Evaluating and Optimizing the Use of Logistic Regression for Tree Mortality Models in the First Order Fire Effects Model (FOFEM)

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BACKGROUND

Wildland fires burn millions of forested hectares annually around the world, affecting biodiversity, carbon storage, hydrologic processes, and ecosystem services largely through fire-induced tree mortality (Bond-Lamberty et al. 2007; Dantas et al. 2016). In spite of this widespread importance, the underlying mechanisms of fire-caused tree mortality remain poorly understood, (Hood et al. 2018). Post-fire tree mortality has been traditionally modeled as an empirical function of tree defenses (bark thickness) and fire injury (crown scorch, stem char) (Ryan and Amman 1996; Woolley et al. 2012). Empirical models are commonly used in fire management to predict fire effects (Reinhardt et al. 1997), from the fine-scale software tools for fire management planning, to process-based succession models (Keane et al. 2011), and global models of the terrestrial carbon cycle (Hantson et al. 2016). Nevertheless, many fire-caused tree mortality models have undergone little evaluation.

We evaluated fire-caused tree mortality models used in the First Order Fire Effects Model (FOFEM) software package (Reinhardt et al. 1997). FOFEM predicts fire-induced tree mortality for 219 tree species in the United States. We evaluated 31 models for 17 tree species using a database we developed that combines fire-induced tree mortality observations from independently collected datasets across the United States. FOFEM uses logistic regression models, which predict the probability of mortality based on tree traits (e.g., bark thickness) and injury levels (e.g., percent of crown volume scorched). A second step, in which trees that have a probability of mortality above a user-defined threshold (e.g., “cutoff”, “decision criteria”) is used to code individual trees as live or dead and predict post-fire stand density and basal area. Land managers who use FOFEM may be interested in optimizing accuracy in predictions of either mortality or survival (Ganio and Progar 2017; Grayson et al. 2017). For example, for post-fire hazard tree removal around recreation sites, may require high accuracy in mortality predictions, so that trees likely to die and become hazards could be identified and removed. Conversely, managers planning a prescribed fire in a long-unburned, old growth stand, may want higher certainty that large legacy trees will survive fire. In a modeling context, we can help support these objectives by identifying the probability thresholds that provide high model accuracy for either sensitivity (correct prediction of dead trees) or specificity (correct prediction of live trees). Thus the objectives of this paper are two-fold. First we evaluate FOFEM performance for species with over 300 individual-

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tree observations with a portion of the dataset that we currently have cleaned and standardized. Second, we demonstrate how the choice of thresholds can be used to optimize the accuracy of predicting either tree survival or tree death.

METHODS

We created a tree mortality data set via published studies and outreach, with the goal of creating a database of fire-caused mortality observations at the tree-level to formally evaluate existing fire-caused tree mortality models. We posted inquiries on numerous mailing lists about unpublished monitoring data, and conducted a literature search of published data on fire injury and mortality and contacted all corresponding authors requesting collaboration and data sharing. This effort produced 40 contributed datasets from state, federal, and university researchers in the United States. Our database currently includes observations of fire injuries and survival or mortality of individual tree observations and site location data from 20 states collected in approximately 400 fires (prescribed and wild) from 1974 to 2016. We are currently standardizing data and documenting metadata for the creation of a publicly available, database of U.S. fire-induced tree mortality. The final database will include observations of approximately 150 tree species, and more than 170,000 trees.

We conducted initial model evaluation using a subset of the final data set that had been standardized, combined, and quality-checked (n = 125,980). Currently, there are 62 species with over 50 observations in our dataset (table 1). The most numerous species in our current database are: *Pinus ponderosa* (ponderosa pine; n = 40,851); *Pinus contorta* (lodgepole pine; n = 15,972); *Pseudotsuga menziesii* (Douglas-fir; n = 14,717); *Abies concolor* (white fir; n = 12,688); *Abies lasiocarpa* (subalpine fir; n = 4,399); and *Calocedrus decurrens* (California incense-cedar; n = 3,079). We limited our model evaluation to species with over 300 individual-tree observations that also included the measurements of fire injury—such as percent crown volume scorched, percent crown length scorched—used in FOFEM (table 2). We evaluated the basic FOFEM logistic regression model (hereafter the “fofem5” model) (Ryan and Amman 1994):

$$\mathrm{P_m} = 1\Big/\left(1 + e^{\left(-1.941+6.316\left(1-e^{-0.3937\mathrm{BT}}\right)-0.000535\left(\mathrm{CVS}^2\right)\right)}\right) \qquad (1)$$

where P_m is the probability of mortality, BT is bark thickness, and CVS is the percent of crown volume scorched. We evaluated the above model separately for each individual species.

For some conifer species, there are additional logistic regression equations implemented in FOFEM that have additional or different predictor variables because they were specifically developed for predicting mortality either prior to prescribed fire (“pre-fire”) or after wildfire for salvage logging decisions (“post-fire”) (Hood and Lutes 2017). We also evaluated these pre-fire and post-fire models; equations for each model are in table 2. The pre-fire models often use projected percent crown scorch (either percent of crown length or percent of crown volume), which cannot be observed prior to the fire. Instead, users either enter anticipated flame length or scorch height, as well as tree height and crown ratio; percent crown scorch is then calculated from these inputs using additional empirical equations. We did not address uncertainty in these other empirical equations, as we used post-fire measurement of percent crown length scorched or percent crown volume scorched in this evaluation.

For each species-model combination we created receiver operating characteristic (ROC) curves, which evaluate sensitivity (i.e., correctly classified positive outcomes/actual positive outcomes; positive outcomes are dead trees) and specificity (i.e., correctly classified negative outcomes/actual negative outcomes; negative outcomes are live trees). We calculated the area under the ROC curve (AUC) for each species-model combination. AUC values ≤0.5 suggest that the model does not perform better than random chance, values between 0.5-0.6 are poor, values between 0.7-8.0 are acceptable, 0.8-0.9 are excellent, and >0.9 are outstanding (Hosmer and Lemeshow 2000). The ROC curve was calculated over a range of probability thresholds at which a tree is classified as dead or alive. We identified the threshold at which the model performed best, based on combined specificity and sensitivity, using the package pROC (Robin et al. 2011) in R (R Development Core Team 2017). For each of these three thresholds we calculated model performance statistics, including the specificity,

Table 1—Species with at least 50 observations in the current dataset.

Species	n	Species	n
Pinus ponderosa	40,851	*Pinus edulis*	337
Pinus contorta	15,972	*Nyssa sylvatica*	315
Pseudotsuga menziesii	14,717	*Pinus elliottii*	313
Abies concolor	12,688	*Quercus gambelii*	305
Abies lasiocarpa	4,399	*Pinus flexilis*	278
Calocedrus decurrens	3,079	*Pinus attenuata*	268
Picea engelmannii	2,830	*Quercus nigra*	266
Pinus lambertiana	2,558	*Prunus serotina*	258
Pinus taeda	2,066	*Juniperus osteosperma*	249
Pinus palustris	1,215	*Pinus resinosa*	240
Pinus albicaulis	1,194	*Quercus stellata*	220
Abies grandis	1,159	*Pinus monticola*	205
Pinus jeffreyi	1,071	*Quercus velutina*	205
Larix occidentalis	1,057	*Acer saccharum*	185
Acer rubrum	1,030	*Pinus coulteri*	182
Quercus garryana	824	*Cornus florida*	158
Tsuga heterophylla	814	*Sequoiadendron giganteum*	156
Juniperus scopulorum	811	*Liriodendron tulipifera*	151
Populus tremuloides	799	*Picea pungens*	142
Quercus prinus	731	*Cornus florida*	140
Quercus laurifolia	638	*Sassafras albidum*	136
Oxydendrum arboreum	628	*Cornus nuttallii*	119
Abies magnifica	577	*Abies amabilis*	111
Pinus echinata	559	*Quercus rubra*	100
Quercus alba	535	*Diospyros virginiana*	93
Quercus kelloggii	469	*Juniperus occidentalis*	74
Quercus falcata	442	*Chamaecyparis lawsoniana*	69
Liquidambar styraciflua	411	*Carya tomentosa*	61
Tsuga mertensiana	376	*Quercus laevis*	61
Quercus coccinea	371	*Alnus incana*	59
Pinus virginiana	370	*Juniperus virginiana*	50

sensitivity, the true positive rate (i.e., correctly classified positive outcomes/predicted positive outcomes), true negative rate (i.e., correctly classified negative outcomes/predicted negative outcomes), and overall accuracy (i.e., correctly classified positive and negative outcomes/total sample size). Using pROC, we also identified two other probability thresholds: the best threshold with ≥80% sensitivity and the best threshold with ≥80% specificity, and calculated model performance statistics for each.

Table 2—Model validation results for species with at least 300 observations in alphabetical order that included relevant predictor variables. Results in bold denote when the best threshold with ≥80 sensitivity or specificity are the same as the best overall threshold. Thres. =Threshold; Sens. = Sensitivity; Spec. = Specificity; TPR= true positive rate; TNR = true negative rate; Acc. = accuracy.

			Best threshold							Sensitivity of at least 0.8				Specificity of at least 0.8			
Species	n	Model	AUC	Thres.	Sens.	Spec.	TPR	TNR	Acc.	Thres.	Sens.	Spec.	Acc.	Thres.	Sens.	Spec.	Acc.
Abies concolor	10,294	Equation 1, see notes below	0.902	0.77	0.78	0.88	0.89	0.77	0.83	0.72	0.8	0.86	0.83	0.77	0.78	0.88	0.83
Abies grandis	1,071	Equation 1, see notes below	0.79	0.78	0.55	0.95	0.94	0.56	0.7	0.23	0.81	0.49	0.69	0.78	0.55	0.95	0.7
Abies lasiocarpa	338	Equation 1, see notes below	0.862	0.81	0.79	0.94	0.96	0.7	0.84	0.72	0.8	0.83	0.81	0.81	0.79	0.94	0.84
Abies magnifica	516	Equation 1, see notes below	0.813	0.96	0.55	0.91	0.78	0.78	0.78	0.7	0.86	0.55	0.66	0.96	0.55	0.91	0.78
Calocedrus decurrens	2,714	Equation 1, see notes below	0.968	0.95	0.88	0.96	0.96	0.88	0.92	0.95	0.88	0.96	0.92	0.95	0.88	0.96	0.92
Larix occidentalis	1,053	Equation 1, see notes below	0.794	0.31	0.75	0.72	0.31	0.94	0.72	0.2	0.81	0.58	0.61	0.63	0.6	0.87	0.83
	1,053	Equation 2, see notes below	0.793	0.26	0.58	0.91	0.52	0.93	0.86	0.05	0.81	0.62	0.65	0.26	0.58	0.91	0.86
	865	Equation 3, see notes below	0.849	0.15	0.7	0.89	0.51	0.95	0.87	0.07	0.8	0.78	0.78	0.15	0.7	0.89	0.87
Oxydendrum arboreum	321	Equation 1, see notes below	0.519	0.85	0.8	0.29	0.15	0.9	0.36	0.85	0.8	0.29	0.36	0.58	0.07	0.94	0.82
Pinus contorta	4,136	Equation 1, see notes below	0.818	0.78	0.6	0.92	0.94	0.53	0.71	0.7	0.8	0.63	0.74	0.78	0.6	0.92	0.71
	4,144	Equation 4, see notes below	0.828	0.35	0.73	0.76	0.86	0.59	0.74	0.33	0.8	0.66	0.76	0.57	0.6	0.89	0.7
	2,275	Equation 5, see notes below	0.922	0.89	0.84	0.92	0.9	0.87	0.88	0.89	0.84	0.92	0.88	0.89	0.84	0.92	0.88

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Table 2 (continued)—Model validation results for species with at least 300 observations in alphabetical order that included relevant predictor variables. Results in bold denote when the best threshold with ≥80 sensitivity or specificity are the same as the best overall threshold. Thres. =Threshold; Sens. = Sensitivity; Spec. = Specificity; TPR= true positive rate; TNR = true negative rate; Acc. = accuracy.

			Best threshold							Sensitivity of at least 0.8				Specificity of at least 0.8			
Species	**n**	**Model**	**AUC**	**Thres.**	**Sens.**	**Spec.**	**TPR**	**TNR**	**Acc.**	**Thres.**	**Sens.**	**Spec.**	**Acc.**	**Thres.**	**Sens.**	**Spec.**	**Acc.**
Picea engelmannii	592	Equation 6, see notes below	0.649	0.74	0.4	0.94	0.94	0.42	0.57	0.26	0.8	0.27	0.64	0.74	0.4	0.94	0.57
	499	Equation 7, see notes below	0.774	0.65	0.65	0.8	0.87	0.51	0.7	…	…	…	…	0.7	0.61	0.82	0.68
	592	Equation 1 or Equation 8, see notes below	0.898	0.73	0.81	0.83	0.92	0.66	0.82	0.73	0.81	0.83	0.82	0.73	0.81	0.83	0.82
Pinus jeffreyi	671	Equation 1, see notes below	0.906	0.57	0.86	0.85	0.8	0.9	0.85	0.57	0.86	0.85	0.85	0.57	0.86	0.85	0.85
	671	Equation 1, see notes below	0.916	0.4	0.8	0.93	0.88	0.87	0.87	0.4	0.8	0.93	0.87	0.4	0.8	0.93	0.87
Pinus lambertiana	2,298	Equation 1, see notes below	0.892	0.52	0.7	0.92	0.92	0.69	0.8	0.25	0.82	0.8	0.81	0.52	0.7	0.92	0.8
	859	Equation 9, see notes below	0.938	0.83	0.84	0.98	0.99	0.81	0.9	0.83	0.84	0.98	0.9	0.83	0.84	0.98	0.9
Pinus ponderosa	29,552	Equation 1, see notes below	0.862	0.8	0.76	0.85	0.63	0.91	0.83	0.67	0.8	0.8	0.8	0.8	0.76	0.85	0.83
	18,659	Equation 10, see notes below	0.827	0.27	0.71	0.82	0.49	0.92	0.8	0.12	0.8	0.68	0.7	0.27	0.71	0.82	0.8
	1,914	Equation 11, see notes below	0.906	0.14	0.83	0.83	0.57	0.95	0.83	0.14	0.83	0.83	0.83	0.14	0.83	0.83	0.83
	31,190	Equation 12, see notes below	0.823	0.35	0.67	0.88	0.71	0.86	0.82	0.07	0.81	0.6	0.66	0.35	0.67	0.88	0.82
	1,914	Equation 13, see notes below	0.91	0.12	0.87	0.8	0.54	0.96	0.81	0.12	0.87	0.8	0.81	0.13	0.86	0.81	0.82

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			Best threshold							Sensitivity of at least 0.8				Specificity of at least 0.8			
Species	**n**	**Model**	**AUC**	**Thres.**	**Sens.**	**Spec.**	**TPR**	**TNR**	**Acc.**	**Thres.**	**Sens.**	**Spec.**	**Acc.**	**Thres.**	**Sens.**	**Spec.**	**Acc.**
Pinus taeda	325	Equation 1, see notes below	0.667	0.57	0.41	0.9	0.64	0.77	0.74	0.35	0.81	0.31	0.46	0.57	0.41	0.9	0.74
Populus tremuloides	493	Equation 1, see notes below	0.705	0.53	0.5	0.81	0.54	0.79	0.72	0.39	0.82	0.39	0.52	0.53	0.5	0.81	0.72
Pseudotsuga menziesii	10,456	Equation 1, see notes below	0.913	0.77	0.78	0.94	0.91	0.85	0.87	0.65	0.81	0.9	0.86	0.77	0.78	0.94	0.87
	10,495	Equation 14, see notes below	0.923	0.69	0.78	0.96	0.94	0.85	0.88	0.48	0.82	0.92	0.87	0.69	0.78	0.96	0.88
	1,442	Equation 15, see notes below	0.902	0.38	0.78	0.87	0.78	0.87	0.84	0.3	0.82	0.82	0.82	0.38	0.78	0.87	0.84
Quercus kelloggii	343	Equation 1, see notes below	0.77	0.89	0.56	0.91	0.78	0.77	0.77	0.65	0.83	0.53	0.64	0.89	0.56	0.91	0.77
Tsuga heterophylla	813	Equation 1, see notes below	0.75	0.34	0.7	0.71	0.83	0.55	0.71	0.28	0.81	0.57	0.73	0.47	0.55	0.83	0.64

Notes:

Equation 1—$P_m = 1/(1 + e^{(-1.941+6.316(1-e^{-0.3937BT})-0.000535(CVS^2))})$

Equation 2—$P_{m(pre)} = 1/(1 + e^{(-(-1.6594+0.0327CVS-0.0489DBH))})$

Equation 3—$P_{m(post)} = 1/(1 + e^{(-(-3.8458+0.1387CVS-0.0004CVS^2+0.6266CKR))})$

Equation 4—$P_{m(pre)} = 1/(1 + e^{(-(-0.3268+0.1387CVS-0.0033CVS^2+0.000025CVS^3-0.0266DBH))})$

Equation 5—$P_{m(post)} = 1/(1 + e^{(-(-1.4059+4.459e^{-6}CVS^3+0.2843CKR^2-0.0485DBH))})$

Equation 6—$P_{m(pre)} = 1/(1 + e^{(-(-0.0845+0.0445CVS))})$

Equation 7—$P_{m(post)} = 1/(1 + e^{(-(-2.9791+0.0405CVS+1.1596CKR))})$

Equation 8— $P_m \geq 0.80$

Equation 9—$P_{m\,(pre)} = 1/(1 + e^{(-(-2.0588+0.000814PCLS^2))})$

Equation 10—$P_{m(pre\ blackhills)} = 1/(1 + e^{(-(1.104-0.156DBH+0.013+PCLS+0.001(DBH)(PCLS)))})$

Equation 11—$P_{m(post\ kill)} = 1/(1 + e^{(-(-3.5729+0.000567(CVK^2)+0.4573CKR+1.6075BTL))})$

Equation 12—$P_{m(pre)} = 1/(1 + e^{(-(-2.7103+0.000004093(CVK^3)))})$

Equation 13—$P_{m(post\ scorch)} = 1/(1 + e^{(-(-4.1914+0.000376(CVS^2)+0.5130CKR+1.5873BTL))})$

Equation 14—$P_{m(pre)} = 1/(1 + e^{(-(-2.0346+0.0906CVS-0.0022(CVS^2)+0.000019(CVS^3)))})$

Equation 15—$P_{m(post)} = 1/(1 + e^{(-(-1.8912+0.07CVS-0.0019(CVS^2)+0.000018(CVS^3)+0.5840CKR-0.031DBH-0.7959BTL+0.0492(DBH)(BTL)))})$

RESULTS AND DISCUSSION

For the 31 models we evaluated, AUC values for most models were ≥0.7, defined as "acceptable" or better, with 20 of the 31 models with AUC values in the good to excellent range (ACU≥0.08) (table 2). Exceptions were an AUC of 0.519 for *Oxydendrum arboreum* (sourwood) using the fofem5 model. This model had notably low specificity and a low true positive rate, meaning that live trees were often classified as dead. *Pinus taeda* (loblolly pine) also had a low AUC of 0.667, and low model sensitivity, meaning that dead trees were often classified as alive. Finally the basic FOFEM model for spruces, which sets mortality to be at least 80%, performed poorly for *Picea engelmannii* (Engelmann spruce) with and AUC of 0.660. Overall model performance was poorer for the deciduous species we assessed (*Oxydendrum arboretum* AUC=0.519; *Populus tremuloides* AUC = 0.705; *Quercus kelloggii* AUC = 0.77).

The pre-fire and post-fire *P. engelmannii* models in FOFEM had higher AUCs of 0.774 and 0.898, respectively. For some species, such as *Larix occidentalis* (western larch), *P. contorta*, *Pinus jeffreyi* (Jeffrey pine), and *P. engelmannii*, *Pinus lambertiana* (sugar pine), the species-specific pre-fire and post-fire models had greater overall accuracy than the fofem5 model (table 2). Species-specific models including those for *P. ponderosa*, and *P. menziesii* did not show distinct improvement over the fofem5 model (table 2). Finally, by adjusting the thresholds, it was possible to get sensitivities above 80% and specificities above 80% for all models except the *P. engelmanni* post-fire model (table 2).

NEXT STEPS

We plan to validate the FOFEM post-fire mortality models for all species with ≥50 observations, once the dataset is fully clean and standardized. Thus we expect to be able to conduct model validation for approximately 70 species. Additional data standardization and data checking will be completed, including for the species-model combinations we presented here, and we will explore whether models perform well or poorly for clades of species or within differing geographic regions. Thus our results should highlight species and regions where new data collect or new model development is needed. Furthermore, we will test whether additional climatic predictors—specifically pre- and post-fire climatic water deficit, precipitation, and vapor pressure deficit—can improve mortality predictions (van Mantgem et al. 2013). For common species for which we have a large number of observations, such as *P. ponderosa* and *P. menziesii*, we will also investigate whether model performance varies regionally or among subspecies, potentially suggesting a need for regionally specific models. Our approach to identifying thresholds and model evaluation results will be incorporated into FOFEM, providing managers with guidance on how to set threshold values to optimize prediction accuracies in a way that mirrors their objectives, and quantifying uncertainty of this widely used decision support tool.

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