A Weibull Analysis of Wood Member Bending Strength

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The analysis of size effect on bending strength of rectangular wood beams based on Weibull's theory of brittle failure has been expanded here to include tapered wood beams under arbitrary loading conditions. The mathematical formulations are expressed in terms of the two parameters in Weibull's model. These parameters must be determined experimentally for each wood species. Numerical data for Douglas-fir beams are presented in graph form, where necessary, for easy application by the design engineer. Design procedures are demonstrated by numerical examples.

Introduction

In the design of many types of wood constructions such as rectangular glued-laminated beams and pallets with unnotched stringers, it has been common practice to assume that their load capacities are dependent mainly upon the bending strength of the structural components. The other stresses are either used to check the adequacy of the design or believed to have less pronounced effect on failure. While this may not be exactly true for some wood constructions such as pitched and curved glulam beams, it is generally agreed that bending failure is one of the very important failure modes. Therefore it is of interest to investigate the bending strength of wood members under various loading conditions.

The bending strengths of different wood species as presented in [11] are based on results of tests on small, clear, straight-grained specimens. It has long been observed that the bending strength of wood structural members decreases with increasing size. This phenomenon was investigated in 1924 by Newlin and Trayer [8], who related this decrease in strength to the depth of the member. The maximum depth considered in their tests was 12 in. The strength-depth theory was extended by Freas and Selbo [6] in 1954 to include beams up to 16 in. deep. As the size of glued-laminated beams continued to increase to depths exceeding 80 in., it was questionable whether the relationship in [6] would still apply. In 1966 Bohannan [2] developed a mathematical theory to explain the size-strength relationship for wood beams using Weibull's statistical theory of strength of materials [13]. The two parameters in the Weibull's model were evaluated from three sets of test data for 2056 clear, straight-grained Douglas-fir beams. The data were adjusted to a 12 percent moisture content and to 0.48 specific gravity. The developed theory was later used by Bohannan [3] to predict the bending strength of three large, clear, glued-laminated Douglas-fir beams (9 in. by 31.5 in. by 600 in. with 21 laminations) to within 2 percent of the average test value.

In [2, 3], only center point loading and/or two point loading were considered. Bohannan [4] also discussed the case of simply supported, rectangular beams under uniform load. As the resulting equations cannot be directly applied to other loading conditions, there is a need to further analyze the effect of size on the bending strength of wood members under the loading conditions most commonly encountered by the design engineer. The present study was originated to perform an analysis to satisfy this need.

In the present analysis, the linear bending stress distribution adopted in [2] is also used. The same stress distribution was assumed by Maki and Kuenzi [7] in their study of the behavior of tapered wood beams of uniformly varying cross section, and verified experimentally in the tension zone by Dewey et al. [5]. Thus the present work also covers both the rectangular and tapered geometries. Due to the geometrical features and/or some special loading conditions, some of the resulting formulas can only be expressed in terms of the single integrals that cannot be reduced to closed form. Numerical techniques must then be used to obtain the solutions for each wood species for which Weibull parameters already have been determined.

Bending Stress Distributions

On the basis of the Bernoulli-Euler theory of bending, that is, the assumption that plane sections before bending remain plane after bending, the following familiar relationship is obtained:

$$\sigma = \frac{M}{S} \tag{1}$$

where σ is the bending stress, M is the bending moment, and S is the section modulus. This relationship, orginally developed for rectangular beams, also holds for tapered beams provided the taper slopes are not too large. For example, in an analysis of a wedge problem in [9] with an extreme slope of 1:4, analogous to a tapered beam problem, an error of only 1.5 percent would be encountered in using the Bernoulli-Euler relationship.

The bending stress for a rectangular beam as shown in Fig. 1A can be written as:

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$$\sigma_x = \frac{12My}{bh^3} \tag{2}$$

and, for a tapered beam with a taper slope falling within the range encountered in usual tapered beam problems, as shown in Fig. 1B, the bending stress is given by:

$$\sigma_x = \frac{12M\left(y - \frac{h}{2}\right)}{bh^3} \tag{3}$$

where σ_x is the bending stress, M is the bending moment, b is the beam width, h is the beam height, and y is the distance from the neutral surface of the rectangular beam, as well as the distance from the nontapered surface of the tapered beam to the point where the stress is σ_x.

For the rectangular beam the maximum tensile value of σ_x will occur at the extreme fiber in the tension zone of the section where M is a maximum; for the tapered beam the section where the maximum tensile value of σ_x will occur has a depth, h, given by [7]:

$$h = \frac{2M\left(\frac{dh}{dx}\right)}{\frac{dM}{dx}} \tag{4}$$

and the maximum value of σ_x is [7]:

$$\sigma_0 = \frac{3}{2bM} \cdot \frac{\left(\frac{dM}{dx}\right)^2}{\left(\frac{dh}{dx}\right)^2} \tag{5}$$

where dM/dx and dh/dx represent the shear force and taper slope at the section, respectively.

Statistical Analysis

The application of the calculus of probability leads to the fundamental law of the statistical theory [13], which can be expressed by

$$\ln(1-F) = -\int_V \eta(\sigma)dv \tag{6}$$

where F is the cumulative probability of failure for any given distribution of stresses, σ, over a volume, V, and η (σ) is a material function.

As described in [2], the statistical theory advanced by Weibull [13] is based on the "weakest link theory." When applied to wood beams, width should not be considered in order to bring about better agreement between theory and observed data. Tucker [10] also made this conclusion in his discussion of the "weakest link theory" and his application of a statistical strength theory to concrete beams. Therefore, in applying equation (6) to wood beams, it is necessary to replace the volume, V, by the area, A, or set the width equal to unity.

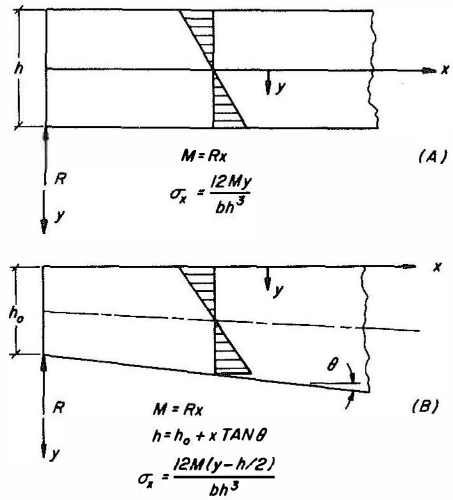

Fig. 1 Coordinate axes and bending stress distributions for: (A) a rectangular beam, and (B) a tapered beam

By putting the material function in the following form [13]:

$$\eta(\sigma) = k\sigma^m \tag{7}$$

where k and m are constants, one obtains from equation (6), with V replaced by A, the following expression:

$$F = 1 - \exp(-B) \tag{8}$$

where

$$B = k\int_A \sigma^m dA \tag{9}$$

in which k can be expressed, according to [2], by

$$k = \frac{2(m+1)^2}{w_0{}^m} \tag{10}$$

where w_0 and m are constants to be determined from test results and m is called the Weibull shape parameter.

For a wood beam under specified loading conditions, it is only necessary to substitute the tensile stress σ_x for σ in equation (9) and to perform the integration to obtain B, which can also be put in the following form, according to equation (8):

$$B = -\ln(1-F) \tag{11}$$

As will be seen later, the expression for B will contain the required strength information.

Rectangular Beam Under Concentrated Loads

For a rectangular beam under concentrated loads, as described in Fig. 2A, the moment diagram in Fig. 2B shows that between any two consecutive loads the moment varies linearly, that is,

Nomenclature

- A = area
- a, a' = lengths
- $\bar{a}$ = $a \tan\theta/h_0$
- $\bar{a}'$ = $a' \tan\theta/h_c$
- B = exponent, see equation 8
- b = beam width
- C_1, C_2 = constants
- d = length
- F = cumulative failure probability
- h = beam height
- h_c = largest height of tapered beam
- h_0 = smallest height of tapered beam
- k = constant, see equation (10)
- L = beam length
- $\bar{L}$ = $L \tan\theta/h_0$
- M = bending moment
- m = Weibull shape parameter
- P = concentrated load
- R_1, R_2 = reactions
- S = section modulus
- V = volume
- w = load per unit length
- w_0 = material constant, see equation (10)
- x, y = coordinate axes
- α = constant
- ζ = characteristic parameter, see equation 19
- η = material function
- θ = taper slope
- σ, σ_x = bending stresses
- σ_0 = maximum bending stress; modulus of rupture

$$M = C_1 + C_2 x \tag{12}$$

where C_1 and C_2 are constants.

Consider the beam section of length d with moment expressed by equation (12). From equations (2), (9), and (10), one obtains

$$B = A\left(\frac{\sigma_0}{w_0}\right)^m \left(\frac{C_1}{C_2 d} + 1\right)\left[1 - \left(\frac{C_1}{C_1 + C_2 d}\right)^{m+1}\right] \tag{13}$$

where A is the area of the beam section and equal to hd and σ_0 is the maximum bending stress in the beam section.

It is seen that equation (13) is true for any values of C_1, C_2, and d. In particular, for $C_1 = 0$ it follows:

$$B = A\left(\frac{\sigma_0}{w_0}\right)^m \tag{14}$$

and using l'Hospital's rule there arises for $C_2 = 0$:

$$B = A\left(\frac{\sigma_0}{w_0}\right)^m (m+1) \tag{15}$$

Equations (14) and (15) agree with results in [2].

The expression for B for the whole beam can be obtained by summing up the contribution due to each section across the length of the beam. For a beam section with nonlinear moment diagram, the foregoing results should still apply if the moment diagram can be approximated with sufficient accuracy by a straight line.

Rectangular Beam Under Uniform Load

The moment expression for the rectangular beam under uniform load as shown in Fig. 3A can be written as:

$$M = \alpha w L x - \frac{w x^2}{2} \tag{16}$$

which is described in Fig. 3B. In equation (16), α is an arbitrary constant. For a simply supported beam, $\alpha = 1/2$; for a two-span continuous beam of length $2L$, the middle support, that is, the right support in Fig. 3A, can be taken as fixed and $\alpha = 3/8$; for a cantilever beam fixed at the right end, $\alpha = 0$.

It is of interest to note that the maximum moment occurs within the length of the beam only when $\sqrt{2} - 1 < \alpha \leq 1$. Within this range, the maximum moment should occur at $x = \alpha L$ with a magnitude of $M_{\max} = w/2\ (\alpha L)^2$. The corresponding maximum stress is

$$\sigma_0 = \frac{3w(\alpha L)^2}{bh^2}, \text{ for } \sqrt{2} - 1 < \alpha \leq 1 \text{ at } x = \alpha L \tag{17}$$

When α falls outside the above range, the maximum absolute moment always occurs at $x = L$, that is, the right end of the beam, with a magnitude of $M_{\max} = |\alpha w L^2 - wL^2/2|$. The corresponding maximum stress is

$$\sigma_0 = \frac{wL^2\,|6\alpha - 3|}{bh^2}, \text{ for } \alpha > 1 \text{ or } \alpha \leq \sqrt{2} - 1 \text{ at } x = L \tag{18}$$

Again, from equations (2), (9), and (10), the following can be obtained:

$$B = \zeta A\left(\frac{\sigma_0}{w_0}\right)^m \tag{19}$$

where the beam area $A = hL$;

$$\zeta = (m+1)\int_{-\alpha}^{1-\alpha}\left|1 - \left(\frac{x}{\alpha}\right)^2\right|^m dx, \text{ for } \sqrt{2} - 1 < \alpha \leq 1 \tag{20}$$

and

$$\zeta = (m+1)\left|\frac{2}{1-2\alpha}\right|^m \int_0^1 \left|\alpha x - \frac{x^2}{2}\right|^m dx,$$
$$\text{for } \alpha > 1 \text{ or } \alpha \leq \sqrt{2} - 1 \tag{21}$$

For $\alpha = 0$, equation (21) yields $\zeta = (m+1)/(2m+1)$.

Equations (20) and (21) cannot be reduced to closed form for an arbitrary value of m. However, it is very convenient to evaluate the integrals contained therein using some numerical techniques such as those presented in [1, 12]. In deducing A or σ_0 for one beam from the values of these parameters for another beam using equation (19), one must be sure that the values of ζ for the beams are properly taken care of.

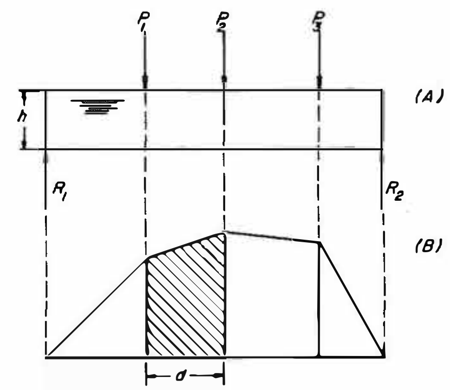

Fig. 2 (A) Rectangular beam under concentrated loads; (B) moment diagram

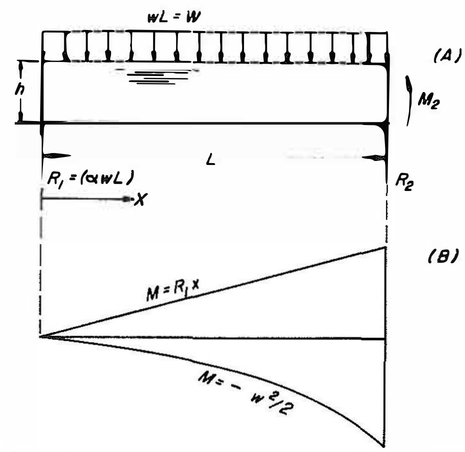

Fig. 3 (A) Rectangular beam under uniform load; (B) moment diagram

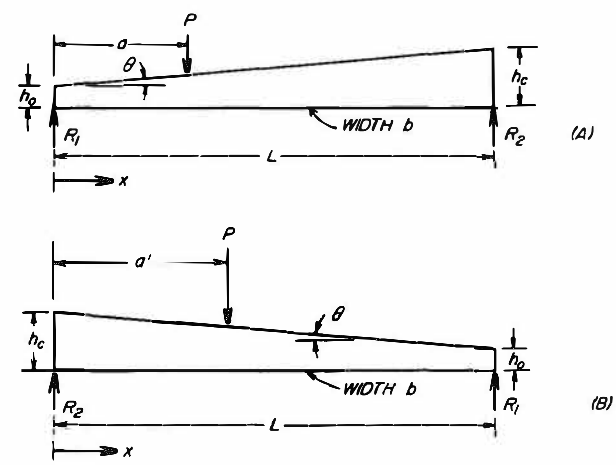

Fig. 4 Simply supported, single-tapered beam under a concentrated load

Tapered Beam Under a Concentrated Load

For a simply supported, single-tapered beam under a concentrated load as shown in Fig. 4, it is convenient to break the analysis into two parts for the two portions of the beam separated by the load point.

(*a*) Left Portion of the Beam in Fig. 4A. The concentrated load, P, is applied at a distance, a, from the small end, where the reaction force is R_1 and the beam height is h_0. Let the taper slope be θ. Then

$$h = h_0 + x\tan\theta \tag{22}$$

and

$$M = R_1 x \quad \text{for } 0 \le x \le a$$

According to [7] it can be shown that the maximum bending stress is

$$\sigma_0 = \frac{6R_1 a}{b(h_0 + a\tan\theta)^2} \quad \text{for } a\tan\theta \le h_0 \text{ and } x = a \tag{23}$$

and it is

$$\sigma_0 = \frac{3R_1}{2bh_0\tan\theta} \quad \text{for } a\tan\theta \ge h_0 \text{ and } x = \frac{h_0}{\tan\theta} \tag{24}$$

Following the same procedures as for the rectangular beams, one obtains

$$B_1 = \zeta A_1 \left(\frac{\sigma_0}{w_0}\right)^m \tag{25}$$

where A_1 is the trapezoidal area of the beam between the small end and the load point and

$$\zeta = \frac{(m+1)}{\bar{a}\left(1 + \frac{\bar{a}}{2}\right)} \left[\frac{(1+\bar{a})^2}{\bar{a}}\right]^m \int_1^{1+\bar{a}} \frac{(x-1)^m}{x^{2m-1}}\,dx, \quad \text{for } a\tan\theta \le h_0 \tag{26}$$

and

$$\zeta = \frac{(m+1)4^m}{\bar{a}\left(1 + \frac{\bar{a}}{2}\right)} \int_1^{1+\bar{a}} \frac{(x-1)^m}{x^{2m-1}}\,dx, \quad \text{for } a\tan\theta \ge h_0 \tag{27}$$

in which $\bar{a} = a\tan\theta/h_0$.

It is of interest to note that in equations (26) and (27), $\bar{a}$ is the only parameter other than the Weibull slope, m, that will determine the value for ζ. Also, the above results can be directly applied to a double-tapered beam as shown in Fig. 5 when the load is applied at the apex of the beam. The two portions of the beam need not be identical, as the above derivations are true for any tapered beam geometry and the B's for all portions of the beam are additive.

(*b*) Left Portion of the Beam in Fig. 4B. The left portion of the beam in Fig. 4B is geometrically identical to the right portion of the beam in Fig. 4A. From Fig. 4B, one obtains:

$$h = h_c - x\tan\theta$$

and

$$M = R_2 x \quad \text{for } 0 \le x \le a' \tag{28}$$

It can easily be shown that the maximum bending stress is

$$\sigma_0 = \frac{6R_2 a'}{b(h_c - a'\tan\theta)^2} \quad \text{at } x = a' \tag{29}$$

Following the same procedures as shown above, it follows:

$$B_2 = \zeta A_2 \left(\frac{\sigma_0}{w_0}\right)^m \tag{30}$$

where A_2 is the trapezoidal area of the beam between the large end and the load point and

$$\zeta = \frac{(m+1)}{\bar{a}'\left(1 - \frac{\bar{a}'}{2}\right)} \left[\frac{(1-\bar{a}')^2}{\bar{a}'}\right]^m \int_{1-\bar{a}'}^{1} \frac{(1-x)^m}{x^{2m-1}}\,dx \tag{31}$$

where $\bar{a}' = a'\tan\theta/h_c$.

Setting $a' = L - a$ in equations (29), (30), and (31), it is evident for the single-tapered beam in Fig. 4A that

$$B = B_1 + B_2 \tag{32}$$

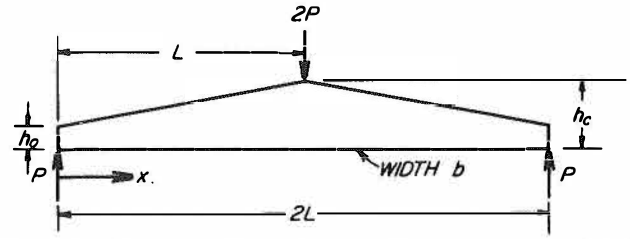

Fig. 5 Simply supported, double-tapered beam under concentrated midspan load

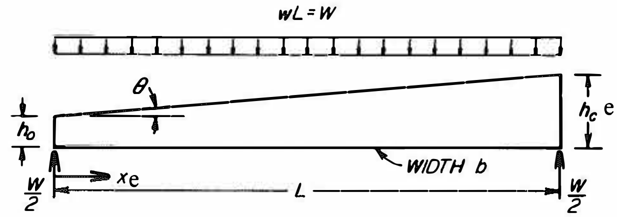

Fig. 6 Simply supported, single-tapered beam under uniformly distributed load

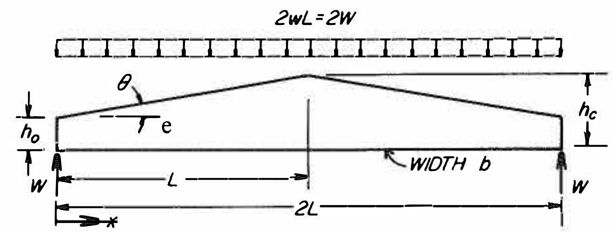

Fig. 7 Simply supported, double-tapered beam under uniformly distributed load

Using equation (11) the load P can be determined, after some algebraic manipulations, for any failure probability, F.

For the case of several concentrated loads acting on a tapered beam, the procedures of derivation remain the same, but the resulting equations will be much more complicated.

Tapered Beam Under Uniform Load

For a single-tapered beam or a double-tapered beam under uniform load, as described in Figs. 6 and 7, the two reaction forces are equal. For the latter case if the two portions on either side of the apex are not of the same length, the two reaction forces can not be equal to the loads acting on the adjacent portions of the beam. Let the left reaction force be αwL, where α is a constant. For the single-tapered beam in Fig. 6, $\alpha = 1/2$; for the double-tapered beam in Fig. 7, $\alpha = 1$. The bending moment at any section x is then

$$M = \alpha wLx - \frac{wx^2}{2} \tag{33}$$

Following [7], it can be shown that the maximum bending stress is

$$\sigma_0 = \frac{3w\alpha^2(h_c - h_0)^2}{bh_0[h_0 + 2\alpha(h_c - h_0)]\tan^2\theta}, \quad \text{at } x = \frac{\alpha L h_0}{h_0 + \alpha L\tan\theta} \tag{34}$$

For $\theta = 0$, equation (34) can be reduced to equation (17).

The expression for B can also be put in following form:

$$B = \zeta A \left(\frac{\sigma_0}{w_0}\right)^m \tag{18}$$

where A is the trapezoidal area between $x = 0$ and L, and

$$\zeta = \frac{m+1}{\bar{L}\left(1 + \frac{\bar{L}}{2}\right)} \left[\frac{1 + 2\alpha\bar{L}}{(\alpha\bar{L})^2}\right]^m \int_1^{1+\bar{L}} \left[\frac{2\alpha\bar{L}(x-1) - (x-1)^2}{x^2}\right]^m x\,dx \tag{35}$$

in which $\bar{L} = L\tan\theta/h_0$.

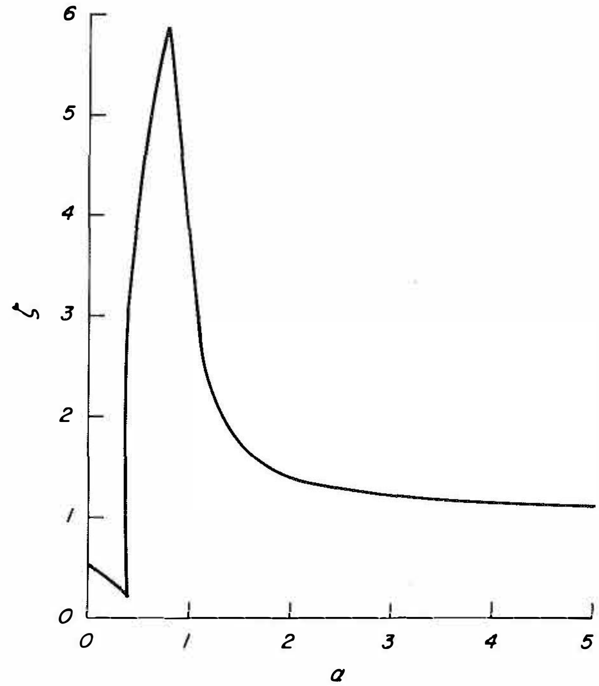

Fig. 8 ζ versus α for rectangular beam under uniform load (equations (20) and (21) with m = 18)

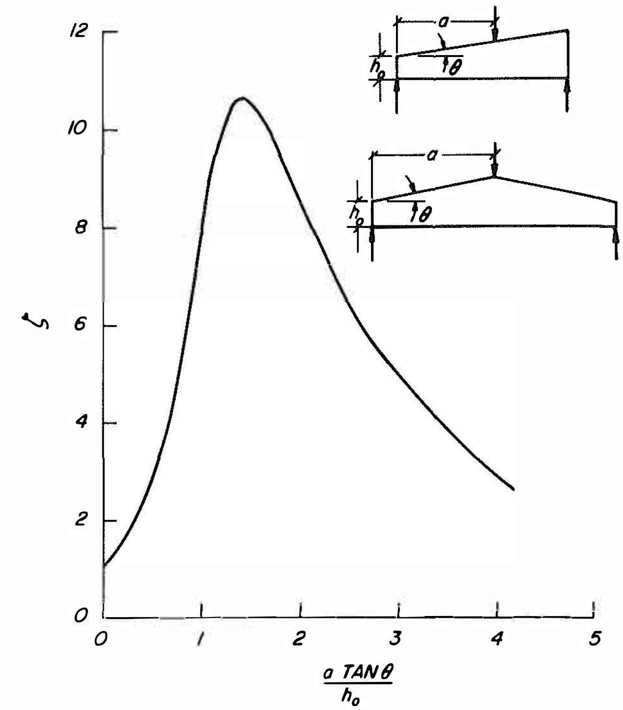

Fig. 9 ζ versus atanθ/h_0 for tapered beam between small end and a concentrated load (equations (26) and (27) with m = 18)

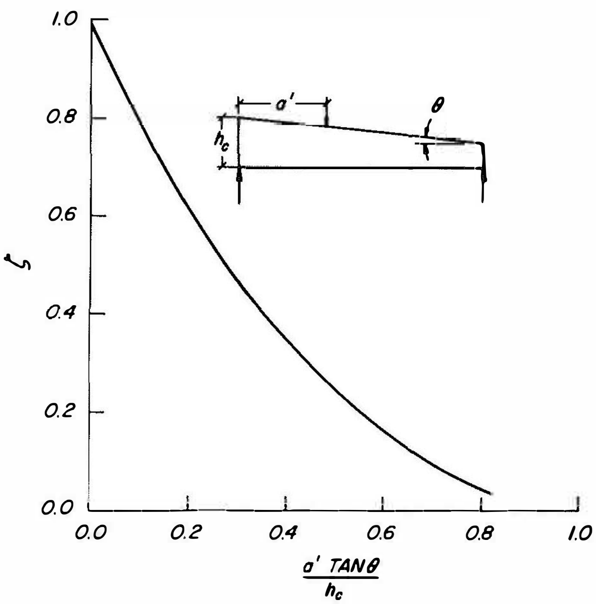

Fig. 10 ζ versus a′tanθ/h_0 for tapered beam between large end and a concentrated load (equation (31) with m = 18)

For the double-tapered beam in Fig. 7, the two portions on either side of the apex are the same. The area A in equation (18) must be doubled and α in equation (35) set equal to unity to obtain the expression for B. When the two portions are not the same or are under different uniform loads, then α will assume a different value in equations (33), (34), and (35). Equation (35) is still valid for the calculation of ζ.

Numerical Results and Design Examples

The equations derived above require the values of m and w_0 determined experimentally for their applications. For dry-use Douglas-fir, it was reported in [2] that $m = 18$ and $w_0 = 15{,}900$ (psi • in.^{3}/m). Therefore the ζ values have been calculated for $m = 18$ for different loading and geometrical conditions.

The ζ values for different α in equations (20) and (21) for a rectangular beam under uniform load as shown in Fig. 3A are plotted in Fig. 8. The plot is also useful when the beam is simply supported and is under a concentrated load in addition to the uniform load, or when the beam is continuous with two spans that are not of the same length or not under the same uniform load. In each of these cases α can actually take any positive value at each of the supports. The corresponding ζ value for the related beam section can simply be read off from the plot.

Figure 9 presents the values of ζ versus ($a \tan \theta / h_0$) for a tapered beam between the small end and a concentrated load at distance, a, from the small end based on equations (26) and (27). As the taper slope, θ, approaches zero, ζ approaches unity. This agrees with the result in equation (14) for rectangular beams. It is seen that the results can be directly applied to double-tapered beams loaded at the apex.

For the portion of the single-tapered beam between the large end and a concentrated load at distance, a', from the large end, the ζ values calculated from equation (31) are presented in Fig. 10 for several values of ($a' \tan \theta / h_c$). Again, as the taper slope, θ, approaches zero, ζ approaches unity, a finding that agrees with the result in equation (14).

The ζ values for the single- and double-tapered beams under uniform load as shown in Figs. 6 and 7, respectively, are calculated from equation (35) and shown in Fig. 11 for several values of ($L \tan \theta / h_0$). As the taper slope, θ, approaches zero, ζ approaches 3.888, which agrees with the results displayed in Fig. 8 for $\alpha = 0.5$ or 1.

To demonstrate the application of the present analysis, it is convenient to include some numerical examples. It is to be noted that the present study does not deal with the various stress reduction factors due to safety requirements, moisture content, load duration, etc. It only discusses the effects of size and loading condition on the bending stress, which must be modified by the stress reduction factors to obtain the allowable stress for design considerations. These effects may or may not be coupled depending on the geometry of the wood member considered as can be seen in the derived equations.

(1) Rectangular Beams. In Table 1 of [2], a group of 210 rectangular beams of the following dimensions were reported: $L = 18$ in. (0.4572 m); $b = h = 1$ in. (2.54×10^{-2} m). The beams were simply supported and under a center point load. The average modulus of rupture was found to be 13,350 psi (92 MPa), which corresponds to a load $P = 494$ lb (2.2 kN).

From equations (11) and (14), one obtains

$$A\left(\frac{\sigma_0}{w_0}\right)^m = -\ln(1-F) \qquad (36)$$

For $A = 1 \times 18 = 18$ in.^{2}, $m = 18$, $w_0 = 15{,}900$ (psi • in.$^{3/m}$), and a failure probability of $F = 0.5$, the stress calculated from equation (36) is $\sigma_0 = 13{,}268$ psi (91.5 MPa), which corresponds to a load $P = 491$ lb (2.18 kN). These are close to the results in [2].

If the failure probability is set at $F = 0.05$ under otherwise identical conditions, the stress calculated will become $\sigma_0 = 11{,}481$ psi (79 MPa) and the corresponding load is $P = 425$ lb (1.89 kN).

Now, if the same beam is under uniform load, w, one can obtain from Fig. 8 for $\alpha = 0.5$, $\zeta = 3.888$. The bending stress calculated from equations (11) and (19) for $F = 0.05$ is $\sigma_0 = 10{,}647$ psi (73.4 MPa) and $w = 43.8$ lb/in. (7.67 kN/m).

In addition, if the above uniformly loaded beam is fixed at the right end, then Fig. 8 will give $\zeta = 0.23$ for $\alpha = 3/8$, and the calculated stress and load will become $\sigma_0 = 12{,}458$ psi (85.89 MPa) and $w = 51.3$ lb/in. (8.98 kN/m), respectively.

(2) Tapered Beams. Consider a tapered beam of the following dimensions: $L = 18$ in. (0.4572 m); $b = h_0 = 1$ in. (2.54×10^{-2} m); $h_c = 3$ in. (7.62×10^{-2} m), which give $\tan\theta = 0.111$.

When a concentrated load is placed at the center, then $a = 9$ in. (0.2286 m) and $a \tan\theta/h_0 = 1$. From Fig. 9 it can be seen that $\zeta = 7.5$. The trapezoidal area of the beam between the small end and the load is $A_1 = 13.5$ in.^{2} (8.71×10^{-3}m^2) and equation (25) gives

$$B_1 = 101.25\left(\frac{\sigma_0}{w_0}\right)^m \qquad (37)$$

From Fig. 10, for $a' = 9$ in. (0.2286 m), $a' \tan\theta/h_c = 0.333$, and $\zeta = 0.428$. The trapezoidal area between the large end and the load is $A_2 = 22.5$ in.^{2} (1.45×10^{-2}m^2), and equation (30) yields

$$B_2 = 9.63\left(\frac{\sigma_0}{w_0}\right)^m \qquad (38)$$

In this particular case, σ_0 in equation (24) equals σ_0 in equation (29), since the two reactions R_1 and R_2 are the same. For the whole beam it follows:

$$(101.25 + 9.63)\left(\frac{\sigma_0}{w_0}\right)^m = -\ln(1-F)$$

and, for the same values of m and w_0 and $F = 0.05$, one obtains $\sigma_0 = 10{,}378$ psi (71.5 MPa) and the applied load $P = 1{,}537$ lb (6.84 kN).

If the same beam is under uniform load then $L \tan\theta/h_0 = 2$ and Fig. 11 gives $\zeta = 2.28$ for $\alpha = 0.5$. The trapezoidal area of the beam is $A = 36$ in.^{2} (2.32×10^{-2}m^2), and equations (11) and (18) give

$$82.08\left(\frac{\sigma_0}{w_0}\right)^m = -\ln(1-F)$$

For the same values of m, w_0, and $F = 0.05$, one finds $\sigma_0 = 10{,}553$ psi (72.76 MPa) and the applied uniform load $w = 130$ lb/in. (22.8 kN/m).

Discussions and Conclusions

Weibull's statistical theory of brittle failure has been used to predict the bending strength of structural wood members considering both the size and loading effects. The mathematical formulations are expressed in the statistical model in terms of the two parameters, m and w_0, which must be determined experimentally for each wood species. The values of these parameters for Douglas-fir have been reported in the literature [2]. These values, therefore, have been used to generate design curves and numerical examples showing a step-by-step procedure to estimate the bending strength of wood beams.

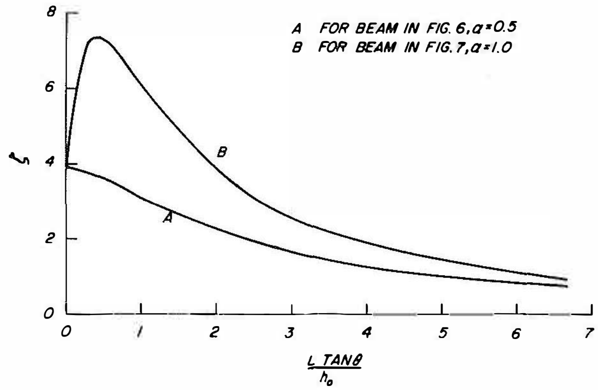

Fig. 11 ζ versus $L\tan\theta/h_0$ for simply supported tapered beam under uniform load (equation (35) with $m = 18$)

The present study presents a general, two-dimensional approach for calculating the bending strength of wood members of any species and under any loading or geometrical conditions. However, only some general loading conditions are considered in generating, where necessary, the design curves for rectangular and tapered beams of Douglas-fir.

In the design of structural wood members, the factors of safety, moisture content, load duration, etc., must all be taken into account in arriving at the allowable bending strength. These factors, however, fall beyond the scope of the present work, which seeks only to provide a rational interpretation of size and loading effects in bending.

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