RESEARCH ARTICLE

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Silviculture

Improved Equations for the Density Management Diagram Isolines of Ponderosa Pine Stands

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This study was conducted to improve estimation of concomitant variables for implementation of a stand density management diagram (SDMD) for ponderosa pine (*Pinus ponderosa* Laws.) in northern California and Oregon. In traditional SDMD, isolines for variables such as stand volume are presented in such a way that uncertainty with estimation is not available. We developed the new top height and stand volume equations, as well as aboveground biomass and percent canopy cover, for building isolines in the SDMD using high-quality data collected from well-managed even-aged stands. The data were selected from the USDA Forest Service's Pacific Southwest Research Station database. A total of 829 observations (from 113 plots across 15 sites in Oregon and California) were used for model construction. In addition, covariance-variance structures of all of the estimated parameters were provided so that users can evaluate the uncertainty associated with predictions. The model validation results indicated that the predictions made from fixed-effects model forms performed better than the current volume equation of SDMD, as well as those from mixed-effects model forms using the population average effect. The proposed equations provide enhanced predictions and additional useful information about managed ponderosa pine stands, including their uncertainty.

Study Implications: Existing stand density management diagrams often present isolines for estimation of volume and dominant height, but these lack sufficient information for estimation of uncertainty associated with these estimates. Here, using data from research plots within even-aged stands in northern California and central Oregon, we present models with estimates of uncertainty for volume, dominant height, biomass, and percent canopy cover. The use of these is illustrated with the standview package in R. The benefit of this approach is that the isolines are actually not needed any longer, as the estimates and their associated uncertainty can be derived for any observed values with the functions provided with no need to eyeball a volume or other estimate from isolines.

Keywords: stand density management, silviculture, stocking diagram, mixed-effects model, fixed-effects model

Stand density management diagrams (SDMDs) have been widely used to achieve forest management objectives by controlling the level of growing stock of even-aged stands (Castedo-Dorado et al. 2009). The SDMD illustrates the relationships among various stand attributes, including tree sizes (e.g., volume, top height), density, and density-dependent mortality (Newton and Weetman 1994), which reflect fundamental concepts of forest stand dynamics (Drew and Flewelling 1979, Jack and Long 1996). Because the diagrams were designed to evaluate stocking or estimate allometric attributes of a stand (Dean and Jokela 1992) on a two-dimensional space, estimation requires minimal information (i.e., two variables). The SDMDs enable foresters to make the biologically, economically, and technologically relevant decisions—particularly initial planting, spacing, and thinning schedules (Newton 1997, López-Sánchez and Rodríguez-Soalleiro 2009)—to meet management objectives (Castedo-Dorado et al. 2009).

SDMDs have been developed in several different forms. Among the earliest examples of an SDMD in North America is that exhibited in the work of Gingrich (1967) for upland hardwoods. Other examples using this particular format are Cochran et al. (1994) and Edminster (1988). The form of this type of SDMD has basal area on the y-axis and tree density (trees per acre [hereafter, TPA] for English units and trees per hectare for metric units) on the x-axis. Another approach using transformations of the same two variables is that of Long et al. (1988) and Long and Shaw (2005), inspired by the work of Reineke (1933). This form uses quadratic mean diameter (QMD) on the y-axis and TPA on the x-axis (in log-log space). Since QMD is itself a transformation of basal area per acre and

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TPA, there is nothing substantive differentiating these two, but this type of SDMD also has higher dimensionality because of the other metrics (e.g., average height or volume) that are often included. The method could be expanded to include as many other yield metrics as desired; the primary limiting factor seems to be that the figures become exceedingly cluttered and difficult to read as additional dimensions are added to this two-dimensional presentation (Farnden 1996). Another method for SDMD development similar to that of Long and Shaw (2005) uses mean tree biomass in place of quadratic mean diameter on the y-axis (again in log-log space). An example of this type of SDMD is the work of Drew and Flewelling (1979). A similar approach to Drew and Flewelling (1979) is that of Newton (2012) who used mean tree volume rather than biomass on the y-axis. As with the pioneering work of Reineke (1933), the driving variables in all of these methods are a combination of average tree size and number of individuals per unit area.

SDMD for ponderosa pine (*Pinus ponderosa* Laws.) of the western United States has been developed by Long and Shaw (2005). On a plane of TPA and QMD, foresters can evaluate density management regimes with the estimation of stand volume and top height (Jack and Long 1996). They use a large data set from the USDA Forest Service Forest Inventory and Analysis (FIA) and apply several selection criteria to screen data from uneven-aged and/or mixed-species stands for their analysis. Although the SDMD has been widely used and contributes to managing ponderosa pine stands, models derived from unmanaged stands data are likely to differ from even-aged managed plots where the SDMDs were originally intended to use. Existing SDMDs lack provision for estimation of uncertainty for estimates of any concomitant variables associated with any pair of size-density observations, without which users cannot evaluate the confidence levels of their predictions. The concomitant variable models are commonly presented with standard errors for parameter estimates, and this is insufficient to assess prediction uncertainty due to the error propagation (McRoberts and Westfall 2013).

The introduction of concomitant variable estimates such as top height or volume in complex graphical representations can cause user-induced interpolation error (Newton 1997). Hence, studies for developed SDMDs have often been constrained to report only a couple of equations, incorporated as isolines. However, with the advances of data visualization software, a diagram can display complex information interactively, even on a two-dimensional plane. Furthermore, graphical interpolation can be eliminated altogether through the use of programmed equations.

Thus, the SDMD also can accommodate additional stand attributes over commonly used ones such as top height and stand volume isolines by taking advantage of software capability. An example of this can be seen in the standview R package (https://www.fs.fed.us/psw/tools/standview/). Aboveground biomass and canopy cover are candidates for demonstrating the expansion of SDMD usage. Estimation of forest biomass can provide useful indications including the amounts of carbon stocks (Ter-Mikaelian and Korzukhin 1997, Ketterings et al. 2001), potential sources of renewable energy (Popescu 2007), forest fuel loads (Agee 1983), and the flow of nutrients and energy in forest ecosystems (Zianis et al. 2005). Canopy cover—defined as the percent forest area occupied by the projection of tree crowns (Burkhart et al. 2018)—provides crucial insights for evaluating stand density (Gill et al. 2000), estimating tree volume and growth (Paletto and Tosi 2009), and achieving silvicultural goals such as regeneration and wildlife management (Jennings et al. 1999, Gill et al. 2000).

Data collected from multiple research trials and/or repeated measures have a hierarchical (multilevel) structure. To account for the hierarchical structure, a mixed-effects modeling approach has been commonly adopted (West et al. 2006). The mixed-effects model can quantify the random values associated with the plots or sites, thus improved prediction performance can be made using the information of random effects (Garber and Maguire 2003). However, if the isolines for SDMDs are constructed through the mixed-effects model, they would necessarily be applied by ignoring the random effects (i.e., assuming the random effects as zero or the population average effect) because the users have insufficient information for the random effect. Previous studies (e.g., Monleon 2003, Robinson and Wykoff 2004) reported that the mixed-effects model with population average effect had a poorer performance than the nonlinear model without random effects (hereafter, referred to as a fixed-effects model). Thus, a comparison between the two modeling approaches can be considered to acquire more improved concomitant variable estimation.

The primary objective of this study is to enhance SDMD for ponderosa pine forests by including uncertainty of predictions and expanding the number of concomitant variables. We constructed the models using the simplest set of predictors from the conventional SDMD (i.e., TPA and QMD). In particular, we focused on refitting the volume and height models for ponderosa pine SDMD using data collected from managed even-aged stands and developing the aboveground biomass and percent crown cover models for additional presentations of stand attributes on the SDMD. In addition, we provided the information required to evaluate the uncertainties of the estimates using the delta method. We compared the prediction performances between fixed-effects and mixed effects models in the absence of a random location effect.

Methods

Database

The data sets for model constructions were extracted from the Forest Service's Pacific Southwest Research Station database of density management installations for ponderosa pine. The selection criteria were the following:

1. Stands (both planted and natural stands) must be even-aged. Stands that clearly exhibit multicohort structures were excluded (Long and Shaw 2005).
2. Stands must be at least 80 percent ponderosa pine by basal area.
3. Stands must be of an age that 80 percent of the trees are above 4.5 feet (137 centimeters; i.e., at least saplings) tall and must have a known (observed) diameter at breast height (DBH). Heights must be measured on at least a 20 percent subsample of trees, or 20 trees per plot (whichever is smaller) covering a range of DBHs within the plot so that a plot-specific height-diameter relationship can be used to derive a dominant height.
4. Plots may be remeasured over time but must meet criteria 1, 2, and 3 for each time period observation to be included. Thus, for example, if a given plot only meets those criteria for the first three observations and subsequent observations do not meet these criteria, only the first three observations are included in the analysis.

A total of 829 observations (from 113 plots across 15 sites with repeated measures) were selected through the selection criteria (Table 1). Most of the sites were located in California; two sites (LM and CC in Table 1) were in Oregon (Figure 1).

Missing heights for ponderosa pine trees were estimated with a plot-level height-diameter equation for each plot at each time period. The height-diameter equations of Larsen and Hann (1987) were used for the other species' missing heights. For each plot, QMD, TPA, top height, stand volume, aboveground biomass, and canopy cover were calculated. For top height, mean height of the 20 tallest trees per acre (HT20; Ritchie et al. 2012) was used because some of the thinned plots had densities too low to present the 40 tallest trees per acre from the plots. Stand volume was calculated using the equations of Walters et al. (1985). Equations from Powers et al. (2013) were used to calculate the aboveground biomass for ponderosa pine. For white fir, Douglas-fir, sugar pine, and incense-cedar, biomass equations from Westman (1987), Brown (1978), Powers et al. (2013), and Jenkins et al. (2004) were used, respectively. Crown projection area (assumed as a circle) for each individual tree was manually computed using the crown width equations of Forest Vegetation Simulator (FVS; Keyser 2008a, 2008b, Keyser and Dixon 2008a, 2008b). Crown projection areas of individual trees were aggregated into stand percent canopy cover following Crookston and Stage (1999).

Model Construction and Validation

Because one of this study's objectives was to develop new concomitant stand attribute equations for use in SDMD application, only the two variables in the SDMD were used for the predictor variables: QMD (inches) and TPA (trees acre^{-1}). Another objective was to compare the prediction performance between the nonlinear mixed-effects model and the nonlinear fixed-effects model when users do not have any information on random effects for the mixed-effects model. The individual plot was treated as a random effect. As a result, two sets of nonlinear models (i.e., mixed- and fixed-effects models) for HT20 (feet), stand volume (ft^3 acre^{-1}), aboveground biomass (tons acre^{-1}), and percent canopy cover were constructed.

The models fitted were as follows:

$$\mathrm{HT20} = 4.5 + \left(\alpha + \beta \cdot \sqrt{\mathrm{TPA}}\right) \cdot \left(1 - e^{\left(\gamma + \frac{\delta}{\sqrt{\mathrm{TPA}}}\right) \cdot \mathrm{QMD}}\right)^{\theta} \quad (1)$$

$$\text{Volume and Biomass} = \mathrm{TPA}^{\alpha} \cdot e^{\left(\beta + \gamma \cdot \log(\mathrm{QMD}) + \delta / \sqrt{\mathrm{TPA}}\right)} \quad (2)$$

$$\text{Canopy Cover} = 100 * \left(1 - e^{\left(\alpha \cdot \mathrm{QMD}^{\beta} \cdot \mathrm{TPA}^{\gamma}\right)}\right) \quad (3)$$

where α, β, γ, δ, and θ are the parameters to be estimated and e is the base of the natural logarithm. Additionally, covariance-variance matrixes were provided to estimate the confidence intervals for predicted values through the delta method, which approximates standard error of the estimate through a Taylor series expansion.

For validation, the 10-fold cross-validation technique was used. Predictions for the mixed-effects models were conducted only with fixed-effects coefficients assuming the random effects as zeros. Root-mean-square errors (RMSEs) were calculated to compare the mixed-effects and the fixed-effects models' prediction performances. Volume prediction by Long and Shaw (2005) was also compared. All analyses were executed using the nlmixed procedure of SAS software (ver. 9.4, SAS Institute, Cary NC).

Results

The aggregated plot-level data contained a wide range of stand attribute metrics (Figure 2). Black Mountain (BV) had the most juvenile stand among the sites; it included the plot with lowest HT20 (7.0 ft), basal area (0.3 ft^2 acre^{-1}), biomass (0.0 tons acre^{-1}), QMD (1.1 inches), stand density index (1), and canopy cover (2 percent) across all of the plot/time measurements (Table 2). Edson Creek contained the plot with the highest HT20 (156.8 ft) and aboveground biomass (151 tons acre^{-1}). The range of TPA for Elliot Ranch was the widest (18–548 trees acre^{-1}), and it included the plot with the highest QMD (36.2 inches). The plots with the highest basal area (338.3 ft^2 acre^{-1}), stand density index (551), and canopy cover (80 percent) were measured at Prattville.

Validation results indicated that the fixed-effects models had better prediction performances than the mixed-effects models for all stand metrics except canopy cover (Figure 3). The fixed-effects model predictions for HT20, volume, and aboveground biomass had lower RMSEs (0.5 ft, 84 ft^3 acre^{-1}, and 0.06 tons acre^{-1}, respectively) than the mixed-effects models (Figure 3). For canopy cover prediction, the fixed-effects model had slightly higher RMSE

Table 1. Data sets for stand density management diagram for ponderosa pine.

Site Code	Site Name	FVS Variant Code	FVS Region	Measurement Period	No. of Plots	Max. No. of Tees†	No. of Observations	Mean Site Index‡ (ft)
AP	Adin Pass	SO	5	1957–2016	1	46	11	65.4
BM	Blacks Mountain	SO	5	1998–2013	2	786	4	80.0
BV	Blacks Mountain	SO	5	2014–2016	30	5,027	2	87.3
CC	Crawford Creek	BM	6	1967–2014	18	2,284	10	76.1
EC	Edson Creek	SO	5	1972–2016	3	177	7	135.2
ER	Elliot Ranch	WS	5	1969–2014	15	2,004	10	153.4
GA	Goose AMA	SO	5	2002–2013	6	1,186	3	116.7
HL	Hog Lake	SO	5	1944–2016	1	82	9	72.7
JC	Joseph Creek	SO	5	1962–2016	3	607	7	85.7
LM	Lookout MT	SO	6	1965–2014	18	1,608	13	94.3
PT	Prattville	CA	5	1971–2016	3	189	8	119.4
SB	Spaulding Butte	SO	5	1972–2016	3	228	10	57.1
SH	Sugar Hill	SO	6	1958–2016	6	1,235	8	90.2
SP	Show Plantation	SO	5	1972–2016	3	290	9	137.3
WM	Washington MT	SO	5	1963–2016	1	42	10	65.9

† Maximum number over the measurement period per site.
‡ Barrett (1978); base age: 100.

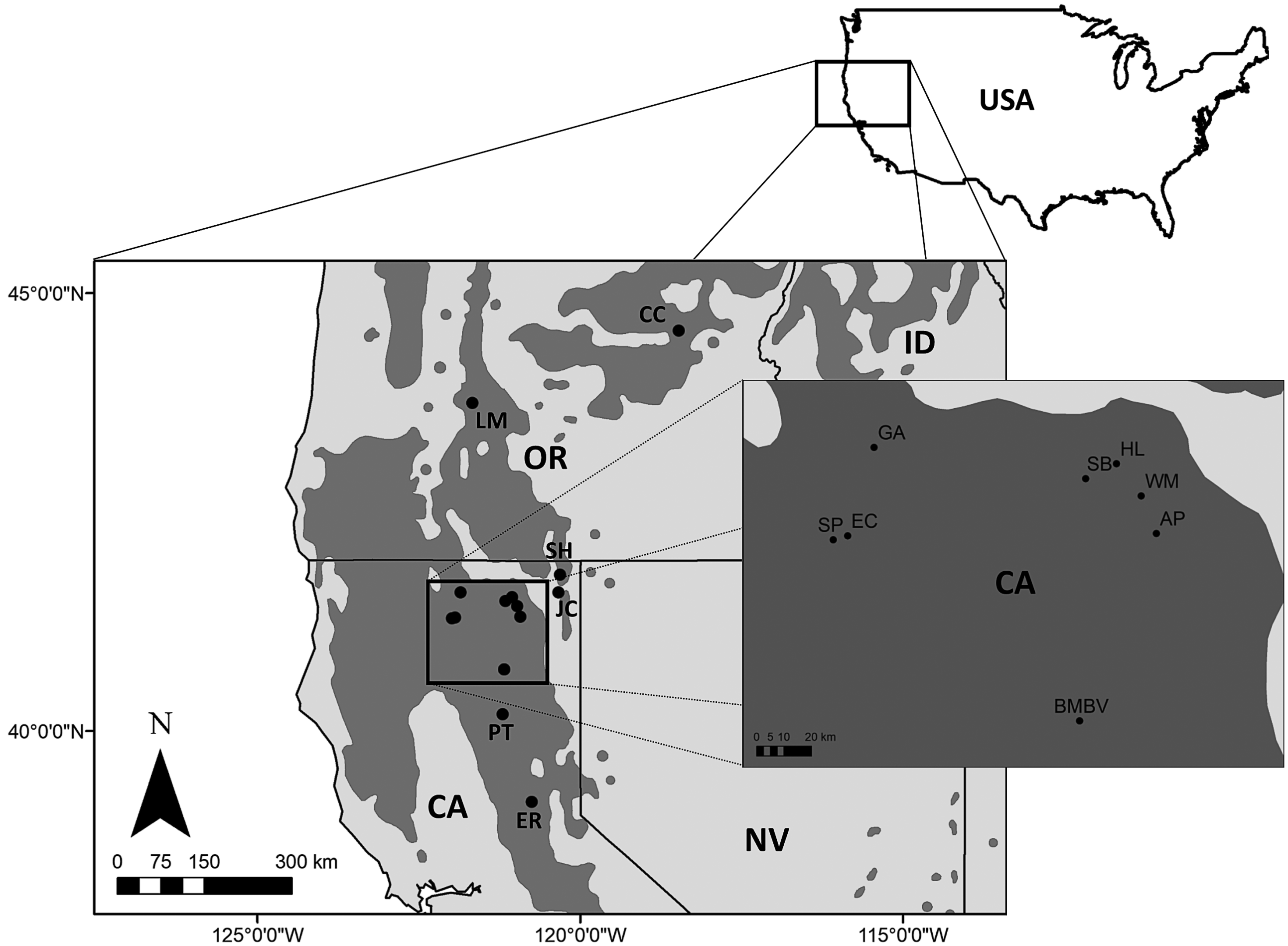

Figure 1. Location of the data sources in this study. Dark gray represents the distribution of ponderosa pine. Site codes are presented in Table 1.

(0.03 percent) than the mixed-effects model. Compared with the volume prediction by Long and Shaw (2005, equation 5), the fixed-effects model showed a lower RMSE by 9.2 percent (Figure 3b). Both aboveground biomass models showed narrow prediction error distributions and extremely subtle differences in prediction performance between mixed-effects and fixed-effects models (Figure 3c) because the biomass equation for dominant species (ponderosa pine) is a function of a single variable (i.e., DBH). Estimated parameters and their covariance structure derived from the fixed-effects models and three-dimensional representation of the relationships among QMD, TPA, and the stand attributes were described in Table 3 and Figure S2, respectively.

Discussion

Previous SDMDs for ponderosa pine developed by Long and Shaw (2005) contained and displayed multiple stand attributes including top height, stand volume, and stand density index. Those attributes were presented as isolines on the plane of QMD and TPA. Farnden (1996) emphasized that other useful stand attributes can also be plotted, such as aboveground biomass and percent crown cover. Aboveground biomass equations can be useful to evaluate the amounts of available resources and the flow of energy and nutrients including the carbon cycle (Parresol 1999). Crown cover can be used to assess the magnitude of competition (Gill et al. 2000) and other ecosystem service values such as wildlife habitat, livestock grazing, water and nutrient cycling, and recreation (Farnden 1996).

The application of isolines in density management diagrams is somewhat limited because figures can become very busy with each additional metric. Any more than two given isolines (e.g., volume and height) and figures become essentially unreadable. Furthermore, isolines require cumbersome and imprecise estimation by eyeball interpolations. By coding these equations into an R package such as standview (https://www.fs.fed.us/psw/tools/standview/; e.g., Figure S1), users can estimate stand metrics such as volume directly from the equations without the need for a rough interpolation. Furthermore, if isolines are abandoned in the graphical presentation, the user is free to estimate other metrics such as those we have included here (canopy cover and biomass).

This study compared the predictive performance between mixed-effects and fixed-effects models, especially when information on the random coefficients was not available. The expected superior prediction performance of mixed-effects models with random coefficients over the fixed-effects model has been widely reported in the literature (e.g., Lappi 1991, Temesgen et al. 2008, Cao and Wang 2014). This is because the random effects in the mixed-effects model can account for the variability among (and/or correlation within) plots through specifying their variance-covariance error structure (Calama and Montero 2004). For the best prediction performance using mixed-effects models, subsampling is commonly used to predict the random effects (Temesgen et al. 2008). However, this procedure requires additional investment and is often impractical for common SDMD users. If predictions are made only from the fixed components in the mixed-effects model because of information unavailability on the random effects, our validation results indicate higher prediction errors than those from the fixed-effects models. This finding is consistent with results published elsewhere (e.g., Monleon 2003, Robinson and Wykoff 2004, Temesgen et al. 2008, Garber et al. 2009) and suggests avoiding a mixed-effects model when there is no information available to estimate the random effects.

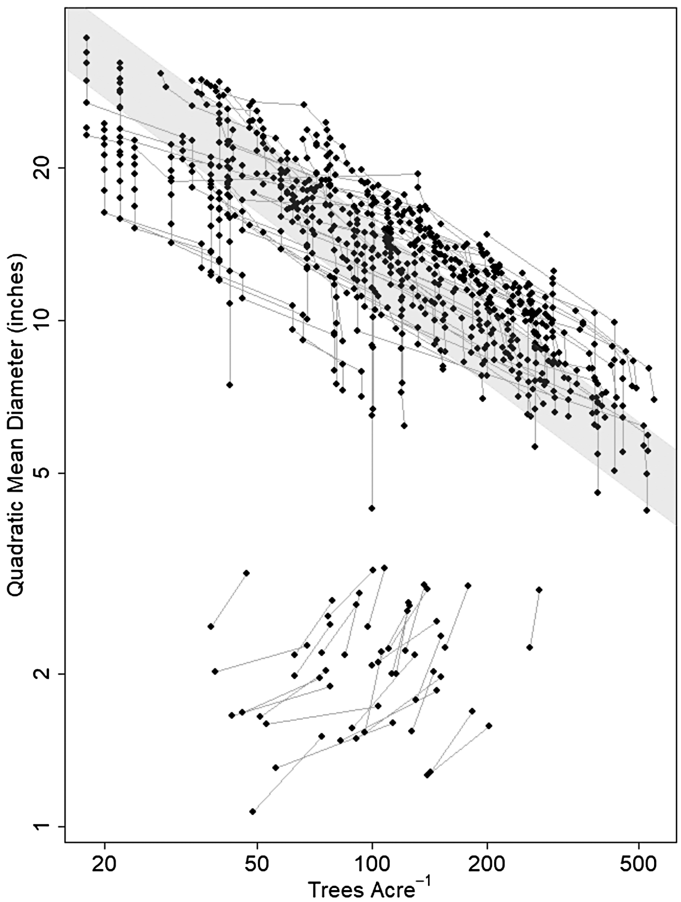

Figure 2. Stand density (trees acre^{-1}) and quadratic mean diameter (inches) of plot data used in this study. Repeated measured observations were connected by lines. Note that both axes are (natural) log-scaled, and gray area represents the management zone (i.e., 30 to 55 percent maximum stand density index; Long and Shaw 2005).

Although Long and Shaw (2005) filtered inappropriate plot data from the FIA database to acquire data close to even-aged, pure ponderosa pine stand conditions, several issues still appeared to remain. First, information about management history may be incomplete in FIA data (particularly for old data; Woudenberg et al. 2010). Stands experiencing recent structural changes may have been included in their analyses. For example, a recently heavily thinned stand would not have the same average size at a given density (Farnden 1996, Jack and Long 1996). Second, from the standpoint of traditional production ecology, since managed stands have been regulated for optimum growing space occupation (Mitchell and Beese 2002), inclusion of data from unmanaged stands may cause underestimation of size metrics such as dominant height as suggested by the underestimation of stand volume by the Long and Shaw (2005) model, particularly in stands with large volume (Figure 3b). Higher irregularity of spatial distribution of trees in unmanaged stands (regular versus aggregated) can also affect prediction performance (Pukkala et al. 2015). These points reinforce the importance of quality data collected from well-managed stands with a known history for SDMD construction as well as other decision support tools (Mowrer 2000).

Using local site index curves can increase the utility of SDMDs, if available. For example, the age of a stand can be determined by top height measurement (Dean and Jokela 1992) or vice versa (Long and Shaw 2005). In the same manner, the timing of thinning and harvesting can also be determined (Barrio Anta and Álvarez González 2005). Since the variability of top height prediction in our model was relatively high (Figure 3a), managers may consider local calibration of SDMD if complementary information about site index and stand age is available. The available site index curves for ponderosa pine throughout the United State regions are listed in the publication of Long and Shaw (2005).

Quantifying the uncertainty of the predicted values and assessing subsequent risks are critical to compare alternative management regimes and to support reasonable decisions (Mowrer 2000). In addition, since most of the stand attributes equations in the SDMD

Table 2. Ranges (minimum–maximum) of the data (throughout the measurements) used for isoline construction. Abbreviations: BA, basal area; CCV, canopy cover; HT20, top height; QMD, quadratic mean diameter; SDI, Reineke's stand density index (based on 1.605 Reineke's term); TPA, trees per acre.

Site Code	Site Name	HT20 (ft)	BA (ft^2 acre^{-1})	TPA (trees acre^{-1})	Biomass (tons acre^{-1})	QMD (inches)	SDI	CCV (%)
AP	Adin Pass	35.6–78.2	25.4–133.8	109–122	7.6–59.3	6.2–15.0	56–209	16–46
BM	Blacks Mountain	57.4–67.6	29.3–53.3	64–73	10.8–21.7	9.0–11.5	56–92	16–23
BV	Blacks Mountain	7.0–14.9	0.3–12.8	38–274	0.0–2.7	1.1–3.2	1–38	2–16
CC	Crawford Creek	43.5–94.4	24.3–196.5	30–528	7.6–78.4	5.5–21.0	50–337	15–58
EC	Edson Creek	118.9–156.8	119.0–258.9	34–80	60.3–150.9	18.2–29.8	172–322	38–58
ER	Elliot Ranch	47.7–143.3	37.4–272.0	18–548	13.2–139.1	7.0–36.2	69–384	19–67
GA	Goose AMA	89.3–122.0	85.8–149.1	49–81	45.8–82.1	16.5–50.8	125–208	29–42
HL	Hog Lake	53.2–82.3	60.0–202.5	137–256	17.5–86.5	6.5–14.7	129–327	32–60
JC	Joseph Creek	66.7–97.3	88.1–189.0	61–283	30.8–81.9	7.5–17.6	138–315	32–59
LM	Lookout MT	65.4–109.2	29.1–197.3	18–380	12.4–89.6	8.0–24.4	44–321	10–51
PT	Prattville	16.0–116.1	9.8–338.3	98–433	2.4–145.4	4.2–17.9	25–551	9–80
SB	Spaulding Butte	20.2–75.7	44.6–208.3	92–525	11.9–85.3	4.2–14.9	85–351	23–64
SH	Sugar Hill	23.9–92.9	12.8–307.2	43–456	4.1–129.4	5.5–23.8	26–497	9–64
SP	Show Plantation	92.0–141.4	84.9–211.2	28–156	38.3–117.7	13.4–30.7	133–278	31–53
WM	Washington MT	36.0–72.8	37.2–125.9	80–120	12.0–55.0	7.5–15.4	76–199	21–44

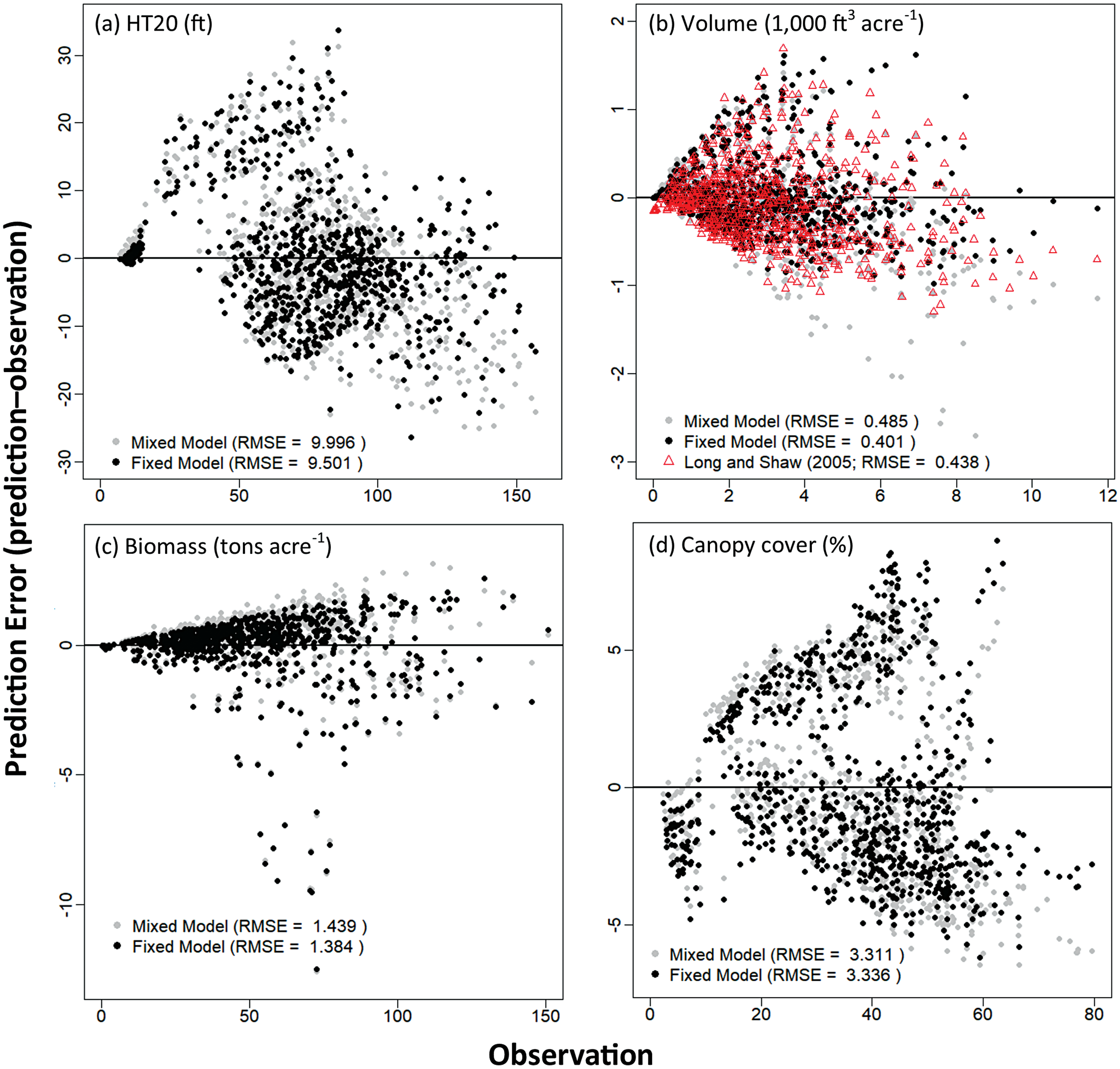

Figure 3. Model validation results for (a) top height, (b) total stand volume, (c) aboveground biomass, and (d) percent canopy cover. RMSE, root-mean-square error.

have nonlinear forms, the inference procedure for linear models is not applicable (Kutner et al. 2005). Common techniques for evaluating uncertainty for predictions from nonlinear modes are empirical bootstrapping (Horowitz 2003) and the delta method (Oehlert 1992, Powell 2007). However, bootstrapping requires resampling (Papke and Wooldridge 2005); thus the delta method has a practical advantage over bootstrapping for SDMD users. This study provides information about the variance-covariance structure of estimated parameters via the delta method (Table 3). With this, forest managers can compute the corresponding confidence intervals of their predictions (for a step-by-step example, refer to the Appendix).

Although this study provides enhanced stand attribute estimates and their uncertainties for the ponderosa pine SDMD, there are some limitations for application in this study. Implementation is restricted by the geographic range of the data (Figure 1) and the range of stand conditions (Table 2). Additional research with uncertainty estimations for concomitant variables will improve our understanding of the concomitant variables in SDMDs. In addition, managers should consider differences in the definitions of top height. In this study, we adopted HT20 as a top height because of the range of densities included; thus the top height might be incompatible with other stand density management tools using HT40 (i.e., the mean height of the tallest 40 trees per acre) or the mean height of dominant and codominant trees (Shaw and Long 2007). With these caveats, these equations may provide enhanced predictions for managed ponderosa pine stands with augmented uncertainty estimates.

Table 3. Estimated model parameters and covariance matrices for top height by equation (1), stand volume and aboveground biomass by equation (2), and percentage canopy cover by equation (3) from the even-aged pure ponderosa pine stands in California and Oregon.

Model	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{\theta}$
Top height	397.6014	22.6343	-0.0185	0.0183	1.3395
Covariance	$\begin{bmatrix} 22{,}191.6149 & & & & \\ 984.0972 & 61.7867 & & & \\ 1.02896 & 0.0514 & 5.0170\cdot 10^{-5} & & \\ -0.7289 & -0.0519 & -4.1658\cdot 10^{-5} & 5.0140\cdot 10^{-5} & \\ -10.5038 & -0.4427 & -5.0039\cdot 10^{-4} & 3.7008\cdot 10^{-4} & 5.7200\cdot 10^{-3} \end{bmatrix}$				
Stand volume	1.0444	-4.6678	3.0070	-1.9141	
Covariance	$\begin{bmatrix} 0.0012 & & & \\ -0.0088 & 0.0684 & & \\ 0.0006 & -0.0055 & 0.0009 & \\ 0.0165 & -0.1154 & 0.0037 & 0.2740 \end{bmatrix}$				
Above-ground biomass	0.9860	-7.1996	2.4713	-0.3093	
Covariance	$\begin{bmatrix} 5.1137\cdot 10^{-5} & & & \\ -3.8277\cdot 10^{-4} & 0.0030 & & \\ 2.4652\cdot 10^{-5} & -0.0002 & 3.6718\cdot 10^{-5} & \\ 7.3145\cdot 10^{-4} & -0.0051 & 1.5833\cdot 10^{-4} & 0.0125 \end{bmatrix}$				
Percent canopy cover	-0.0002	1.3605	0.9486		
Covariance	$\begin{bmatrix} 3.0615\cdot 10^{-10} & & \\ 3.6894\cdot 10^{-7} & 4.7635\cdot 10^{-4} & \\ 1.9069\cdot 10^{-7} & 2.1390\cdot 10^{-4} & 1.2739\cdot 10^{-4} \end{bmatrix}$				

Supplementary Materials

Supplementary data are available at *Forest Science* online.

Supplement 1. Stand density management diagrams and figures of model estimations from Table 3.

Appendix: Example of Calculating Standard Error for Prediction with Delta Method and Application

Assume that we want to calculate the standard error of stand volume prediction when QMD = 10.5 inches and TPA = 180 trees acre^{-1}.

Define $\mathbf{X}_i$ as the vector of the $p-1$ explanatory variables for ith observation and γ as the vector of q parameters, then the ith observation $\mathbf{Y}_i$ can be rewritten with the corresponding error ε_i as follows:

$$\mathbf{Y}_i = f(\mathbf{X}_i, \gamma) + \varepsilon_i$$

where:

$$\mathbf{X}_i = \begin{bmatrix} X_{i1} \\ X_{i1} \\ \vdots \\ X_{iq} \end{bmatrix} \gamma = \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \vdots \\ \gamma_{p-1} \end{bmatrix}$$

Through the delta method, the approximated variance of volume prediction $\hat{\mathrm{Y}}$ **is**

$$\mathrm{Var}\left[\hat{\mathrm{Y}}\right] \cong [\partial\hat{\mathbf{Y}}/\partial\hat{\gamma}]'\mathbf{V}[\partial\hat{\mathbf{Y}}/\partial\hat{\gamma}]$$

where $\left[\partial\hat{\mathbf{Y}}/\partial\hat{\gamma}\right]$ is the first-order partial derivative of the equation and $\mathbf{V}$ is the (asymptotic) variance-covariance matrix of $\hat{\gamma}$.

Our volume equation is (equation 2)

$$\mathrm{volume} = \mathrm{TPA}^{\alpha} \cdot e^{\left(\beta+\gamma\cdot\log(\mathrm{QMD})+\delta/\ \overline{\mathrm{TPA}}\right)}$$

The 4 × 1 matrix of estimated coefficients $\hat{\gamma}$ is given as (Table 3)

$$\hat{\gamma} = \begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \\ \hat{\gamma} \\ \hat{\delta} \end{bmatrix} = \begin{bmatrix} 1.0444 \\ -4.6678 \\ 3.0070 \\ -1.9141 \end{bmatrix}$$

Then the estimate for volume is 2,172.5 ft^3 acre^{-1}. The $\left[\partial\hat{\mathbf{Y}}/\partial\hat{\gamma}\right]$ is obtained as follows:

$$\left[\frac{\partial\hat{\mathbf{Y}}}{\partial\hat{\gamma}}\right] = \begin{bmatrix} \widehat{\partial\mathrm{vol}/\partial\hat{\alpha}} \\ \widehat{\partial\mathrm{vol}/\partial\hat{\beta}} \\ \widehat{\partial\mathrm{vol}/\partial\hat{\gamma}} \\ \widehat{\partial\mathrm{vol}/\partial\hat{\delta}} \end{bmatrix} = \begin{bmatrix} \mathrm{TPA}^{\hat{\alpha}} \cdot \log\hat{\alpha} \cdot e^{\left(\hat{\beta}+\hat{\gamma}\cdot\log(\mathrm{QMD})+\hat{\delta}/\ \overline{\mathrm{TPA}}\right)} \\ \mathrm{TPA}^{\hat{\alpha}} \cdot e^{\left(\hat{\beta}+\hat{\gamma}\cdot\log(\mathrm{QMD})+\hat{\delta}/\sqrt{\mathrm{TPA}}\right)} \\ \mathrm{TPA}^{\hat{\alpha}} \cdot \log\hat{\alpha} \cdot e^{\left(\hat{\beta}+\hat{\gamma}\cdot\log(\mathrm{QMD})+\hat{\delta}/\sqrt{\mathrm{TPA}}\right)} \\ \mathrm{TPA}^{\hat{\alpha}} \cdot \frac{1}{\sqrt{\mathrm{TPA}}} \cdot e^{\left(\hat{\beta}+\hat{\gamma}\cdot\log(\mathrm{QMD})+\hat{\delta}/\sqrt{\mathrm{TPA}}\right)} \end{bmatrix}$$

Thus,

$$\begin{bmatrix} 180^{1.0444} \cdot \log(1.0444) \cdot e^{\left(-4.6678+3.0070\cdot\log(10.5)+-1.9141/\sqrt{180}\right)} \\ 180^{1.0444} \cdot e^{\left(-4.6678+3.0070\cdot\log(10.5)+-1.9141/\sqrt{180}\right)} \\ 180^{1.0444} \cdot \log(10.5) \cdot e^{\left(-4.6678+3.0070\cdot\log(10.5)+-1.9141/\sqrt{180}\right)} \\ 180^{1.0444} \cdot \frac{1}{\sqrt{180}} \cdot e^{\left(-4.6678+3.0070\cdot\log(10.5)+-1.9141/\sqrt{180}\right)} \end{bmatrix} = \begin{bmatrix} 11,277.77 \\ 21,71.74 \\ 51,06.58 \\ 161.87 \end{bmatrix}$$

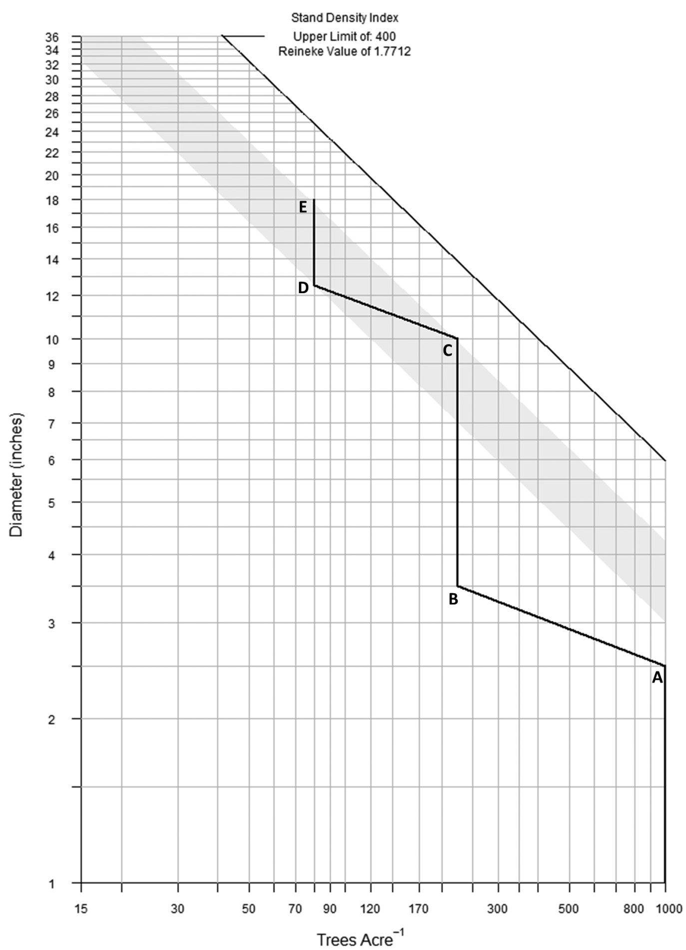

Figure A1. Example of application of stand management diagram to the management regime given by Long and Shaw (2005). Letters (A–E) indicate stand conditions illustrated in Table A1, and gray area represents the management zone (i.e., 30 to 55 percent maximum stand density index; Long and Shaw 2005).

From Table 3, **V** is given as follows:

$$\mathbf{V} = \begin{bmatrix} 0.0012 & -0.0088 & 0.0006 & 0.0165 \\ -0.0088 & 0.0684 & -0.0055 & -0.1154 \\ 0.0006 & -0.0055 & 0.0009 & 0.0037 \\ 0.0165 & -0.1154 & 0.0037 & 0.2740 \end{bmatrix}$$

Therefore, the variance of the predicted value is estimated as

$$\mathrm{Var}\left[\hat{Y}\right] \cong [\partial\hat{\mathbf{Y}}/\partial\hat{\gamma}\,]'\mathbf{V}[\partial\hat{\mathbf{Y}}/\partial\hat{\gamma}\,]$$

$$= \begin{bmatrix} 11277.77 & 2171.74 & 5106.58 & 161.87 \end{bmatrix} \begin{bmatrix} 0.0012 & -0.0088 & 0.0006 & 0.0165 \\ -0.0088 & 0.0684 & -0.0055 & -0.1154 \\ 0.0006 & -0.0055 & 0.0009 & 0.0037 \\ 0.0165 & -0.1154 & 0.0037 & 0.2740 \end{bmatrix} \begin{bmatrix} 11277.77 \\ 2171.74 \\ 5106.58 \\ 161.87 \end{bmatrix}$$

$$= 333.88$$

Finally, the standard error (SE) of prediction is derived through square root of the variance:

$$\mathrm{SE}\ [\hat{Y}] = \sqrt{\mathrm{Var}\ [\hat{Y}]} \cong \sqrt{333.88} = 18.27\ (\mathrm{ft}^3\ \mathrm{acre}^{-1})$$

The standard errors for all other variables can be calculated through the same procedure.

The example of a density management regime given by Long and Shaw (2005) can be reillustrated (Figure A1). The management regime assumed the following: initial 1,000 TPA, 18-inch diameter at end of rotation, a single precommercial thinning, and a single commercial thinning while maximizing site occupancy and minimizing density-dependent mortality (i.e., Figure 5 from Long and Shaw 2005). Note that in this example, we used 1.7712 of Reineke term (Oliver and Powers 1978) and an assumed maximum stand density index of 400, and the management zone was set to 30 and 55 percent of maximum stand density index (Long and Shaw 2005). The stand attribute predictions and their standard errors corresponding to five different stand conditions (i.e., A–E in Figure A1) were calculated in Table A1.

Table A1. Stand summary for the example of density management regime (marked as A–E) illustrated in Figure A1. Abbreviations: BA, basal area; HT20, top height; QMD, quadratic mean diameter; SE, standard error; TPA, trees per acre.

Stand	TPA	QMD (in)	BA (ft² acre⁻¹)	Volume (ft² acre⁻¹)		HT20 (ft)		Biomass (tons acre⁻¹)		Canopy Cover (%)	
				Estimates	SE	Estimates	SE	Estimates	SE	Estimates	SE
A	1,000	2.5	34.1	189	9.9	21	1.0	6.5	0.08	38.6	0.5
B	225	3.5	15.0	102	5.0	21	0.8	3.4	0.04	17.1	0.1
C	225	10.0	122.7	2,404	95.2	67	0.5	45.2	0.63	54.2	0.7
D	80	12.5	71.5	1,465	56.5	68	0.6	27.9	0.41	32.8	0.4
E	80	18.0	141.4	4,385	165.4	101	0.7	68.7	1.06	47.9	0.7

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