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Estimating maximum stand density for mixed-hardwood forests among various physiographic zones in the eastern US

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A B S T R A C T

Quantifying maximum stand density is important to evaluate the potential stand density of the target population in forest management. However, most of the past research in the eastern US was mainly focused on planted monocultures, coniferous forests, a few commercially important species at the stand level, or in a particular geographic region. This study aimed to estimate the maximum stand density for mixed-hardwood forests across physiographic zones in the eastern US between two decades (1996–2009 and 2010–2021). Data used in analyses were collected from the US national forest inventory established and maintained by the USDA Forest Service's Forest Inventory and Analysis (FIA) program.

Results showed that the slope of the self-thinning lines varied among forest types. Estimating maximum stand density index (SDI_{max}) from size-density relationships produced more precise estimates than using SDI-size curves. Among all forest types, elm-ash-cottonwood (*Ulmus-Fraxinus-Populus*) showed consistent SDI_{max} estimates whereas other forest types varied by regions. New England had considerably higher SDI_{max} in aspen-birch (*Populus-Betula*), oak-hickory (*Quercus-Carya*) and oak-pine (*Quercus-Pinus*) forests than other physiographic zones. Most of the combinations showed consistent SDI_{max} between two time periods. Only six combinations showed a significant gain (4–14% increase), which was likely driven by the growth of the same dominant species groups. The findings of this work provided not only additional insights of maximum stand density in the region, but also a methodology for forest ecologists and managers to quantify SDI_{max} for a variety of forest types.

1. Introduction

Maximum stand density at a specified average tree size is commonly used to measure the degree of competition and site occupancy in a forest, which is important to assess stocking level, predict forest growth, and estimate stand carrying capacity (Burkhart & Yang, 2022). In stand-level forest management, maximum stand density can be used as a baseline to evaluate the potential stand density of the target population in order to apply appropriate silvicultural treatments (e.g., thinning) (Newton, 1997; Sterba, 1987). Past studies have been devoted to various types of forests, from single-species, even-aged monocultures to mixed-species, uneven-aged forests (e.g., Curtis, 1970; Shaw, 2000; Pretzsch and Biber, 2016; Condés et al., 2017). Due to increasing concerns over the impact of climate change and fire disturbance on forest health and dynamics, recent research has expanded to a larger scale (e.g., state- and country-levels) (Woodall et al., 2005; Woodall and Weiskittel, 2021).

Stand density index (SDI) defined by Reineke (1933) was widely used as a standardized measure to compare stand density and relative stocking level among stands, which was calculated based on quadratic mean diameter and number of trees per unit area. However, SDI was originally developed for single-species, even-aged populations. It has been reported that using quadratic mean diameter as a measure of average tree size may be inappropriate for uneven-aged, multi-species populations (e.g., Andrews et al., 2018; Shaw, 2000; Zeide, 1983). To better quantify the complex stand structure, Zeide (1983) proposed an alternative definition of mean diameter, denoted as Reineke's diameter ($\overline{D}_R$), to substitute quadratic mean diameter in SDI. Stage (1968) suggested using a summation method to calculate SDI, which can account for the contributions of various classes of trees in the stand. Long and Daniel (1990) indicated that the summation method is a more general expression of SDI than the original form proposed by Reineke (1933). As shown by Shaw (2000), calculating SDI with Reineke's diameter resulted in an identical SDI calculated by the summation method. More discussion about the orginal SDI and other versions can be found in Ducey

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and Larson (2003) and Ducey (2009). To determine maximum stand density index (SDI_{max}), one approach is to estimate SDI_{max} from maximum size-density relationships (self-thinning lines) (e.g., Weiskittel and Kuehne, 2019; Woodall and Weiskittel, 2021). Another approach is to calculate SDI of all observations and then estimate SDI_{max} from SDI-stand variable (e.g., SDI-age) curves (e.g., Burkhart and Yang, 2022). However, to our knowledge, the efficacy of both approaches has not been explicitly studied in forestry literature, especially for multi-species stands across a wide range of physiographic regions.

Mixed-hardwood forests in the eastern US play an important role in providing timber products, carbon sequestration, wildlife habitats, and ecosystem services. Sustainability of mixed-hardwood forests has received increased concern because of decreasing trends of certain commercial species, such as oaks (*Quercus* spp.), observed in the region (Oak et al., 2015). Recently, the impact of climate change and natural disturbances on forest growth and yield has received increasing attention. Thus, an accurate estimate of maximum stand density is important to assess potential stand density for forest management. However, most of the past research in the eastern US was mainly focused on planted monocultures, coniferous forests, or a few commercially important species at the stand level or in a particular geographic region (e.g., Binkley, 1984; Stout and Nyland, 1986; Zhang et al., 1995; Ducey and Knapp, 2010; Yang and Burkhart, 2017; Weiskittel and Kuehne 2019). Burkhart and Yang (2022) reported an increase of productivity between two loblolly pine generations in the southeastern US, which likely resulted from increased levels of atmospheric CO_2, more intensive forest management and improved genetics. Woodall and Weiskittel (2021) reported that relative density of US forests has shifted to higher levels over the past decades, but the temporal change of maximum stand density for different types of mixed-hardwood forests in the eastern US has not been fully investigated.

Therefore, the overall objective of this study was to estimate the maximum stand density of the mixed-hardwood forests for six different forest types across six physiographic zones in the eastern US. Specifically, maximum stand density index (SDI_{max}) estimated from maximum size-density relationships was compared to that estimated from SDI-Reineke's diameter curves. We further compared the differences of SDI_{max} between 1996 and 2009 (Time 1) and 2010–2021 (Time 2) to provide additional insights about the change of maximum stand density over time. The results of this study would provide not only a baseline of maximum stand density in the region, but also a methodology for forest ecologists and managers to quantify SDI_{max} for a variety of forest types.

2. Materials and methodology

2.1. Physiographic zone and condition class

Data used in analyses were collected from the US national forest inventory established and maintained by the USDA Forest Service's Forest Inventory and Analysis (FIA) program. Permanent plots were installed throughout the natural growing range of the mixed-hardwood forests in the eastern US, which encompassed six physiographic zones: New England (NE), Appalachain Plateau (AP), Valley and Ridge (VR), Interior Plains (IP), Interior Highlands (IH), and Mississippi Alluvial plain (MI). The boundaries of the zones were delineated with the classification made by Fenneman and Johnson (1946) and the map of the FIA survey units (FIA survey units are listed in Burrill et al. (2021)). This study area covered over 142 million hectares and encompassed a wide range of geologic formations, topographies, soil types, climatic conditions, and associated vegetation. Central to the study area is the Appalachian Mountain Belt running northeast to southwest, where many peaks reach elevations greater than 1,800 m (the highest peak, Mt. Mitchell, reaches 2,037 m), with associated highlands and piedmont areas. The study area also included adjacent areas of rolling uplands, interior plateaus, and at its southerwestern extreme, portions of the Ouachita and Ozark Plateaus. The climate of this region of the United States is primarily temperate, with much of the area's weather influenced to varying degrees depending on distance from the Atlantic Ocean and Gulf of Mexico. We focused on forests that consisted of primarily decidous broadleaf trees, sometimes mixed with lesser components of evergreen broadleaf and conifer species. The spatial distribution of the six zones is shown in Fig. 1.

Each permanent plot contained four 7.3-m (24-ft) fixed-radius subplots and four 2.1-m (6.8-ft) nested, fixed-radius microplots. On each subplot, all trees with diameter at breast height (DBH) greater than or equal to 12.7 cm (5 in.) were measured, whereas on each nested microplot, trees with 2.5 cm (1 in.) $\leq$ DBH $<$ 12.7 cm (5 in.) were measured. At each installation, the sampled area (the area of the permanet plot) was classified based on site conditions, such as land use, forest type, stand size, regeneration status, tree density, stand origin, ownership group, and disturbance history. If there was more than one distinct condition, the plot would be mapped by condition class even though the boundaries cut through subplots or microplots. Each permanent plot was remeasured every 5–7 years. The details of sampling design, plot layout, and estimation procedures can be found in Bechtold and Patterson (2005), USDA (2020), and Pugh et al. (2018), respectively.

2.2. Forest type and stand characteristics

All observations (i.e., remeasurements) from the FIA plots within the six physiographic zones were queried from the FIA database. According to forest typing algorithm used by FIA (Burrill et al., 2021), six primary forest types of the mixed-hardwood forests were identified, which included oak-gum-cypress, oak-hickory, oak-pine, aspen-birch, elm-ash-cottonwood, and maple-beech-birch. To better capture the forest type at each installation in this study, a forested condition class on a plot was treated as the basic sampling unit. Stand characteristics, including quadratic mean diameter, number of trees per ha, basal area per ha, were computed from all live trees (live trees tallied on subplots and microplots) on a "condition class" rather than a "plot." For example, two distinct condition classes on a plot would have two separate values for a given stand characteristics (i.e., two observations). In other words, each plot's condition was treated as an individual sample of trees. Sampling units with irregular observations (e.g., disturbances, or increasing number of trees per ha over time) were removed. A summary of stand characteristics for all combinations of forest types and physiographic zones is given in Table 1.

2.3. Maximum stand density index

To estimate SDI_{max}, two approaches were compared in this study. In the first approach, SDI_{max} was obtained from a maximum size-density relationship line, whereas the second approach was based on SDI-$\overline{D}_R$ curves.

2.3.1. Slope coefficient

Regardless of approaches, the slope coefficient is an essential component to calculate SDI. In general, the slope coefficient of −1.605 reported by Reineke (1933) was used (e.g., Andrews et al., 2018; Ducey and Knapp, 2010; Shaw, 2000; Zeide, 1983). However, it has been shown that the slope coefficient varied among species, environment and management treatments (VanderSchaaf and Burkhart, 2007). In this study, rather than using a fixed value, the slope coefficient was estimated from maximum size-density relationships (i.e., a linear equation of quadratic mean diameter and number of tree per ha on a log–log scale). That is,

$$LnN = a_{ij} + b_i Ln\overline{D}_q \tag{1}$$

where *Ln* is natural logarithm, *N* is number of trees per ha, $\overline{D}_q$ is quadratic mean diameter in cm, *i* is forest type, *j* is physiographic zone, *a*

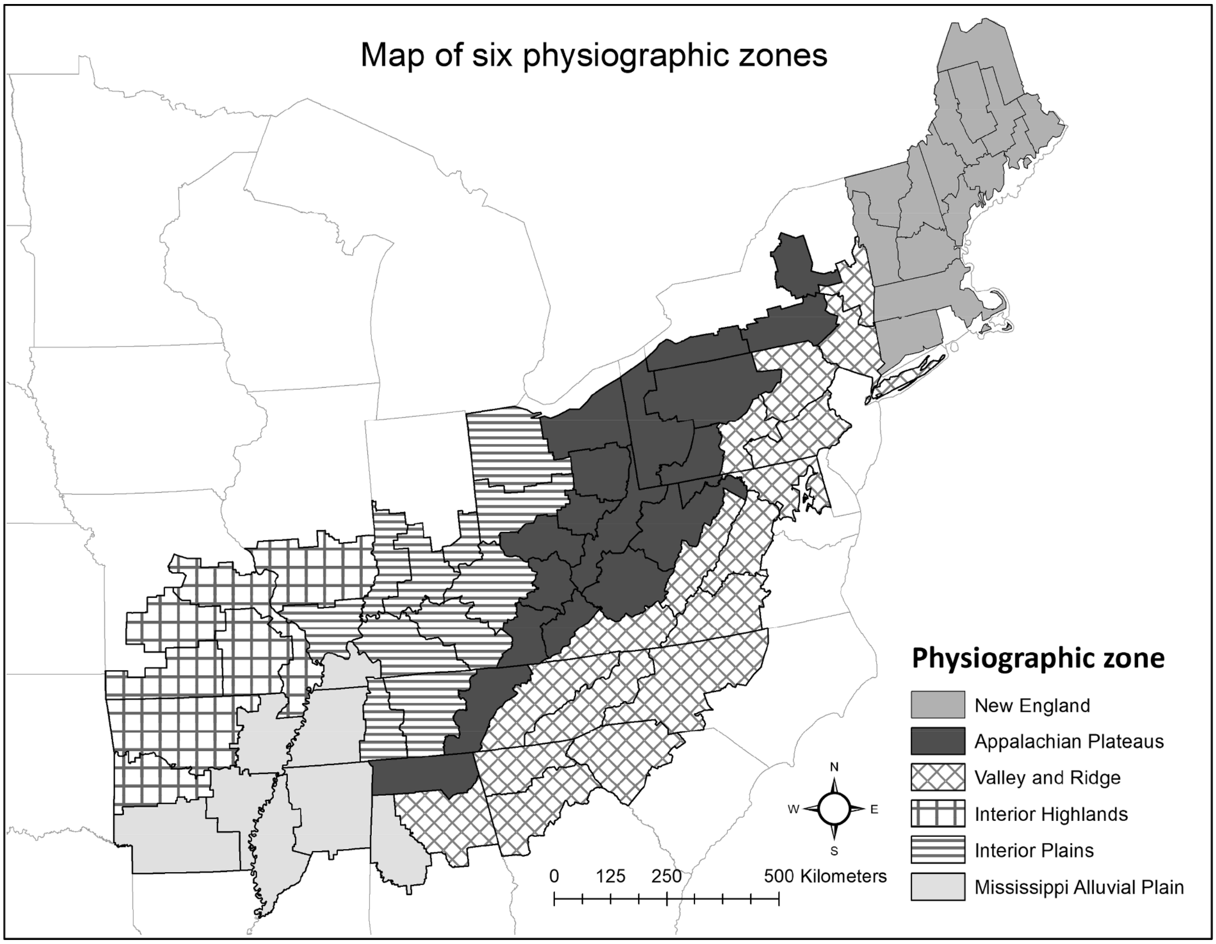

Fig. 1. Spatial distribution of six physiographic zones in the eastern US.

and b are the intercept and slope coefficients, respectively. In the preliminary analysis, the slope coefficient (b) did not significantly vary among physiographic zones for a given forest type. Thus, a consistent b was applied to all physiographic zones for a given forest type.

2.3.2. Approach I: SDI_{max} estimated from maximum size-density relationship lines

Reineke's diameter ($\overline{D}_R$) proposed by Zeide (1983) as an alternative formulation of mean diameter was calculated as:

$$\overline{D}_R = \left[\frac{1}{N}\sum_k n_k d_k^b\right]^{\frac{1}{b}} \tag{2}$$

where N is number of trees per ha, b is the slope coefficient in Eq. (1), d_k is DBH of tree k in cm, and n_k is number of trees per ha represented by tree k. Then, $\overline{D}_q$ was subtituted with $\overline{D}_R$ in Eq. (1), and the equation was refitted to update the intercept coefficient (a) for each combination of forest type and physiographic zone. Maximum stand density index (SDI_{max}) was computed by inserting $\overline{D}_R = 25.4$ cm (10 in.) into Eq. (1) given the estimates of coefficients.

2.3.3. Approach II: SDI_{max} estimated from SDI-$\overline{D}_R$ curves

Given $\overline{D}_R$ in Eq. (2), SDI[1] was determined as:

$$SDI = N\left(\frac{\overline{D}_R}{25.4}\right)^{\frac{1}{b}} \tag{3}$$

where all variables have been defined above. To estimate SDI_{max}, the SDI-$\overline{D}_R$ equation was fitted using the nonlinear quantile regression. That is,

$$SDI = c_1 e^{-c_2\left(\frac{1}{\overline{D}_R}\right)} \tag{4}$$

where c_1 and c_2 are model coefficients. SDI_{max} was computed by inserting $\overline{D}_R = 25.4$ cm (10 in.) into Eq.(4) given the estimates of coefficients.

2.3.4. Coefficient estimation

Coefficient estimation included two stages: In the first stage, the goal was to estimate the slope coefficient (b) in Eq. (1) for a given forest type. The estimated b was then used to compute $\overline{D}_R$ and SDI for all sampling units within the same forest type. In the second stage, data with the same forest type were split by physiographic zones. SDI_{max} was estimated for each physiographic zone using the approaches described above. In addition, quantile regression with cluster bootstrapping was used to handle the correlation among repeated measurements. Cluster bootstrapping is a nonparametric resampling algorithm for clustered data when the assumption of independent observations is not appropriate (Field and Welsh, 2007; Ren et al., 2010). Unlike the basic bootstrap algorithm, observations in cluster bootstrapping are selected in groups or clusters, not individuals. The procedure mimics the data generation mechanism of remeasurements; thus, the correlation structure of

[1] Notably, in this study, SDI is referred to as the maximum stand density index based on Reineke's diameter (Zeide, 1983), not the original version of SDI defined by Reneike (1933) (i.e., $SDI = f(N, \overline{D}_q)$).

Table 1
Summary statistics of quadratic mean diameter ($\overline{D}_q$), Reineke's diameter ($\overline{D}_R$) and trees per ha (TPH) among physiographic zones (New England, NE; Appalachain Plateau, AP; Valley and Ridge, VR; Interior Plains, IP; Interior Highlands, IH; Mississippi Alluvial plain, MI) and forest types. 95% confidence interval (CI) was calculated from 1,000 bootstrap samples for a given combination. The symbol, "-", means no observation or insufficient sample was collected.

	NE									AP								
	$\overline{D}_q$	95% CI		$\overline{D}_R$	95% CI		TPH	95% CI		$\overline{D}_q$	95% CI		$\overline{D}_R$	95% CI		TPH	95% CI	
Aspen-birch	15.9	15.5	16.3	10.9	10.6	11.2	2828	2655	2995	19.1	18.3	19.8	14.3	13.5	15.0	1228	1067	1400
Elm-ash-cottonwood	21.4	20.4	22.3	16.5	15.5	17.6	975	856	1096	24.8	23.9	25.8	18.4	17.3	19.4	721	646	802
Maple-beech-birch	21.5	21.2	21.7	15.0	14.8	15.3	1942	1874	2008	25.6	25.4	25.9	19.8	19.5	20.0	987	955	1018
Oak-gum-cypress	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Oak-hickory	25.1	24.7	25.6	18.2	17.8	18.8	1042	991	1101	24.7	24.6	24.9	17.7	17.5	17.9	990	969	1010
Oak-pine	25.2	24.6	25.7	17.6	16.9	18.2	1222	1134	1315	21.9	21.3	22.4	15.0	14.3	15.7	1204	1124	1292

	VR									IP								
	$\overline{D}_q$	95% CI		$\overline{D}_R$	95% CI		TPH	95% CI		$\overline{D}_q$	95% CI		$\overline{D}_R$	95% CI		TPH	95% CI	
Aspen-birch	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Elm-ash-cottonwood	25.3	24.6	26.0	19.3	18.6	20.0	680	616	749	23.6	22.8	24.3	18.0	17.3	18.8	651	591	714
Maple-beech-birch	25.3	24.9	25.7	19.9	19.4	20.4	911	856	968	24.9	24.3	25.7	18.5	17.8	19.4	706	647	766
Oak-gum-cypress	25.9	24.9	26.9	19.0	18.0	20.1	762	676	860	–	–	–	–	–	–	–	–	–
Oak-hickory	24.6	24.4	24.8	18.0	17.8	18.2	941	924	960	23.4	23.2	23.7	16.3	16.0	16.6	908	878	939
Oak-pine	20.3	20.1	20.6	13.7	13.4	13.9	1406	1363	1454	19.1	18.4	19.8	13.2	12.5	14.0	1046	959	1138

	IH									MI								
	$\overline{D}_q$	95% CI		$\overline{D}_R$	95% CI		TPH	95% CI		$\overline{D}_q$	95% CI		$\overline{D}_R$	95% CI		TPH	95% CI	
Aspen-birch	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Elm-ash-cottonwood	24.8	23.9	25.7	19.0	18.1	20.0	642	578	712	23.7	22.9	24.5	18.4	17.6	19.4	724	662	794
Maple-beech-birch	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Oak-gum-cypress	24.6	22.9	26.4	19.2	17.2	21.3	960	794	1133	26.4	25.6	27.1	19.8	19.1	20.5	827	772	885
Oak-hickory	22.8	22.6	23.0	15.7	15.5	15.9	1044	1018	1069	21.5	21.1	21.9	14.8	14.4	15.2	1050	1007	1096
Oak-pine	20.8	20.4	21.2	14.2	13.8	14.6	1201	1141	1264	19.2	18.8	19.7	12.5	12.1	12.9	1386	1314	1469

observations within each cluster is retained.

Each stage had five identical steps. As mentioned above, the only difference was the data used in each stage. Data were divided by forest type in the first stage, and then further split by physiographic zone in the second stage.

(1) Step 1: Selection of observations within the self-thinning stage.

In stand development, size-density trajectories generally consist of two major stages: the density-independent stage (stage I) where the mortality is independent of stand density, and the density-dependent stange (stage II, also known as self-thinning stage) where the mortality is induced by tree competition. VanderSchaaf and Burkhart (2008) applied a segmented regression function to estimate the average tree size when stands transitioned from stage I to stage II. In this study, sampling units where self-thinning occurred were selected using the segmented regression function proposed by VanderSchaaf and Burkhart (2008), which can be written as:

$$LnN = b_1 I_1 + \left[b_1 + b_2 * \left(Ln\overline{D}_q - c\right)^2\right] I_2 \quad (5)$$

where b_1 and b_2 are model coefficients, I_1 and I_2 are indicator variables, and c is the join point of segments to indicate the occurance of self-thinning. When $Ln\overline{D}_q < c$, $I_1 = 1$ and $I_2 = 0$, whereas when $Ln\overline{D}_q \geq c$, $I_1 = 0$ and $I_2 = 1$. The coefficients in the segmented regression model (Eq. (5)) were estimated using nlsLM function (Elzhov et al., 2016) in R. Then, observations greater than or equal to the estimated join point (c) were selected for the next step.

(2) Step 2: Preparation of a bootstrap sample.

The cluster bootstrap technique was used to account for the repeated measurements for a given sampling unit (i.e., condition class). In this study, sampling units served as clusters and were randomly selected with replacement to construct a bootstrap sample.

(3) Step 3: Cofficient estimation.
Stage I: The bootstrap sample was used to estimate the slope coefficient of Eq.(1) using the quantile regression (quantile = 0.90) (Koenker, 2021) in R.
Stage II: In the first approach, the bootstrap sample was used to estimate the intercept coefficient using the quantile regression (quantile = 0.90) (Koenker, 2021) in R. In the second approach, Eq.(4) was fitted with the bootstrap sample using the nonlinear quantile regression (quantile = 0.90) (Koenker, 2021) in R. Regardless of approaches, the estimated coefficients were used to compute SDI_{max}.
(4) Step 4: Steps 2 and 3 were repeated 1,000 times (i.e., 1,000 bootstrap samples).
(5) Step 5: Point estimates, the upper and lower bounds of 95% confidence intervals (CI) of model coefficients and SDI_{max} were computed by the median, 2.5% and 97.5% quantiles of 1,000 bootstrap samples.

The above steps were implemented with the entire data. To further examine the change of maximum stand density over decades, all data were split into Time I (1996–2009) and Time II (2010–2021) for a given combination of physiographic zone and forest type. The procedure listed above was followed in coefficient estimation. In the first approach of estimating SDI_{max}, a consistent slope estimate was used for a given forest type, while a separate intercept coefficient (a) was obtained to build the maximum size-density lines for Time I and Time II, respectively.

3. Results and discussion

3.1. Comparison of maximum stand density index estimated by two different approaches

3.1.1. Slope coefficient

Reineke (1933) reported that the slope coefficient (*b*) was approximate to −1.605 for a variety of species. As shown in Fig. 2, the slope coefficient for four of the six forest types was significantly different from −1.605, meaning that calculating SDI with a predefined, universal *b* may not be appropriate for mixed-hardwood forests in the eastern US. Notably, the slope was estimated at around −2 for aspen-birch (95% CI: [−2.35, −1.93]) and maple-beech-birch groups (95% CI: [−2.13, −1.95]), which was smaller than other forest types. The steeper slope implied that the mortality of trees was higher or growth was slower than mortality in the self-thinning stage (see Fig. 3). Results varied in oak abundant forests. In forests where oaks were mixed with hickories or pines, the slope coefficient was slightly lower than −1.605. However, based on 95% confidence interval, the slope was not statistically different from −1.605 in the mixture of oaks, gums and cypresses. The results from elm-ash-cottonwood forests were similar to oak-gum-cypress forests. Although a couple of forest types did not show a significant difference from the value suggested by Reineke (1933), the point estimate of the slope coefficient for all forest types was lower than −1.605 (see the black dots in Fig. 2). Similar findings were reported by Woodall and Weiskittel (2021) where the slope coefficient of −1.797 was estimated using plot-level FIA data from the entire US.

3.1.2. Approach I versus approach II

In general, the differences in SDI_{max} between the two approaches were not statistically significant based on 95% confidence intervals. Two exceptions were found in NE and VR where SDI_{max} estimated by approach II was higher than approach I (see maple-beech-birch forests in NE, and oak-hickory forests in VR in Fig. 4). However, it was noted that the approach II (SDI_{max} estimated by the SDI-$\overline{D}_R$ function) produced appreciably wider confidence intervals than the approach I (SDI_{max} estimated from maximum size-density relationship lines). Namely, the second approach yielded less precise estimates than the first one. An extreme case was found for aspen-birch in Appalachian Plateau. The width of the confidence interval from approach II was more than double that of approach I (see Fig. 4). Burkhart and Yang (2022) implemented the second approach for loblolly pine, which provided reliable estimates of SDI_{max}. However, stand age was used to build the SDI-age equations instead of average tree size like this study. For single-species plantations,

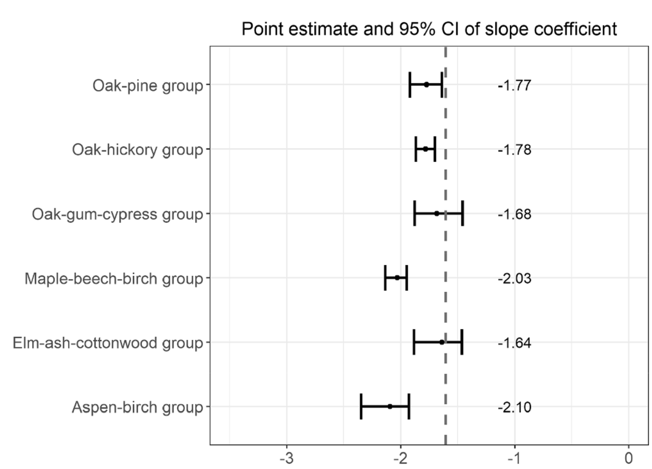

Fig. 2. The point estimate (values shown) and 95% confidence interval (horizontal bars) of the slope coefficient (*b*) for six forest types. The vertical dashed line represents the slope coefficient of −1.605 reported by Reineke (1933).

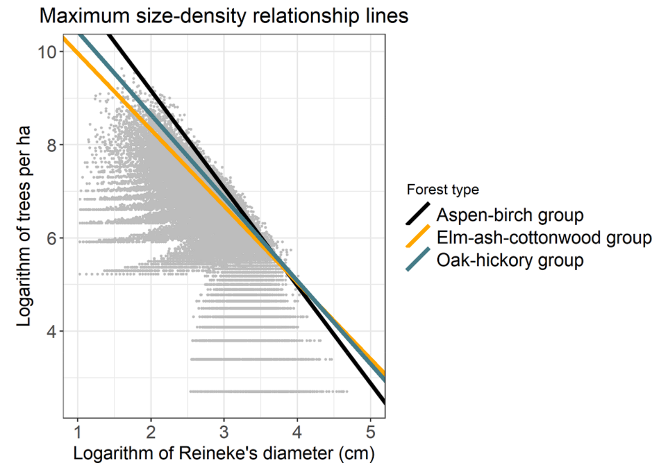

Fig. 3. Maximum size-density relationship lines among three forest types. Grey dots represent observations.

SDI generally increases with increasing stand age, and then reaches to an upper aysmptote. However, stand age has nebulous meaning in the context of uneven-aged or mixed-species forests (Burkhart and Tomé, 2012), so it was an unsuitable variable in this study. Additionally, age is not usually recorded in inventories for natural mixed-hardwood forests. Thus, $\overline{D}_R$ was used as an independent variable instead, but the results were still not ideal. In fact, other stand variables (e.g., average canopy height) were tested as a surrogate for stand age in the prelimiary investigation, but the relationships were weaker than SDI-$\overline{D}_R$. In short, statistical evidence was clear that estimating SDI_{max} from maximum size-density relationships performed better than the alternative option, so discussion and comparison in the following sections will be focused on the results from approach I.

3.2. Comparison of maximum stand density between physiographic zones and forest types

As shown in Fig. 4, New England, Appalachian Plateau and Valley and Ridge had five forest types, while other regions had four of the six types. Elm-ash-cottonwood, oak-hickory and oak-pine can be found in all physiographic zones, while aspen-birch forests were only observed in New England and Appalachian Plateau. Among all forest types, elm-ash-cottonwood showed consistent SDI_{max} estimates, which were not statistical differences among six physiographic zones ($\alpha = 0.05$). A similar pattern was found in the oak-gum-cypress group, but this forest type was only recorded in three physiographic zones. Based on 95% confidence intervals shown in Fig. 4, New England showed significantly greater SDI_{max} in aspen-birch (average 41% higher), oak-hickory (average 12–35% higher) and oak-pine forests (average 23–46% higher). Compared to other regions, a substantially lower SDI_{max} was found for maple-beech-birch forests in Interior Plains, which implied a lower stand carrying capacity in the region. Fig. 5 illustrated the physiographic zones with the highest and lowest SDI_{max} for a given forest type. Maple-beech-birch and oak-pine forests between New England and Interior Plains showed the greatest diffference. The SDI_{max} in New England was 54% higher than Interior Plains (Fig. 5). Oak-pine forests in New England had significantly higher maximum stand density than other regions.

Notably, the SDI_{max} of the New England region reported in this study was lower than the findings by Weiskittel and Kuehne (2019). The discrepancies may be due to different data sources, data screening processes, and quantiles used in model fitting. In the preliminary study, the quantiles of 90%, 95% and 99% were examined, which were commonly used in the literature (Burkhart and Tomé, 2012; Weiskittel and Kuehne, 2019; Woodall et al., 2005). Ninety-five percent quantile was selected

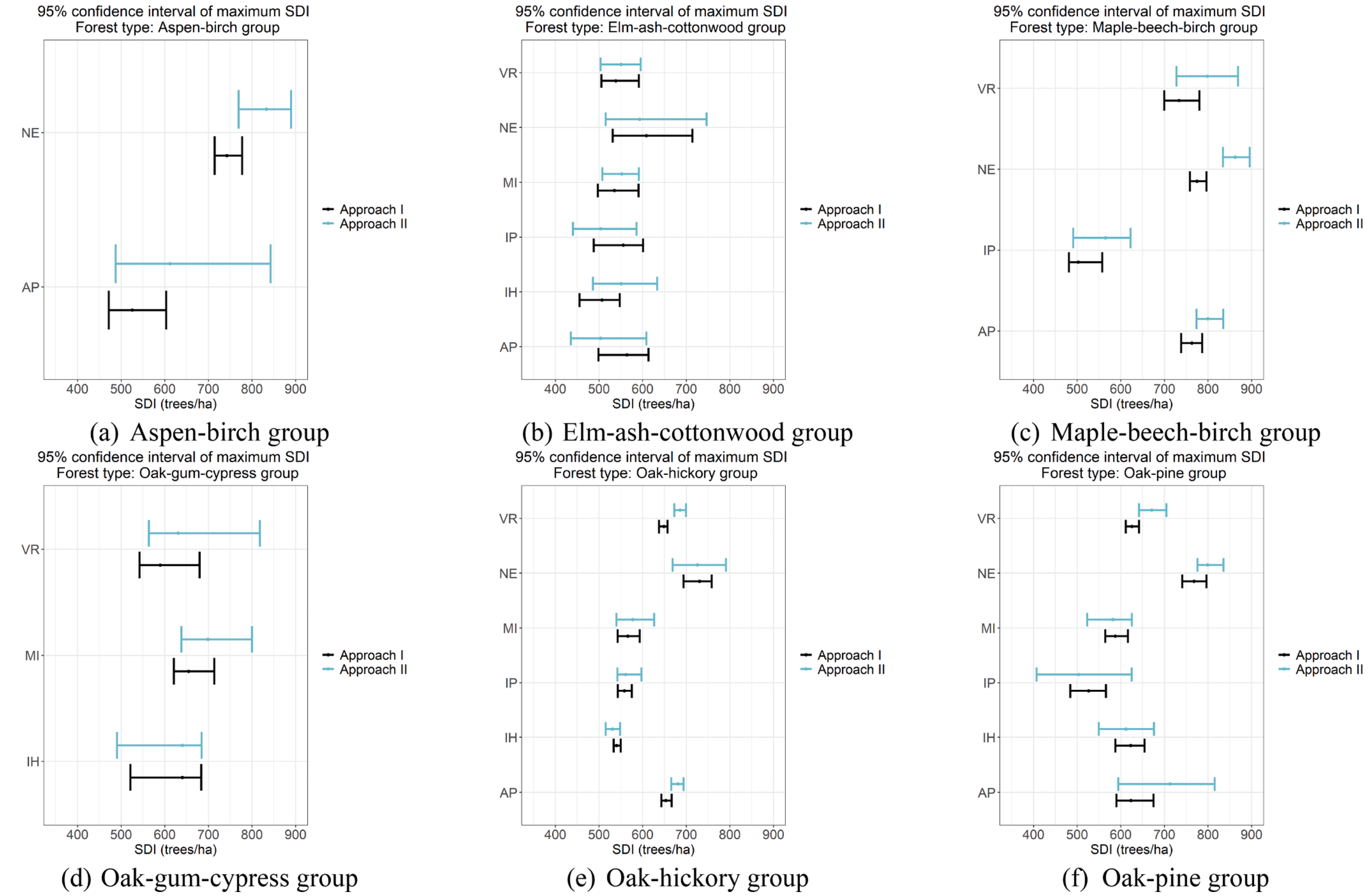

Fig. 4. The 95% confidence interval of the maximum SDI (SDI_{max}) estimated from two approaches for forest types and physiographic zones (New England, NE; Appalachain Plateau, AP; Valley and Ridge, VR; Interior Plains, IP; Interior Highlands, IH; Mississippi Alluvial plain, MI).

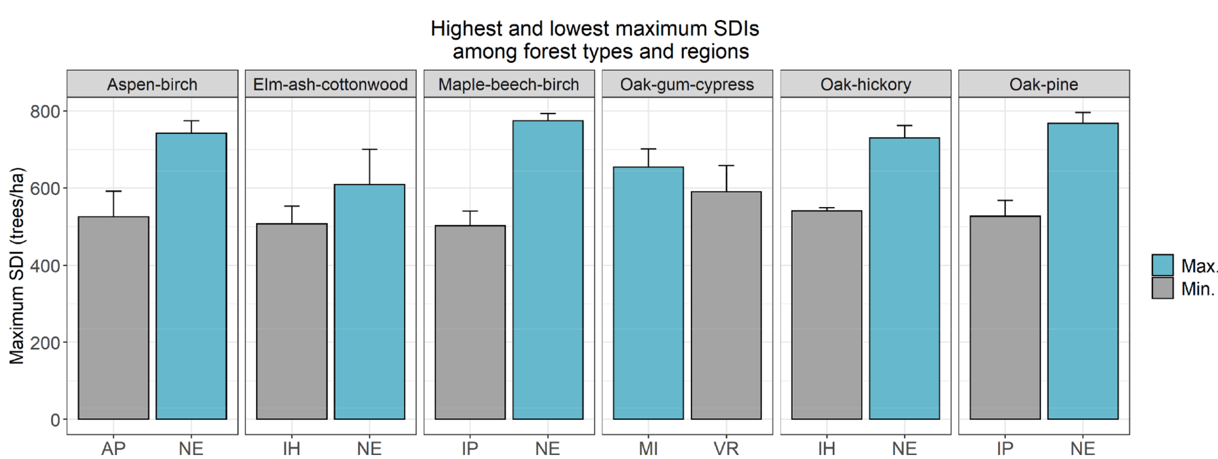

Fig. 5. Highest and lowest maximum stand density index (SDI_{max}) observed in physiographic zones for a given forest type. Physiographic zones included New England (NE), Appalachian Plateau (AP), Valley and Ridge (VR), Interior Plains (IP), Interior Highlands (IH) and Mississippi Alluvial plain (MI). Error bars represent margin of error from 95% confidence interval.

because it produced the most stable and reasonable estimates across all combinations.

3.3. Comparison of maximum stand density between Time 1 (1996–2009) and Time 2 (2010–2021)

SDI_{max} between Time 1 (1996–2009) and Time 2 (2010–2021) was calculated by approach I. Due to insufficient observations, aspen-birch group in Appalachian Plateau in time 2 was dropped. In general, we did not find a significant gain or loss in SDI_{max} between two time periods for most of the combinations (see 95% confidence intervals in Table 2). For elm-ash-cottonwood and oak-gum-cypress types, none of the groups were observed to have a significant change ($\alpha = 0.05$). Only six combinations showed significantly higher SDI_{max} at Time 2 than Time 1, which included aspen-birch in New England, maple-beech-birch and oak-hickory in Appalachian Plateau, oak-hickory in Valley and Ridge, as well as oak-hickory and oak-pine in Mississippi Alluvial plain. Among the six combinations, the rate of increase ranged from 4 to 14%.

Table 2
SDI_{max} estimate and 95% CI among forest types and physiographic zones (New England, NE; Appalachian Plateau, AP; Valley and Ridge, VR; Interior Plains, IP; Interior Highlands, IH; Mississippi Alluvial plain, MI). "Overall" represents the overall estimate computed from the total number of observations for a given combination, while Time 1 (1996–2009) and Time 2 (2010–2021) represent the estimates computed from the observations taken before or after the year of 2010, respectively. The symbol, "-", means no observation or insufficient sample was collected.

	NE									AP								
	Overall			Time I			Time II			Overall			Time I			Time II		
	SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI	
Aspen-birch	743	715	778	661	693	726	756	785	814	526	472	604	457	538	684	–	–	–
Elm-ash-cottonwood	609	531	714	533	588	697	516	680	787	564	499	614	490	583	631	490	563	645
Maple-beech-birch	775	759	797	737	766	787	767	784	811	763	738	787	704	727	745	774	799	829
Oak-gum-cypress	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Oak-hickory	730	694	758	658	714	742	703	751	776	653	643	667	624	635	644	666	680	697
Oak-pine	768	741	797	724	751	780	750	789	824	623	590	675	564	622	682	581	630	686

	VR									IP								
	Overall			Time I			Time II			Overall			Time I			Time II		
	SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI	
Aspen-birch	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Elm-ash-cottonwood	538	505	591	531	567	618	451	516	570	556	488	601	465	525	571	486	571	613
Maple-beech-birch	733	700	780	682	715	785	699	739	783	502	481	558	477	496	558	482	548	626
Oak-gum-cypress	590	542	680	534	616	652	527	572	725	–	–	–	–	–	–	–	–	–
Oak-hickory	649	638	657	627	637	648	653	663	675	559	543	575	542	559	583	550	563	583
Oak-pine	626	611	642	597	613	626	621	645	674	527	484	566	465	516	546	460	536	622

	IH									MI								
	Overall			Time I			Time II			Overall			Time I			Time II		
	SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI		SDI_{max}	95% CI	
Aspen-birch	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Elm-ash-cottonwood	507	455	548	403	509	604	459	507	548	535	497	591	508	581	633	481	520	568
Maple-beech-birch	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Oak-gum-cypress	640	521	684	–	–	–	–	–	–	655	621	714	631	661	739	594	650	704
Oak-hickory	541	534	550	533	541	552	530	544	555	566	543	594	525	553	566	568	607	627
Oak-pine	623	588	655	609	635	664	545	582	655	587	564	617	528	553	582	587	625	654

To further investigate the increase of maximum stand density in the six combinations and understand which species might be driving these changes, proportion of species group at the sampling unit level was examined. For a given combination at a time, five genus groups with the greatest percent basal area per ha were selected. The percent basal area per ha (percent BA, %) of the genus group was calculated as:

$$\text{Percent BA}(\%) = \frac{\text{BA}}{\sum \text{BA}} * 100\% \quad (7)$$

where BA is basal area per ha (m^2/ha) for the genus group, and $\sum$BA is the sum of basal area per ha (m^2/ha) calculated from all groups on a sampling unit. Then, the average percent BA (%) was calculated by averaging the percent BA of all sampling units for a given genus group.

Overall, the top four to five dominant genus groups were consistent between two time periods among all combinations. We did not find an appreciable increase or decrease of percent basal area for the dominant groups where most of the changes were within 5% (see Fig. 6). Specifically, in New England, the proportion of birches (*Betula* spp.), firs (*Abies* spp.), maples (*Acer* spp.), and spruces (*Picea* spp.) were slightly higher at Time 2, but poplars (*Populus* spp.) decreased about 2% in aspen-birch forests (Fig. 6a). In maple-beech-birch forests in Appalachian Plateau, maples remained the most abundant genus group, which accounts for more than 35% in both periods. Other species, such as cherries (*Prunus* spp.), ashes (*Fraxinus* spp.) and beech (*Fagus* spp.) were recorded, but they were between 5 and 15% (Fig. 6b). In oak-hickory forests, the same four genus groups were found in Appalachian Plateau and Valley and Ridge. The change of species proportion was similar in both regions. However, the abundance of oaks was higher in Valley and Ridge (about 35%), whereas maples had a greater percent basal area in Appalachian Plateau (about 20%). Cherries, pines (*Pinus* spp.), yellow-poplars (*Liriodendron* spp.) and hickories (*Carya* spp.) were around 5–15% (Fig. 6c and 6d). Lastly, from Time 1 to Time 2, pines, oaks, sweetgums (*Liquidambar* spp.), hickories and elms (*Ulmus* spp.) were the dominant groups in oak-hickory and oak-pine forests in Mississippi Alluvial plain (Fig. 6e and 6f). In Time 2, elms outperformed maples and junipers (*Juniperus* spp.) in oak-hickory and oak-pine forests, respectively. Given the nearly constant species composition, it implied that the increase of SDI_{max} in the six combinations was driven by the growth of the same species groups.

3.4. Further discussion

Given the sample plots were representative of forest management and silvicultural practice for the forest types and regions at the time, the findings of this work provided a region-level measure of maximum stand density. Unlike loblolly pine plantations in the southeastern US reported by Burkhart and Yang (2022), we did not find a significant increase of maximum stand density over time for mixed-hardwood forests in the eastern US. One possible reason could be that forest management in hardwood stands did not change much during this period of time, nor is it as intensive as plantation management (e.g., lacking thinning, site preparation, and other silvicultural treatments). The variations of SDI_{max} among forest types, regions and time periods can be affected by various confounding factors, such as species composition, anthropogenic activity, environment, and climate. The FIA data used in this study were not

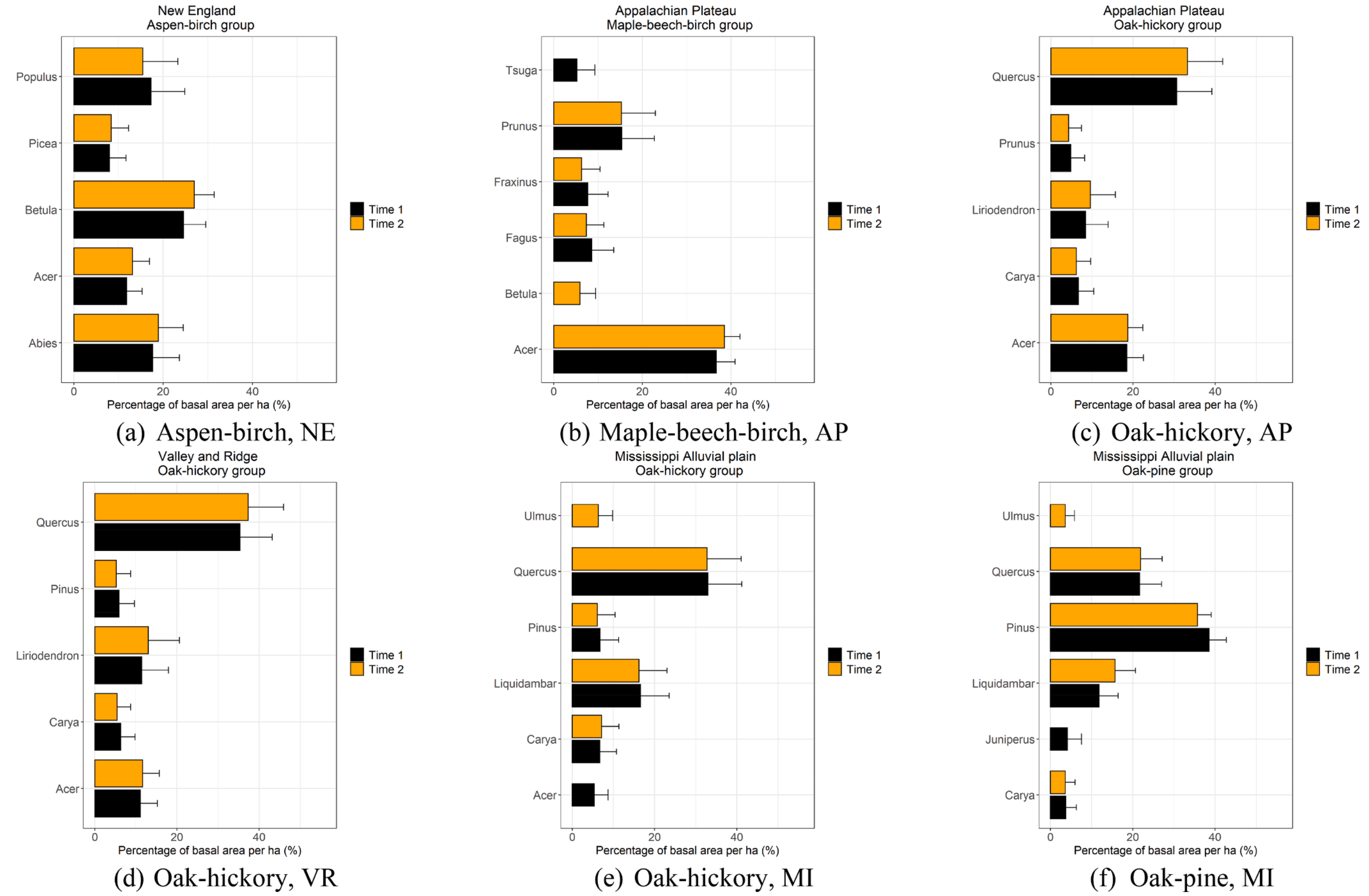

Fig. 6. The top five species with the greatest percent basal area per ha at the sampling unit level between Time 1 (1996–2009) and Time 2 (2010–2021) among six combinations of forest types and physiographic zones (New England, NE; Appalachain Plateau, AP; Valley and Ridge, VR; Mississippi Alluvial plain, MI). Error bars represent standard error.

collected from a single designed experiment or forests with a single management objective, which makes it difficult for causal inference. However, several studies have been focused on assessing the relationships between maximum stand density and other factors, which provided insights on potential influences (Andrews et al., 2018; Ducey and Knapp, 2010; Weiskittel and Kuehne, 2019; Woodall et al., 2005).

In addition, a nonparametric method was applied to estimate the precision of parameters in this study. The method accounted for the correlation among repeated measurements. It was found that parametric statistical procedures, such as linear quantile mixed-effect model, or first-difference model, remain the most commonly implemented approaches in the literature (e.g., Andrews et al., 2018; VanderSchaaf and Burkhart, 2007; Woodall and Weiskittel, 2021). However, the parametric methods require the assumption that observations follow certain probability distributions. Meeting such assumptions could be challenging, especially with data collected from a variety of forest types and regions. In those cases, nonparametric methods, as distribution-free methods, provide an alternative option to quantify the precision of estimates. Common limitations of the nonparametric methods (e.g., weaker testing power, more heavily data driven, and weaker extrapolation than parametric approaches) mean users should be cautious when drawing inference from them, especially when sample sizes are small. Although this study was focused on the regional scale, assessing potential density to make stand density management decisions is still the most common application of stand density measures in practice. The framework of mixed-effect modeling has proven useful in providing site- or species-specific estimates while accounting for the correlated structure of observations (e.g., Andrews et al., 2018). Incorporating cluster bootstrap or other resampling techniques into mixed-effect model could be worth exploring. Future study is suggested to investigate the efficacy of other newly-emgering algorithms, such as the mixed-effect random forest algorithm (Hajjem et al. 2014).

4. Conclusion

In short, this study demonstrates a modeling approach to quantify maximum stand density. Variable slope coefficients were found among forest types, which implied a predefined, universal value may not be appropriate when calculating SDI for a variety of forest types. Estimating SDI_{max} from size-density relationships is suggested for mixed-hardwood forests because it produced more precise estimates than using SDI-$\bar{D}_R$ curves. Cluster bootstrap provides an alternative approach to quantify the uncertainty of estimates.

Among all forest types, elm-ash-cottonwood showed consistent SDI_{max} estimates whereas other forest types varied by regions. New England had significantly higher SDI_{max} in aspen-birch, oak-hickory and oak-pine forests than other physiographic zones. Most of the combinations showed consistent SDI_{max} between Time 1 (1996–2009) and Time 2 (2010–2021). Only six combinations showed a significant gain (4–14% increase), which was likely driven by the growth of the same dominant species groups. This study furthered the investigation by Woodall and Weiskittel (2021) to provide additional insights of the baselines for various forest types and physiographic zones. With increasing CO_2 concentrations and changing climate projected in the region over the next decades (Burkhart et al., 2018), proper stand density management is critical to optimize the resilience and carbon sequestration of hardwood forests.

CRediT authorship contribution statement

Sheng-I Yang: Conceptualization, Formal analysis, Methodology, Writing – original draft. **Thomas J. Brandeis:** Conceptualization, Data curation, Methodology, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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