

Biometrics

The Wrapper Model for Multiobjective Forest Monitoring Systems

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Abstract

Large multiobjective forest monitoring efforts such as the USDA's Forest Inventory and Analysis Program and other National Forest Inventory (NFI) systems are usually described to the public in terms of the relationship of their sample designs to the land base of interest. Sometimes the third dimension of time is included in the description of the sample design. Additionally, there is a tendency to favor descriptions that arguably support design-based views of the used estimation systems. The accompanying justification usually relies on an assumption that design-based estimators are "objective," whereas model-based estimators are "subjective." This article posits that this argument misses the mark because design-based estimation begins with the assumption that a probability sample exists and that the sample observations have been obtained without error. In most large sampling efforts, it is known that violations of the sample design not only might, but will, occur. Additionally, some objectives in a multiobjective inventory require estimation of attributes of tangentially related populations. Here we unify estimation methods into an overall theoretical framework, or "wrapper" model, that allows for the recognition, acknowledgement, and accounting for nonsampling errors and imperfect sample frames into estimation systems capable of combining all observations available to analysts of these large inventory systems.

Study Implications: This study describes a theoretical framework, named a "wrapper" model, that gives developers of national forest inventory and monitoring systems a more complete way to describe the assumptions and information sources contributing to the outputs from those systems. Rather than the usual description of the outputs in terms of the sample design and corresponding estimation system, these elements are recognized simply as (albeit major) components within the wrapper model.

Keywords: forestry, inventory, monitoring, estimation, modeling

Usually, forest inventory and monitoring efforts are described in their documentation and presentation to the public in terms of the sample design and the estimation system being used. Although this approach is certainly reasonable for inventory efforts covering up to a moderately sized land area (say 10^7 hectares) over a short time span (say less than one year), more extensive inventory and monitoring efforts require a more complete description for a full understanding. This is true because the more extensive that an inventory effort is, the higher the probability that any particular observation will have been obtained "off-design" or will have been drawn from a greater error distribution than what can be attributed to pure sampling error. Although all NFI programs make extreme efforts to provide a full description of all the information underlying the development of the resulting estimates, this full description is usually provided "as needed" in field guides, database documentation and publications, estimation documentation, and so forth. The larger and more complex that a monitoring effort becomes, the more likely it is that this full understanding will only be achieved by users who have persevered through thousands of pages of disparately sourced descriptions.

Below, we provide a unifying theory to consolidate ideas that appear piecemeal in these descriptions, and in the litera-

ture into an overall framework, which we will call a "wrapper" model. We view the wrapper model as a general idea or term, rather a specific mathematical model or family of models. Any model, defined for the estimation system of a specific monitoring effort, that is intended to allow practitioners to recognize, acknowledge and account for nonsampling errors and imperfect sample frames while combining all the available observations constitutes a wrapper model.

The wrapper model approach is intended to provide a better start for the consumer of NFI products than the usual description of national forest inventory systems in survey sampling terms that rely on standard survey sampling and design-based estimation theory. The justification for the design-based argument relies on the premise that design-based estimators are "objective," whereas model-based estimators are "subjective." The failing of design-based estimation lies in the fact that it begins with the assumption that a sample design exists and that the sample observations have been obtained without error and with respect to that design. We note that assumptions themselves are subjective, and when invoked, most are known to be not entirely true; but it is hoped that they are "true enough." In extensive multiobjective inventories, nonsampling errors are often treated as anomalies which are either ignored or miti-

gated through the application of ad-hoc or, even worse, black-box solutions. Coupling this with the fact that it is known beforehand that violations of the sample design not only might, but will, occur, suggests that the most basic understanding of standard survey sampling theory, in which one defines a population, samples it and then forms estimators based solely on the sample outcome, is inadequate as an overall framework for the description of extensive forest monitoring programs. This has long been recognized by sampling theorists as an issue that should always be acknowledged and addressed (Raj 1968, Cassel et al. 1977, Särndal et al. 1992). Minimally, an overall framework must be model-based, as it must allow us to acknowledge and account for nonsampling errors and imperfect sample frames. Below, we first present a simple model for a complete sampling strategy, then show how all the observations are related to it. Next, we discuss a simple “wrapper” model for the combination of within-design and off-design observations. That is, we build the wrapper model from the inside out, starting with the sample design at its core and then attach information from all other sources as it does (or does not) relate to the core. The ideas that we are exploiting here are not new in statistics, as many model-based, and model dependent estimation systems have been described in the past, a thorough examination of which can be found in Särndal et al. (1992), however they are not yet commonly acknowledged as a requirement for a minimally sufficient description of the products of an NFI.

The Sampling Strategy

A complete sampling strategy consists of the pair (d, e) where d is the sampling design for a probability sample on the population (U), and e is the estimation system. In its most basic form, this is a simple concept. How do the facts that during the execution of most multiobjective forest sampling efforts, the underlying population U will change and exceptions to the design will occur fit into this strategy? The exceptions to which we refer minimally include nonsampling errors (e.g., when a sample is not observed because of physical barriers or human error) and imperfect sample frames for some populations of interest (e.g., in multiobjective inventories different estimands will be associated with different underlying populations, which have simultaneously been observed with a single sample design). It is only through models that we can begin to provide an explanation of these otherwise implicit linkages. In the “Missing Observations” and “Imperfect Sample Frames” sections below, we provide examples of how models are used to address exceptions to design-based descriptions of large area forest inventories. The extension to the model-based viewpoint to modern-day extensions of NFIs as continuous monitoring systems is further reinforced in “The Three-Dimensional Population” section, along with a description of how Theil’s mixed estimator (Theil 1963) could serve as an encompassing or overall “wrapper” model. The ideas expressed in the introduction and “The Three-Dimensional Population” section are then illustrated in a simple heuristic example in the section “A Heuristic Example.”

Missing Observations

Much work has been done to accommodate missing observations. In cases such as these, the sampled population is completely covered and the sample frame is intact, but a value is not obtained for certain elements of the frame. Common causes

of missing observations in national forest monitoring programs are a landowner’s denial of access to sample locations, hazardous conditions on the location, and data transmission errors.

When design-based estimators are used in the presence of exceptions to the design, the original design, d , is unfulfilled and can itself be modeled as a partition of fulfilled and unfulfilled parts: $d = \{d_f, d_u\}$, and the estimation system (e) acts on $d' = \{d_f, \hat{d}_u\} = \{d_f, m_f(\hat{o}_u)\}$, rather than on the original design, d , where $m_f(\hat{o}_u)$ is the model or set of models intended to fulfill d_u , potentially from observation set $\hat{o}_u$.

The various methods that have been used to fulfill d_u can be classified by whether d_u can be assumed to have arisen at random. If d_u can be assumed to have arisen at random, then one might proceed by either assuming that d' is exchangeable with d , or $d' = d$, or alternatively, that $m_f(\hat{o}_u) = d_u$. In practice, these assumptions are often made liberally, and without adequate consideration to the effects of the substitution. For instance, the latter equality is implicitly being assumed when protocol allows reattempts at any missing observation. Each of these assumptions becomes more problematic and less tenable with the passage of time.

If d_u cannot be assumed to have arisen at random, then one should proceed by assuming d_f and d_u were drawn from different subpopulations of $U = \{U_f, U_u\}$. In this case, given that U_u has not yet been sampled, of the many approaches that have been suggested for estimation of variables in U_u , only a few might be considered to be design based. Those would involve the clear identification of both U_f and U_u , an acknowledgement that it was U_f that was actually sampled originally, rather than U , and a subsequent targeted attempt to sample U_u (Roesch et al. 2012). In practice, methods are often employed, which involve the use of an assumed, but sometimes unstated and unsupported, model. These models often relate the observed sample outcomes of U_f to the unobserved attributes of the subpopulation U_u . For instance, attempting to fulfill d_u through subsamples of d_f requires a model, and therefore is model based. Ratio or regression estimation (Pathak 1964, Rao 1966), simple or multiple imputation approaches (Rubin 1987), the Pólya posterior approach (Magnussen and Köhl 2002, Magnussen et al. 2004, 2010, Meeden and Lazar 2006, Roesch et al. 2012), and the use of external information in general, are all examples of model-based approaches. Discussions specific to the National Forest Inventory system in the United States can be found in Patterson et al. (2012), Patterson and Goeking (2012), and Roesch et al. (2012).

Imperfect Sample Frames

Less often discussed is the problem of the imperfect sample frame; that is the frame that does not really divide the population entirely and uniquely into the expected, knowable units, or the frame that indistinguishably includes units from other populations. An imperfect frame can result when the population is defined in general for many variables of interest but not defined specifically for each of them, as in large forest inventory and monitoring efforts such as the USDA’s Forest Inventory and Analysis Program (FIA) and other National Forest Inventory (NFI) systems. For example, the frame may be defined in the two dimensions of land area while observations are made on variables that change measurably during a single observation period. In these cases, the limited definition of the frame can hinder our ability to provide estimates of growth or change in particular forest attributes, and also our

ability to provide estimates of instantaneous values of forest attributes. A further complication can arise when estimates of growth in a forest attribute are made from successive observations that cover varying lengths of time. To address these issues, we discuss an example of the sample design as a model for analyses of forest monitoring data.

An example arises from an area-controlled sample design started in 1998 by the USDA Forest Service's FIA Program in the United States. A simple representation of the design is given in Figure 1. Briefly, the entire land area was divided into a grid of hexagonal cells of approximately 6,000 acres each. A systematic pattern was applied to the grid to divide it into p panels. (In the eastern United States, $p = 5$.) In turn, one panel per year was intended to be measured. After all panels have been measured in p years, the sequence of panel observations reinstitutes. All estimation procedures are developed as dependent on panel membership. Because a panel is intended to be measured each year, ideally the design would result in predictable, approximately equal length, overlapping, measurement intervals, as seen in the relationship between Cycle 1 and Cycle 2 in Figure 1.

Often, the collection of ground observations is delayed for a myriad of reasons, a few of which include lapses in funding, unexpected human resource shortages, and significant weather events. This results in a phenomenon known as panel creep, in which interval lengths might slowly increase, and is illustrated by the series of x 's in Figure 1. In other cases, a wealth of resources might exist, and ground observations might be made more rapidly than originally intended, resulting in shortened interval lengths as depicted by the series of squares in Figure 1. This is usually done intentionally, with the intent of improving instantaneous estimates while down weighting the importance of change and growth estimates.

The Three-Dimensional Population

A partial solution to problems arising from the continuous observation of a dynamic population is given in Roesch (2008) in the form of an alternative viewpoint, that of the three-dimensional population and sampling frame, consisting of the two dimensions of land area and the third dimension of time. In a perfect world, using that viewpoint as a basis for analysis appeared, theoretically, to solve the problem. One interference by the real world on the three-dimensional view lies in the fact that, during actual data collection, the random-

ization mechanism will often apply only partially to the temporal dimension of the population. There really is not a good solution for this in design-based estimation, but a model-based solution is available, as shown here.

Initially, suppose we are interested in a particular measure of annual growth for all live trees in a particular area, such as cubic meters of wood growth per hectare, per year. Figure 2 might be used as a depiction of the population of such a particular value of interest. The face of each three-dimensional column in the figure represents a hexagonal segment of land area while the length of the columns represents a time segment of interest. Each column, therefore, represents a spatial-temporal population unit.

Next, we consider how the panelized sample design described in the introduction acts on the population represented in Figure 2. A "perfect" panel sample of growth, at a coarse scale, is one in which, for all observations, the year in which the observation was to be made according to the sample design (Inventory Year) is equal to the year that it was actually measured (Measurement Year), and there is only one value obtainable for a particular unit in any particular year. The resulting sample of growth intervals is depicted in Figure 3. Note that even in a perfect sample our scale of interest is different from our scale of observation, so the frame does not uniquely and completely subdivide the population into sample units and therefore annual growth estimates will always be model dependent because the individual annual values are comingled within each sample unit. At a finer scale, the actual time of observation within a year can vary, as can the observed value, and the length of the observed growth intervals will therefore have some variance, a model or assumption is required to combine observations of different interval lengths and temporal position. Another model is required to associate the observed set of growth observations to a particular year or set of years. These models often go unacknowledged and unrecognized because they have arisen and been developed as standard practices.

An imperfect panel sample of growth, at a coarse scale, is characterized by having some (and occasionally many) observations for which the inventory year is not equal to the actual measurement year, and interval lengths have greater variation. This requires us to rethink what has actually been observed with respect to annual growth. If we annually partition a set of "growth" observations, we see that multiple intervals have "observed" growth in a particular year (see Figure 3), while any particular observation interval usually spans or "observes" an aggregate of full-year and partial-year growth intervals (Figure 4).

A simple solution would be to devise a scale-appropriate model to partition the growth or change observed by each interval and allocate those partitions to their appropriate annual or subannual growth periods. One would define a scale-appropriate intra-annual growth model, such as the example in Figure 5, for beginning-of-year growth and end-of-year growth to partition the interval's observed growth, preferably limiting a partial year's growth to the growing season. We represent the estimated partition of growth interval j for year t as $\hat{g}_{tj}$.

As noted, in the first and last year of an interval observation, only a partial year's growth may be estimated by $\hat{g}_{tj}$. We will point out that the rate of observation may also vary through the year, such as when seasonal field crews are hired in the summer to supplement full-time field crews who make observations year-round. When this is the case, it should also be considered

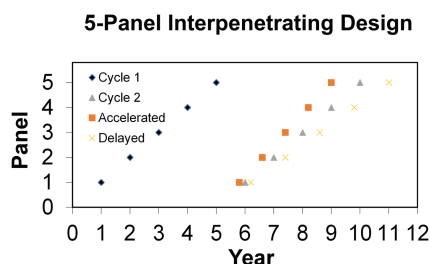


Figure 1. A temporal depiction of two cycles of the interpenetrating panel design with five panels. The expected mean time of panel observations by design are given for cycle 1 (diamond) and cycle 2 (triangle). The actual mean time of delayed panel observation for cycle 2 is given in the presence of panel "creep" (x), and in the presence of panel "acceleration" (square).

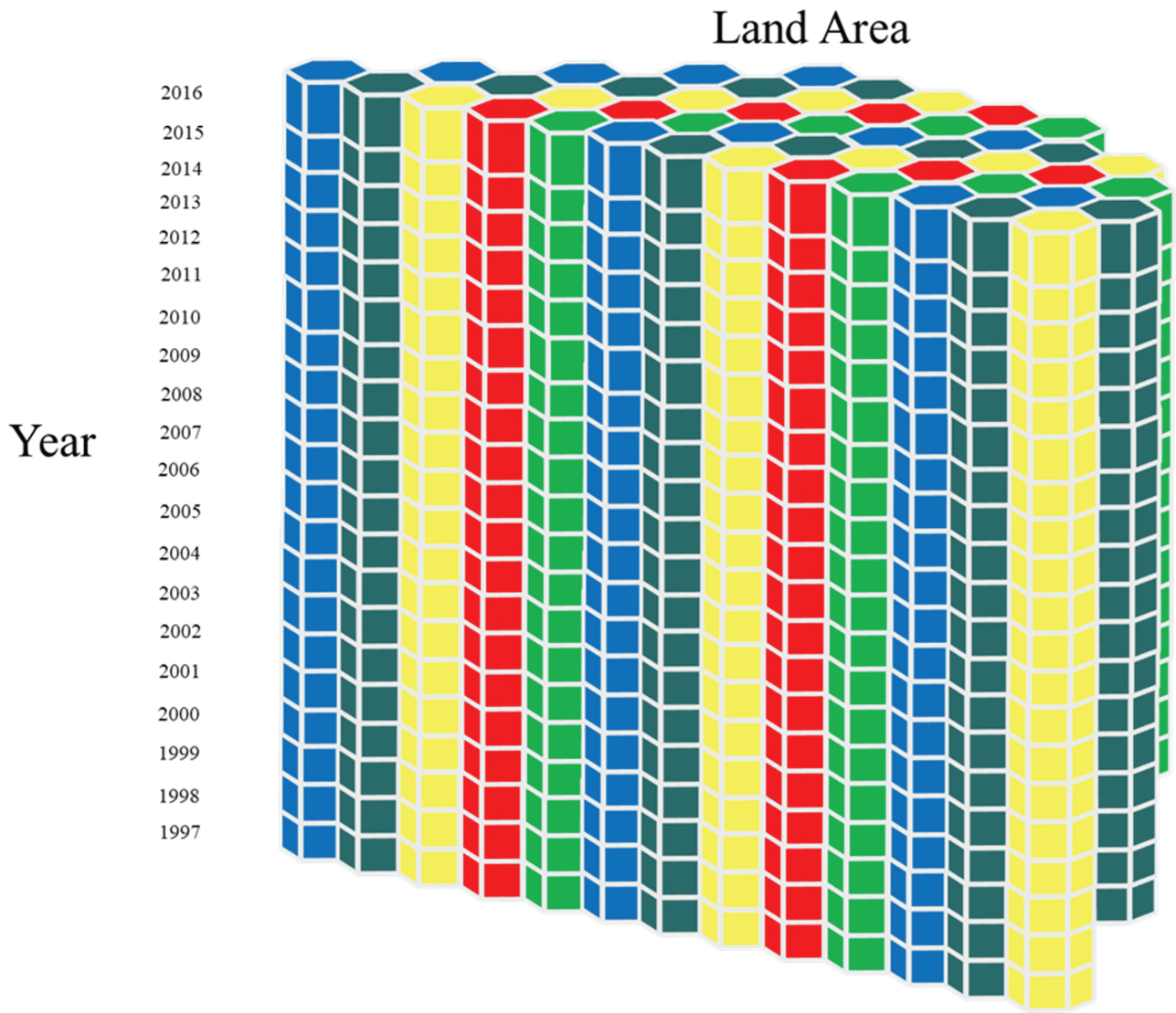


Figure 2. A population of annual values divided into five panels. Each panel is represented by a unique color.

when evaluating a partial year's growth. It would be convenient to assume that the rate of observation remains constant, in which case the known inclusion probability of each interval observation is uniformly distributed through all the growing seasons and partial growing seasons spanned by the interval (Roesch 2018, 2019). That is, one would model partitions of the observations with the proportion of each year's growing season that they intersect. This leaves the question of how these annual partitions should be used once they have been obtained. For instance, we could simply sum the n_t weighted partition estimates over the entire area for a particular year:

$$\hat{G}_t^P = \sum_{j=1}^{n_t} \hat{p}_{tj} \quad (1)$$

where $\hat{p}_{tj} = \frac{\hat{g}_{tj}}{\alpha_{tj}}$ is the estimate of growth for partition j at time t divided by the proportion of growth interval spanned by partition j for year t .

Noting that $\hat{G}_t^P$ is actually a composite of design components and off-design components, one might prefer to state

explicitly a model relating the off-design components to the design components. One useful model forms the basis for the well-known composite estimator, such as one in which the modeled design components, $d' = \{d_f, \hat{d}_u\}$, contribute to estimator 1 (ξ_1), and the off-design components (d_o) inform a second estimator (ξ_2). Here, one (or both) of the component estimators may be biased so we use the inverse of an estimate of each mean-squared error (mse), rather than each variance, to weight the components. The inverse mean squared error composite estimator is then formed:

$$\hat{G}_t^C = w(\xi_1) + (1 - w)\xi_2 \quad (2)$$

where

$$w = \frac{1/mse(\xi_1)}{\sum_{r=1}^2 1/mse(\xi_r)} \quad (3)$$

Either $\hat{G}_t^P$ or $\hat{G}_t^C$ could be taken at face value or their progression through time could be constrained using a

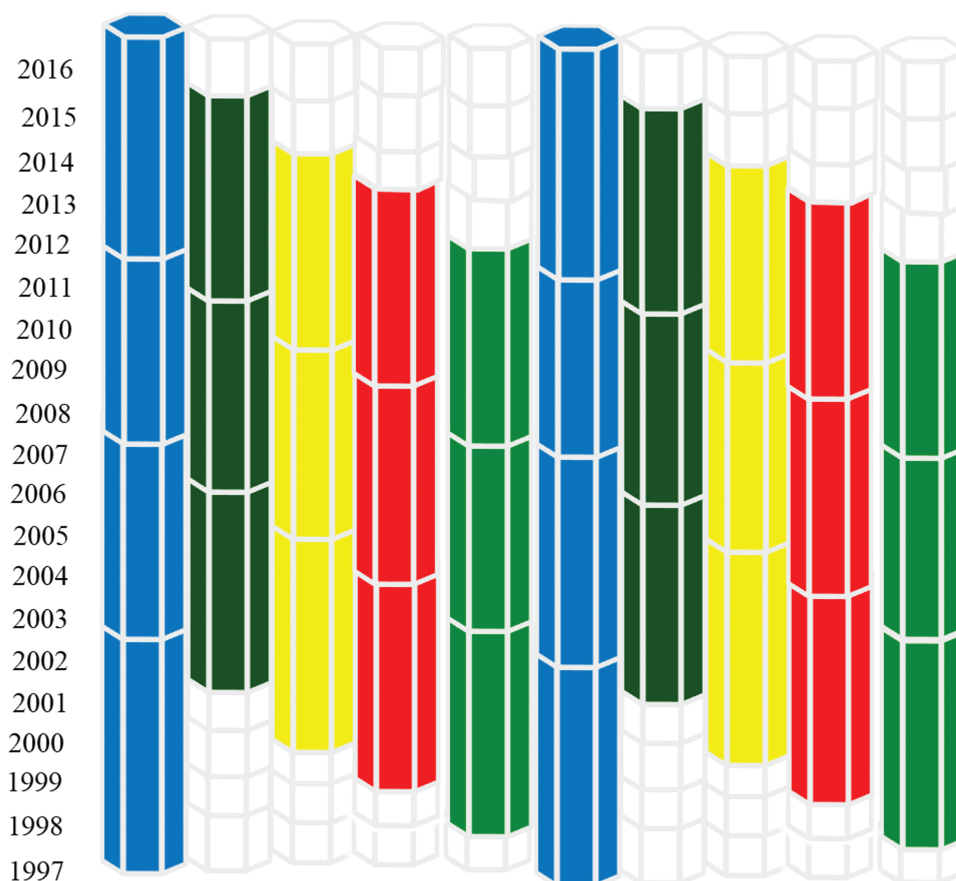


Figure 3. A “Perfect” Panel Sample of growth for the population in Figure 2. For all observations: Inventory Year = Measurement Year, and all interval lengths are the same. Each segmented solid represents the observation of growth between instantaneous observations (represented by the white lines.)

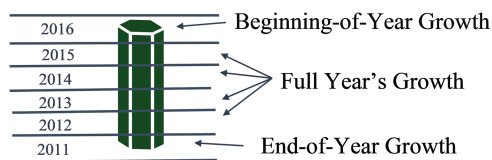


Figure 4. The partitioning of an area-based growth interval observation made over years 2011 to 2016, into annual growth or partial annual growth by year.

wrapper model, such as Theil’s Mixed Estimator (Theil and Goldberger 1961, Theil 1963, Van Deusen 1996, 1999). Although the focus was different, the mixed estimator was used similarly as a second filter following an initial compilation of moving average estimators in a system that was called dual-filter estimation in Roesch (2017). The mixed estimator is a generalized least squares procedure that allows us to consider any kind of extraneous information, including models arising from introspection. The interested reader is referred to Theil (1971, p. 347–352). One way to use the mixed estimator as a wrapper model is to start with an appropriately weighted composite estimator such as $\hat{G}_t^C$ and assume that all information relating the design components to the off-design components is captured in $\hat{G}_t^C$. Here we use the more direct approach and enter the $\hat{G}_t^P$ estimates directly into the mixed estimator. To simplify the no-

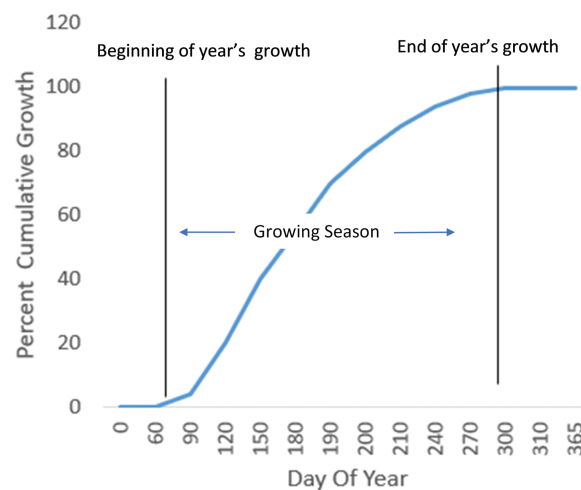


Figure 5. An example intra-annual cumulative growth plot.

menclature, we let $\hat{G}_t = \hat{G}_t^P$ and combine an observation model: $\hat{G}_t = \beta_t + e_t$, where β_t is an unknown coefficient and e_t is iid $(0, \sigma_t^2/n_t)$, with a transition model, r , such as a third-derivative constraint model:

$$\beta_t - 3\beta_{t-1} + 3\beta_{t-2} - \beta_{t-3} = v_t; \quad v_t \text{ iid } (0, p\sigma_t^2/m_t) \quad (4)$$

where p is a parameter controlling the strength of the constraints, an optimal value of which can be estimated

by various methods, such as maximum likelihood (as shown in Van Deusen [1999]), and m_t is the number of estimated annual partitions for year t . Then let, $k = 3$, $\mathbf{R}$ be the $(T-k) \times T$ constraint matrix, $\mathbf{G} = [\hat{G}_1, \dots, \hat{G}_T]'$, $\mathbf{e} = [e_1, \dots, e_T]'$ and $\mathbf{v} = [v_{k+1}, \dots, v_T]'$. Σ is the covariance matrix of $\mathbf{G}$, with the vector $\mathbf{e}$ on the diagonal. Ω is assumed to be a scaled sub-matrix of Σ .

Combine everything into the mixed model

$$\begin{bmatrix} \mathbf{G} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{R} \end{bmatrix} \beta + \begin{bmatrix} \mathbf{e} \\ \mathbf{v} \end{bmatrix} \quad (5)$$

A thorough discussion of the solution to this mixed estimation model is given in Van Deusen (1999). The reason that the mixed estimator can be used as a wrapper model is that different error models than $\mathbf{e}$ and $\mathbf{v}$ can be incorporated into Ω and Σ to account for nonsampling errors and imperfect sample frames.

A Heuristic Example

In this heuristic example we show several estimators that have appeared previously in the literature and might be used as a building block within a wrapper model (i.e., as ξ_1 , ξ_2 , or $\hat{G}_t^C$ in equation [2]), and show how each estimator would contribute differentially to the information used by the mixed estimator, especially when only a short series of data is available. We note that each estimator requires different models and levels of models to complete a wrapper model.

We use a sampling simulation to provide a heuristic example of how the oftentimes implicit assumptions used in wide-area inventories affect a number of presumably well-known estimation approaches. That is, we do not consider the following to be a thorough test of the estimation systems but rather fodder for a discussion of the effects of some of the practical decisions that are sometimes unspecified in the design and therefore left up to the discretion of managers or data gatherers.

Sampling Scenarios

We describe and examine five scenarios below, namely the “Perfect,” “Fixed Order,” “Annual Random,” “Accelerated,” and “Decelerated” scenarios.

Perfect

Often when estimators are compared in forest inventory sampling simulations, a “perfect” sample outcome is implicitly assumed, although this outcome is never achieved in practice. The “perfect” panel sample assumes that we can somehow observe the sum of each entire year’s growth for five specific years. That is, for each panel j , an observation is made of five complete years of growth, say years i through $i+4$. After the fifth panel’s five-year growth observation, the panel sequence reinitiates and a new observation is available every year. Observations are often grouped as if this scenario had been effected, while observations are usually made somewhere in the continuum between two extremes, on the one end is a fixed-order scenario in which plot observations are always made in the same order throughout the year, and on the other end is a completely random order throughout the year.

Fixed Order

In practice, the order of plot observation is usually left up to a field crew or a supervisor’s discretion, after logistical

considerations. We obtain one scenario by assuming that all plots are observed in the same “fixed order” throughout the year, each year that the panel is observed. We will call this the fixed order scenario. We simplify the simulation by first ordering all rows in the population panel and then assume each selected row is measured during the year at a time when the ratio of the population order number divided by the panel population size times the total annual growth would be observed.

Under a fixed order assumption, the observed growth observation in each row would usually constitute the sum of two partial and four full years’ growth: the end of year growth during the first year and the beginning of year growth during the sixth year and full year’s growth for each of the intervening years.

Random Order

Additionally, we investigate a scenario that assumes plot observations are random within inventory year. In this scenario, similar to the fixed scenario, we calculate the proportion of each year’s growth in each row (or each plot) that has occurred prior to the observation, although in this case the proportion is randomly selected using the R function *runif* on a [0,1] interval.

Logistical considerations will invariably result in a temporal plot-observation pattern falling somewhere between the two extremes of the fixed-order and random scenarios.

Accelerated and Decelerated

Finally, we examine accelerated and decelerated observation scenarios, also under the fixed-order assumption. Specifically, under the accelerated scenario, we observe the five panels in three years while keeping the order of sample observations between and within panels constant. Under the decelerated scenario (also known as panel creep when it is unplanned) we observe the five panels in six years while keeping the order of sample observations between and within panels constant. For convenience, we repanel the observations to six panels by year actually observed.

Sampling Simulation

For the simulation, we sampled one of the publicly available populations described in Roesch (2019), specifically population 13,001, derived from sample data for FIA survey unit 1 in the state of Georgia, USA. For the interested reader, a full description of the population construction is given in Roesch (2019). To construct the population, the data from each forest sample plot observation were used to create a row with a sequence of 21 successive, annual values, for each of five compatible variables of interest, commonly referred to as the components of growth, although only one of those variables, live growth of wood volume ($\text{m}^3\text{ha}^{-1}\text{year}^{-1}$), was used in this simulation. We used sampling simulations on this population that composed of 1,000 iterations of a random sample of size 250 for each of five panels.

In this example the partition estimator given above as $\hat{G}_t^P$ is used as input into the mixed estimator under a third-derivative constraint model (Van Deusen 1999) given in equation (4). This estimator assumes a growth function between sample observations and then combines the growth proportions assigned to each year. Here, for the growth function, we use a simple linear interpolation through the growing seasons spanned by each pair of sample observations in a population row. We will call this resulting estimator $\hat{G}_t^{PMix}$.

The Sampling Simulation

At each of the 1,000 iterations, we sample the population with a random sample of rows, of size 250 for each panel, using the R function *sample*. Then we assume nonsampling errors (measurement and estimation error) have a Normal distribution of mean 0 and standard deviation of $0.05 \times (\text{population mean})$, using the R function *rnorm*.

Over the 1,000 iterations of 250 samples for each panel, we calculate the bias, the variance and the mean squared error for each estimator and each year under the alternative scenarios. We compare the outcome from the perfect scenario for a system with five annual panels to the other, arguably more realistic, potential scenarios.

Figure 6 shows the simulation results for the mean annual growth under the five sampling scenarios described above: Perfect, Fixed Order, Random Order, Accelerated Fixed Order, and Decelerated Fixed Order. The annual mean volume growth ($\text{m}^3\text{h}^{-1}\text{year}^{-1}$) from the simulation is plotted against the population mean trend in the upper row of graphs. The bias under each scenario is given in the second row, while the corresponding variances are given in the third row and the mean squared errors are shown in the fourth row. The panel mean values on the left-hand side of Figure 6 constitute the initial observations on which all other growth estimators are built. Single panel means are not usually used as final estimates because they will often have too high of a variance, and therefore other estimators are introduced to combine observations from multiple panels and draw strength from the information in the other panels.

The left-hand side of Figure 6 could be viewed as showing the cost, and sometimes the benefit, of an imperfect sample frame. Observations are not made instantaneously each year, which is essentially what the Perfect sampling scenario is assuming, so the growth intervals for the other four scenarios will usually be partially “off-frame” for the population and target sample. At first blush, one might think that the Perfect scenario should always be superior to the other scenarios, at least with respect to having the lowest bias and MSE. We see that is usually, but not always, the case in these results. The reason that this is not always the case is simply because the desired estimation interval was one-fifth of the length of the observation interval. In the cases where another scenario shows a lower bias or MSE than the Perfect scenario, that scenario is the Accelerated scenario, which has a consistent set of shorter observation intervals, closer to the target estimation interval, than the other three imperfect scenarios.

The right-hand side of Figure 6 gives the simulation results for the $\hat{G}_t^{\text{PMix}}$ estimator of annual mean volume growth ($\text{m}^3\text{h}^{-1}\text{year}^{-1}$) under each of the five sampling scenarios after the 1,000 iterations of samples of size 250 for each panel. As in left-hand side of Figure 6, the annual estimates are plotted against the population annual mean trend in the upper row; the biases in those estimates are shown in the second row, the variances are in the third row and the mean squared errors are in the bottom row.

In comparing the two graphs in the top row of Figure 6, we note that the noticeable deviation of the panel means from the population mean trend in the final three years is reduced only slightly by the third-derivative constraint model in $\hat{G}_t^{\text{PMix}}$, an observation that is borne out by comparing the two bias graphs in the second row of the figure. Bias in the annual panel means arises from two predominant sources. The first is that because panels subdivide a population, as the number of panels increases, the more likely it is that one or more panels will have a no-

ticeably different mean value than the population mean value. The second source is the existence of an unknown underlying nonlinear trend in the population.

The third row of Figure 6 makes two important points. The first is that the accelerated scenario will invariably lead to lower variance annual estimates, as seen in the left-side graph. The second is that $\hat{G}_t^{\text{PMix}}$, like most sound methods of combining panels, can significantly reduce the variances of estimates. The bottom row of Figure 6 shows us that while the use of $\hat{G}_t^{\text{PMix}}$ did not lower the annual MSE's seen in panel for year in which they were already quite low, it did significantly lower the annual MSEs when they were high, as in the final years of the estimation period.

This example does not and is not intended to demonstrate the effectiveness or superiority of $\hat{G}_t^{\text{PMix}}$ as an estimator of annual growth under a panelized design, as only a much broader investigation of error structures and sample sizes can begin to establish any estimator's effectiveness.

In this heuristic example we have shown an estimation approach that has appeared previously in the literature and can be adapted to the requirements of a wrapper model. We note that any estimator requires different models and levels of models to complete a wrapper model. We also note that many initial estimators, in addition to $\hat{G}_t^p$, can be incorporated into the mixed estimator to effect its role as a wrapper model.

Discussion

To fully understand the products of an extensive forest inventory or monitoring system, one must understand how those products have arisen. Most often, the explanation of how estimates are achieved in these extensive NFI systems is incomplete. The description offered is usually in terms of the sample design's relationship to the land base, and sometimes to the temporal period, of interest. Estimation is usually described with respect to the sample design, using what are known as design-based estimators. As we have noted, design-based estimation begins with the assumptions that a probability sample has been drawn and that all the observations have been obtained with respect to that design and without error. Unavoidable and difficult to avoid violations of the sample design will occur, and a full description of how these violations will be handled should be included in a description of any inventory or monitoring effort. We have explored an overall framework, using the idea of a “wrapper” model, that is intended to allow us to recognize, acknowledge and account for nonsampling errors and imperfect sample frames when deriving estimates of forest growth. A fully developed wrapper model should be capable of combining all the observations available to an inventory's analysts.

Forest growth is an inherently nonlinear phenomenon, (see, for example, Avery and Burkhardt 1983, chapters 14 and 15), although many forest inventory algorithms in NFI systems rely implicitly on linearity assumptions. These linearity assumptions are often useful and pragmatic; however, their limitations often go untested. For instance, the class of sample designs being considered in this article (temporally overlapping panelized designs) does not have an inherent assumption of linearity, but both the implementation of the design and the choice of data aggregation methods almost always do have linearity assumptions. This work recognizes that the “realized” sample is often different from what was intended by

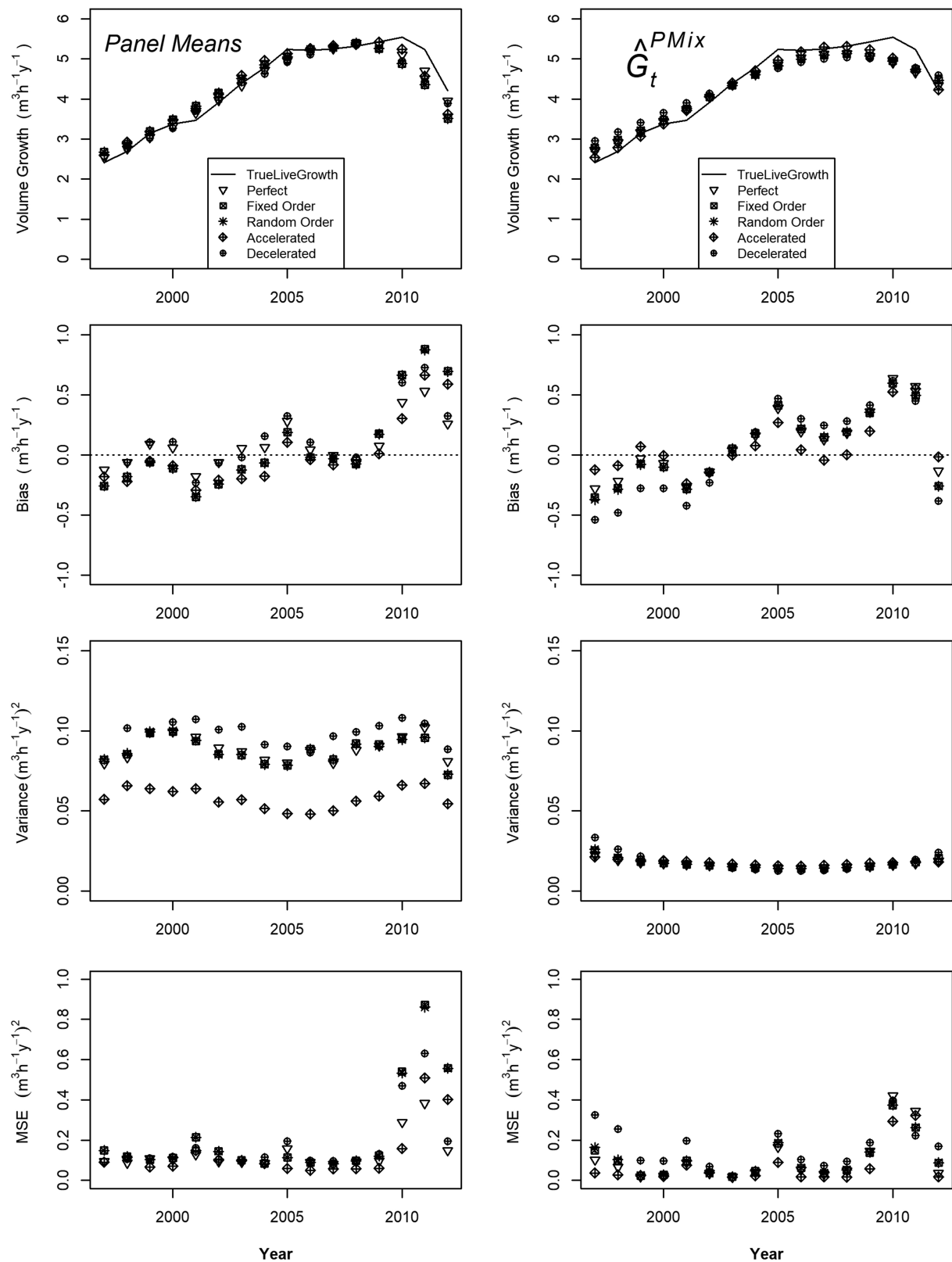


Figure 6. Simulation results for annual growth from the centralized panel means (left) and $\hat{G}_t^{PMix}$ (right) from 1,000 samples of size 250 from each panel under each of the five sampling scenarios (Perfect, Fixed Order, Random Order, Accelerated Fixed Order, and Decelerated Fixed Order). The estimated annual volume growth ($\text{m}^3\text{h}^{-1}\text{year}^{-1}$) for each year, from the simulation, is plotted against the population mean trend in the top row, the simulation bias for the annual panel means and $\hat{G}_t^{PMix}$ are in the second row, the variances are in the third row, and the mean squared errors (MSE) are in the bottom row.

the sample design, while the theory justifying or underlying most forest monitoring estimation systems relies solely on a description of the sample design rather than on the “actual” or “realized” sample. In the actual sample, some of the observations were intended by design while others were unintended. For instance, some observations are made later than intended while others are made earlier than intended. We would argue that, for estimation purposes, growth is too important to allow the intended time of observation to overrule the actual time of observation. This argument of temporal fidelity is problematic for sample design purists focusing solely on the spatial relationships of the panel design, but less problematic than aggregating temporally mismatched observations.

Conclusions

In many large inventory efforts, it is useful to consider the sample design, d , to be a model because d' is almost always substituted for d . Even when “design-based” estimators have been invoked, the sampling strategy is still appropriately viewed as model-based because models have been used to fulfill the design. Furthermore, the effects of the substitution of d' for d cannot be evaluated if the substitution is not recognized. Formally defining the sample design as a model and coupling that model inside of a wrapper model incorporating all observations can assist in incorporating and understanding off-frame information. Given that we know that violations of the sample design will occur and that some objectives in a multiobjective inventory will require estimation of attributes of tangentially related populations, a more inclusive framework is required. In this article, we have unified theories that have appeared piecemeal in the literature into an overall framework, or “wrapper” model, allowing us to account for nonsampling errors and imperfect sample frames in estimation systems that are capable of combining all observations available to the forest monitoring effort.

Acknowledgments

Three anonymous reviewers provided valuable comments that contributed to the improvement of this manuscript. This research was funded by the US Department of Agriculture, Forest Service, Southern Research Station, Forest Inventory and Analysis Program.

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