSE approximation assumes D is large and ignores all $o(D^{-1})$ terms, i.e. terms ultimately smaller than D^{-1} (Molina *et al*. 2009). In this sense, MSE estimates from the *R* sae package are second-order unbiased or 'nearly' unbiased and tend to improve with increasing D, asymptotically approaching unbiasedness as $D \to \infty$ (Li and Lahiri 2010). Although slightly different formulations are used to achieve the desired statistical properties, the package is able to calculate MSEs for SFH EBLUPs estimated by either maximum likelihood or REML.

Generalized variance function

It is common practice to use a generalized variance function (gvf) to smooth the sampling variances (Rao and Molina 2015). The need for a gvf arises due to the instability of $v(\overline{Y_d})^{PS}$ when the domain of interest is small and hence may have too few observations to be reliably estimated. Westfall *et al*. (2011) suggested that the *PS* estimator can produce biased estimated variances when there are fewer than 10 observations per stratum and a total sample size <75 observations. In our analysis, $v(\overline{Y_d})^{PS}$ was used for all survey unit-level SAE applications while a gvf was developed for county-level SAE applications. The gvf distributes the regionwide variance of each estimate at the regional level to each county. The variance of the estimated total for each parameter for the region was:

$$v\left(\hat{Y}_{\mathrm{region}}\right)^{PS} = \sum\nolimits_d^D v\left(\hat{Y}_{d_{\mathrm{survey\ unit}}}\right)^{PS} \tag{17}$$

The gvf for county d for each parameter (E is the total areal extent of the population) was then:

$$\mathrm{gvf}_d = v\left(\hat{Y}\right)^{PS}\frac{n_{d_{\mathrm{county}}}}{\sum n_{d_{\mathrm{county}}}}E_{d_{\mathrm{county}}}^{-2} \tag{18}$$

where E_d is the total areal extent of the county-level domain. In this manner we sum the variance of the total across the study area, distribute the total variance to county d based on the proportion of total plots (across all counties) in county d, and then convert the variance of the total back to the variance of the mean per domain.

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Table 2 Description of models used for both the FH and SFH approaches

Model	Y_d	X variables								
		TPO_{total}	TPO_{sw}	TPO_{swpulp}	$TPO_{swsolid}$	TPO_{hw}	TPO_{hwpulp}	$TPO_{hwsolid}$	TCL	Interaction
1	Total removals	x							x	x
2	Total softwood removals		x						x	x
3	Softwood removals (dbh 12.7–21.6 cm)			x						
4	Softwood removals (dbh > 21.6 cm)				x					
5	Total hardwood removals					x			x	x
6	Hardwood removals (dbh 12.7–31.8 cm)						x			
7	Hardwood removals (dbh > 31.8 cm)							x		

Models and Parameters of Interest

We considered seven *Y* variables (Table 2) for testing the FH and SFH models. These models were used to estimate removal volumes in total, by species group, and by species group and merchantability class (pulp or solid wood). The **X** variables arising from the TPO data reflect only the portion of forest removals going to timber products and hence should be less than removals from the forest. Conversely, the TCL variable accounts for TCL caused by both removals and mortality both within and outside of forest land uses. Hence, the TCL should account for more than just volume removals on forest land uses. When the TCL variable was included in a model, a multiplicative interaction term with the TPO variable was also included. Model forms were constructed with auxiliary data that were meaningfully related to the parameter of interest. For example, it would be expected that the total removals estimated from the TPO data and the removals estimated from the TCL data are both meaningful relationships. Certain auxiliary data; however, are not expected to be meaningfully related to all parameters of interest. For example, in predominately pine growing regions, hardwood pulpwood removals estimated using TCL data alone are not necessarily meaningful relationships despite what fit statistics may indicate. In addition to constructing models with logical relationships, initial testing suggested that simple models had fewer convergence problems; therefore, four of the seven models were based on a single explanatory variable and the remaining three were based on two explanatory variables and their interaction (Table 2). The models in Table 2 were parameterized using both the FH and SFH approaches with both counties and survey units as the domains, based on the full sets of panels for 2007 and 2009 removal years and for single panels having removal years in 2007, 2009, 2011 and 2013. The combinations of model-dependent variables (7), modelling approaches (2), measurement panel time frames (6) and spatial domains (2) made a total of 168 distinct model parameterizations developed in the test application.

Evaluating results

Evaluation of sampling and estimation approaches requires a known population so that MSE and bias can be quantified; however, a known population is rarely at hand. To overcome this issue, Goerndt *et al*. (2011, 2013) treated the full set of sampled plots as the population, then subsampled from that population at varying intensities to evaluate the efficacy of various SAE techniques. Magnussen *et al*. (2017) used a more traditional approach, where the MSE $(\bar{Y})^{EBLUP}$ and $v(\bar{Y})^{PS}$ were reported for each modelling type and the ratio $MSE(\bar{Y})^{EBLUP}/var(\bar{Y})^{PS}$ was calculated for each domain and then averaged across domains to evaluate the reduction in variance for different SAE techniques. Here we took a similar approach to Magnussen *et al*. (2017).

For simplicity, we refer to results under the post-stratified estimator as '*PS*', the Fay-Herriot as FH, and the spatial Fay-Herriot as SFH. We focused on the standard error,

$$SE\left(\bar{Y}\right)^{PS} = \left[v\left(\bar{Y}\right)^{PS}\right]^{0.5} \tag{21}$$

of the *PS* estimates and the root mean squared error,

$$RMSE\left(\bar{Y}\right)^{EBLUP} = \left[MSE\left(\bar{Y}\right)^{EBLUP}\right]^{0.5} \tag{22}$$

of the FH and SFH rather than the variance and MSE because the errors expressed in the same measurement units as the parameter of interest are more directly interpretable. To address our objective questions 1 and 2 (Q1—Can precision of survey unit and county-based removal estimates be improved?; Q2—How does incorporating spatial correlation influence the results?), we

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examined the distribution of the standard error ratio (SER),

$$\mathrm{SER} = \mathrm{RMSE}\left(\bar{\mathrm{Y}}\right)^{\mathrm{EBLUP}} / \mathrm{SE}\left(\bar{\mathrm{Y}}\right)^{\mathrm{PS}} \tag{23}$$

where a value <1 indicated increased precision of the EBLUP estimate.

We also used the likelihood ratio test (LRT) to compare the FH models to the SFH models to address question 2. When making these comparisons the ratio was assumed to follow a chi-squared distribution with 1 degree of freedom. The degrees of freedom were based on the difference in the number of estimated parameters between the two models. The SFH models only had one addition parameter arising from the estimation of spatial correlation coefficient ($\hat{\rho}$).

To address question 3 (What are the magnitudes and directions of changes in estimates?), we graphically examined the relationships between $\bar{Y}^{\mathrm{EBLUP}}$ and $\bar{Y}^{PS}$. We further examine whether county-scale SFH estimates summed within survey units were similar to *PS* survey unit estimates. By definition $\sum_{c\in su} E_c \overline{Y}_c^{PS} = E_{su} \overline{Y}_{su}^{PS}$ where E_c= is the areal extent of county *c* and E_{su}= the areal extent of survey unit *su* and hence one would expect

$$\overline{\overline{Y}}^{\mathrm{EBLUP}} = \frac{\sum_{c\in su} E_c \overline{Y}_c^{\mathrm{EBLUP}}}{\sum_{c\in su} E_c} \sim \overline{Y}_{su}^{PS}. \tag{24}$$

We graphically examined the relationship expressed in equation (24) and further examined whether $\overline{\overline{Y}}^{\mathrm{EBLUP}}$ was within 1 standard error of $\overline{Y}_{su}^{PS}$. While we do compare estimates, it is important to note that comparing the *PS* estimates to the FH or SFH estimates does not quantify bias. To address question 4 (Given that the precision of survey unit *PS* estimates based on a full set of panels is generally considered adequate can we achieve similar precision at finer spatial scales (counties) and finer temporal scales (single panel estimates)?), several approaches were used. First, we assumed that the precision (standard error of the mean) of full panel *PS* estimates at the survey unit scale were sufficient to serve as benchmarks. The precision of survey unit removal estimates based on single panels was then compared graphically to precisions estimated from full panel *PS* estimates from 2007 and 2009. The precision of the county removals based on full and single panels was compared to the maximum SE and the third quartile of the survey-unit *PS* estimates based on full sets of panels. This comparison was based on the notion that full-set-of-panel estimates at the survey unit-level are of adequate precision. Comparing county-level estimates to the upper quartile of the distribution for 'adequately' precise estimates indicates the degree to which county-level estimates' precision approached that of the full panel, survey unit estimates.

Results

Question 1: to what degree can the precision of FIA survey unit scale (aggregates of counties ∼ 2–4 million ha in area) and county-scale (∼ 6700–410 000 ha) removal estimates be improved?

The first question focused on whether the precision of county and survey unit estimates of removals could be increased. Our results suggest that the FH estimates consistently increased the precision of removal estimates at both the county and survey unit scales for each *Y* parameter examined (Figure 2). With one exception, the SER was <1 for the county and unit-level estimates constructed with single and full sets of panels. FH estimates at the survey unit scale based on a full sets of panels led to median SERs between 0.40–0.93 with estimates of total softwood removals having the lowest median SER. Survey unit FH estimates based on a single panel (Figure 2; column 4) had a median SER ranging from 0.33–0.59, with softwood removals for pulpwood (dbh 12.7–21.6 cm) exhibiting the lowest median SER. Additionally, other estimates such as survey unit total removals, total softwood removals, and softwood removals for pulpwood FH estimates based on single panels showed improvement over their corresponding *PS* estimates, with the SER interquartile ranges (IQR) of 0.24–0.53, 0.21–0.62, and 0.21–0.75, respectively (Figure 2). County-scale results were similar to survey unit scale results, where FH estimates were more precise than corresponding *PS* estimates. For example, the median SER for total removals was 0.46 for full panel estimates and 0.44 for single panel estimates. In general, the FH estimates displayed the largest gains in precision when the *PS* estimate was less precise. This finding was expected because of the shrinkage factor as described in equation 10. Though expected, the finding does highlight the opportunity to increase the precision of parameter estimates when the domain of interest has a relatively large variance of the *PS* estimate.

Question 2: how does incorporating spatial correlation among domain means to influence the results?

Our second question focused on examining the effect of incorporating spatial correlations. Based on the LRT, the SFH model was significantly better than the FH model ($P<0.001$) in all cases. The critical value for $P<0.001$ was 10.83 and the minimum LTR for models across the survey unit scale was LRT=11.35 (P=0.00076). The maximum LRT for the survey unit scale was 148.5. At the county level scale, the LRT ranged from LRT=399.1 to LRT=2692.3.

Our analysis of SER offered further insights into the effect of incorporating spatial correlations. In general, the SFH estimates behaved similarly to the FH estimates in that they showed improved precision over *PS* estimates in nearly all combinations tested (Figure 2). Survey unit-level SFH precision results were in most cases similar to FH results, regardless of whether they were based on single panels or full panel datasets (Figure 2; columns 3 and 4). Similarity here refers to the pattern of overlapping IQRs observed between SFH and FH results for a combination of species group, size-class, panel set, and spatial domain. The same pattern of similar precision improvements for SFH or FH based methods was noted for county-level results when single

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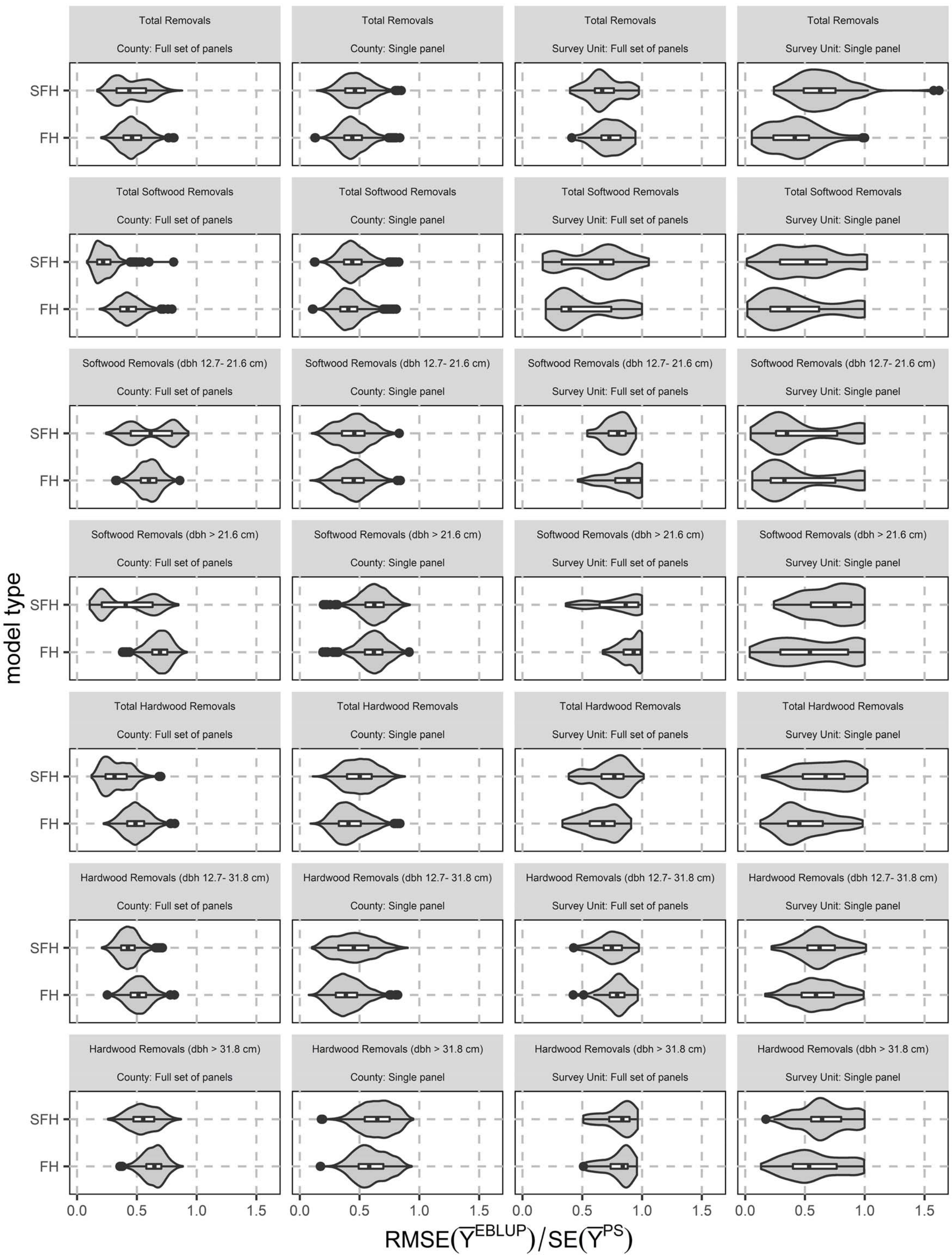

Figure 2 Violin plots displaying the distribution of SERs for the FH estimates and the SFH estimates for each *Y* parameter at the county and survey-unit scales based on a single panel and a full set of panels. The full panel estimates are pooled for 2007 and 2009. The single panel estimates are pooled for 2007, 2009, 2011 and 2013. Values on the X-axis below 1 indicate an improvement in estimate precision compared to the *PS* estimate.

panel data were used (Figure 2, column 2). A somewhat distinct pattern of non-overlapping IQRs emerged for total softwoods, softwoods (dbh > 21.8 cm), and total hardwood removals calculated from full panel sets based on county-level domains. For these parameters, SFH precision was notably higher than the precision of corresponding FH model results (Figure 2; column 1,

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rows 2, 4, and 5). A maximum SER value (1.62) was noted in SFH estimates for total removals based on a single data panel (Figure 2; column 4, row 1) indicating that in some cases, the SFH model was difficult to fit at the survey unit scale where there were relatively few domains. The maximum SFH SER values (1.0) were noted in 6 of 7 parameters when single panel survey unit-level data were used, and in 3 of 7 parameters when full panel set data were used (Figure 2, Columns 3 & 4).

Question 3: what are the magnitudes and directions of changes in estimates?

The third question focused on magnitudes and directions of changes in estimates for different populations and parameters. Because the true population values are unknown, we cannot address bias of the FH and SFH estimates. Here we focus only on FH results to simplify the presentation, noting that FH and SFH removal estimates were similar throughout. Generally, the larger the support sample (larger n_d) the better the alignment between *PS* and FH estimates (e.g. 2009 estimates as shown in Figure 3). Here we use the term align to reflect a tendency for the estimates to fall on the 1:1 relationship, where poor alignment does not necessarily reflect bias. For example, estimates across *Y* parameters at the survey unit scale based on full sets of panels exhibited the best alignment. The relationship between $\bar{Y}^{EBLUP}$ and $\bar{Y}^{PS}$ was more variable across *Y* parameters at the survey unit scale based on a single panel. For survey unit scale total hardwood removals, hardwood removals (dbh 12.7–31.75 cm), and hardwood removals (dbh > 31.75 cm) based on a single panel there was a departure from the 1:1 line as $\bar{Y}^{PS}$ increased. At the county-scale, there was also a departure from the 1:1 relationships. At small values of $\bar{Y}^{PS}$ the $\bar{Y}^{EBLUP}$ was typically greater than $\bar{Y}^{PS}$. At large values of $\bar{Y}^{PS}$ the $\bar{Y}^{EBLUP}$ was typically less than $\bar{Y}^{PS}$. This pattern was more pronounced for single panel estimates as comparted to full panel estimates.

To gain additional insight into potential systematic county-level estimation issues we compared the FH and SFH $\bar{\bar{Y}}^{EBLUP}$ (equation 24) to $\bar{Y}^{PS}$ across survey units and noted that lack of alignment evident in individual county estimates (Figure 3) was reduced when county-level estimates were aggregated to the survey unit-level (Figure 4). Notably, SFH $\bar{\bar{Y}}^{EBLUP}$ for total removals based on a full sets of panels were largely aligned with corresponding survey-level $\bar{Y}^{PS}$ across the full range of *PS* estimates. Roughly 79 per cent of the SFH $\bar{\bar{Y}}^{EBLUP}$ estimates for total removals (full sets of panels) were within one standard error (SE($\bar{Y}^{PS}$), full sets of panels) of $\bar{Y}^{PS}$. Examination of single panel SFH $\bar{\bar{Y}}^{EBLUP}$ for total removals showed that a comparatively modest 46 per cent of the estimates were within one standard error of $\bar{Y}^{PS}$. When viewed across all *Y* parameters combined, $\bar{\bar{Y}}^{EBLUP}$ values for the full set of panel estimates were within 1 standard error of the *PS* estimate 68 per cent of the time. For single panel estimates SFH $\bar{\bar{Y}}^{EBLUP}$ was within one standard error of the *PS* estimates 35 per cent of the time, raising some concerns of systematic estimation issues for county-level SFH estimates based on single panels of data. Results for the FH model (not shown here) were similar in terms of graphical alignment to the SFH results but $\bar{\bar{Y}}^{EBLUP}$ was within 1 standard error of the *PS* estimate 56 per cent and 36 per cent of the time for full-set-of-panel and single panel estimates, respectively.

Question 4: given that the precision of survey unit post-stratified direct estimates based on full sets of panels is generally considered adequate, can we achieve similar precision at finer spatial scales (county estimates) and finer temporal scales (single-panel estimates)?

Our final question focused on whether county removals estimates (based on full sets of panels, and single panel) and survey unit removal estimates (based on a single panel) could be constructed with the same precision as *PS* estimates from a full set of panels. At the survey unit scale, we found that single-panel estimates of *Y* parameters based on the FH model typically had an estimated precision comparable to that of *PS* estimates based on a full set of panels. For example, 2009 *PS* single panel estimates of total removals based on the FH model had smaller standard errors than 2009 *PS* estimate standard errors based on full panels in 60 per cent of survey units (Figure 5). Typically, RMSE($\bar{Y}^{EBLUP}$) was larger than SE($\bar{Y}^{PS}$) when SE($\bar{Y}^{PS}$) was relatively small. For example, some survey-unit single panel RMSE($\bar{Y}^{EBLUP}$) FH 2009 estimates for total hardwood removals were larger than the SE of full panel *PS* 2009 estimates when the removal volume was <0.13 m^3 ha^{-1}.

We compared the precision of the county-level FH and SFH estimates to the precision of the corresponding maximum and 3rd quartile of survey-unit full panel estimates for each parameter. Because of the results from question 1, we focused on the precision of estimates from the SFH model to examine the county-level precision for single panels and full sets of panels as compared to survey unit full panel estimates. This analysis assumed that survey-unit full panel estimates generally provide adequate precision for removals. For county-level, single panel estimates mean RMSE($\bar{Y}^{EBLUP}$) across counties was larger than the maximum corresponding SE($\bar{Y}^{PS}$) from the survey-unit full panel estimates for all seven removal parameters examined except for hardwood pulpwood removals (Table 3). At the county-scale based on full sets of panels, the mean SFH RMSE($\bar{Y}^{EBLUP}$) was less than the maximum SE($\bar{Y}^{PS}$) in corresponding estimates for, total softwood removals, total hardwood removals, and hardwood pulpwood removals (dbh 12.7–31.8 cm). In all three of these comparisons 99.5 per cent or more counties had a RMSE($\bar{Y}^{EBLUP}$) that was less than or equal to the maximum SE($\bar{Y}^{PS}$) for corresponding estimates. In contrast only one parameter, total softwood removals, produced estimates based on full sets of panels with greater than 50 per cent of counties having RMSE($\bar{Y}^{EBLUP}$) values < 3rd quartile of observed survey-unit SE($\bar{Y}^{PS}$).

Generalized variance function

We used a gvf for direct county-level estimates. This was done for two reasons. First, the gvf was used to adjust for potential unstable direct variance estimates. Second, removals were a rare

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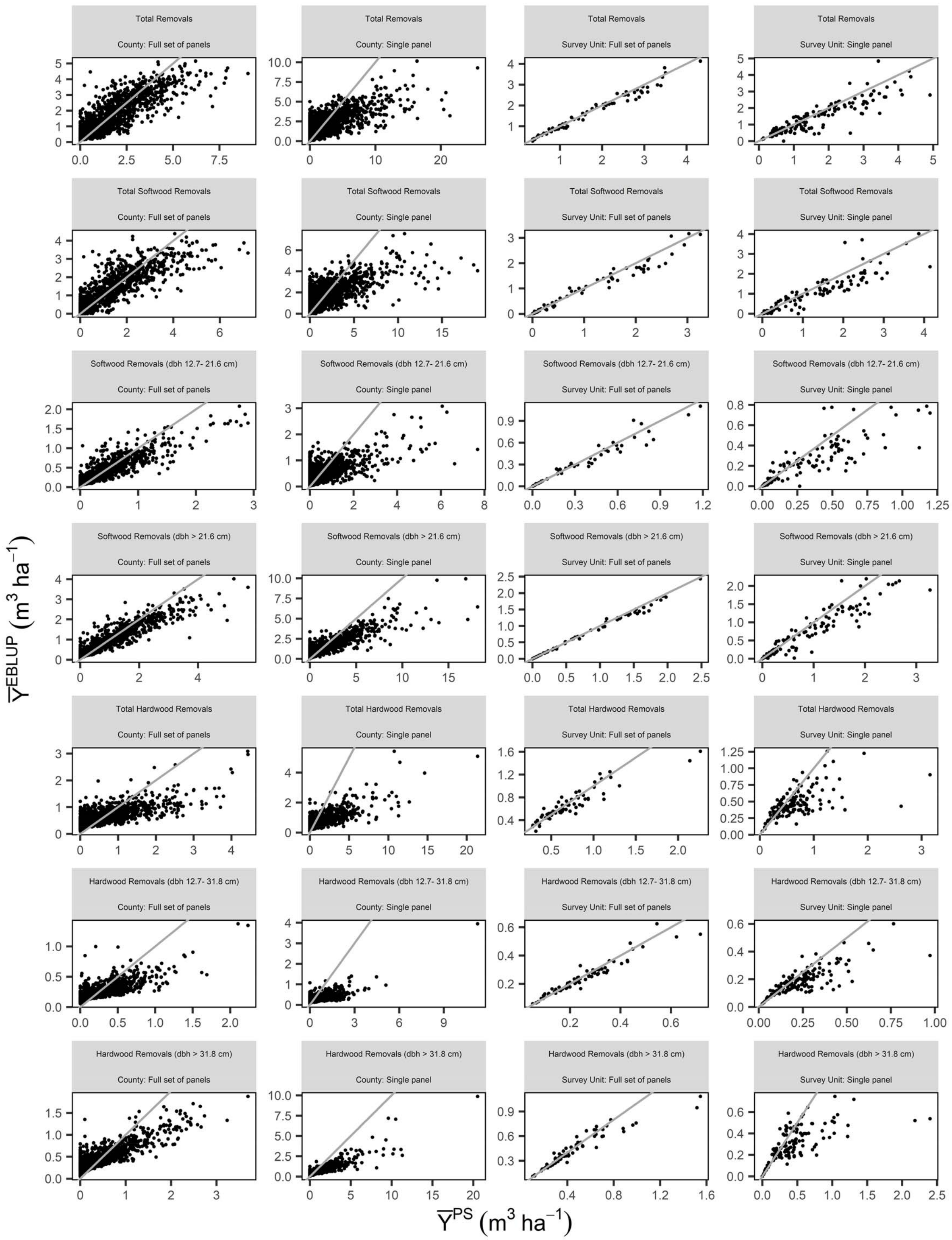

Figure 3 The Year 2009 estimates of each *Y* parameter based on the *PS* estimate ($\bar{Y}^{PS}$) and the FH estimate ($\bar{Y}^{\mathrm{Eblup}}$) at the survey-unit and county-scale based on a single panel and a full set of panels. This solid gray line denotes the 1:1 line.

event (often < 2 per cent of an area). This created a situation when using county-level domains where one may have n_d = 10, for example, but no removals were observed and yet the auxiliary data suggested there were unobserved removals in the domain. In this case, the direct estimate and the variance of the estimate were zero. Based on equation (9), the direct estimate was retained because of its apparent precision (i.e. zero variance). We used a gvf to overcome both of these issues.

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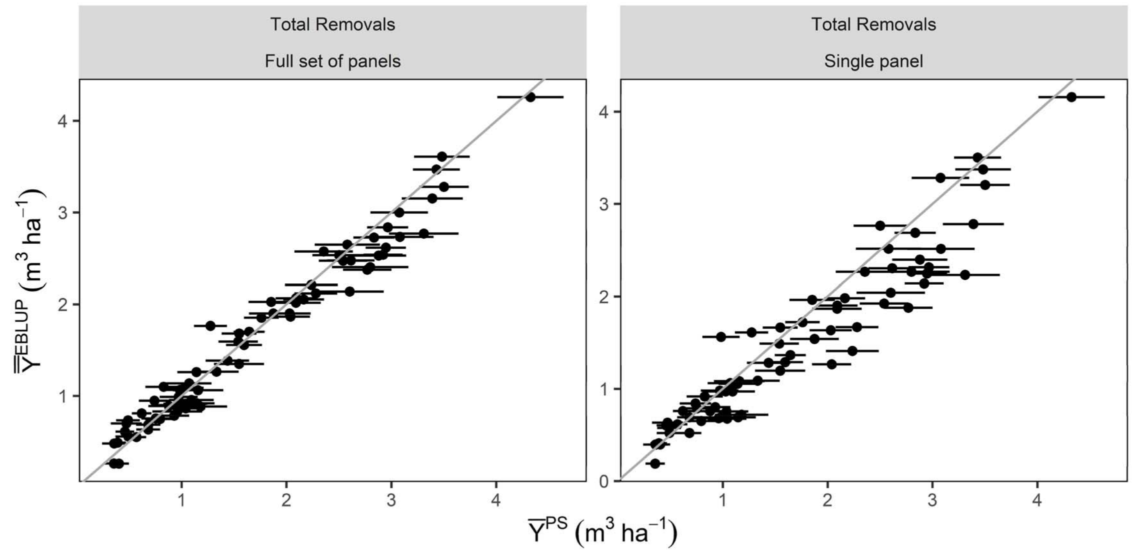

Figure 4 Full set of panels and single panel county-level SFH estimates summarized to the survey unit-level ($\bar{\bar{Y}}^{EBLUP}$) versus survey unit-level *PS* estimates based on a full set of panels ($\bar{Y}^{PS}$). Estimates are for 2007 and 2009. The gray diagonal line is the 1:1 line. The horizontal black lines are SE($\bar{Y}^{PS}$).

Our gvf followed expectation with respect to decreasing standard error with increasing n (Figure 6). For example, we examined the standard error of the estimate of total removals for 2009 single panel estimates and found that when n=25 the average standard error of the estimate was 1.26 m^3 ha^{-1} yr^{-1}. When n=100 the standard error of the estimate was 0.64 m^3 ha^{-1} yr^{-1}. This result suggests that a sample of four times as many plots resulted in half the standard error which conforms to the notion that a 4x sampling intensification decreases the standard error by half (Burkhart, *et al*. 2019).

Discussion

The objective of this research was to answer four questions. We found (1) that in nearly all cases, the use of FH and SFH models increased the precision of both county and survey unit estimates for the seven-volume removal parameters examined. (2) While the incorporation of spatial correlation through the SFH model in most cases improved model fits, it generally did not increase the practical significance of the precision of estimates of removal parameters compared to the FH model. Exceptions were noted mainly in county-level full panel parameters for which SFH estimates showed modest improvements in precision over non-spatial, full panel FH estimates. (3) Estimates across removal parameters were influenced to varying degrees using the SAE approaches. At the survey-unit scale based on a full set of panels, *PS* estimates generally aligned with those using FH and SFH model approaches. Relationships became more variable as the number of observations decreased in domains being compared. Because removal parameters are not known for the spatial and temporal scales involved here, bias could not be addressed directly; however, systematic differences between *PS* estimates of removals and SAE results were noted for individual counties when single panel data were used. (4) We found that survey unit, single panel estimates of *Y* parameters could generally be estimated with precision similar to that of survey unit, full panel estimates. In contrast, county level, full panel estimates of *Y* parameters could be estimated with precision at or below the maximum observed standard errors for *PS* estimates based on survey unit full panel datasets about 65 per cent of the time (on average).

While we focused on removal estimates rather than point-in-time status estimates (e.g. growing stock), our results agree with other investigations into the performance of area-level SAE models (e.g. Mauro *et al*. 2017, Magnussen *et al*. 2017). Within the context of the FIA programme, our results suggest that survey unit estimates of removal volumes can be constructed with similar precision using a single panel of remeasurements (i.e. ∼ 20 per cent of the total number of remeasurements) using either the FH or SFH approach compared with the full panel, *PS* estimate. This is important because it allows for more current estimates of removals to be constructed, increasing the temporal resolution of removal estimates. Also, the precision of county-level removal estimates based on full sets of panels, while not as precise as survey-unit full-panel-set estimates, was increased when the auxiliary information examined was used to form EBLUPs with FIA data. This approach provides an opportunity to perform analyses at spatial resolutions finer than what FIA *PS* estimators currently support. Although the exact limits of spatial resolutions that may support various types of strategic planning information needs were not identified here, results indicate that spatial resolutions intermediate between county and survey unit-levels (e.g. woodshed analysis and strategic forest management planning) may be adequate for a wider range of analysis needs than are currently thought to be supported.

Our results regarding spatial models differed somewhat from Magnussen *et al*. (2017) who found the methods of Chandra

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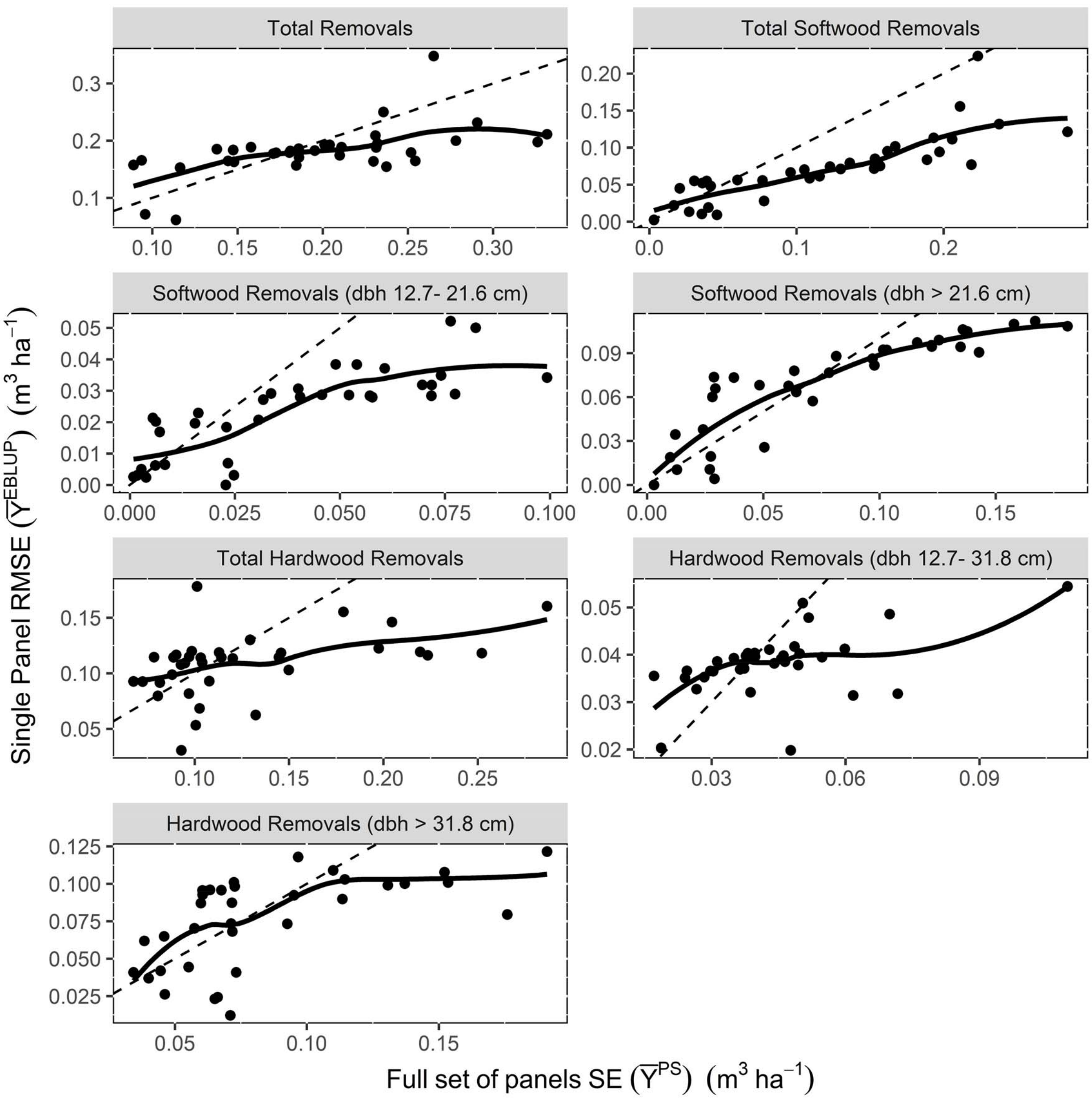

Figure 5 The Year 2009 survey unit scale single panel RMSE($\bar{Y}^{EBLUP}$) vs 2009 survey unit scale full set of panels SE($\bar{Y}^{PS}$) for the FH estimates. The solid curves were developed using locally estimated scatterplot smoothing and are only intended to guide interpretation. The dashed line is the 1:1 line.

et al. (2012) that account for spatial autocorrelation to provide further increases in precision over basic area-level FH models. Using the Petrucci and Salvati (2006) approach as implemented by Molina and Marhuenda (2015), we only observed consistent increases in precision at the county-scale based on a full set of panels when incorporating spatial correlation. This matched our expectation that county-level full panel estimates based on SFH were more likely to provide increases in precision than survey-unit-level estimates. There were only 35 survey units, and spatial correlation among adjacent survey units failed to explain much of the error structure leading to minimal improvements compared to non-spatial removal estimates.

Many of the comparisons and tests we examined to identify potential gains of SFH versus non-spatial FH estimators depended on MSEs estimated using analytical approximations which have been the subject of considerable interest in SAE research (Molina *et al.* 2015). As noted by Molina *et al.* (2015), second-order unbiased approximations for FH EBLUP MSEs were developed first by Prasad and Rao (1990), and furthered by others including Datta and Lahiri (2000) and Das *et al.* (2004). Second-order unbiased approximations for SFH EBLUP MSEs have garnered ongoing attention as well (Petrucci and Salvati 2006, Torabi and Jiang 2020). The FH and SFH MSE approximations implemented in the sae package are second-order unbiased and have been tested in simulation studies and compared to parametric and nonparametric bootstrap estimates (Molina *et al.* 2009, Marhuenda *et al.* 2013). Other tests and properties related to FH and SFH variance estimates are subjects of ongoing interest including significance tests and interval coverage rates for area random effects (Datta *et al.* 2011, Molina *et al.* 2015). While these were not directly pursued here, they remain topics of interest for ongoing research.

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Table 3 Comparison of RMSE($\bar{Y}^{\mathrm{EBLUP}}$) for county-level SFH small area estimates to SE($\bar{Y}^{PS}$) for survey unit full panel estimates for 2007 and 2009

Y	Survey unit full set of panels Count SE (Y^{direct})		County full set of panels RMSE (Y^{EBLUP})			County single panel RMSE (Y^{EBLUP})		
	3rd quartile ($m^3\ ha^{-1}$)	Maximum ($m^3\ ha^{-1}$)	Mean ($m^3\ ha^{-1}$)	P_{3q} %	P_{max} %	Mean ($m^3\ ha^{-1}$)	P_{3q} %	P_{max} %
Total removals	0.24	0.36	0.49	0.0	48.2	1.10	0.0	0.0
Total softwood removals	0.19	0.29	0.18	52.7	99.5	0.83	0.0	0.0
Softwood removals (dbh 12.7–21.6 cm)	0.06	0.11	0.17	0.0	13.0	0.27	0.0	0.0
Softwood removals (dbh > 21.6 cm)	0.12	0.19	0.22	26.9	49.7	0.81	0.0	0.0
Total hardwood removals	0.14	0.31	0.19	15.0	99.9	0.61	0.0	0.0
Hardwood removals (dbh 12.7–31.8 cm)	0.05	0.12	0.09	0.0	99.9	0.23	0.0	23.8
Hardwood removals (dbh > 31.8 cm)	0.10	0.20	0.21	0.0	44.0	0.51	0.0	0.0

P_{3q} and P_{max} are the proportions of county-level RMSE EBLUP values smaller than the survey unit full panel SE($\bar{Y}^{PS}$) 3rd quartiles and maxima, respectively. County-level estimates were based on the SFH model.

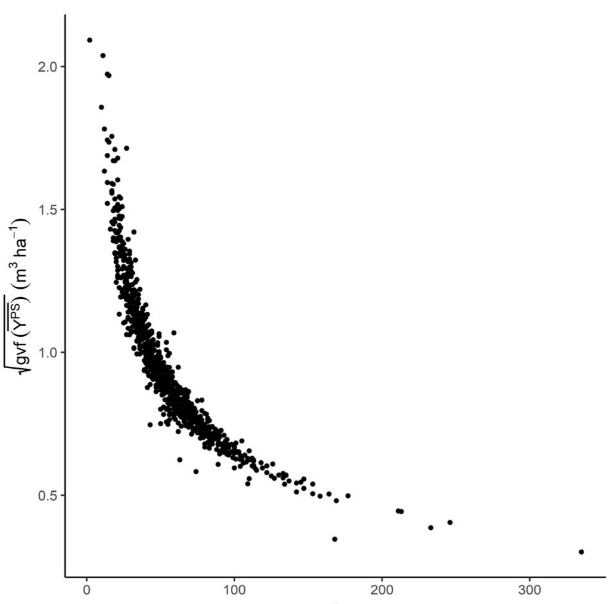

Figure 6 The standard error of total annual removals for 2009 using the gvf.

As part of this research, we attempted to understand how employing the FH and SFH approaches would affect removal estimates. The survey unit results based on a full set of panels exhibited the expected behaviour, i.e. minor random offsets from the 1:1 line and only in survey units with the largest *PS* estimates. However, the results at the county-scale (full sets of panels and single panels) were more difficult to interpret. At first glance, the county-scale results presented in Figure 3 may seem problematic in that the FH and SFH estimates tended to be larger than the *PS* estimate when the *PS* estimate was small, while the FH and SFH estimates tended to be smaller when the *PS* estimate was larger. The pattern suggests a smoothing effect. However, removals are relatively rare events (typically <2 per cent yr^{-1}). Given the sample size at the county level (50 plots on average for a county using a full set of panels and 9 plots on average using a single panel), we would expect county-level *PS* estimates to deviate farther from actual removals than survey unit-level estimates would. In many cases there were no removals recorded in a county according to the plot-level observations, yet the auxiliary data suggested otherwise. For example, in the average county with nine plots measured in a single panel and a 5 per cent removal rate (unknown in reality), it is expected that a removal would be observed on less than one plot. The *PS* estimates, in this case, would underestimate removal volume if no removals were observed and overestimate removals if a removal was observed. Our use of the gvf appears to have led to a greater reliance on the model and hence the synthetic estimate. It seems improper to conclude that FH and SFH estimates are biased given the analyses performed here. There was some indication that the county-level single panel parameter estimates based on the FH and SFH approaches systematically either overestimated or underestimated the parameters of interest, but bias could not be addressed as part of this research. Bias can only be assessed when the true population parameters are known. Additional research into this question is warranted.

Being linear models, the FH and SFH require auxiliary data that are linearly related to the variable of interest. In this research, we used TPO data and TCL data as auxiliary data. These two auxiliary data sources had a moderate to strong linear correlation with observed inventory removals. For example, the TCL data had correlations with total inventory removals ranging from r = 0.93 at the survey-unit scale based on a full set of panels to r = 0.58 at the county-scale based on a single panel. The TPO data had correlations with total inventory removals ranging from

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r = 0.94 at the survey-unit scale based on a full set of panels to r = 0.74 at the county-scale based on single panels. Given the potential for gains shown here and elsewhere for the SAE approach in general, other datasets, particularly from a remote sensing perspective, should be examined. One example arises from Moisen *et al*. (2016). Their work relies on the time series of Landsat imagery, where forest disturbances are identified and attributed by causal agents (e.g. harvest, fire). The use of these data streams in small area applications may prove fruitful. We acknowledge that the TCL data record TCL regardless of land use or driver of the change. Testing remotely sensed datasets with enhanced thematic resolution (e.g. Sentinel 2 and image-derived point clouds (Mura *et al*. 2018, Hawryło and Wężyk 2018)) could improve precision under SAE applications for components of change estimation. This is particularly important in other parts of the US where multiple disturbances occur in AOI. However, when considering developing estimation tools for the FIA user community, it is important to have relatively stable auxiliary data (in terms of availability and contemporaneity).

Our focus was on the FH and SFH approaches; however, there are extensions of these methods that should also be examined. The FIA programme is a continuous inventory, and new data are available each year. As the FIA time series grows in length over time, models that also include temporal correlation (Rao and Molina 2015) should be examined. Further, the research presented here considers a collection of univariate *Y* parameters, but there is an expectation on the part of the practitioner that the corresponding estimates are additive. For example, hardwood removals of trees 12.7–31.75 cm dbh and hardwood removals of trees >31.75 cm dbh should sum to total hardwood removal estimates. Likewise, softwood removal of trees 12.7–21.59 cm and softwood removals of trees >21.59 cm dbh should sum to total softwood removals. Total hardwood removals and total softwood removals should sum to total removals. Under a univariate Y approach, additivity is not guaranteed. Multivariate extensions to the FH and SFH models that will allow for the estimation of more than one *Y* parameter using SAE techniques (Rao and Molina 2015) should be considered.

Conclusions

Precise estimates of forest inventory parameters are important for making informed decisions and conducting meaningful analyses. The applied research presented here is the first example of SAE techniques for components or change estimation and further, the analysis is performed across a broad region which makes the results informative to broad-scale inventory programmes interested in expanding their estimation approaches. Our results suggest that area-level, SAE techniques can leverage auxiliary information to improve estimate precision at finer spatial and temporal scales. Specifically, survey-unit scale FH estimates of removals based on a single panel of remeasurements exhibited sufficient precision indicating that the temporal resolution of estimates at the survey unit scale can be meaningfully increased. Additionally, county-scale SFH estimates of removals based on a full set of panel remeasurements generally exhibited sufficient precision indicating that the spatial resolution of removal estimates can be meaningfully increased.

Conflicts of Interest

None declared.

Data Availability

The datasets were derived from sources in the public domain: Forest Inventory and Analysis inventory data used to construct post-stratified direct estimates are available online at: https://apps.fs.usda.gov/fia/datamart/. Landsat-derived Tree cover loss (TCL) data are available online at: https://data.globalforestwatch.org/. Timber Product Output (TPO) survey data are available online at: https://www.fia.fs.fed.us/program-features/tpo/.

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