



Estimating change in annual timber products output using a stratified sampling with certainty design

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Abstract

A key aspect in understanding patterns in wood demand and harvesting activities is monitoring of timber products output by wood processing facilities. Estimation of change from year-to-year is necessary but is complicated due to shifts in the population as well as changing strata over time. Taking independent samples each year eases complexity, yet suffers from relatively large sampling error in comparison to other designs that take advantage of the covariance arising from correlated samples. In this study, a design intended to maximize the precision of the change estimate by retaining the initial sample to the extent possible was analyzed. Several approaches to estimating the covariance, with the primary challenge being that sometimes only a single sample unit occurred in both samples within a given stratum. Variance underestimation and overestimation were encountered depending on the covariance method. The best outcome was attained using a measure-of-size variable at the population level to approximate the covariance. However, this approach overestimated the variance by 11% in a Monte Carlo simulation. The simulation results suggested a 14% reduction in the standard error of the estimate was attainable from correlated samples relative to independent samples. Due to the challenges introduced for estimating the covariance for changing populations and strata over time, the value of relatively small reductions in sampling error need to be considered in the context of introducing complex and potentially unreliable covariance estimation methods.

Keywords Covariance · Finite population · Roundwood consumption · Tree harvest

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1 Introduction

Monitoring of timber products output (TPO) is a key aspect of understanding current forest composition and structure and for projections of future forest condition and trends (Chen et al. 2015; Wear et al. 2016). There are numerous drivers that affect amounts and types of outputs, including resource availability, material costs, labor costs, competition, opportunity costs (choosing which product to manufacture), and product demand—all of which may be subject to regional, national, and international influences (Nanang 2010; O'Brien and Bringezu 2018). The complex interactive effects of these factors emphasize the need for continual TPO monitoring efforts to accurately assess market roundwood demand in a timely manner such that management and policy responses can be effectively formulated and implemented (du Toit et al. 2018; Alberdi et al. 2016). A primary example of a significant shift in TPO was the substantial decrease in roundwood consumption associated with broader economic decline in 2008–2009 (Pepke 2010; Howard and Westby 2009). Ongoing TPO monitoring was also instrumental in capturing the magnitude and timing of the subsequent forest products industry resurgence (Brandeis and Hodges 2015; Howard and McKeever 2016).

These trends in forest product markets also influence tree harvest patterns and resultant effects on forest composition and structure (Luppold and Pugh 2016; Chaudhary et al. 2016). Trees of desired species and size characteristics needed for production of wood products are more likely to be actively managed (Crow et al. 2002). For naturally regenerated multi-species stands, active management creates disturbances that can alter typical development regimes and produce both short- and long-term effects on species assemblages and tree size distributions (Angers et al. 2005; Hall et al. 2003). Stands in planted forests are often monocultures managed for specific products; thus the species diversity and structural characteristics are heavily controlled from the outset (Rubilar et al. 2018). In either case, it is clear that substantial manipulation of forested lands is done in response to the supply demands of the forest product sector.

In the U.S., TPO monitoring has been conducted since 1948 by the Forest Inventory and Analysis (FIA) program of the U.S. Forest Service (Bentley and Johnson 2011). The methods used to construct TPO estimates have varied in subsequent decades; with the most recent approach relying on a census of all primary wood using mills/facilities—defined as those processing roundwood in log, bolt, or chipped form (Bentley and Johnson 2011). From each mill, information is gathered on wood volumes received by product (e.g., saw log, pulpwood, veneer logs, poles, and logs used for composite board products), species group (i.e., hardwood, softwood), and county of origin. Because pulp mills tend to be of large size and few in number, these facilities are censused annually, while manufacturers of other products are censused on a periodic basis with varying frequencies. To move toward a more efficient and consistent approach to TPO monitoring, FIA implemented a stratified sample-based approach beginning in 2019. This change will increase efficiency as fewer mills will need to be contacted; in particular the effort to sample the numerous small mills will decrease considerably. Improved consistency is gained via annual data collection, analysis, and reporting instead of periodic assessments with varying time intervals.

The methods for constructing point-in-time TPO estimates under the stratified sample design are described in Coulston et al. (2018).

Ongoing implementation of the prescribed design would entail drawing an independent sample each year. While this approach is considered desirable for the change estimate as changes in the population composition would be accounted for, the precision of the estimate is likely to be poor. An alternative approach would be to remeasure as many sample units as possible as this maximizes the precision of change estimates where positive correlation between samples is present. Thus, it is worthwhile to evaluate the potential gain in precision of change estimates under an alternative sample design where the mills selected into the initial sample are retained for measurement in the successive sample. In this scenario, the primary issue is adequately estimating the covariance for two correlated samples (Wood 2008; Nordberg 2000). The procedures can be fairly straightforward in relatively simple sampling environments, however difficulty increases considerably for more complex scenarios (Knottnerus and van Delden 2012). In particular, finding appropriate and well-behaved variance estimators can be elusive when populations and/or strata fluctuate in successive measurements (Schreuder et al. 1993; Berger 2004). In the context of a sample-based TPO monitoring program, the population of mills changes year-to-year as some existing mills close and new mill operations are established. Thus, the population elements available for selection into the sample change over time. These population shifts also can affect the construction of strata used to conduct the sampling effort, resulting in some mills having different stratum assignments over time.

As key outputs of a TPO monitoring program are estimates of change, accounting for shifts in populations and strata is of considerable importance to making valid inferences, i.e., providing interval estimates of change that reflect the uncertainty present due to sampling. The proposed sample design for the U.S. includes both fully enumerated and sampled strata (Coulston et al. 2018), where contributions to the estimated variance only arise from sampled strata. When some of the same mills are purposely retained for remeasurement, there is a need to account for the covariance between sampled stratum means when the same mill(s) appears in two successive samples (Berger 2004). An issue arises when only one mill is in common between strata, which precludes the use of the traditional covariance estimator that requires at least two observations (Knottnerus and van Delden 2012). Considering these circumstances, the objectives of this study are: (1) develop the overall statistical framework for estimating change from successive TPO samples, (2) outline a method for combining strata that are dynamic over time, (3) evaluate various approaches to estimating the covariance that addresses some strata having only a single element in common to correlated samples, and (4) assess tradeoffs in precision of change estimates between independent and correlated sampling strategies.

2 Methods

2.1 Data

Two TPO datasets covering 12 states in the southeastern U.S. were used in this study (Fig. 1). The first arises from the 2011 TPO census of this area which collected data from $N = 1363$ mills. Information from each mill included amount of roundwood consumed by county of origin and primary output product. The second dataset is similar to that of 2011, but contains data collected in 2013 from $N = 1374$ mills (Table 1). For this study, the intended sampling rate was 40% of the population, which corresponds approximately with the level of effort used in previous TPO studies in the region (Johnson et al. 2011). To approximate this sampling goal, strata were developed within each state and primary product group using a measure-of-size (MOS) variable (Coulston et al. 2018). MOS is an approximate indicator of mill output, which in practice is obtained from prior mill receipt data or other information sources. For the purposes of this study, the MOS for each mill in the 2011 dataset was based on the 2013 consumption and vis-versa for the 2013 dataset. The MOS for mills that were only active in 2011 or 2013 were based on a statistical model. Each mill in the population was assigned to a single stratum. All mills with a MOS greater than ten million cubic feet of roundwood consumption were sampled with certainty. In these data, all pulpwood mills and composite panel mills were sampled with certainty from strata having either 1 or 2 mills (Table 1). For the remaining product classes, strata containing only 1 or 2 mills were sampled with certainty, whereas a sample of size 2 was taken in strata having at least 3 mills.

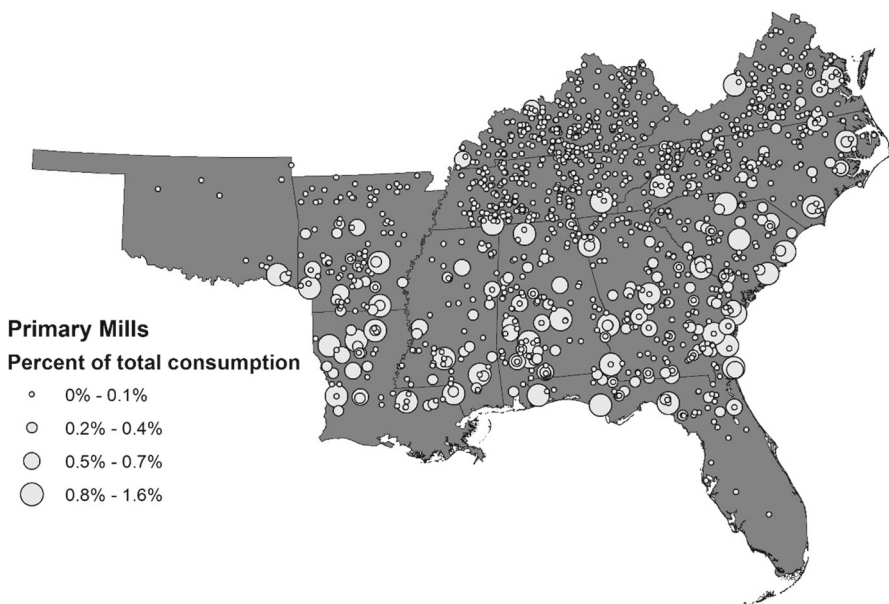


Fig. 1 Primary mill locations and relative size in the twelve-state study area of the southeastern United States

Table 1 Summary statistics for mill receipts (MCF = 1000 cubic feet) in the 2011 and 2013 mill populations across a 12-state area in the Southern U.S.

Primary product	Mill receipts (MCF)			# Strata		
	# Mills	Mean	Std. dev	Sampled	Enumerated	Total
2011						
Bioenergy/Fuelwood	42	3575.72	5692.90	2	28	30
Composite panel	20	14,927.36	8673.85	0	20	20
Miscellaneous	48	707.48	831.93	6	11	17
Poles	44	1140.28	905.72	7	14	21
Posts	22	401.38	1140.38	1	17	18
Pulpwood	76	48,222.12	24,727.94	0	74	74
Saw logs	1055	2022.19	4100.27	103	171	274
Veneer logs	56	6956.60	7825.49	6	27	33
All products	1363			125	362	487
2013						
Bioenergy/Fuelwood	56	4796.38	7652.93	6	20	26
Composite panel	22	17,424.48	9501.40	0	22	22
Miscellaneous	45	760.77	911.33	6	11	17
Poles	44	1279.58	992.56	8	14	22
Posts	21	385.17	972.41	1	16	17
Pulpwood	77	46,887.88	23,443.72	0	76	76
Saw logs	1060	2203.98	4523.27	107	165	272
Veneer logs	49	7714.39	8485.26	4	29	33
All products	1374			132	353	485

There were 1300 mills in common to both populations

2.2 Estimation

Assessing change in a population parameter is often accomplished by evaluating observed data at two points in time. Two primary methods are used: (1) finding the difference between population estimates generated at each time point, or (2) calculating the differences for each sample unit and directly estimating the difference. The former approach has the desirable property of the difference estimate matching the difference between the two time-point estimates, but requires explicit calculation of the covariance between two samples if they are not independent. The latter approach affords more computational simplicity as the covariance is implicitly accounted for; however, the direct difference estimate and the time point difference are usually unequal due to various factors. The former approach was used in this study as there is full utilization of all observations in each sample, whereas the latter approach can only use sample units having an observation at both time points.

2.3 Time 1

For the initial (T1) sample, the population is stratified into $h = 1$ to H strata using the measure of size (MOS) protocol described in Coulston et al. (2018). Each stratum h has a population size of N_h and a corresponding sample size denoted n_h . Strata containing only one or two population elements ($N_h = 1, 2$) are completely enumerated ($n_h = N_h$). Strata having $N_h \geq 3$ population elements are subjected to a simple random sample (SRS) of $n_h = 2$. As each mill receives roundwood from one or more sources, the observed value for the mill is the sum of all sources. In notational form, this value is y_{ih} = sample observation from mill i in stratum h . The unbiased estimators of total roundwood consumption and associated variance for stratum h are

$$\hat{Y}_h = N_h \frac{\sum_{i=1}^{n_h} y_{ih}}{n_h} \quad (1)$$

$$v(\hat{Y}_h) = (1 - f_h) N_h^2 \frac{\sum_{i=1}^{n_h} (y_{ih} - \bar{y}_h)^2}{n_h(n_h - 1)}, \quad (2)$$

where $f_h = n_h/N_h$ is the finite population correction factor for stratum h (non-zero variances are only from sampled strata) and $\bar{y}_h$ is the sample mean of stratum h .

The estimated population total roundwood consumption at Time 1 is calculated from

$$\hat{T}_1 = \sum_{h=1}^H \hat{Y}_h \quad (3)$$

and the variance estimator for the estimated total is

$$v(\hat{T}_1) = \sum_h^H v(\hat{Y}_h). \quad (4)$$

The percent standard error is obtained from:

$$SE\%(\hat{T}_1) = \frac{\sqrt{v(\hat{T}_1)}}{\hat{T}_1} 100 \quad (5)$$

Generally, the sampling design was conceived to provide relatively small variance. This is due, in part, to the commonly encountered phenomenon that the stratum variance increases as the magnitude of the mean response increases (e.g., Figure 2 in Coulston et al. 2018). In the context of this study, strata composed of large mills would tend to have relatively large variances compared to strata having smaller mill sizes. Thus, the strata with large mean values are (1) sampled with certainty, which nullifies the potentially large contribution to the variance that would occur if the strata were sampled, or (2) are subject to large sampling fractions (e.g., $N_h = 3$ and $n_h = 2$)

where there is a considerable effect of the finite population correction factor (see (2)) in reducing the variance.

2.4 Time 2

For a successive Time 2 (T2) sample, the same estimation process as Time 1 is used; noting that the T2 population being sampled may have changed since T1. Also, the MOS for mills in the T2 population that were also in the T1 population may be different. These MOS differences and the addition of MOS for new mills in the T2 population will alter the MOS distribution from that found at T1, thus it is likely the T2 stratification will result in differing assignments of population elements to strata in comparison to T1. Thus, the notation is slightly altered to indicate the T2 stratification, in which $l = 1$ to L strata are constructed. The stratum-level estimators analogous to (1) and (2) above are

$$\hat{Y}_l = N_l \frac{\sum_{i=1}^{n_l} y_{il}}{n_l} \quad (6)$$

$$v(\hat{Y}_l) = (1 - f_l) N_l^2 \frac{\sum_{i=1}^{n_l} (y_{il} - \bar{y}_l)^2}{n_l(n_l - 1)} \quad (7)$$

It is straightforward to proceed to the population total estimator $\hat{T}_2$ and its variance estimator $v(\hat{T}_2)$ by summing across strata as in (3) and (4).

2.5 Time 2 – Time 1

Repeated sampling of the population over time provides the ability to estimate change. Given the formulae shown above, the estimated difference $\hat{\Delta}_{\hat{T}_2 - \hat{T}_1}$ is:

$$\hat{\Delta}_{\hat{T}_2 - \hat{T}_1} = \hat{T}_2 - \hat{T}_1 \quad (8)$$

The variance expression for the $\hat{\Delta}_{\hat{T}_2 - \hat{T}_1}$ estimate can be complex due to the potential need to account for the nonzero covariance that may occur when the same sample units are present in both T1 and T2 samples. In the context of stratified estimation, estimates from stratum h at T1 may be correlated with estimates from stratum l at T2. When the samples at two points in time are selected independently there is no covariance by definition and the presence of in-common sample units is due to random chance. However, for the sampling scenario that exploits remeasurement of the same mills to maximize the precision of change estimates (denoted MP), mills sampled at T1 and still in the population at T2 are included in the T2 sample. In some cases, sampled T2 strata may only have one mill due to closure of mills from the T1 sample. To obtain $n_l = 2$ for these strata, an additional mill is randomly chosen from the population of mills for each stratum. Once the sample units for the T2 stratification are established, a new stratification scheme is developed that incorporates the existing strata from T1 and T2 (Schreuder et al. 1993). A key property of this new stratification is the retention of

consistency with the T1 and T2 estimations, i.e., the same results are given using either the original or new stratification. The new stratification is based on the combinations of T1 and T2 strata where the new stratification contains $m = 1$ to M strata with $M \geq \max(H, L)$. Estimates of covariance will need to be calculated for sampled strata having mills in common to both samples. The covariance equals zero for strata that were sampled with certainty or where no sample unit overlap occurs.

Computational solutions broadly applicable to nonzero covariance situations are needed. In cases where the same two sample units comprise a sampled stratum at both T1 and T2, it is straightforward to calculate the covariance using the standard formula (e.g., Schreuder et al. 2004, p. 16). However, under a paradigm of $n_h = n_l = 2$ and a changing population that precludes repeating the exact same sample at T2, it will sometimes be encountered that only one sample unit is in common. In this case, the parent stratum is divided into two revised sub-strata—one containing the common sample unit and the other having the remaining sample unit that only appears in either the T1 or T2 sample. Using T1 as an example, the population size of a split stratum ($N_{h'}$) is approximated from the parent population stratum size (N_h) and the ratio of the number of sample units in the revised strata ($n_{h'}$) to the number of sample units in the parent stratum (n_h):

$$N_{h'} = N_h \frac{n_{h'}}{n_h} \quad (9)$$

and analogously for the revised T2 stratum sizes

$$N_{l'} = N_l \frac{n_{l'}}{n_l} \quad (10)$$

When no sample units from a T1 stratum are selected into a T2 stratum, the adjusted sizes are zero; when all sample units in a T1 stratum comprise a T2 stratum, the original stratum sizes are retained, i.e., $N_{h'} = N_h$, $N_{l'} = N_l$. This is straightforward in the case with only 1 sample unit in common as the result is an equal division of the parent stratum size, i.e., $n_{h'} = n_{l'} = 1$ and $n_h, n_l = 2$.

An example of the revised stratification derived from the original T1 and T2 samples and stratification is shown in Table 2. For both the T1 and T2 samples, each stratum has $n_h = n_l = 2$ and population sizes N_h and N_l , respectively; with overall population sizes of $N = 37$ for T1 and $N = 61$ for T2. There were nine mills in the T2 sample that also occurred in the T1 sample and these are identified by having a stratum assignment at both times. For these situations, the revised stratification for change estimation divides these based on the formulae above, and this division results in $M = 14$ total strata having samples of size n_m . For the MP design in which mills from the T1 sample are retained when still in the T2 population, calculation of a nonzero covariance will be needed for $m = 3, 4, 5, 6, 7, 8$, and 9 (Table 2).

When the revised stratum only has one sample observation, typical formula-based estimation of the stratum sample variance is precluded. Under the paradigm of enforcing consistent estimations with the original T1 and T2 stratifications, the revised strata retain the original sample variance from the parent stratum. Thus, the calculation

Table 2 Example for T1 and T2 samples and their aggregation into $M = 14$ change strata with revised sample (n_m) and population ($N_{h'}$, $N_{l'}$) sizes

Mill #	Time 1		Time 2		Time 2 – Time 1			
	Stratum h	N_h	Stratum l	N_l	Stratum m	n_m	$N_{h'}$	$N_{l'}$
1	1	2	1	2	1	2	2.0	2.0
2	1	2	1	2	1	2	2.0	2.0
3	2	3	–	–	2	1	1.5	0.0
4	2	3	2	3	3	1	1.5	1.5
5	3	5	3	3	4	1	2.5	1.5
6	3	5	4	4	5	1	2.5	2.0
7	4	8	4	4	6	1	4.0	2.0
8	4	8	5	9	7	1	4.0	4.5
9	5	19	6	12	8	1	9.5	6.0
10	5	19	7	28	9	1	9.5	14.0
11	–	–	2	3	10	1	0.0	1.5
12	–	–	3	3	11	1	0.0	1.5
13	–	–	5	9	12	1	0.0	4.5
14	–	–	6	12	13	1	0.0	6.0
15	–	–	7	28	14	1	0.0	14.0

pertinent to the revised stratification is to estimate the covariance term needed for estimating the variance of the estimated difference in stratum m .

The variance of $\hat{\Delta}_{\hat{T}_2 - \hat{T}_1}$ is expressed generally as:

$$v(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1}) = \sum_{h=1}^H N_h^2 E(\bar{y}_h - \bar{Y}_h)^2 + \sum_{l=1}^L N_l^2 E(\bar{y}_l - \bar{Y}_l)^2 - 2 \sum_{m=1}^M N_m^2 E(\bar{y}_m - \bar{Y}_m)^2 \quad (11)$$

The first term of (11) can be shown as $\sum_{h=1}^H N_h^2 S_h^2 (1 - \frac{n_h}{N_h}) / n_h$ with $S_h^2 = \sum_{i=1}^{N_h} (y_{ih} - \bar{Y}_h)^2 / (N_h - 1)$, the second term analogously as $\sum_{l=1}^L N_l^2 S_l^2 (1 - \frac{n_l}{N_l}) / n_l$ with $S_l^2 = \sum_{i=1}^{N_l} (y_{il} - \bar{Y}_l)^2 / (N_l - 1)$, and the last term being equivalent to $\sum_{m=1}^M N_m^2 S_m^2 (1 - \frac{n_m}{N_m}) / n_m$ with $S_m^2 = \sum_{i=1}^{N_m} (y_{im} - \bar{Y}_m)^2 / (N_m - 1)$. The definition of terms is generalized using ‘.’ notation where . refers to h, l , or m to indicate the different stratifications and mill assignments to strata. Subsequently, $\bar{Y}.$ = population mean in stratum . and $y_{i.}$, $n.$, and $N.$ are as previously defined. The estimate of variance is obtained by substituting the sample means $\bar{y}.$ for $\bar{Y}.$ and sample sizes $n.$ for $N.$ in the calculations of s_h^2 , s_l^2 , and s_m^2 that indicate sample-based estimates of these parameters and lead to the overall $v(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1}) = v(\hat{T}_1) + v(\hat{T}_2) - 2cov(\hat{T}_1, \hat{T}_2)$.

The third term of [11] corresponds with $\sum_{m=1}^M N_m^2 s_m^2 (1 - \frac{n_m}{N_m}) / n_m = \sum_{m=1}^M \text{cov}(\hat{T}_{1m}, \hat{T}_{2m}) = \text{cov}(\hat{T}_1, \hat{T}_2)$ where calculations are required for strata having sample units in common under the MP sampling approach. To simplify notation at the stratum level, these calculations will be referred to as $\text{cov}(\hat{T}_{1m}, \hat{T}_{2m})$ hereafter. Calculation of $\text{cov}(\hat{T}_{1m}, \hat{T}_{2m})$ can be a complex undertaking as the selection probability of a given sample unit may be different from T1 to T2 due to differences in N_h and N_l . One method to approximate the covariance component is to use the simple random sample (SRS) formulation from Wood (2008) to obtain an unbiased estimate of $\text{cov}(\hat{T}_{1m}, \hat{T}_{2m})$ within each stratum m .

$$\text{cov}_W(\hat{T}_{1m}, \hat{T}_{2m}) = \frac{n_c \left(1 - \frac{\pi_2}{\pi_{2|1}}\right)}{\pi_1 \pi_2 \left(1 - \frac{\pi_1 \pi_{2|1}}{\pi_2 n_1}\right)} \left\{ (\bar{y}_1 \bar{y}_2)_c - \frac{\pi_1 \pi_{2|1} n_{2(1)}}{\pi_2 n_c} \bar{y}_{s_1} \bar{y}_{s_{2(1)}} \right\} \quad (12)$$

The right-hand-side variables, whose values are specific to stratum m , are $\pi_1 =$ first-order inclusion probability for a sample unit at T1 ($= n_h/N_h$), $\pi_2 =$ first-order inclusion probability for a sample unit at T2 ($= n_l/N_l$), $\pi_{2|1} =$ the conditional inclusion probability that a sample unit is selected at T2 given that it was selected at T1, $n_l =$ number of samples at T1, $n_c =$ number of sample units in common to T1 and T2, $n_{2(1)} =$ number of T2 sample units present in the T1 population, $\bar{y}_{s_1} =$ mean of the T1 sample, $\bar{y}_{s_{2(1)}} =$ mean of T2 sample units present in the T1 population, and $(\bar{y}_1 \bar{y}_2)_c =$ mean crossproduct of samples in common to T1 and T2. As the ratios $\frac{\pi_2}{\pi_{2|1}}$ and $\frac{\pi_1 \pi_{2|1} n_{2(1)}}{\pi_2 n_c}$ cannot be expressed using known population and sample sizes, they are estimated using the Hidiogrou et al. (1995) approximation for $\frac{\pi_2}{\pi_{2|1}} = \frac{n_h n_l N_c}{N_h N_l n_c}$ and $E\left(\frac{\pi_1 \pi_{2|1} n_{2(1)}}{\pi_2 n_c}\right) = 1$ (Wood 2008) to result in the following computational form to estimate the within-stratum covariance:

$$\text{cov}_W(\hat{T}_{1m}, \hat{T}_{2m}) = \frac{n_c \left(1 - \frac{n_h n_l N_c}{N_h N_l n_c}\right)}{\frac{n_h n_l}{N_h N_l} - \frac{n_c}{N_h N_c}} \left\{ (\bar{y}_1 \bar{y}_2)_c - \bar{y}_{s_1} \bar{y}_{s_{2(1)}} \right\}, \quad (13)$$

where $N_c =$ number of population units in common to T1 and T2. Strict unbiasedness of (13) is achieved only under certain sample unit rotation schemes, however, only a small amount of bias is expected for most applications (Wood 2008). Note the estimator (13) is computationally tractable for the case of $n_m = 1$. Having the stratum covariance from (13), the variance estimator for each stratum m was:

$$v_W(\hat{\Delta}_{\hat{T}_{2m} - \hat{T}_{1m}}) = v(\hat{T}_{1m}) + v(\hat{T}_{2m}) - 2\text{cov}_W(\hat{T}_{1m}, \hat{T}_{2m}) \quad (14)$$

and the corresponding overall variance is calculated as:

$$v_W(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1}) = \sum_{m=1}^M v_W(\hat{\Delta}_{\hat{T}_{2m} - \hat{T}_{1m}}) \quad (15)$$

An alternative may be to use the usual SRS covariance formula; however, valid calculation requires a minimum of $n_m = 2$. To circumvent this problem when $n_m = 1$, it is often suggested that data be borrowed from ‘nearby’ strata to increase the sample size (Singh and Sedransk 1988). The most simplistic implementation of this approach is to perform the calculation using all sample elements in common across all sampled strata. In this case, it is sensible to estimate the overall sample correlation (ρ) as:

$$\rho = \frac{cov(T_{1(s)}, T_{2(s)})}{\sqrt{v(T_{1(s)})}\sqrt{v(T_{2(s)})}}, \quad (16)$$

where $T_{1(s)}$ and $T_{2(s)}$ refer to the respective mill receipt observations from units occurring in both the T1 and T2 samples, $v(T_{1(s)}) = \sum_{i=1}^{n_c} (y_{1i} - \bar{y}_1)^2 / (n_c - 1)$, $v(T_{2(s)}) = \sum_{i=1}^{n_c} (y_{2i} - \bar{y}_2)^2 / (n_c - 1)$, and $cov(T_{1(s)}, T_{2(s)}) = \sum_{i=1}^{n_c} (y_{1i} - \bar{y}_1)(y_{2i} - \bar{y}_2) / (n_c - 1)$. The calculated value of ρ is then used in the estimator:

$$v_\rho(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1}) = v(\hat{T}_1) + v(\hat{T}_2) - 2\rho\sqrt{v(\hat{T}_1)}\sqrt{v(\hat{T}_2)} \quad (17)$$

When ρ is obtained in a manner consistent with the sample design, the estimator in (17) is unbiased (Gregoire and Valentine 2008, p. 238). However, in this case, the underlying stratified design was ignored and thus an implicit bias in the estimator is introduced. Further, the reliability of the estimated ρ may be questionable, depending on the number of units in common to both the T1 and T2 sampled strata. A more reliable estimate of correlation may be obtained by using the population elements in common sampled strata. This approach would require a variable containing information for all population elements and which has high correlation with the attribute of interest (mill receipts). In this study, MOS meets these criteria and could serve as a proxy for improved estimates of correlation based on more observations. Further, the increase in observations allows for implementation at the stratum level. Similar to (16), ρ_{MOS} for sampled strata would arise from:

$$\rho_{MOS} = \frac{cov(T_{1m(p)}, T_{2m(p)})}{\sqrt{v(T_{1m(p)})}\sqrt{v(T_{2m(p)})}}, \quad (18)$$

where $T_{1m(p)}$ and $T_{2m(p)}$ refer to the respective MOS from units occurring in both the T1 and T2 population for sampled strata.

An implementation of ρ_{MOS} would be to implement the unbiased variance estimators for calculation of the stratum variances (19) and the subsequent overall variance (20):

$$v_{MOS}(\hat{\Delta}_{\hat{T}_{2m} - \hat{T}_{1m}}) = v(\hat{T}_{1m}) + v(\hat{T}_{2m}) - 2\rho_{MOS}\sqrt{v(\hat{T}_{1m})}\sqrt{v(\hat{T}_{2m})} \quad (19)$$

$$v_{MOS}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1}) = \sum_{m=1}^M v_{mMOS}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1}) \quad (20)$$

2.6 Monte Carlo simulation

To evaluate the alternative variance estimation approaches described above, Monte Carlo simulations were conducted using the roundwood consumption data from the census of mills in 12 states across the southeastern U.S. during the years 2011 (T1, $N = 1363$) and 2013 (T2, $N = 1374$) (Table 1). Estimations were evaluated for the maximum precision (MP) design that retains T1 mills to the extent possible. A total of $r = 1$ to 5000 replications were conducted. At each iteration, an approximate 40% sample was drawn from both the T1 and T2 populations, wherein all strata having $N_h = N_l \leq 2$ were fully enumerated and strata with $N_h = N_l > 2$ were sampled randomly such that $n_h = n_l = 2$. The simulation output consisted of estimated change across all product categories combined and associated estimates of variance. For comparative purposes, the variance for an independent samples scenario where $cov(\hat{T}_1, \hat{T}_2) = 0$ was calculated as $v_I(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1}) = v(\hat{T}_1) + v(\hat{T}_2)$. The MP design was evaluated using the T2 strata assignments and drawing randomly as needed from other T2 mills in the same stratum to attain $n_l = 2$ for sampled strata. This was necessary to account for mills in the T1 sample that had dropped out of the T2 population. Also, changes in the stratification occasionally produced $n_l > 2$ and T1 sample units needed to be randomly dropped from the sample to maintain $n_l = 2$. Under this sampling paradigm, three variance estimators were evaluated, namely $v_W(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$ (15), $v_\rho(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$ (17), and $v_{MOS}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$ (20). Upon completion of 5000 replications, the mean values of $v_W(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$, $v_\rho(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$, $v_{MOS}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$, and $v_I(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$ were calculated along with the Monte Carlo (MC) variance of the distribution of the change estimates

$$v_{MC}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1}) = \frac{\sum_{r=1}^{5000} [(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})_r - (\bar{\Delta}_{\hat{T}_2 - \hat{T}_1})]^2}{4999}, \quad (21)$$

where $\bar{\Delta}_{\hat{T}_2 - \hat{T}_1}$ is the mean estimated change value over the 5000 MC replications. The MC percent sampling error is:

$$SE\%(\bar{\Delta}_{\hat{T}_2 - \hat{T}_1}) = \frac{\sqrt{v_{MC}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})}}{\bar{\Delta}_{\hat{T}_2 - \hat{T}_1}} 100 \quad (22)$$

3 Results and discussion

The motivation for calculating $v_{MC}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$ was to establish a basis for comparison with $v_I(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$, $v_W(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$, $v_\rho(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$, and $v_{MOS}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$. When no covariance was accounted for, $v_I(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$ was too large and had a corresponding sampling error of about 8.5%. Under the MP design, the three proposed approaches to estimating the variance, to wit $v_W(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$, $v_\rho(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$, and $v_{MOS}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$, exhibited discernable differences due to the various covariance approaches. The sampling errors were

approximately 8.5% using $v_W(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$, 6.5% using $v_\rho(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$, and approximately 7.6% based on $v_{MOS}(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$. The sampling error based on $v_{MC}(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ was 7.3%.

In terms of the actual variances of $v_I(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$, $v_W(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$, $v_\rho(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$, and $v_{MOS}(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$, the largest positive differences with $v_{MC}(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ were for $v_I(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ and $v_W(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ where the overall variance was overestimated by roughly 37% (also see discussion below). Results obtained for the other methods showed $v_\rho(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ underestimated by 22%, and $v_{MOS}(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ was 11% too large. These results suggest that ρ overestimated the true correlation, which may have occurred due to ignoring the underlying stratified sampling design. Conversely, ρ_{MOS} underestimated the correlation and suggests that using MOS as a proxy for mill receipt observations may produce unsatisfactory results. The average ρ and ρ_{MOS} from the simulation were 0.81 and 0.28 respectively. Using $v_{MC}(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$, $v(\widehat{T}_1)$, and $v(\widehat{T}_2)$, an approximate population correlation can be empirically calculated as 0.52. That is, correlations near this value would result in close agreement between $v_{MC}(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ and the formula-based approaches.

In addition to the relatively large overestimation, estimates of $v_W(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ where highly erratic from sample to sample. One issue was the occurrence of some $v_W(\widehat{\Delta}_{\widehat{T}_{2m} - \widehat{T}_{1m}}) < 0$, which caused ultimately $v_W(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1}) < 0$ for about 8% of the samples drawn in the simulation. Obtaining variance estimators that consistently provide positively-valued results has been an on ongoing challenge in population/strata change scenarios (Berger 2004; Nordberg 2000). A related issue was the occurrence of extremely large values of $v_W(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$; nearly 3% of these values would be considered outliers based on a Median + 1.5 $\times$ IQR (Interquartile Range) threshold (Tukey 1977). The resulting distribution of $v_W(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ across the 10,000 samples in the simulation was positively skewed such that the mean of $v_W(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ was exceedingly large compared to $v_{MC}(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ and was essentially the same as $v_I(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$. Although the Wood (2008) covariance estimator can accommodate the $n_m = 1$ circumstance that often occurs under the prescribed sampling paradigm, the $cov_W(\widehat{T}_{1m}, \widehat{T}_{2m})$ appear to be poorly estimated, prone to high instability, and can result in $v_W(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1}) < 0$. These factors suggest that, in general, too little information is provided by only one or two observations to make reliable covariance estimates. For these reasons, operational implementation of this approach is not recommended.

The underestimation of the variance using $v_\rho(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ suggests this approach cannot be endorsed. Further investigation into the performance of $v_\rho(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$, which is primarily predicated on the estimation of ρ , indicated a strong relationship with the stratification that arises from the level of sampling effort. Because the strata sample sizes are fixed, the sampling effort dictates the number and size of strata. For example, reducing the sampling effort produces fewer strata with larger N_m . Increases in N_m results in relatively fewer enumerated strata and more sampled strata. The converse is true when the sampling effort increases. Generally, more sampled strata produces more information upon which to estimate ρ . However, this does not ensure accurate estimation of ρ as both over and underestimation was encountered as the sampling effort varied. Thus, $v_\rho(\widehat{\Delta}_{\widehat{T}_2 - \widehat{T}_1})$ should not be considered a reliable method.

Although $v_{MOS}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$ overestimated the variance, it may be considered the best approach studied here in that it reflects the intuitive reduction in comparison to independent samples. $v_{MOS}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$ provides a conservative estimate in comparison to $v_{MC}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$; however, there is a sacrifice in that the overestimation will hamper findings of statistical significance and produce confidence intervals that are too wide. As expected, ρ_{MOS} was relatively stable to sampling effort changes in stratification as all the population values of MOS are used. An important consideration for using MOS is how well the MOS represents actual mill receipts. If the MOS and receipts are well-correlated, MOS-derived correlations between two samples should mimic well the mill receipt relationships. However, the strength of the correlations cannot be assessed until after the sample has been taken and reliance on the $v_{MOS}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$ estimator may be misguided unless careful attention is given to obtaining an accurate MOS for each mill.

A trade-off to consider is the larger variance under the simplicity of an independent sample paradigm in comparison to a smaller variance associated with the complexity of the MP design in a changing population/strata framework. Of particular note for the latter method is the need to estimate covariance in situations where only one correlated observation is present in a sampled stratum. This situation requires the use of non-standard approaches that may be difficult to implement operationally and/or may provide poor results. Also, in this study two consecutive time points were considered but other comparative scenarios such as T1 to T3 or T1 to T4 could be envisioned where further complications would arise in tracking population and strata changes across multiple time periods. In regard to sample unit selection, the MP approach is similar in concept to sampling with partial replacement and would be subject to the same difficulties of tracking sample units over time and the associated complexity in estimation methods (Scott 1986). Thus, implementation of the MP approach implies increasingly complex processes over time.

Other approaches to variance estimation could also be used, e.g., the bootstrap (Efron 1979) or jackknife methods (Tukey 1958); although any alternative methods would be subject to the same dilemmas such as small within-stratum sample sizes. Consider the simplest usage of the bootstrap approach for sampled strata having $n_m = 2$: only four resampling outcomes are possible, two of which would be identical and equal to the mean of the original sample. Given the bootstrap method would be drawing from a frame composed of two sample units, the bootstrap variance estimate would likely perform poorly under the proposed sampling design. A secondary concern for using resampling methods is the computational burden, which may produce unacceptable delays in delivering outputs in situations where a large amount of data are present or many estimates are being requested, e.g., for many combinations of classification variables.

While these results pertain to a real-world market outcome, is it useful to consider how other population structures and market trend scenarios may impact the results. One circumstance of interest would be different trend scenarios such as considerable decreases or increases in mill output (Woodall et al. 2012). These situations could be considered from a multiplier perspective on the T2 observations, e.g., a downturn of 20% would essentially be similar to multiplying the T2 values by 0.8 and similarly

by values larger than 1.0 to represent increased outputs. It seems that this would have little effect on the outcome as variances and covariances would scale accordingly and there is no reason that the estimators themselves would behave differently. The primary way by which the magnitude of the variances would be affected is to have more or less heterogeneity within sampled strata (changes in fully enumerated strata would have no effect). While theoretically possible, in practice it would be unlikely to occur systematically as strata are constructed within product categories and mill state location. Thus, owing to the same product and close geographic proximity, it is unlikely that mills in the same stratum would be disparately affected by market conditions.

4 Conclusion

The shift in TPO data collection from a periodic census to an annual sample allows for timelier monitoring of TPO trends. A primary difference is the sample-based approach, which requires specification of a sampling design and associated estimators for population parameters such as total roundwood consumption. Estimation of change resulting from repeated measurements must also conform to the sample design, where proper estimation of variances is needed to make valid inferences. The use of independent samples simplifies variance estimation, at the cost of poorer precision relative to designs that take advantage of correlated observations. Assuming $v_{MC}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$ adequately represents the variance, increases in precision were found under the design that maximized remeasurement of the initial sample; however considerable complexity was encountered as the population and strata therein change over time as well as the inability to estimate covariance via standard methods. For the proposed MP design, the best approach to approximating the covariance was based on the $v_{MOS}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$ formulation with the resultant 11% overestimation in comparison to $v_{MC}(\hat{\Delta}_{\hat{T}_2 - \hat{T}_1})$.

For ongoing efforts such as TPO monitoring, annual sampling and reporting requirements often make it necessary to rely on automated data analysis processes. Processing algorithms must be robust under a wide range of input circumstances. The covariance estimation approaches investigated in this study bring a higher level of operational complexity that may be challenging to incorporate into automated processes in an efficient and reliable manner. Thus, imposition of monitoring designs that effectively subvert the covariance complication may be particularly appealing from an implementation perspective. Selecting independent samples each year is considerably different in degree of implementation difficulty in comparison to the MP design. In summary, additional research is needed to further explore other methods that more accurately reflect the true variance, increase precision over independent samples, and do not incur substantial additional complexity in analytical processes.

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References

- Alberdi I, Michalak R, Fischer C, Gasparini P, Brändli UB, Tomter SM, Kuliesis A, Snorrason A, Redmond J, Hernández L, Lanz A (2016) Towards harmonized assessment of European forest availability for wood supply in Europe. *Forest Policy Econ* 70:20–29
- Angers VA, Messier C, Beaudet M, Leduc A (2005) Comparing composition and structure in old-growth and harvested (selection and diameter-limit cuts) northern hardwood stands in Quebec. *For Ecol Manag* 217(2–3):275–293
- Bentley JW, Johnson TG (2011) Mississippi's timber industry—an assessment of timber product output and use, 2009. Resource bulletin SRS-181. USDA Forest Service, Southern Research Station, Asheville
- Berger YG (2004) Variance estimation for measures of change in probability sampling. *Can J Stat* 32(4):451–467
- Brandeis C, Hodges DG (2015) Forest sector and primary forest products industry contributions to the economies of the southern states: 2011 update. *J Forest* 113(2):205–209
- Chaudhary A, Burivalova Z, Koh LP, Hellweg S (2016) Impact of forest management on species richness: global meta-analysis and economic trade-offs. *Sci Rep* 6:23954. <https://doi.org/10.1038/srep23954>
- Chen W, Xu D, Liu J (2015) The forest resources input–output model: an application in China. *Ecol Ind* 51:87–97
- Coulston JW, Westfall JA, Wear DN, Edgar CB, Prisley SP, Treiman TB, Abt RC, Smith WB (2018) Annual monitoring of U.S. timber production: rationale and design. *Forest Sci* 64(5):533–543
- Crow TR, Buckley DS, Nauertz EA, Zasada JC (2002) Effects of management on the composition and structure of northern hardwood forests in Upper Michigan. *Forest Sci* 48(1):129–145
- du Toit B, Malherbe GF, Lambrechts H, Naidoo S, Eatwell K (2018) Market analysis to assess timber products from dryland woodlots and farm forests in South Africa. In: Revermann R, Krewenka KM, Schmiedel U, Olwoch JM, Helmschrot J, Jürgens N (eds) *Climate change and adaptive land management in southern Africa—assessments, changes, challenges, and solutions*. Klaus Hess Publishers, Göttingen and Windhoek
- Efron B (1979) Bootstrap methods: another look at the Jackknife. *Ann Stat* 7:1–26
- Gregoire TG, Valentine HT (2008) *Sampling strategies for natural resources and the environment*. Chapman and Hall/CRC, Boca Raton
- Hall JS, Harris DJ, Medjibe V, Ashton PMS (2003) The effects of selective logging on forest structure and tree species composition in a Central African forest: implications for management of conservation areas. *For Ecol Manag* 183(1–3):249–264
- Hidiroglou MG, Särndal C-E, Binder D (1995) Weighting and estimation in business surveys. In: Cox BG, Binder DA, Chinnappa BN, Christianson A, Colledge M, Kott PS (eds) *Business survey methods*. Wiley, New York
- Howard JL, McKeever DB (2016) U.S. forest products annual market review and prospects, 2012–2016. Research note FPL–RN–0343. USDA Forest Service Forest Products Laboratory, Madison
- Howard JL, Westby R (2009) U.S. forest products annual market review and prospects, 2005–2009. Research note FPL–RN–0313. USDA Forest Service, Forest Products Laboratory, Madison
- Johnson TG, Bentley JW, Howell M (2011) The South's timber industry—an assessment of timber product output and use, 2009. Resource Bulletin SRS–182. USDA Forest Service, Southern Research Station, Asheville
- Knottnerus P, van Delden A (2012) On variances of changes estimated from rotating panels and dynamic strata. *Surv Methodol* 38(1):43–52
- Luppold W, Pugh S (2016) Diversity of the eastern hardwood resource and how this diversity influences timber utilization. *For Prod J* 66(1):58–65
- Nanang DM (2010) Analysis of export demand for Ghana's timber products: a multivariate co-integration approach. *J for Econ* 16(1):47–61
- Nordberg L (2000) On variance estimation for measures of change when samples are coordinated by the use of permanent random numbers. *J off Stat* 16(4):363–378
- O'Brien M, Bringezu S (2018) European timber consumption: developing a method to account for timber flows and the EU's global forest footprint. *Ecol Econ* 147:322–332
- Pepke E (2010) *Forest products annual market review 2009–2010* (No. 25). United Nations Publications, Geneva
- Rubilar RA, Allen HL, Fox TR, Cook RL, Albaugh TJ, Campoe OC (2018) Advances in silviculture of intensively managed plantations. *Curr for Rep* 4(1):23–34

- Schreuder HT, Li J, Scott CT (1993) Estimation with different stratification at two occasions. *Forest Sci* 39(2):368–382
- Schreuder HT, Ernst R, Ramirez-Maldonado H (2004) Statistical techniques for sampling and monitoring natural resources. General technical reports. RMRS-GTR-126. USDA Forest Service, Rocky Mountain Research Station, Fort Collins
- Scott CT (1986) An evaluation of sampling with partial replacement. Use of auxiliary information in natural resource inventories. Use of auxiliary information in natural resource inventories. Society of American Foresters, Maryland, pp 74–79
- Singh B, Sedransk J (1988) Variance estimation in stratified sampling when strata sample sizes are small. *Sankhyā Indian J Stat Ser B* (1960–2002) 50(3):382–393
- Tukey JW (1958) Bias and confidence in not-quite large samples. *Ann Math Statist* 29:614
- Tukey JW (1977) Exploratory data analysis. Addison-Wesley series in behavioral science: quantitative methods. Addison-Wesley, Reading
- Wear DN, Prestemon JP, Foster MO (2016) US forest products in the global economy. *J for* 114(4):483–493
- Wood J (2008) On the covariance between related Horvitz-Thompson estimators. *J off Stat* 24(1):53–78
- Woodall CW, Ince PJ, Skog KE, Aguilar FX, Keegan CE, Sorenson CB, Hodges DG, Smith WB (2012) An overview of the forest products sector downturn in the United States. *For Prod J* 61:595–603

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