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Site Index Models for Tree Species in the Northeastern United States

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Quantitative measures of site productivity provide critical information for management of forests. In this study, models were developed to predict site index (the average height of dominant/codominant trees at a specified base age) using data that encompassed the entire northeastern United States and a wide range of stand conditions. Regression analyses were conducted for 22 tree species/groups, with models utilizing breast-height age being presented. For 15 remaining species/groups having relatively low frequency of occurrence, empirical evaluations were performed to determine which of the 22 sets of model coefficients provided the best predictions. Comparisons with site index models currently being used in the region indicated a smaller range of site index values using the models developed in this study, primarily due to higher predicted values for poorer sites. Thus, replacement of existing models with the models presented here will produce different results in data processing and growth projection systems that use site index as an input variable. However, because of the recency and breadth of the underlying data, the models developed in this study should more accurately reflect the site productivity of forest stands across the region.

Keywords: dominant height, forest inventory, base-age invariant, mixed models.

For nearly a century, site index has served as the primary indicator of site quality for forest stands in the United States (Spring 1917, Frothingham 1918). Site index is most commonly assessed via height/age relationships for dominant/codominant trees that show no signs of previous damage or suppression (Avery and Burkhart 2002). Site quality has considerable influence in guiding management decisions (Wall 2012) and making growth and yield projections (Weiskittel et al. 2011). Because of the importance of site quality estimates, a considerable amount of research has been conducted on this topic. Early efforts consisted of developing a base height/age curve, which was then scaled proportionally to apply to sites of lower or higher quality (Bruce 1926). This anamorphic method implies that the shape of the curve is identical for all sites. Subsequently, more sophisticated methods were conceived to allow increased flexibility, such as polymorphic curves whose shapes could vary among stands (Stage 1963, Beck 1971). Numerous methodological approaches and advancements have been subsequently presented (e.g., Bailey and Clutter 1974, Biging 1985, Lappi 1991, Cieszewski and Bailey 2000, Wang et al. 2008b); Gregoire (2011) lists nearly 290 citations related to site quality research from 1970 from 2010.

Although the most straightforward applications of site index are for even-aged, monospecific stands, it is often computed for uneven-aged, multispecies conditions where the outcome is dependent on the selection of the site tree species (Shifley 1987). The interpretation of site index for these stands is problematic under the traditional definition (Avery and Burkhart 2002), and practical usage often requires additional complexity (Dixon and Keyser 2008). Other approaches to quantifying site index have been investigated, such as using climatic and/or physiographic predictors (Ung et al. 2001, Nigh et al. 2004). However, site index calculated from height/age data continues to be the most common implementation. The height/age methodology is used nationwide by the Forest Inventory and Analysis (FIA) program of the US Department of Agriculture (USDA) Forest Service, which calculates site index for all forest conditions present in the sample (O'Connell et al. 2015). In the Northeastern region, the models developed by Scott and Voorhis (1986) are used to estimate site index at a base age of 50 years. These models were based on data sources ranging from Frothingham (1914) to Curtis and Post (1964), in which the data were extracted from tables, curves, or models published therein (see Table 1 in Scott and Voorhis 1986). In some cases, these data sources covered a limited geographic range, e.g., the Curtis and Post (1964) data for four species were from a portion of the state of Vermont. In addition, the range of ages and site qualities for some species may not have fully represented the population distribution across the region (see Table 4 in Scott and Voorhis 1986). Model coefficients were presented for 19 species; however, FIA in the Northeastern region recognizes a host of other species as being suitable for site index determination, and thus application of the Scott and Voorhis

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Table 1. Summary of site tree information by species across 13 states in the Northeastern United States.

Group	Species	*n*	Height (ft)			Age (yr)		
			Minimum	Mean	Maximum	Minimum	Mean	Maximum
1	Balsam fir (*Abies balsamea*)	1106	20	47	100	15	48	120
2	Norway spruce (*Picea abies*)	4	50	58	72	35	50	62
2	White spruce (*Picea glauca*)	176	21	50	84	14	50	110
2	Black spruce (*Picea mariana*)	158	21	42	68	15	62	133
2	Blue spruce (*Picea pungens*)	1	36	36	36	31	31	31
2	Red spruce (*Picea rubens*)	1427	20	55	106	15	70	178
3	Red pine (*Pinus resinosa*)	131	18	62	103	15	45	104
4	Shortleaf pine (*Pinus echinata*)	12	22	61	96	18	67	114
4	Pitch pine (*Pinus rigida*)	16	37	52	75	23	67	103
4	Loblolly pine (*Pinus taeda*)	191	26	68	111	14	43	108
4	Douglas-fir (*Pseudotsuga menziesii*)	2	24	31	38	15	16	17
5	Eastern white pine (*Pinus strobus*)	1497	19	66	117	15	52	197
6	Virginia pine (*Pinus virginiana*)	177	20	57	89	15	46	115
7	Eastern hemlock (*Tsuga canadensis*)	702	23	63	118	20	77	200
8	Red maple (*Acer rubrum*)	2433	24	68	119	11	54	128
8	Silver maple (*Acer saccharinum*)	58	32	69	117	15	45	94
9	Sugar maple (*Acer saccharum*)	1952	32	72	120	17	63	154
10	Yellow birch (*Betula alleghaniensis*)	386	32	62	114	11	61	115
11	Paper birch (*Betula papyrifera*)	230	27	57	99	16	51	99
12	Bitternut hickory (*Carya cordiformis*)	118	39	77	120	20	57	115
12	Pignut hickory (*Carya glabra*)	9	52	78	107	40	70	106
12	Shagbark hickory (*Carya ovata*)	150	35	77	115	21	68	206
12	Mockernut hickory (*Carya alba*)	1	81	81	81	42	42	42
13	American beech (*Fagus grandifolia*)	174	22	70	135	21	66	141
14	White ash (*Fraxinus americana*)	2167	24	70	125	10	51	135
14	Black ash (*Fraxinus nigra*)	46	29	53	106	20	51	108
14	Green ash (*Fraxinus pennsylvanica*)	290	26	66	106	11	45	164
15	Yellow-poplar (*Liriodendron tulipifera*)	1237	24	84	147	10	45	118
16	Eastern cottonwood (*Populus deltoides*)	35	35	70	111	16	36	98
16	Bigtooth aspen (*Populus grandidentata*)	113	33	66	102	16	40	114
16	Quaking aspen (*Populus tremuloides*)	175	25	59	100	10	34	116
17	Black cherry (*Prunus serotina*)	343	34	78	125	15	54	107
18	White oak (*Quercus alba*)	1320	22	76	128	17	74	188
19	Scarlet oak (*Quercus coccinea*)	329	27	66	105	16	57	106
19	Post oak (*Quercus stellata*)	2	29	30	31	21	46	70
20	Chestnut oak (*Quercus prinus*)	695	29	70	120	16	74	160
21	Northern red oak (*Quercus rubra*)	1918	30	74	128	1	63	172
22	Black oak (*Quercus velutina*)	529	30	72	125	15	58	120
23	Atlantic white-cedar (*Chamaecyparis thyoides*)	22	30	60	79	33	73	148
24	Eastern redcedar (*Juniperus virginiana*)	35	20	36	52	28	45	88
25	Larch spp. (*Larix* spp.)	25	25	59	92	15	35	59
26	Tamarack (native) (*Larix laricina*)	63	27	53	81	19	53	115
27	Scotch pine (*Pinus sylvestris*)	75	23	48	90	14	34	85
28	Northern white-cedar (*Thuja occidentalis*)	132	18	42	97	20	75	122
29	Black walnut (*Juglans nigra*)	39	36	57	82	15	36	65
30	Sweetgum (*Liquidambar styraciflua*)	106	38	79	117	15	58	127
31	Southern red oak (*Quercus falcata*)	32	45	73	100	27	66	115
31	Cherrybark oak (*Quercus pagoda*)	3	56	68	89	70	81	100
32	Shingle oak (*Quercus imbricaria*)	11	32	57	75	20	35	68
33	Pin oak (*Quercus palustris*)	88	34	70	119	15	43	81
34	Black locust (*Robinia pseudoacacia*)	129	25	56	92	10	32	76
35	American basswood (*Tilia americana*)	118	39	78	119	22	55	102
36	American elm (*Ulmus americana*)	160	29	54	110	15	35	120
36	Slippery elm (*Ulmus rubra*)	1	94	94	94	24	24	24
37	Sweet birch (*Betula lenta*)	10	62	71	85	35	75	112
37	Gray birch (*Betula populifolia*)	1	40	40	40	20	20	20
37	Hackberry (*Celtis occidentalis*)	10	40	60	87	20	43	62

(1986) models requires that numerous species be assigned to 1 of the 19 sets of published coefficients.

Given the availability of newer data across the region and the prospect that tree height/age relationships may have changed over the last 50 to 100 years (Bazzaz et al. 1990, Westfall and Amateis 2003), a reevaluation of site index prediction is warranted. Specifically, the objectives of this study include the following: development of site index models using data that better encompass the extant geographic, age, and site quality conditions and reflect current environmental factors; provision for an empirical basis to assign less-common species to a set of model coefficients; and comparison of results to the Scott and Voorhis (1986) models currently in use.

Methods

The data used in this study were collected on FIA sample plots from 1999 to 2013 in 13 states across the Northeastern United

States (Maine, Vermont, New Hampshire, New York, Rhode Island, Connecticut, Massachusetts, New Jersey, Pennsylvania, Ohio, West Virginia, Delaware, and Maryland). FIA sample plots are distributed across the landscape in a quasi-systematic pattern that provides an even spatial coverage of the area for each year of measurement (Reams et al. 2005). The sampling intensity is approximately 1 plot per 6,000 acres (2,400 ha) of area. Plots having forested conditions are visited, and numerous measurements are taken, including those needed to estimate site quality. Specifically, trees suitable for site index determination are identified and observations of species, total height, and breast-height age are recorded (USDA Forest Service 2007). Thus, the observations related to site quality are site tree-specific and may not represent all species and ages present on the plot. In some cases, more than one forested stand occurs within the plot area, and separate site trees are measured for each stand. Although the same plots often received repeated visits every 5 to 7 years, measurements for site index determination were usually only taken once. Thus, no growth data on site trees are available. A summary of these data is provided in Table 1.

The lack of growth data as well as actual site index values for most of the data restricts the choices of analytical methods. For example, site index cannot be used as the response variable in a regression modeling context. Because of the successful use of the Chapman-Richards model (Richards 1959) in numerous previous studies (Carvalho and Parresol 2005, Wang et al. 2008, Nigh 2015), the form of the model chosen for this study was

$$H_{jk} = 4.5 + \beta_0(1 - \exp(-\beta_1 A_{jk}))^{\beta_2} + \varepsilon_{jk} \quad (1)$$

where H_{jk} and A_{jk} are the tree height and age, respectively, of the site index tree for stand j on plot k, β_0–β_2 are parameters to be estimated from the data, and ε_{jk} are random residual errors. This model produces a base-age invariant site index curve. The base-age invariance is due to the lack of base-age specification before model fitting. It can also be viewed as the so-called "guide curve" for an anamorphic site index model system (Biging 1985).

In the spirit of Bailey and Clutter (1974), any of the model parameters β_0–β_2 may be recast as being site-specific. By using contemporary statistical methods, site-specific parameter estimates can be obtained using mixed-effects modeling techniques (Vonesh and Chinchilli 1997, Ni and Zhang 2007), which implies that Model 1 may take the form

$$H_{jk} = (\beta_0 + \theta_{1jk})(1 - \exp((-\beta_1 + \theta_{2jk})A_{jk}))^{\beta_2 + \theta_{3jk}} + \varepsilon_{jk} \quad (2)$$

where $\theta_{ijk} \sim N(0, \sigma\sigma^2_{\theta_i})$ are the estimated random-effects parameters for stand j on plot k.

Initial investigations using Model 2 suggested that the only significant random-effects parameter was θ_{1jk} (i.e., $\sigma\sigma^2_{\theta_1} \neq 0$ at $\alpha = 0.05$), resulting in anamorphic site index models. To further explore the potential for more complex modeling outcomes, the unobserved "growth intensity factor" concept of Cieszewski and Bailey (2000) was considered. A key advantage to this method is the ability to simultaneously address both variable asymptotes and polymorphism via the growth factor. Krumland and Eng (2005) applied this methodology to several candidate model specifications, including the Chapman-Richards model shown in 1. For this study, the formulation of the base model differed slightly from the CR2 specification given by Krumland and Eng (2005):

$$H_{jk} = 4.5 + (\beta_0 + X_{jk})(1 - \exp(-\beta_1 A_{jk}))^{\beta_2 + \beta_3 X_{jk}} + \varepsilon_{jk} \quad (3)$$

where X_{jk} is the site-specific parameter. Note that the single site-specific parameter now affects both the asymptote and the shape of the resultant model. An important deviation from the approach described by Cieszewski and Bailey (2000) is that instead of proceeding to construct the final model via substitution, Model 3 was fitted to the data where the site-specific parameter X_{jk} was considered a random effect with $X \sim N(0, \sigma^2_X)$. Heteroscedasticity in the ε_{jk} was addressed via the distribution described by $\varepsilon_{jk} \sim N(0, \exp(\lambda A_{jk}))$, where λ is a parameter estimated from the data.

With use of Model 3, estimates of site index (S) for stand j on plot k (S_{jk}; base age 50 years) were acquired using the estimated fixed-effects parameters, the site-specific parameter, and the value of 50 for the age predictor variable:

$$S_{jk} = 4.5 + (\beta_1 + X_{jk})(1 - \exp(-\beta_1 50))^{\beta_2 + \beta_3 X_{jk}} + \varepsilon_{jk} \quad (4)$$

Goodness-of-fit statistics were assessed via the concordance correlation (Vonesh et al. 1996):

$$r_c = 1 - \frac{\sum_{jk}(H_{jk} - \hat{H}_{jk})^2}{\sum_{jk}(H_{jk} - \bar{H})^2 + \sum_{jk}(\hat{H}_{jk} - \hat{\bar{H}})(H_{jk} - \hat{\bar{H}}) + n(\bar{H} - \hat{\bar{H}})^2} \quad (5)$$

where $\hat{H}_{jk}$ is the model prediction, H is the mean model prediction, $\bar{H}$ is the mean observed height, and n is the number of observations. The r_c statistic spans the interval between -1 and $+1$, with $r_c = 1$ indicating a perfect fit to the data and $r_c \leq 0$ suggesting considerable lack of fit.

Model validation was conducted using 20% of observations randomly withheld from the fitting process for each species/group. The validation process required prediction of the site-specific parameter X_{jk} for each observation. Given the observed data and the estimated global parameters, the site-specific component is then predicted from Vonesh and Chinchilli (1997):

$$\hat{X}_{jk} = DZ'(ZDZ' + R)^{-1}(y - Mb) \quad (6)$$

where $\hat{X}_{jk}$ is the predicted site-specific parameter for stand j on plot k, $Z = FB$ (B is the regression design matrix for the site-specific parameter, F is the matrix of partial derivatives with respect to each fixed-effects parameter), R is the variance of residual errors, D is the variance of random effects, y is the observed tree height, M is the regression design matrix for fixed-effects parameters, and b is the vector of fixed-effects parameters. The predicted values for the validation data are then obtained as

$$H_{jk} = 4.5 + (\beta_1 + \hat{X}_{jk})(1 - \exp(-\beta_1 A_{jk}))^{\beta_2 + \beta_3 \hat{X}_{jk}} \quad (7)$$

Statistical assessment was accomplished via calculation of the mean residual for the validation data $\bar{\varepsilon}_v = \Sigma(H_{jk} - \hat{H}_{jk})n_v$ and testing the null hypothesis ($\alpha = 0.05$) that $\bar{\varepsilon}_v = 0$.

Model 3 was fitted separately to data from species/species groups 1–22 that encompassed 38 of the 57 species found in the data (Table

Table 2. Results of regression analyses for Model 3 for 22 species groups.

Group	β_0	β_1	β_2	β_3	λ	σ_X^2	r_c
1	55.02 (4.66)	0.0287 (0.014)	0.7966 (0.29)	−0.0221 (0.009)	0.05452 (0.003)	70.50 (13.80)	0.97
2	57.40 (1.17)	0.0389 (0.005)	1.4957 (0.23)	−0.0261 (0.007)	0.03923 (0.002)	96.86 (8.66)	0.98
3	79.50 (5.61)	0.0457 (0.010)	1.9477 (0.41)	−0.0159 (0.012)	0.05497 (0.010)	180.02 (63.75)	1.00
4	81.43 (4.28)	0.0467 (0.011)	1.3033 (0.29)	−0.0097 (0.009)	0.07813 (0.004)	151.31 (54.26)	0.99
5	82.69 (2.49)	0.034 (0.004)	1.3968 (0.16)	−0.0283 (0.006)	0.05143 (0.002)	117.85 (17.92)	0.91
6	61.66 (2.09)	0.0875 (0.019)	3.2058 (1.32)	−0.0458 (0.036)	0.06785 (0.007)	96.23 (30.69)	0.91
7	71.90 (4.72)	0.0203 (0.006)	0.8385 (0.18)	−0.0111 (0.005)	0.04012 (0.002)	126.87 (21.65)	0.97
8	89.29 (3.69)	0.0188 (0.003)	0.7197 (0.06)	−0.0103 (0.002)	0.05157 (0.002)	100.06 (15.45)	0.98
9	97.64 (6.68)	0.0142 (0.004)	0.6634 (0.07)	−0.0110 (0.003)	0.05218 (0.001)	65.01 (12.89)	0.96
10	72.75 (3.79)	0.0222 (0.005)	0.6940 (0.07)	−0.0091 (0.003)	0.02713 (0.004)	121.02 (20.43)	1.00
11	59.26 (3.82)	0.0332 (0.017)	0.5767 (0.21)	−0.0368 (0.015)	0.05850 (0.005)	59.51 (29.48)	0.95
12	85.11 (3.21)	0.0391 (0.007)	1.3240 (0.24)	−0.0205 (0.008)	0.05328 (0.003)	93.40 (19.71)	0.95
13	131.05 (18.98)	0.0061 (0.002)	0.6040 (0.07)	−0.0038 (0.011)	0.04377 (0.011)	233.31 (535.42)	0.98
14	91.99 (3.41)	0.0217 (0.003)	0.7561 (0.06)	−0.0094 (0.002)	0.06328 (0.001)	119.98 (18.01)	0.96
15	103.16 (2.96)	0.0385 (0.005)	1.0919 (0.13)	−0.0230 (0.005)	0.07267 (0.002)	98.64 (19.69)	0.97
16	87.90 (8.64)	0.0184 (0.007)	0.5340 (0.09)	−0.0057 (0.005)	0.04080 (0.010)	183.04 (84.52)	1.00
17	107.28 (8.69)	0.0182 (0.005)	0.7543 (0.08)	−0.0282 (0.013)	0.05520 (0.003)	43.85 (32.32)	0.98
18	80.65 (1.85)	0.0321 (0.005)	1.0805 (0.22)	−0.0247 (0.008)	0.04351 (0.002)	100.66 (15.11)	0.97
19	67.75 (2.41)	0.056 (0.015)	1.3487 (0.48)	0.0100 (0.009)	0.04172 (0.008)	279.81 (39.48)	1.00
20	67.34 (0.77)	0.0911 (0.016)	3.9519 (1.67)	0.0229 (0.026)	0.03269 (0.003)	172.07 (12.81)	0.99
21	81.94 (2.32)	0.0276 (0.005)	0.7529 (0.13)	−0.0266 (0.006)	0.05407 (0.001)	72.68 (13.28)	0.94
22	80.14 (1.85)	0.0422 (0.005)	1.2955 (0.18)	−0.0022 (0.004)	0.03014 (0.009)	180.89 (24.24)	1.00

Statistics of $r_c = 1.00$ represent values greater than 0.995. Estimates for β_3 in italics were not statistically different from zero at $\alpha = 0.05$.

1). The species/species groups were adapted from existing classifications used by FIA. As is common in large-area analyses using forestry data, there remain a number of species/species groups (23–37; Table 1) for which model fitting was problematic or there was a lack of sufficient data to adequately fit the model. Two primary options are then considered: add these data to another group and fit the model to the wider range of species; or assign the species/group to the model coefficients developed for another group. Although neither of these options provides an optimal solution, the latter approach was chosen to not contaminate the parameter estimates for the well-represented species in the data. Often, assignment of these remaining species is based on taxonomic classification (e.g., family and genus) or other perceived similarities that may or may not result in the best match to existing model results. In this study, each set of coefficients obtained for the major species/groups were evaluated for their prediction ability for the unassigned species/groups. The analysis was conducted unconditionally, i.e., the set of coefficients that provided the best prediction was selected, and conditionally where the remaining hardwood species could only be assigned to coefficients developed for other hardwood species and likewise for softwoods. The criterion for determining the best harmonization was the minimum mean-squared prediction error (MSE):

$$\text{MSE} = \frac{\sum \varepsilon_{jk}^2}{n} \tag{8}$$

To better understand how models fitted to recent FIA data compare with the existing site index estimates, comparisons were made between the predicted values from model 4 and the site index values in the FIA database based on Scott and Voorhis (1986). Differences in ranges of predicted values for site index were evaluated graphically and *t*-tests were conducted to determine whether the mean differences ($\bar{D}$) statistically deviated from 0 ($\alpha = 0.05$):

$$\bar{D} = \frac{\sum (\hat{SI}_{jk}^{\text{FIA}} - \hat{SI}_{jk}^{\text{SV}})}{n} \tag{9}$$

where the FIA and SV superscripts reference the predictions from Model 4 and those from Scott and Voorhis (1986) (hereafter referred to as SV), respectively. As the differences in predicted site index values are primarily due to differences in the underlying data, differences in age ranges used in this study and those reported by SV were examined. The comparisons between the two studies were only conducted for species for which there was exact or very close alignment with the groupings used by SV. Specifically, the species/groups used in the comparative assessment were 1, 2, 5, 6, 9, 10, 13, 14, 15, and 17.

Results

Regression analysis results are shown in Table 2. The model parameter estimates β_0–β_2 and λ were significantly different from 0 ($\alpha = 0.05$) in all cases; however, the β_3 parameter was not statistically significant for 8 of the 22 groups. The fact that σ_X^2 was always statistically different from 0 indicates a considerable range of variation not explained by the mean response model consisting of the fixed effects only (Figure 1). The variance of the ε_{jk} increased with A_{jk} in all cases, and the rate of increase was indicated by the value of λ (Table 2). These results show that heteroscedasticity varies substantially among the different species/groups. In particular, species/groups 4 (*Pinus* spp.), 6 (Virginia pine), and 15 (yellow-poplar) exhibited notable increases in error variance as age increased. Other species/groups such as 10 (yellow birch), 20 (chestnut oak), and 22 (black oak) had relatively little increase with age. Overall, there was increased uncertainty associated with model predictions as age increases. Model fit as assessed by the r_c statistic indicated values $\geq$0.95 for all species/groups except 5 (eastern white pine), 6 (Virginia pine), and 21 (northern red oak), with the overall average $r_c = 0.97$. Based on the determinations of McBride (2005), $r_c < 0.90$ is indicative of poor agreement between the observations and predictions, whereas $0.90 \leq r_c < 0.95$ is considered moderate agreement, and $0.95 \leq r_c$ is judged to be substantial agreement. Thus, most of the results suggest moderate to substantial agreement between observed and predicted values.

Figure 1. Maximum and minimum height/age curves (dashed lines) based on random effects from model 3, the mean response model (solid line), and observed data points (circles) for species group 3 (red pine).

Table 3. Validation results for Model 3 by species group.

Group	$\bar{\epsilon}_v$	σ_{ϵ_v}
1	−0.60	2.69
2	0.36	3.19
3	−1.07	2.44
4	−0.85	7.40
5	−0.70	3.09
6	−0.43	3.01
7	0.57	4.00
8	−0.22	3.21
9	−0.33	4.14
10	−0.30	0.79
11	−0.07	4.51
12	0.50	4.65
13	−0.78	1.92
14	−0.19	4.67
15	−0.37	4.47
16	−0.56	0.74
17	−1.24	2.85
18	0.13	3.73
19	0.21	0.84
20	0.20	2.26
21	−0.47	5.10
22	0.01	0.81

The model validation results showed that mean residuals ($\bar{\varepsilon}_v$) ranged from −1.24 ft (−0.4 m) for group 17 (black cherry) to 0.57 ft (0.2 m) for group 7 (eastern hemlock) over the 22 species/groups (Table 3). Ninety-one percent (20/22) of the validation analyses had mean residuals between −1.0 and 1.0 ft (0.3 m). The standard deviations of the residuals (σ_ε^2) ranged from 0.74 for group 16 (*Populus* spp.) to 7.40 for group 4 (*Pinus* spp.). Given that the results show only two groups having a mean error of greater than 1.0 ft (0.3 m), the models can be judged to provide accurate predictions for new observations.

For the 15 species/groups not included in the regression analyses, the use of Model 3 provided the basis for assigning these species to coefficients developed for other species/groups. Specifically, the set of coefficients that minimized the value of 8 was selected to be the best match for these species (Table 4). The unconditional results were indicative of no restrictions for matching, and sometimes the outcome lacked strict adherence to hardwood/softwood classifications. Specifically, group 25 (larch spp.) was associated with group 22 (black oak), group 26 (tamarack) was matched with group 11 (paper birch), and group 30 (sweetgum) was assigned to group 4

Table 4. Conditional and unconditional assignment of coefficients to 15 species/groups not included in regression analyses.

	Unconditional				
Assignment	Group	n	$\bar{\epsilon}$	MSE	Assigned group
Unconditional	23	22	−0.17	17.04	7
	24	35	−6.31	52.96	2
	25	26	0.28	56.04	22
	26	63	−1.73	84.91	11
	27	75	−1.30	21.29	5
	28	132	−7.82	108.97	1
	29	39	−0.27	34.97	19
	30	106	2.79	40.57	4
	31	35	2.04	35.15	19
	32	11	0.48	6.75	18
	33	88	0.04	14.07	17
	34	129	−0.71	24.52	21
	35	118	−0.61	47.85	17
	36	161	−0.27	46.02	18
	37	21	−0.17	4.72	19
Conditional	25	26	−1.85	56.67	4
	26	63	−4.06	123.21	6
	30	106	−0.01	44.01	17

Table 5. Summary statistics for differences in predicted site index between the SV (Scott and Voorhis 1986) models and Model 4.

Group	n	$\bar{D}$	$\Pr > \lvert t \rvert$
1	895	5.93	<0.001
2	1,416	10.65	<0.001
5	1,180	6.63	<0.001
6	146	1.81	<0.001
9	1,579	11.39	<0.001
10	306	4.40	<0.001
14	2,024	1.40	<0.001
15	1,009	2.04	<0.001
16	258	−8.26	<0.001
18	1,053	5.08	<0.001

$\Pr > |t|$ values <0.05 indicate statistically significant differences at the 95% confidence level.

(*Pinus* spp.). These results suggest that occasionally interspecies height/age relationships can be similar even though tree growth characteristics are markedly different; although groups 25 and 26 are classified as a softwood, they exhibit the hardwood-like characteristic of being deciduous. Because of the characteristic differences between hardwood and softwood species, it is usually considered inappropriate to conflate outcomes across these natural demarcations. As such, the conditional analysis constrained assignment of coefficients to be within the associated hardwood/softwood classification. This restriction resulted in groups 25 and 26 being matched with groups 4 (*Pinus* spp.) and 6 (Virginia pine), respectively (Table 4), with group 30 being reassigned to group 17 (black cherry). The constrained results produced increases in absolute bias of approximately 1.6 ft (MSE increase 1.1%) and 2.3 ft (MSE increase 45.1%), for groups 25 and 26, respectively. For group 30, the absolute bias was decreased by 2.8 ft (MSE increase 8.5%).

Comparisons between the predicted site index values from Model 4 and those from SV for analogous species/groups based on paired t-tests showed statistically significant ($\alpha = 0.05$) differences for all 10 of the species/groups assessed (Table 5); although it could be argued that some of the smaller differences are not of considerable practical significance. Two results that were particularly noteworthy were for groups 2 (*Picea* spp.) and 9 (sugar maple), for which the

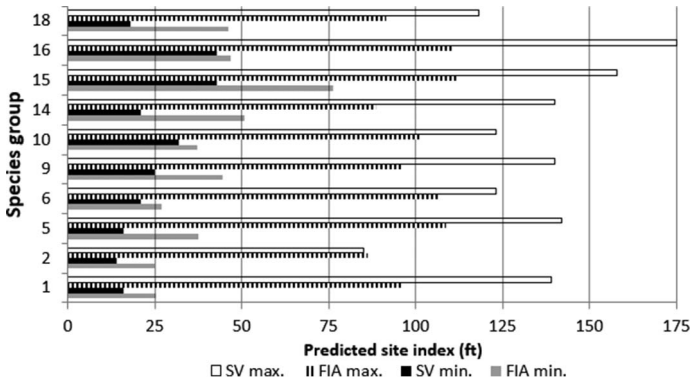

Figure 2. Maximum and minimum predicted site index (base age 50) for Scott and Voorhis (1986) (SV) and Model 4 (FIA) from this study.

mean differences exceeded 10 ft (3.0 m). Also of interest was the finding that all groups exhibited higher mean predicted values using Model 4, with the exception of group 16 (*Populus* spp.), which had a mean site index that was nearly 8.3 ft (2.5 m) smaller than those predicted from the SV models. The general trend of higher mean predicted site index values for models developed in this study is probably due to increases in predicted site index values for poor- to moderate-quality sites that comprise much of the data (Figure 2). It is also notable that the SV models predicted more extreme large values of site index, with the maximum SV predictions averaging nearly 35 ft. (10.7 m) larger than the maximum values from Model 4 over the 10 species/groups examined. Thus, the overall range of site index predictions was smaller when the new models were used. This could be due to the true range of site index values being smaller or possibly due to a tendency to overestimate site index for poorer sites and underestimate site index for better sites. Sabatia and Burkhart (2014) found this behavior for both parametric and nonparametric model fitting methods, with the parametric method appearing to be more susceptible to the phenomenon. Thus, investigation of alternative model fitting procedures that can potentially alleviate this issue should also be considered, e.g., the iterative evaluation (IE) method (Krumland and Eng 2005).

Discussion

The general Chapman-Richards model formulation performed well, considering that the primary fixed-effects parameters (β_0–β_2) were statistically significant ($\alpha = 0.05$) for all 21 species/groups (Table 2). The generally large concordance correlations and the validation mean residuals being relatively small (Table 3) suggests that the models provide an accurate depiction of the height/age relationships. The inclusion of the 4.5 intercept produced slightly better fit statistics than a no-intercept model. Although a zero slope is implied at a height of 4.5 ft, this aspect is generally not a concern unless the site tree is very young. To maintain consistency throughout, the model specified in 3 was applied to all species/groups; however, in some cases the β_3 parameter was nonsignificant, which would suggest only asymptotic variability among sites (anamorphism). Generally, anamorphic models have been found to inadequately represent the actual site-to-site variation in height/age relationships (Nord-Larsen 2006, Weiskittel et al. 2011).

Two matters of note arose during the analysis: first, bias in the model due to the use of only cross-sectional data (Walters et al. 1989); and second, the apparent lack of polymorphism for some species/groups. The use of cross-sectional data can be acceptable if there is a similar mean site index across all age classes in the data (Avery and Burkhart 2002). To determine the potential for bias in this study, a simple linear regression was conducted to determine whether there was a statistically significant correlation between site quality and age, i.e., $\widehat{SI}_{jk} = \phi_0 + \phi_1 A_{jk}$. For the 22 species/groups examined, the estimated slope parameter (ϕ_1) was not statistically different from 0 at the 95% confidence level (Table 6) with the exception of group 17 (black cherry). It is proposed that the random plot locations and large geographic extent of these data generally resulted in a sample that covered a wide range of site quality/age combinations.

Table 6. Test for significant slope in $\widehat{SI}_{ik} = \phi_0 + \phi_1 A_{ik}$ to determine site quality representation across age classes.

Group	Pr > |t|
1	0.99
2	0.99
3	0.52
4	0.47
5	0.27
6	0.54
7	0.67
8	0.45
9	0.68
10	0.76
11	0.88
12	0.36
13	0.89
14	0.07
15	0.38
16	0.37
17	0.01
18	0.56
19	0.28
20	0.68
21	0.67
22	0.64

Pr > |t| values >0.05 indicate that the slope parameter (ϕ_1) is not statistically different from 0 at the 95% confidence level.

In regards to the second issue, each height/age pair provides the self-referencing (Northway 1985) that shifts the curve asymptote ($\beta_0 + \theta_{jk}$); however, in the absence of a statistically significant β_3 parameter, the remaining fixed-effects parameters (β_1, β_2) define the mean curve shape properties across all sites (anamorphism). Given the wide range of site conditions encountered in these data, it is plausible to assume that curve shape truly varies among sites; however, for some species/groups, it was not detected from a statistical standpoint. This could be due to factors such as sample size or simply that anamorphic curves adequately describe the site differences in the population. Regardless, the β_3 parameter estimate was retained in model application for all species/groups. As expected, a cursory analysis showed that better predictive accuracy was attained through inclusion of β_3, even when it was not statistically significant.

For species/groups not used in the model-fitting exercise, assignment to an existing set of model coefficients provided an empirical basis for guidance on site index prediction when these species are encountered. This analysis is considered to be a substantial improvement over that of the SV article, which did not address the assignment of other species to existing coefficients. The results presented a potentially contentious issue regarding broad species classifications, i.e., hardwood and softwood. As noted earlier, foresters are often not

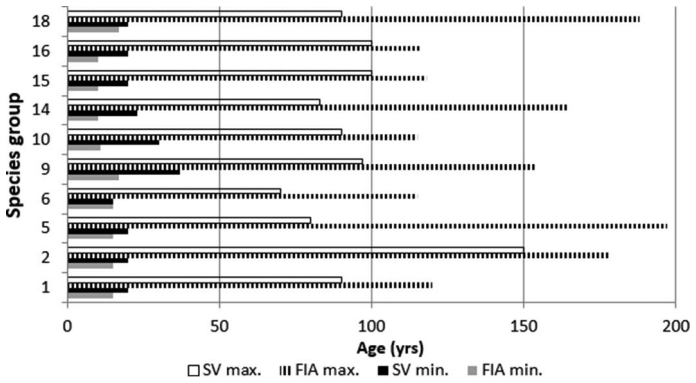

Figure 3. Maximum and minimum observed stand age for Scott and Voorhis (1986) (SV) and the data (FIA) used in this study.

inclined to intermingle these two species classes for a number of reasons (growth form differences being the primary objection). However, in the context of modeling height as a function of age, there may be less opposition as the primary focus is on height/age trajectories that could be similar for some hardwood and softwood species. The general recommendation is to use the unconditional results as those assignments provided the best fit to the data based on the minimum MSE criteria; however, it was notable that the conditional results for group 30 had only slightly larger MSE and also eliminated the prediction bias. The conditional results may be applied if strict adherence to hardwood/softwood classifications is desired.

The results shown in Figure 2 suggest that application of the models developed in this study may produce substantially different predictions of site index than SV values. As noted earlier, these differences may be attributable to a combination of altered environmental conditions, the area of geographic coverage in the data, and/or the range of site quality/age combinations. Although environmental and geographic variations would be difficult to quantify, an examination of stand ages used in each study is straightforward. With the exception of group 6 (Virginia pine), the minimum stand ages used in this study were younger than those in the SV data (Figure 3). The maximum stand ages found in the SV data were often considerably younger than those encountered across the breadth of FIA plots. For example, 5 of the 10 groups examined had maximum stand age exceeding 150 years in the FIA data, whereas the corresponding maximum ages in the SV data were less than 100 years. The most dramatic difference was for group 5 (eastern white pine), for which the FIA maximum age was 197 years compared to the SV maximum of 80 years (Figure 3). As such, application of the SV models to current FIA data resulted in considerable extrapolation beyond the range of the SV age data and possibly imparted biased estimates of site index for some stands.

In addition to changes in actual site index values resulting from using these models in place of those from SV, there can be a ripple effect as data processing and compilation routines that depend on SI may produce different results, e.g., subsequent land productivity classifications (O'Connell et al. 2015). These differing results can affect current estimates of forest resources as well as estimates of change over time. Potentially, there could also be an effect on individual tree volume predictions if the volume models used site index as an input variable (Hahn and Hansen 1991). There could also be nontrivial effects on simulation systems such as those that project future conditions. For example, it is not uncommon for FIA data to be transformed for use in the Forest Vegetation Simulator (FVS). In particular, the Northeast variant of the FVS utilizes site index information in tree growth models, and thus changes in SI will translate into different tree growth rates and subsequent projections of future conditions (Dixon and Keyser 2008). The full extent of the implications for new SI predictions in existing data is unknown, and the sensitivity of systems that use SI information should be fully evaluated.

The methods described in this article followed the traditional approach to estimating site quality via height and age information. Alternative strategies have been investigated, including the use of climate, physiographic, and other environmental factors (Bontemps and Bouriaud 2014). These methods may require additional stand information and/or information sources other than inventory data. Further research is needed to evaluate whether the use of these alternative predictors can provide predictions of similar or superior accuracy as those derived from height/age data.

Conclusion

The models developed in this study should provide representative predictions of site index for forests in the Northeastern United States. The geographic extent of forests in the region was encompassed, which included a wide range of stand height/age combinations and site index tree species coverage such that model extrapolations should be few in application. Future implementation of these models in lieu of the SV (Scott and Voorhis 1986) models for FIA data in the Northeastern United States will result in a smaller range of SI values across the region. The subsequent effects on data compilation processes and growth projections that use these values are currently unknown and require additional investigation specific to each system. Although changes in existing data due to prediction model differences can be disconcerting, it is proposed that these changes will more accurately reflect the site productivity of forests across the region.

In recent years, there has been an expanding body of research on how forests are affected by various environmental influences. Although some studies have focused on specific agents, it is often difficult to realize how various interrelated factors ultimately affect site quality and inherent stand development processes. For example, the combined effects of precipitation patterns and amounts, temperature regimes, nitrogen deposition, and ambient CO_2 can have various effects on forest ecosystems that are difficult to quantify holistically. Further, the magnitude of dominant height growth response to these factors is unknown. As such, FIA should consider remeasurement of site index trees to afford ongoing assessment of potential changes in forest productivity over time. Such an effort would also provide additional data to further test the long-term applicability of the models presented in this article.

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