

A Method for Designing a Fire Weather Network

FRANCIS M. FUJIOKA

Pacific Southwest Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture, Riverside, CA 92507

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ABSTRACT

Fire weather stations used to be located only where observers were available. Now, remote automatic weather stations can sample weather without manual assistance, virtually anywhere. Locating them, however, remains a problem. An objective method was developed for designing a fire weather network. The design process begins with the identification of the area to be sampled, the relevant meteorological variables, target meteorological fields, and interpolation (or other analysis) method to be used. Using a least-squares criterion, optimum networks for a varying number of stations are computed with a constrained optimization algorithm. The optimum error sum of squares, thus obtained by interpolation, is used to help choose an optimum network. Simulated fire rate-of-spread fields were analyzed with optimum networks varying in size from one to seven stations. Preliminary results were obtained for the design of a fire weather network for southern California.

1. Introduction

In the United States, the Forest Service (U.S. Department of Agriculture), the Bureau of Land Management (U.S. Department of Interior), and numerous state and local organizations use weather data as input to the National Fire Danger Rating System (Deeming et al., 1977) to guide resource protection activities against the threat of wildfire. The required data have been disproportionately sparse in the vast regions where the agencies have fire protection responsibilities, because weather station site selection has historically been governed by the availability of observers. Now that remote automatic weather stations (RAWS) can observe weather virtually anywhere, without manual assistance, the agencies plan to have several hundred RAWS operational by 1990. Some agencies have yet to decide where—indeed, how—stations will be placed.

This note describes a method for designing a meteorological network to minimize analysis errors, and its application to the design of a fire weather network.

2. Theoretical framework

The theoretical basis for the method derives from the Kuhn–Tucker theory (Kuhn and Tucker, 1951), which outlines sufficiency conditions for the solution of constrained nonlinear optimization problems. The principles may be applied broadly to a variety of environmental sampling problems.

The explicit design objective is to minimize the error sum of squares of interpolation, S , by choosing observation sites, $\mathbf{X}$, optimally. Function S is called the objective function:

$$S(\mathbf{X}) = \sum_i [T_i - F_i(\mathbf{X})]^2 \quad (1)$$

where I is the point set of intersections of a grid, whose elements are denoted by i ; T_i is the value of the target weather field at the point i ; $\mathbf{X}$ is the $n \times 2$ matrix of coordinates for the n observation sites in the network area (n = design size); and $F_i(\mathbf{X})$ is the interpolated field value at i , obtained from distance-weighted interpolation of the target field values observed at $\mathbf{X}$. The elements of $\mathbf{X}$ are called design variables. The set of $\mathbf{X}$ that minimizes S is called a local solution, unless it minimizes S uniquely; it is then called a global solution. The elements of $\mathbf{X}$ may be subject to equality and inequality constraints, which delineate the geographic region from which observation sites are chosen.

The target weather field, the interpolation function, and the constraint functions are assumed to satisfy certain mathematical regularity assumptions, first-order differentiability in particular. The observations are assumed to be error free. Under these conditions, the existence of a local solution is guaranteed by the Kuhn–Tucker theorem (Phillips et al., 1976). Unfortunately, the theorem does not explain how the solution is found—a complex undertaking requiring computerized methods (Fletcher, 1981).

3. Design process

Given prespecified meteorological conditions, called target fields, the method locates stations to minimize analysis errors by an objective criterion (least-squares). The method requires identifying 1) the geographic region over which meteorological analysis is performed, and within which observation sites will be designated; 2) the algorithm used to analyze the weather field, given the observations; and 3) a quantitative description of

the target weather field—the meteorological conditions for which the network must be tuned. A framework is thus established for assessing the efficacy of meteorological networks objectively (error sum-of-squares), and for determining the number and locations of observing sites required for a given application.

The design method will first be illustrated for a simulated target field, representing the potential rate of fire spread in a hypothetical, grass-covered area subject to marine and continental weather effects. Assume that the design function is driven by the need for better spatial resolution of fire spread potential under pre-defined weather conditions. The rate of spread (ROS) is evaluated from three surface weather inputs—dry-bulb temperature, relative humidity, and windspeed—plus fuels and topography data (Rothermel, 1972). The ROS model is a natural weighting function for the weather variables, in that it determines the relative importance of each meteorological variable to the decision process requiring the weather information.

For this example, the target ROS field was structured so that the global optimum solution to the design problem was known beforehand, and optimization results could easily be checked:

$$T(x, y) = \sum_{m=1}^7 w_m R_m \quad (2)$$

where $T(x, y)$ is the ROS value at an arbitrary point (x, y) , R_m is a positive value at point m of seven pre-specified points, and w_m is a weighting function of the distance d_m between (x, y) and point m :

where

$$w_m = \exp(-ad_m^2) / \left(\sum_{k=1}^7 w_k \right) \quad (3)$$

$$d_m^2 = (x - x_m)^2 + (y - y_m)^2,$$

and a = a positive constant. If the coordinates at the gridpoint i are denoted as (x_i, y_i) , then T_i may also be expressed as $T(x_i, y_i)$, and calculated from (2). The target field (Fig. 1) rises gradually in the direction of increasing x , crests near the middle of the field, then rapidly drops to a minimum at the far edge. The waveform reflects the response of the ROS function to simulated conditions of uniformly moist and cool maritime air on the west side, increasing windspeeds and decreasing humidity toward the middle, and decreasing windspeeds toward the east (Fujioka, 1983).

The interpolation function is similarly a distance-weighted function of point values of the field variable, except that the interpolation function varies with the design size, n . Distance-weighted interpolation is a common tool of meteorological analysis (Goodin et al., 1979). The term F_i is the interpolated field value at gridpoint i :

$$F_i = \sum_{j=1}^n w_{ij} R_j \quad (4)$$

where

$$w_{ij} = \exp(-ad_{ij}^2) / \left(\sum_{k=1}^n w_{ik} \right) \quad (5)$$

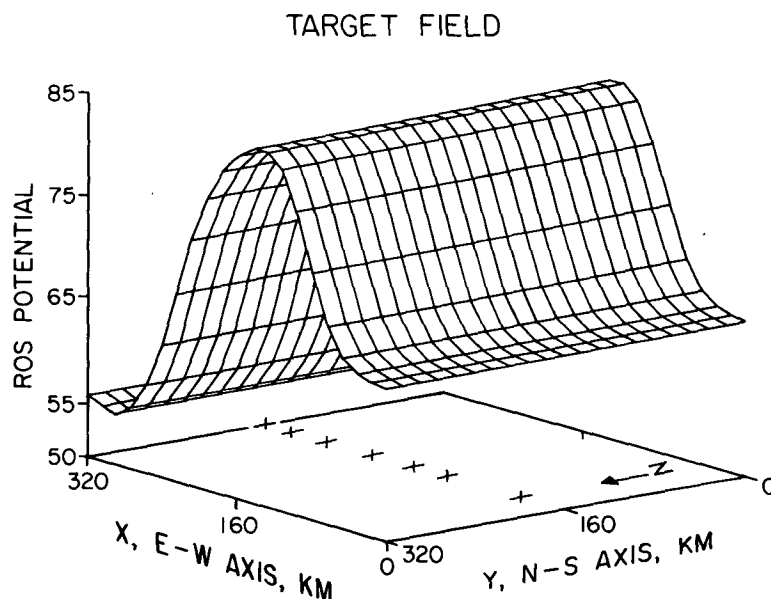
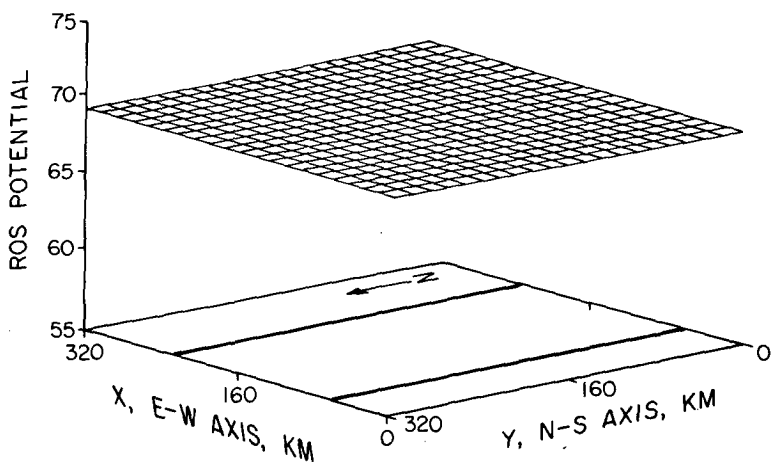
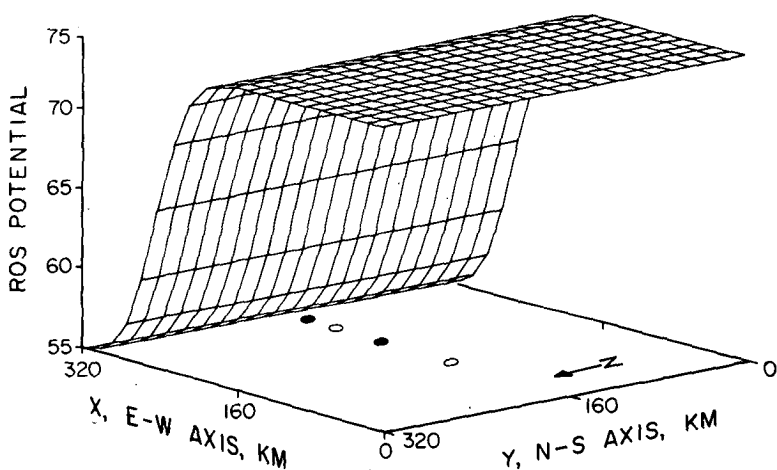


FIG. 1. The target field function, $T(x, y)$, representing potential rate of spread of a fire at (x, y) , was structured from simulated weather, fuel, and topographic conditions (Fujioka, 1983). The global optimum network for this problem is obtained by placing a station at each of the seven locations marked by an X.

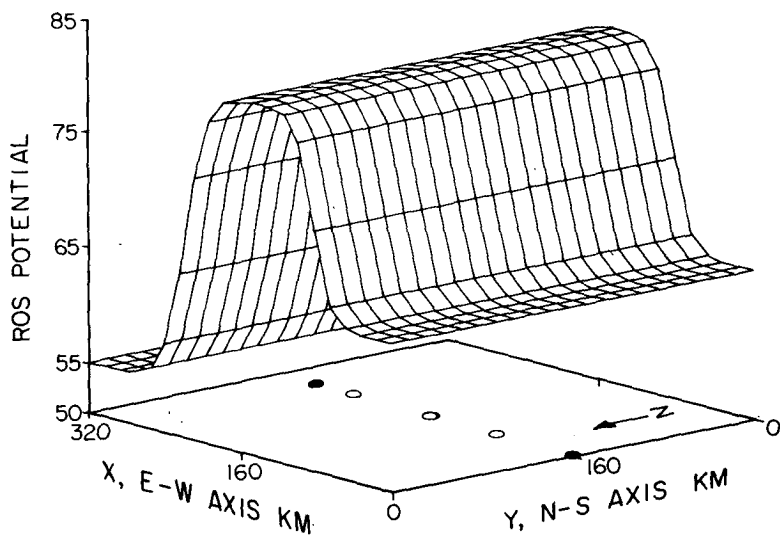
A. ONE-STATION OPTIMUM ANALYSIS



B. TWO-STATION OPTIMUM ANALYSIS



C. THREE-STATION OPTIMUM ANALYSIS



and d_{ij} is the distance between gridpoint i and observing site j . Because (5) is functionally identical to (3) when n is seven, the optimum solution for this problem is the one that locates stations at the seven points indexed by m in (2) (Fig. 1).

Optimum (least-squares) designs were computed for network sizes ranging from two to seven stations, with a quasi-Newton constrained nonlinear optimization algorithm (Gill and Murray, 1976). The design variables, $\mathbf{X}$, were constrained to the interval (0, 320), which represents the range of x and y coordinates in the network area. The parameter space of the objective function is thus a hypercube in $2n$ -dimensional space.

The errors of interpolation were evaluated at the intersections of a 21×21 grid (16 km spacing on both axes) superimposed on the region. The error sum of squares was normalized by the variance statistic of the grid-point target field values. The quasi-Newton algorithm was used to find a minimum, $\mathbf{X}^*$, that optimized the design for a particular n .

Finally, the design method was used to generate a fire weather network plan for southern California. Optimization runs were limited to a network size of 50 stations, because it was not known how optimization algorithms would perform on these problems; a 50-station design was considered a maximum problem size. The target field characteristics were determined from case studies of local fire weather conditions, of which the Santa Ana is perhaps the most notorious. Even in a moderate Santa Ana, fire danger is enhanced by brisk foehn winds, which also create dry and relatively warm weather in the Los Angeles basin.

The Fire Weather Index—FWI (Fosberg, 1978)—was used to depict the fire potential induced by Santa Ana conditions over the area. Like the ROS function, FWI is calculated from dry-bulb temperature, relative humidity, and windspeed; however, unlike ROS, FWI does not require fuels information, which was lacking. Among FWI, ROS, and several other fire indices, FWI was statistically a good indicator of fire potential (Haines et al., 1983).

Exponentially weighted interpolation [Eqs. (4) and (5)] of FWI was performed on a 71×31 grid (6.4 km spacing) over southern California. The objective function was evaluated on the subset of grid points that did not fall in the ocean. Preliminary results of a network optimized for a Santa Ana scenario will be discussed.

4. Results

The simulation results are best appreciated by examining the impact of the optimum designs on the analyses as the number of stations was increased (Fig. 2).

Under the least-squares criterion, the best location for one station is where the target field equals the mean of the grid point values of the target field, because the mean minimizes the error sum of squares. With one station, the interpolation gives a uniform field, everywhere equal to the station ROS value (Fig. 2a). Clearly, a one-station design gives no opportunity to resolve spatial variability. Because the mean is bounded by the minimum and maximum grid point values, the optimum one-station solution will generally correspond to an infinite number of locations: the intersection of the plane of the mean and the target field. In this example, the plane of the ROS grid point mean intersected the ROS surface along two lines. The optimum error sum of squares for subsequent designs was normalized by the one-station optimum error, to indicate the reduction in error effected by additional stations.

Adding a second station reduced the normalized error sum of squares substantially, to 0.368. The optimum two stations were on the east-west line bisecting the network area. One was located just beyond the crest of the target field and the other in the trough, about 48 km east of the first. The placement was effective in resolving the trough on the east side of the target field (Fig. 2b).

Assuming that the optimum design is one which reduces the normalized error to less than 5% with the least number of stations (an arbitrary criterion), the optimum configuration for this problem consisted of three stations. The normalized error under the three-station solution was 0.019, or about 2% of the one-station error. Again, the optimum locations lay along the east-west centerline, an axis of symmetry for the target field. The interpolation function uses the symmetry to advantage in producing the analyzed field with corresponding symmetry. Under the three-station optimum design, one station was on the y -axis of the network area, the second was nearly centered on the area, and the third was near the bottom of the target field trough. This placement improved on the two-station design by resolving the rise in the target field to

FIG. 2. Interpolated fields, as depicted from (locally) optimum station locations. In (b) and (c), the optimization process was started with initial station locations marked by the open circles, and ended with optimum station locations marked by the solid circles. (a) The one-station optimum analysis was the horizontal plane corresponding to the mean target field value computed from the grid points. The bold lines in the XY plane are the loci of optimum one-station locations. (b) The two-station optimum analysis reduced the error sum of squares considerably, but did not resemble the target field. (c) The three-station optimum analysis was best, because it essentially duplicated the target field with the least number of stations. The point at (160, 160) was coincidentally an initial and final station location (overlapping circles).

its crest, on the west side of the network area (Fig. 2c). The essential features of the target field were substantially duplicated by the analysis with the optimal three-station design.

Design sizes greater than three did not improve significantly on the error sum of squares of the optimum three-station network. In some cases the optimum designs contained station doublets: two stations in the same location. These cases suggest that the weighting of such a location be doubled, with respect to the rest of the observations, but otherwise appear to have no practical significance.

The simulation showed no advantage in spacing the stations uniformly through the field for each network size. For example, a three-station network was configured so that the three stations were on the east–west centerline, partitioning the region into three equal rectangles (Fig. 2c). This produced a normalized error of 0.298, more than 15 times the optimum three-station error of 0.019. The error was even worse for the four-station, centric systematic configuration (Ripley, 1981), consisting of a station in the center of each quadrant of the network area; at 0.799, the error was nearly 160 times larger than the four-station optimum error. The analysis under this configuration missed the ridging of the target field near the center and suggested instead a terraced field declining from west to east.

The design process was further tested by altering the set of coefficients $\{R_i\}$ of the target field, making the target field values uniformly higher than in the first field, and smoothing out the trough on the east. The result of the optimization process was an optimum error function that approached zero much faster than it did with the original target field. The normalized error for the optimum two-station design was 0.028, a 97% reduction in error from the one-station optimum. The rapidity of the drop in the error function was probably due to the simpler spatial structure of the altered target field compared to the original. Oddly, the two optimum locations were on the southern boundary of the network area, on the west side of the field. Apparently, the optimization algorithm did not necessarily prefer the east–west centerline in this case, and smoothing out the trough reduced the necessity of having stations on the east side of the field, as in the previous two-station optimum.

The design method was then tested by combining the two target fields in the objective function. In this case, the optimum design would minimize the error sum of squares under both conditions. The objective function is the sum of the errors under each field:

$$S(\mathbf{X}) = \sum_I \sum_{j=1}^2 a_j [T_{ij} - F_{ij}(\mathbf{X})]^2. \quad (6)$$

This kind of adjustment allows consideration of variability in the target field. The $\{a_j\}$ can be used to weight the target fields, but that was not done here. The op-

timum error function was calculated for the combined fields (Fig. 3); this function also converged asymptotically with at least three stations in the network.

The target fields for southern California were far more complex than those of the preceding examples. The resultant designs are still under study, but a network optimized for a Santa Ana scenario is offered as an example.

The moderate Santa Ana condition that was chosen for a target field model induced an FWI field with areas alternating between high and low FWI values (Fig. 4). The spatial variations of FWI were highly correlated with the spatial distribution of winds. The source of Santa Ana winds is usually a high in the Great Basin, but the mountains surrounding Los Angeles channel the flow, creating alternating zones of high and low windspeeds over the area. For example, the core of high FWI values running southwest to northeast from Santa Ana (SNA) to Strawberry Peak (SY) was due to the wind channeling through the Santa Ana Canyon (from which the event gets its name).

A variety of optimization algorithms—quasi-Newton and conjugate gradient types—were tested for the southern California design problem (Jacobsen and Brucker, 1985). None performed uniformly better than the others; in fact, when the rate of improvement of the objective function decreased with one algorithm, switching to an algorithm from the other class often accelerated the rate of minimization. The algorithms were constrained to choosing station locations within the rectangular region in Fig. 4.

With some exceptions, the 50 station locations chosen for the Santa Ana target field were scattered over the network area (Fig. 4), except for station doublets at Victorville (VCV) and Palm Springs (PSP). Two other stations were located close to each other southeast of San Bernardino, within a windshadow formed by the San Bernardino Mountains. All three pairs were on the low side of an FWI slope, within a relatively short distance of the area where the FWI gradients were steepest. At least one station was located within or near each closed contour that delimited an FWI peak/depression. Three stations were placed off the coast, although the users would more than likely find these locations impractical. With the exception of Victorville, Palm Springs, and Beaumont (BUO), no first-order station in the area came within 10 km of a station located by this design.

5. Discussion

At this writing, analysis is continuing on the southern California design results, but it is already apparent that although the method has appeal, work needs to be done on various aspects, one of which is defining target fields. A comprehensive objective function, which weights the various manifestations of the target field, is needed. An ideal framework for the design process would be one in which the objective function measures the deficiency

OPTIMUM ERROR FUNCTION FOR COMBINED FIELDS

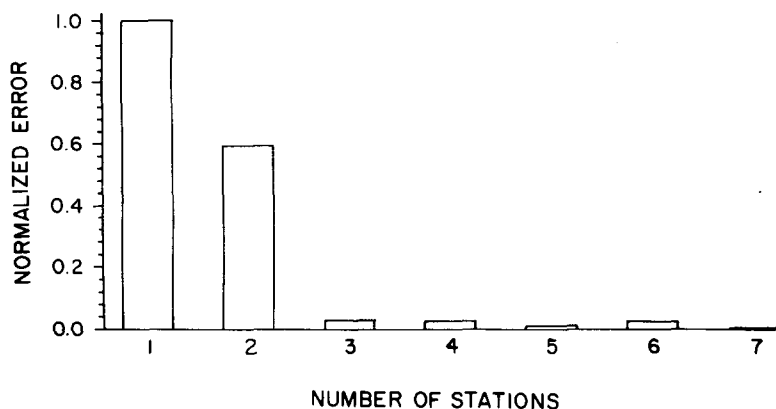


FIG. 3. The optimum error function for the combined target fields, normalized by the sum of the respective second central moments, converged asymptotically after the addition of a third station.

of the network with respect to the requirements of the decision process. Equation (6) could be modified in such a way that the $\{a_j\}$ provide a weighting of risk associated with different target fields. The design process would then be aimed at minimizing the weighted error expressed by, say, a Riemann-Stieltjes integral:

$$S(\mathbf{X}) = \sum_i \int_V (T(\mathbf{v}) - F(\mathbf{X}))^2 da(\mathbf{v}) \quad (7)$$

where $\mathbf{v}$ is a vector that parameterizes the target field T and V is the domain of $\mathbf{v}$, on which we define a multivariate weight function da that expresses a relative

weight for the target field $T(\mathbf{v})$. But this assumes considerable knowledge about the target field. In fact, if one can assume that the target field can be characterized by a (weakly) stationary spatial stochastic process with known mean and autocovariance functions, then that knowledge can drive not only the design, but also the interpolation, $F(\mathbf{X})$ (Eddy, 1974). In some situations, such as the present one, lack of data limits our ability to define target fields. In such cases, a sequential design process might be implemented, whereby target fields will be reevaluated as data becomes available, and the network will be reconfigured, as warranted.

This study showed the feasibility of using nonlinear

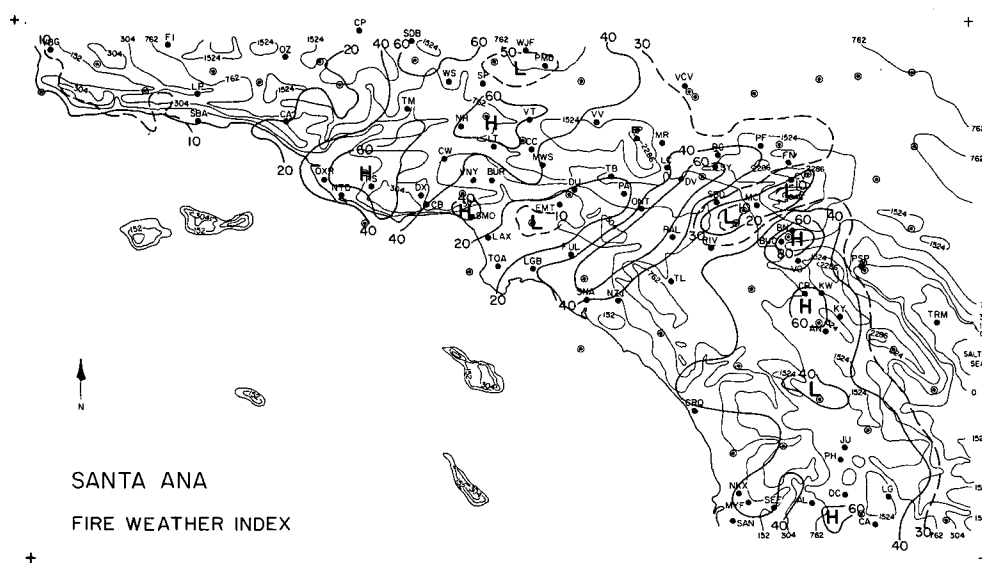


FIG. 4. Target Fire Weather Index field (dark contours) induced by moderate Santa Ana conditions over southern California, and corresponding optimum 50-station network. Optimum locations are denoted by circled dots, existing locations by solid dots.

optimization algorithms for meteorological network design, but more experience with the process is required. Nonlinear optimization with many parameters is never clearcut, and the imposition of constraints just adds to the complexity. The objective function of the Santa Ana example contained several local minima, which could not be handled effectively by a single algorithm. Even the design in Fig. 4 must be considered a locally optimum solution. A better understanding of the convergence properties of the optimization algorithms might also improve the efficiency of the design process. For example, one might speculate whether the appearance of station doublets in the optimized designs is peculiar to the tail of the error function, as it was in the simulation (Fig. 3); if it is, the desired network size for southern California might be less than 50. (Optimization runs are planned for smaller design sizes.)

The least-squares criterion is a measure of the average error in the system. Another potentially important statistic in design is the maximum absolute error of interpolation. In certain applications, it may be more important to keep this error below a certain level. The least-squares criterion ameliorates the maximum absolute error only to the extent that the optimum design satisfactorily reduces the mean-square error. Optimum error analysis for the first simulated target field showed, in that case, that the least-squares criterion did not improve the maximum absolute error beyond a network size of three.

6. Conclusions

Meteorological network design can be cast as a nonlinear optimization problem with constraints. This is a useful way of approaching the problem, because it considers the elements important to the decision process requiring the sampling. The procedure requires the user to systematically address the relevant meteorological and analytical aspects: the meteorological component of the problem is defined in target fields; and the effectiveness of a particular data analysis technique is examined by error analysis. The error analysis is also used to determine the appropriate network size for an application, if the admissible error level is predefined. Selection of the appropriate optimum network size determines the optimum network configuration. Geographic scope of the design process is defined by the user through constraint functions.

Further study is required on both the practical and theoretical implications of optimum designs generated this way. The mathematical stability of these solutions and their validity must be considered when the assumptions are violated. Otherwise, the method is quite general in its basic requirements and may prove useful to other design applications for sampling a spatially continuous field.

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